

# **LINEAR OPTIMIZATION MODEL THAT MAXIMIZES THE VALUE OF PORK PRODUCTS**

January 2012

**Kamilla Reynisdóttir**

Master of Science in Decision Engineering





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Reykjavík University

**M.Sc. RESEARCH THESIS**





# **Linear optimization model that maximizes the value of pork products**

by

Kamilla Reynisdóttir

Research thesis submitted to the School of Science and Engineering  
at Reykjavík University in partial fulfillment of  
the requirements for the degree of  
**Master of Science in Decision Engineering**

January 2012

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January 2012

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# **Linear optimization model that maximizes the value of pork products**

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## **Abstract**

The fishing industry in Iceland has applied methods of engineering, such as optimization research and mathematical modeling, into the fish processing plants with a great success. The fishing industry have reached a great progress in terms of utilizing available raw material and developing mathematical models that assist with complex decision making within the fish processing plants. The agricultural industry has not applied these methods as much. This is very likely because the fishing industry has weighted more in the Icelandic economy in terms of gross domestic product (GDP) and is one of the most important export industries in Iceland.

It is nevertheless very important for the agricultural industry to take advantage of methods of engineering. Consumers are doing more demand on the manufacturer of lower product price, but at the same time, of higher product quality. It is therefore important to use before mentioned methods to achieve better utilization of production and increasing the profitability of available raw material at each time.

The goal with this research is to maximize the product value of pork products, by developing a model that displays the optimal production plan for a given planning horizon. The model is designed in a way that it can be implemented on any kind of carcass: poultry, cattle, sheep, horses, calf and etc, by using different data. The model can also be implemented on different kind of production to make a production plan. It could be a meat processing or any other food processing. That is however not tested in this research.

A linear programming model to support decision making is presented, in order to find the optimal production plan for pork, by maximizing the product value. Data from Sláturfélag Suðurlands was applied to the model. Results from comparison between the model and reality indicated that a linear programming model is an efficient approach to solve a production planning problem. The model provides higher revenues than actually for the same production quantity. By applying sensitivity analysis on certain parameters

in the model, it can be seen what products are important to emphasize on in order to maximize profit.

**Keywords:** Production planning, optimization, linear programming, food industry, meat processing, decision support system, sensitivity analysis.

# Línulegt bestunarlíkan sem hámarkar verðmæti svínaafurða

Kamilla Reynisdóttir

Janúar 2012

## Útdráttur

Sjávarútvegurinn á Íslandi hefur nýtt sér aðferðir aðgerðagreiningar, bestun og líkanagerð, með miklum árangri. Því hefur sjávarútvegurinn náð langt hvað varðar nýtingu á afurðum og þróun á reikniforritum sem aðstoða með ákvarðanatöku. Landbúnaðurinn hins vegar hefur ekki nýtt sér þessar aðferðir í eins miklum mæli. Mjög líklega er það vegna þess að sjávarútvegurinn hefur vegið meira í íslensku atvinnulífi, hvað varðar verga landsframleiðslu og er ein mikilvægasta útflutningsgrein landsins.

Það er þó engu að síður jafn mikilvægt fyrir landbúnaðinn að nýta sér þessar aðferðir. Neytendur gera sífellt meiri kröfur til framleiðanda um lægra vöruverð en á sama tíma, um meiri gæði. Það er því mikilvægt að nýta sér áðurnefndar aðferðir til að ná fram betri nýtingu í framleiðslu ásamt því að auka arðsemi á því hráefni sem er til umráða hverju sinni.

Í þessu verkefni verður leitast eftir að hámarka afurðaverðmæti svínaafurða, með því að þróa reiknilíkan sem birtir hagkvæmustu framleiðsluáætlunina hverju sinni. Líkanið er hannað á þann hátt að það er hægt að nýta það fyrir hvaða kjöttegund sem er og jafnvel annarskonar hráefni úr matvælaiðnaðinum, með því að nota önnur gögn. Prófanir á því verða hinsvegar ekki gerðar í þessu verkefni.

Línulegt bestunarlíkan verður kynnt ásamt helstu niðurstöðum af samanburði við gögn fengin frá Sláturfélagi Suðurlands. Niðurstöður gefa til kynna að línulegt bestunarlíkan hentar vel fyrir áætlanagerð í framleiðslu sem þessari. Líkanið gefur hærri tekjur með framleiðslu á sama magni og var í raun. Með því að beita næmigreiningu á ákveðna þætti í líkaninu má sjá hvaða vörur er mikilvægt að leggja áherslu á í því skyni að hámarka hagnað.

Lykilorð: Landbúnaður, matvælaiðnaður, kjötframleiðsla, línuleg bestun, framleiðsluáætlun, reiknilíkan, ákvarðanatökuaðferðir, næmigreining.



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# Contents

<b>List of Figures</b>	<b>xi</b>
<b>List of Tables</b>	<b>xii</b>
<b>1 Introduction</b>	<b>1</b>
<b>2 Literature review</b>	<b>5</b>
2.1 Iceland . . . . .	5
2.2 Denmark . . . . .	7
2.3 International . . . . .	8
<b>3 The meat plant</b>	<b>11</b>
3.1 Flow of carcasses through the meat plant . . . . .	11
<b>4 Mathematical model</b>	<b>13</b>
4.1 Model characteristics . . . . .	13
4.2 Problem description . . . . .	14
4.3 Abstract model . . . . .	15
4.4 Mathematical formulation . . . . .	15
4.4.1 Notation and model variables . . . . .	16
4.4.2 Objective function . . . . .	18
4.4.3 Constraints . . . . .	18
4.5 Mathematical model verification . . . . .	20
4.6 Model limitations and simplifications . . . . .	22
4.6.1 Limitations . . . . .	22
4.6.2 Simplifications . . . . .	22
<b>5 Data</b>	<b>25</b>
5.1 Division of pork carcass . . . . .	28
5.2 Data discrepancy . . . . .	29

<b>6</b>	<b>Results</b>	<b>31</b>
6.1	Comparison between actuality and model . . . . .	31
6.1.1	Production same in model and actually . . . . .	32
6.1.2	Model producing more than actually . . . . .	33
6.1.3	Limitations in lower limit constraint . . . . .	35
6.1.4	Limitations in upper limit constraint . . . . .	35
6.1.5	Comparison of revenues . . . . .	37
6.2	Sensitivity analysis . . . . .	38
6.2.1	Level 1 . . . . .	38
6.2.2	Level 3 . . . . .	38
6.2.3	Demand . . . . .	44
6.3	Summary . . . . .	46
<b>7</b>	<b>Conclusion</b>	<b>47</b>
	<b>Bibliography</b>	<b>49</b>
<b>A</b>		<b>53</b>
A.1	Network diagram . . . . .	53
<b>B</b>		<b>55</b>
B.1	Lower- and upper limit of production . . . . .	55
<b>C</b>		<b>59</b>
C.1	The code written in MPL . . . . .	59



# List of Figures

1.1	Meat production in Iceland from 1983 to 2009. . . . .	2
3.1	Product flow . . . . .	12
4.1	Abstract model. . . . .	16
4.2	Illustration of the model using simple data set. . . . .	21
5.1	Diagram of pork . . . . .	29
6.1	Production and inventory level for Shank - Actual vs. Model . . . . .	33
6.2	Production and inventory level for Lard - Actual vs. Model . . . . .	33
6.3	Production and inventory level for Loin without crackling - Actual vs. Model . . . . .	34
6.4	Production and inventory level for Ham 1 - Actual vs. Model . . . . .	34
6.5	Production and inventory level for Loin speck - Actual vs. Model . . . . .	35
6.6	Production and inventory level for Side - Actual vs. Model . . . . .	36
6.7	Production and inventory level for Cut off - Actual vs. Model . . . . .	36
6.8	Production and inventory level for Bacon ham - Actual vs. Model . . . . .	37
6.9	The parts of the carcass in order of importance . . . . .	39
A.1	Network diagram . . . . .	54



# List of Tables

4.1	Indices used in the optimization model. . . . .	16
4.2	Sets in the optimization model. . . . .	17
4.3	Decision variables in the optimization model. . . . .	17
4.4	Data and constants used in the optimization model. . . . .	17
5.1	Pork products sorted from the lowest value to the highest. . . . .	26
5.2	The division of pork carcass into parts and raw material. . . . .	29
6.1	Initial inventory level. . . . .	32
6.2	Shadow prices for products from the raw material Side. . . . .	40
6.3	Shadow prices for products from the raw material Loin in slices of meat. .	41
6.4	Shadow prices for products from the raw material Loin and Loin chop. . .	43
6.5	Shadow prices for products from the raw material Saddle and Fore saddle.	44
6.6	Shadow prices for products from the raw materials Shoulder, Ham and Bayon ham. . . . .	45
6.7	Sensitivity analysis for demand. . . . .	46
B.1	Lower limit of production. . . . .	56
B.2	Upper limit of production. . . . .	57
B.3	Upper limit of production. . . . .	58



# Chapter 1

## Introduction

For centuries Iceland's main industry has been the fishing industry. For the past decades aluminum and travel industry have been growing considerably and for the past years pharmaceutical and ferrosilicon production have also been entering the manufacturing market in Iceland. The manufacturing industry is the second largest industry after the service industry with 47% of total turnover and 22.2% of total number of employees in the year of 2010. Within the manufacturing industry the food industry is the second largest, with 11.5% of total turnover and 36.1% of total number of firms. The meat processing plants are 4.3% of the total food manufacturing firms [23].

Planning in manufacturing is allocating limited resources into products that meet customer demand over a certain planning horizon. The objective is often to develop a production plan that minimizes total production cost and maximizes the possible revenues from producing a certain amount of raw material while meeting customer demand. There may be opportunities within the meat processing plants in Iceland to take advantage of optimization methodology and mathematical modeling in their production planning. The range of meat products has been rapidly increasing in recent years with increased product development, innovation in production technology and new solutions in packaging. Figure 1.1 shows how the demand for poultry and pork has increased vastly in recent years [22]. The main reason for the increase is because it is less expensive than its substitutes. Poultry has also been popular because of increased health awareness in recent years and poultry meat contains less fat than other meat products. Lamb and beef have however remained stable considering growth in production, as can be seen in Figure 1.1.

Companies in the field of meat processing should consider taking advantage of the increase in demand and use quantitative method in their production planning. Decision

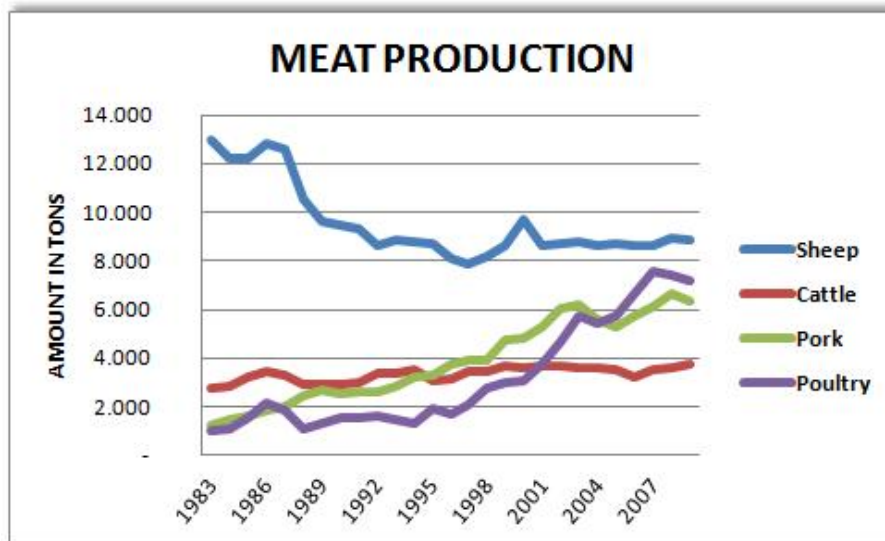


Figure 1.1: Meat production in Iceland from 1983 to 2009.

making in the production process might be taken in more systematic way by using a quantitative method in production planning. As for example:

- What products to produce?
- How much to produce?
- What to do with products in excess of demand?
- How to increase utilization of available raw material?

For a complex manufacturing company it is not enough to use experience and rules of thumb in production planning. Manufacturing companies that consist of more than one plant and wish to have consistency in production planning between plants, might want to apply some kind of quantitative methods that can take complexity in manufacturing into account. Methods like linear programming, simulation, probability modeling and inventory- and operation management are examples of quantitative methods that could be applied in complex manufacturing processes in order to assist with complex decision making.

Studies have shown that methods of engineering, as operation research, have return increase in efficiency within companies. The aim with this research is to estimate whether linear optimization can be used as a tool in making production plans for a meat processing plant. A mathematical model will be presented which is a linear programming model that maximizes the product value of pork products by finding the optimal production plan for pig production, for a given amount of carcasses and a forecasted demand in a given planning horizon. The model is developed in cooperation with a meat processing plant,

Sláturfélag Suðurlands (SS), positioned in Hvolsvöllur. The meat plant is integrated with a slaughterhouse located in Selfoss. Total amount of slaughtered pigs in the slaughterhouse was 11.504 pigs, with a total value of 222.1 million ISK over the year of 2010 [21]. Approximately 85% of these carcasses are supplied to the meat plant under study, for further processing [6]. The total production of pork products in 2010 at the meat plant was 762.098 kg. These pork products are used in further product mix at the meat plant. Data from the meat processing plant is applied to the model for verification. The model is designed in a way that it can be implemented for any kind of carcass by using different data. The model requires that forecasted demand for the planning horizon is prepared. One of the challenges in the development process of the model was to apply real data set to the model. Real data sets often include discrepancies as difficulties to conclude exactly about employees experience and rules of thumb that have evolved within companies.

The paper is structured as follows: In section 2 we briefly review studies about linear optimization and how the method has been used in the food industry, in Iceland and internationally. In section 3, the meat plant under study will be presented. In section 4, the problem will be defined and the mathematical model introduced; the objective function, constraints and variables. Mathematical model validation will also be performed and introduced in section 4. In section 5, the dataset will be presented. In section 6, computational results will be described and in section 7, the conclusions of this work and perspectives for future research are presented.





## Chapter 2

### Literature review

Methods of engineering like mathematical modeling and optimization methods have been applied to some extent in the food, fishing and agricultural industry in Iceland. Most effort has however been applied to the fishing industry. Few projects have been done considering optimization methods for the agricultural industry in Iceland. These methods have however been applied in other purposes than for production scheduling, as to improve product processing and optimize sorting of raw material. Methods of engineering have also been used to some extent in pharmaceuticals and aluminum industry. Quite a few projects in the fishing industry have been supported by AVS, a research fund for the fishing industry in Iceland [1] and RANNIS - the Icelandic centre for research, which supports research studies, technical development and innovation for the science and technology community in Iceland [18]. Funds and support centre for the agriculture are not as accessible as for the fishing industry in Iceland.

Optimization methods have also been applied to some extent in the food industry internationally and especially in Denmark. In this chapter we will touch on the projects that have been done in the food industry in Iceland, Denmark and internationally.

#### 2.1 Iceland

A few projects have been done in Iceland using optimization methods in both fishing industry and agriculture. The very first major project in Iceland in the field of operational research was in the year of 1966 [14]. The project was cooperation between an Icelandic and Danish team and the objective was to develop a computer program that simulated the fishing and landing of herring in time.

In [8] a mathematical model for the operation of dairy farms in Iceland was designed (1980) to solve the overproduction problem in agriculture and especially on dairy farms. Emphasis was placed on describing the conjunction between fertilizers, concentrate feed, production and cost as well as hay quality. This was cooperation between The Agricultural University of Iceland and The Farmers Association of Iceland. The main results were that the quality of hay is crucial regarding production cost of milk. The model also showed that the most sensible way to compensate for the loss of revenues because of the contraction in production was to ensure the quality of hay.

In [13] LÍndal (2005) designed an optimization model to evaluate how to improve the way of processing different carcasses. The model was designed to analyze how to allocate available meat at each time, based on certain time that it takes to produce each product. The model's constraints are manpower and the production time. The input is the quantity of meat processed per year.

More effort has been applied to operations research within the fishing industry in Iceland. The ambition within the fishing industry to increase the total value of seafood has increased in recent years. Here we will touch on the main projects that have been applied in the fishing industry.

In [12], Jensson developed a linear programming model (1988). The model maximized the profit of fish processing firms. The goal was to develop a decision support system for a production manager in fish processing firms, regarding an overall decision making in the production plant, as staff scheduling and what to produce at each time. The model was tested in a fish processing plant in Reykjavík and was found to be very useful for the production managers.

In [17] a model was developed (1999) by Gunnarsson and Jensson to help with decision making for the most profitable classification of fish. The model also calculates the highest possible product value of available fishing in process at each time. The model uses mathematical optimization methods to maximize product value of capelin, herring and shrimps based on their size distribution.

Margeirsson et al. collected data from 2002 to 2006 on fillet yield, gaping and parasites [25]. Rúnarsson (2006) utilized this data set to develop a linear optimization model that maximized the profit of an Icelandic company in the field of fish processing [20]. The objective of the project was to analyze the data and show how to utilize the data to increase productivity and profit of companies within the fishing industry. The model assists with decision making regarding how to choose catching areas and to evaluate how much to pay for leased quota. An aggregate plan was made for one year where the year was divided

into four seasons and the Icelandic fishing territories into 13 areas. In [7] Guðmundsdóttir (2007) developed a short term aggregate plan for these seasons mentioned before. The results of the aggregate plan were then used in a mixed integer linear programming model. The model maximized the total operational profit of catching and processing cod.

Margeirsson (2008) wrote a Ph.D. thesis where he used the same data set as Rúnarsson. Models obtained from statistical analysis of the data was utilized to construct a linear optimization model for easier decision making when catching and processing Icelandic cod [25]. The model was designed to increase the value of Icelandic cod products and to maximize economic yield of quota with minimal cost. Results obtained from the models indicated that the profit from the value chain of cod can be increased by catching fish in a certain areas and by managing the season of the catch.

To the best of our knowledge, companies within the food industry in Iceland are not utilizing in a major way any methods of engineering, like optimization models, in their production planning. It is more of employee's knowledge and experience that is used in decision making. There is however, to the best we know, one meat processing plant in Iceland that have implemented a deboning line from Marel food system into their meat plants. The system keeps track of all orders and information about forecasted demand. Carcasses are then processed after these information's. If there are no orders in the system, the carcasses are processed in order to maximize the utilization. The deboning line was implemented in the year of 2002 and productivity increased of 200-300% in 6 months. Employees decreased in the same time and manufacturing bases decreased from three into two, now positioned in Akureyri and Húsavík [19].

## 2.2 Denmark

The Danish pig industry is among the leading pig industries in the world which explains the great amount of work that has been done in the operations research field within the pig industry in Denmark. The Danish Meat Research Institute has been very active in sponsoring projects in this field. One of the latest projects in Denmark in operation research within the pig industry was done by Toke Koldborg Jensen and Niels Kjærsgaard in 2010 [27]. A mixed integer programming model was designed for computing optimal sorting groups based on parameters describing the carcasses, slaughter weight and fat layers of the carcasses. Models based on the one presented here are relevant for the pork sector to gain new insight into the optimal sorting of pig carcasses in order to improve the profit.

In [15] Niels Kjærsgaard (2008) wrote a Ph.D. thesis on optimization of the raw material use at Danish slaughterhouses. The thesis included four papers addressing different optimization problems of major importance for Danish slaughterhouses. Part of his work was to reveal the value of improved measurements in a pig slaughterhouse. In the second part of his research Niels examined the value of a general increase in slaughter weight for pigs. The model used the entire pig and product weight for all potential products which were estimated based on slaughter data and a simulated measuring errors. In the third part of the research he dealt with limitations in production and stock and the effect on the profitability of the slaughterhouses. The model was further refined to take the physical conditions in the equalization room into consideration. At last Niels evaluated different sorting criteria and strategies using mathematical programming. Here the same model was used as in [27] but in this case it evaluated different sorting strategies and at the same time a graphical tool was implemented to help the slaughterhouses design their sorting strategies.

In [4] Claus Fertin (1995) presented a multi-period linear programming model as the basis for designing support systems for hog slaughterhouses. The model delivered an economical optimization of the slaughterhouse decision problems by comparing sales, products, quantity, demand and prices with the constraints of available raw material and product limitations. A sensitivity analysis was also done to be able to make changes in the use of raw material and manpower along with the production itself.

In [26] Svend Rasmussen (1992) also presented a multi-period linear programming model which formed a base for a decision support system for a hog slaughterhouse. Rasmussen describes a mathematical short term and linear programming model supported by vector-tensor notation. The model also yielded shadow prices on raw materials, products and capacity. The shadow prices for the raw material determined the fair out price to farmers. It was also used as a tool for analyzing economic relationships between the slaughterhouse and the farmers. The decision support system supported daily decision making concerning production, storage and sale.

## 2.3 International

Various projects have been done globally in different sectors of agriculture processing. In [3] an application of dynamic programming was used by Clark and Kumar (1978) in Australia to plan beef production. The determination of optimal feeding and marketing strategies for beef cattle grazing pasture was formulated as a dynamic programming

problem. The model was developed to incorporate variable live weight gains and sale yard prices, and took into consideration buying and selling cost. The model delivered what the policy of feeding and marketing should be pursued in order to maximize gross income over  $n$  periods. The results obtained in the study have found acceptance among those with expertise in agriculture.

In [5] Glen developed a method (1983) for determining the optimal feeding policy for a pig production unit, since the market value of pigs produced is determined by the weight and composition of the carcass. The optimal feeding policy involves feeding least cost rations throughout the fattening period. A dynamic programming model was developed to determine the sequence of these least cost rations to produce pigs of specified final weight and carcass composition at the least cost. The method was used as a tool for planning the operations of pig producers.

In [2] a linear programming model was developed by Benseman (1986) in New Zealand to find the most profitable daily production schedule of various powder, casein, cheese and butter products. The model incorporates seasonal fluctuations in whole milk availability and quality as well as transportation costs, factory capacities and cost and product-yields and prices. The model was implemented in the largest company in the world in handling solely milk products, at that time. The company spent considerable time and money developing the production planning model. The model saved the company over \$1 million per year by better utilizing downstream processing. The company also closed a few factories after the model highlighted the actual cost of running them. The model assisted long-term planners in deciding where, when and what type of new factories to build.

In [11] Niemi wrote a Ph.D thesis (2006) where she developed a dynamic programming model that optimized the feeding pattern of pigs until slaughter. The model takes into account the genetic characteristics of pigs and return feeding decision and the timing of slaughtering. Results indicated that producer could benefit from improvements in the pig's genotype. Animals with improved genotype can reach optimal slaughter maturity quicker and give leaner meat than animals of poor genotype.

In [16] Ohlmann and Jones (2008) presented an mixed integer programming model for optimal pork marketing. As pigs reach marketable weights, a pork producer must devise a marketing strategy to determine when to sell pigs, which pigs and how many pigs to sell and to which packer. The model was developed to help with these decisions. It determined the marketing strategy for a pork producer that maximizes expected annual profit. The work demonstrated the importance of the opportunity cost of the finishing facility in determining marketing plans that maximizes annual profit.

As can be seen, a great amount of work has been done in the field of operations research with a great success within the agriculture and fishing industry in Iceland and internationally. It is therefore relevant to propose a linear programming model to solve the problem in this research. The aim with the model is to maximize product value of pork products for a given input and forecasted demand. The decision variables are continuous and the model is linear, which makes it easy to solve and implement for this problem. The solution will give a good picture of how to optimize the production plan at the meat plant.

## Chapter 3

### The meat plant

SS is one of the leading meat processing companies in Iceland with a market share of 19% in total slaughtering and its market share in production and sale of processed meat products is between 30% - 70% and up to 80% by commodities [24]. Within the meat processing plant, cattle, horses, pigs and sheep are processed further into products. The meat plant has a wide range of products and therefore a great opportunity to utilize a very high proportion of their raw material. The overall process of product flow can be seen in Figure 3.1. The figure shows the product flow from farmer to retailer. In this research, we are only working with the third phase, from Figure 3.1.

#### 3.1 Flow of carcasses through the meat plant

The meat plant is integrated with a slaughterhouse located in Selfoss and supplied with carcasses from this slaughterhouse. When these carcasses arrive at the meat plant from the slaughterhouse, they go on a conveyor belt to a carcass chiller. Then it goes to the cutting room, where the carcasses are cut into parts and then further into raw material. Raw material is then allocated into five different departments within the meat plant. Following departments processes the raw material further into products according to a short term plan for demand that is made within the meat plant.

- **Readymade meals.** SS is a leading company in Iceland in producing readymade meals, named 1944. Readymade meals are processed in this department along with products that is fried or boiled.
- **Packing A.** Products that are boiled, as slices of meat and hot dogs are packed in this department.

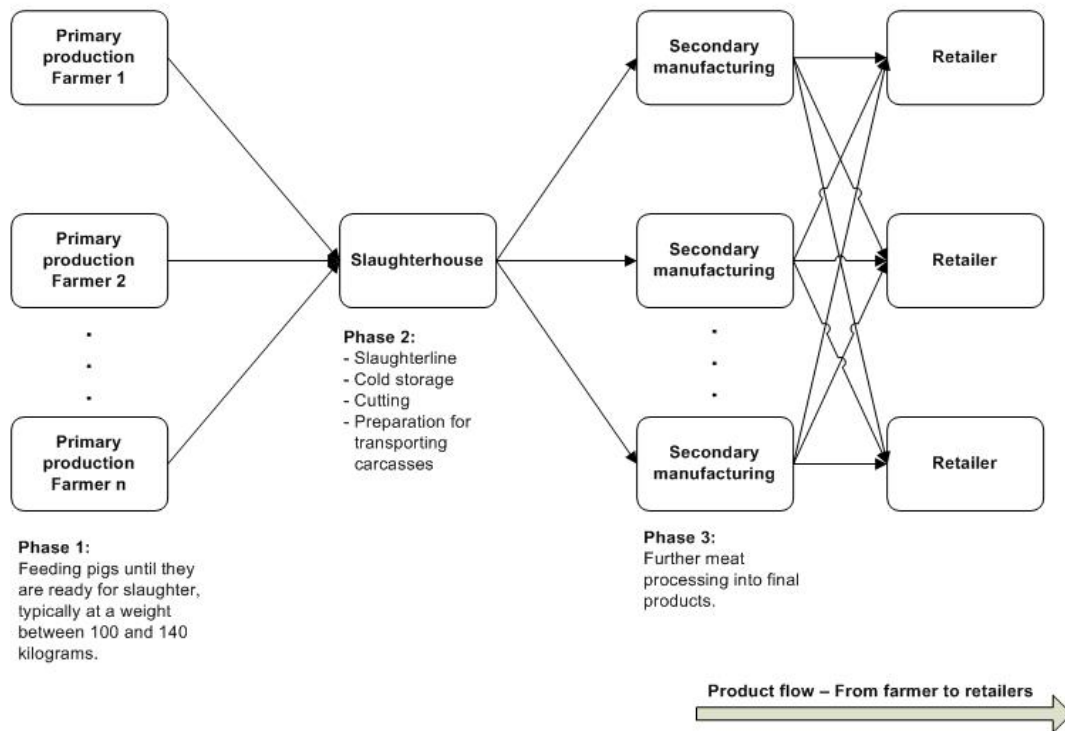


Figure 3.1: Product flow - from farmer to retailer.

- **Sausage and force meat production.**
- **Freezing plant and packing B.** Packing of all raw materials, as marinated meat products and salted meat takes place here. The freezing plant takes care of serving frozen products into the processing plant.
- **Blood sausage making and meat hypodermic.** Processing of blood sausage, smoked meat and products like salami takes place in this department.

In the meat processing plant under study the production schedule is currently only made for one week at a time and often it follows what is available in stock for their product selection. No long-term planning is done. This is likely to create a production plan which does not necessarily return the highest profit possible. Due to different seasonal patterns it would be very helpful to be able to change the production plan to meet the demand for different products during different seasons. A linear decision model might make decision making regarding production planning and resource allocation a lot easier for the meat plant under study. The goal of the model is to maximize the product value of pork products by finding the optimal production plan for pork production. The considered planning horizon is one year.



## Chapter 4

# Mathematical model

Planning is a very important tool in production, by planning strategically in advance, firms and operations are more likely to operate more smoothly and get higher returns. Operations research methods in planning are an excellent way to capture the increasing complexity in production plants and to help with decision making relating to what products and how much to produce at each time in more optimal way.

The complexity in production plants increases as the range of products increase. A complex production plan makes it more difficult to keep track of the overall process and to make an optimal production plan. Seasonal pattern in supply, demand and changes in consumer behavior also make it difficult for the planner to reach performance and improvements in production planning. It is very common that short term planning is used in production planning and companies are thereby not getting the optimal plan with the best results available. Short term planning can be very important in instances like when orders come in with a short notice. It's however necessary for manufactures to make a long term planning as well. A linear optimization model will be presented in this chapter. The goal with the model is to make a production plan that maximizes product value of pork products while meeting customer demand. The main decision variables describe the flow of the raw material through the model, production and inventory level and necessary beginning inventory.

### 4.1 Model characteristics

The model presented is a linear programming model that consists of an objective function which is being optimized, decision variables whose optimal values are to be found and

constraints which restricts the range of a variable's possible value. Linear programming requires linearity in the equations. In a linear equation, each decision variable is multiplied by a constant coefficient with no multiplying between decision variables and no nonlinear functions such as logarithms. A common type of application of linear programming involves allocating resources to activities. The amount available of each resource is limited, so the allocation of resources to activities must be made carefully. Linearity requires the following assumptions:

- **Proportionality.** *The contribution of each activity to the value of the objective function is proportional to the level of the activity.* [10].
- **Additively.** *Every function in a linear programming model is the sum of individual contributions of the respective activities.* [10].
- **Divisibility.** *Decision variables in a linear programming model are allowed to have any values, including non-integer values, which satisfy the functional and nonnegative constraints.* [10].
- **Certainty.** *The value assigned to each parameter of a linear programming model is assumed to be a known constant.* [10].

In reality the four assumptions of the linear programming almost never hold completely. Therefore sensitivity analysis often needs to compensate for the violation of the assumptions. It is also important to examine the four assumptions for the problem under study to analyze how large the disparities are, because if any of the assumptions are violated in a major way, it might be more useful to apply an alternative model to the problem [10].

## 4.2 Problem description

The model presented in this research is a linear programming model as previously mentioned and all decision variables are continuous. The model is set up as a network diagram. The model is however solved with linear programming using MPL 42 with the solver Gurobi 4.2.1. The model can be used in sensitivity analysis, which is in this case for the following:

- To find out what products to concentrate on in a marketing point of view in order to increase demand. This is done by analyzing shadow prices for a given constraints.

- The shadow price is the price of producing one extra unit of a given limited resource. It can also indicate whether it pays off to add capacity if possible [10].
- To find bottlenecks in the manufacturing process.
- Assist with decision making in pricing products.

The model is designed in a way that it can be implemented for any kind of carcass: poultry, cattle, sheep, horse, calf and etc., by applying different data in the model. The model might also be implemented on different kind of production to make a production plan. It could be for a meat processing plant or any other food processing plant. This is however not tested in this research, but would be interesting to do as a next step in the process.

### 4.3 Abstract model

The abstract model shows how the input of raw material flows through the model, as can be seen in Figure 4.1. The first node represents the input, which is the amount available for production in each time period. Level 1 represents the division of the input into different parts. Level 2 represents how the parts divide into raw material. Level 3 represents how the raw material is allocated into products. Last node represents the output of the model, which are the revenues obtained from producing a certain amount of input. Every node in each level enters every single node at the next level. The nodes are indexed with  $i, j$  and  $k$  and the legs between the nodes are the decision variables,  $x_i, y_{i,j}$  and  $z_{j,k}$ . The decision variables all represent the same which is the proportion of the material that goes to the next level. There are lower and upper limits on each node and they all lie on the interval between 0 and 1. These limits control how the raw material flows through to the next level. All constraints are equilibrium constraints.

### 4.4 Mathematical formulation

The model aims to maximize the profit that comes from processing a certain amount of input in each time period. The mathematical programming model of the present production planning problem under study will be introduced in following sections.

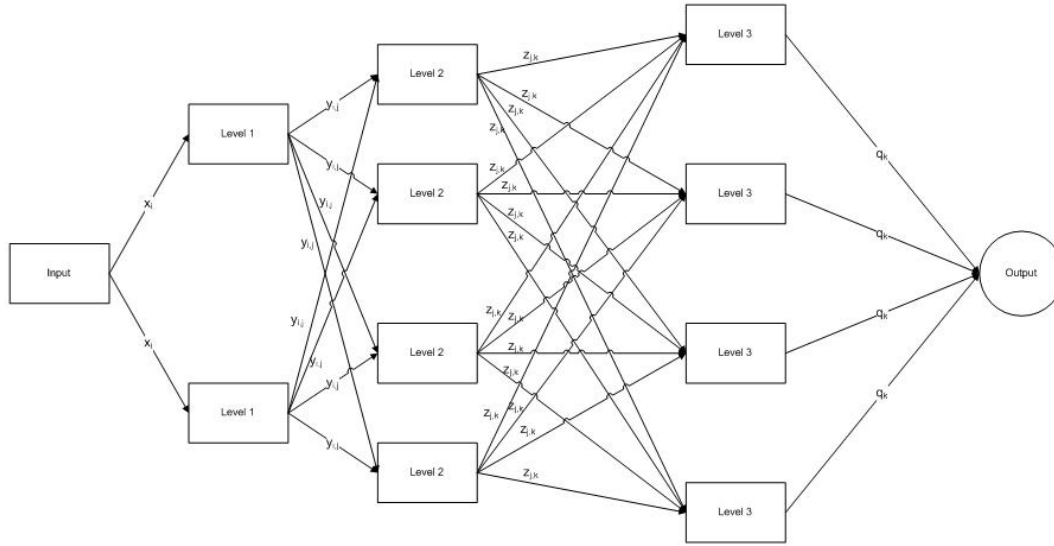


Figure 4.1: Abstract model.

<i>Index</i>	<i>Definition</i>
$i$	Division of the carcass in parts.
$j$	Raw material.
$k$	Products.
$t$	Planning horizon (months).

Table 4.1: Indices used in the optimization model.

#### 4.4.1 Notation and model variables

The linear programming model is based on a number of indices, parameters, decision variables and constraints. The indices that represent division of the carcass through the model are listed along with definitions in Table 4.1. The model returns values for eight decision variables, which are listed in Table 4.3 along with definitions. The first three decision variables represent the three levels in the model and return the amount of raw material that flow through the levels. Other decision variables represent production, inventory level, how much to buy of extra raw material from vendor and necessary beginning inventory level. The necessary inventory level was added as a decision variable because of discrepancies in the data set, which will be described further later. All data and constants are listed in Table 4.4 along with definitions.

<b>Set</b>	<b>Feasible sequence of allocation of parts/ raw material</b>
<i>IJ</i>	(1, 1), (2, 2), (2, 3), (2, 4), (3, 5), (4, 6), (5, 7), (6, 8), (6, 9), (7, 10)
<i>JK</i>	(1, 4), (1, 5), (1, 6), (1, 9), (1, 22), (2, 2), (2, 4), (2, 7), (2, 9), (2, 14) (2, 22), (3, 7), (3, 9), (3, 16), (3, 22), (4, 4), (4, 7), (4, 9), (4, 15), (5, 4) (5, 7), (5, 9), (5, 18), (5, 22), (6, 1), (6, 2), (6, 4), (6, 8), (6, 9), (6, 13) (6, 17), (6, 20), (6, 22), (7, 4), (7, 8), (7, 9), (7, 18), (7, 19), (7, 20) (7, 22), (8, 1), (8, 2), (8, 4), (8, 8), (8, 9), (8, 10), (8, 11), (8, 20) (8, 22), (9, 1), (9, 2), (9, 3), (9, 4), (9, 8), (9, 9), (9, 12), (9, 20) (9, 22), (10, 9), (10, 21), (10, 22)

Table 4.2: Sets in the optimization model.

<b>Decision variables</b>	<b>Definition</b>
$x_{t,i}$	Amount (Kg.) of part i in time period t.
$y_{t,i,j}$	Amount (Kg.) of raw material j from part i in time period t.
$z_{t,j,k}$	Amount (Kg.) of product k from raw material j in time period t.
$prod_{t,k}$	Production of product k in time period t.
$inv_{t,k}$	Inventory level of product k in time period t.
$beginv_k$	Necessary inventory level at the beginning of the planning horizon.
$level1_{t,i}$	Amount (Kg.) of extra raw material bought from vendor in time period t and added to the model in level 1.
$level3_{t,k}$	Amount (Kg.) of extra raw material bought from vendor in time period t and added to the model in level 3.

Table 4.3: Decision variables in the optimization model.

<b>Data</b>	<b>Definition</b>
$input_t$	Weight (Kg.) of total carcasses to process in time period t.
$v_k$	Value (ISK) of final product k.
$part_i$	Proportion of the division of input in part i.
$demand_{t,k}$	Demand for product k in time period t.
$lowerlimit_{j,k}$	Proportion of the lower limit of production of product k from a raw material j.
$upperlimit_{j,k}$	Proportion of the upper limit of production of product k from a raw material j.
$c1$	Cost of buying additional product in level 1 from vendor.
$c3$	Cost of buying additional product in level 3 from vendor.
$ic$	Cost of holding inventory.
$pd$	Product deterioration (%).

Table 4.4: Data and constants used in the optimization model.

#### 4.4.2 Objective function

The goal of the linear programming model is to maximize product value of pork products by finding the optimal production plan as previously mentioned. The objective function 4.2 consists of three components, which are as following:

- **Production.** Production level and product value for all products in each time period.
- **Inventory.** Inventory level and inventory cost for all products in each time period.
- **Extra raw material bought from vendor.** How much to buy of extra raw material in each time period and the cost of buying it.

How much to buy of extra raw material is measured in weight (kg.) of products in each time period. The product value, inventory cost and the cost of buying extra raw material is expressed in ISK per kilogram. The objective function of the proposed model is to maximize revenues as follows:

$$\begin{aligned} Max = & \sum_t \sum_k \sum_i (v_k \cdot prod_{t,k} - ic \cdot inv_{t,k} - c1 \cdot level1_{t,i} - c3 \cdot level3_{t,k}) \quad (4.1) \\ & - \sum_k (inv_{t_{end},k} \cdot v_k \cdot dp) \end{aligned}$$

The objective function 4.2 maximizes the total yearly revenue from producing a certain amount of raw material in each time period along with minimizing inventory level and how much extra raw material to buy from the company's vendor. The model does not take into account production cost. We assume 30% product deterioration at the end of the planning horizon to prevent the model to collect unnecessary inventory over the planning horizon.

#### 4.4.3 Constraints

Constraints in a mathematical model serve as boundaries to the solution space. A feasible solution is a solution for which all the constraints are satisfied while an infeasible solution is a solution for which at least one constraint is violated. An optimal solution is a feasible solution that has the most favorable value of the objective function [10]. Constraints of the proposed model can be seen here below.

### Constraints for first level

$$x_{t,i} = part_i \cdot input_t + level1_{t,i} \quad \forall_{t,i} \quad (4.2)$$

Constraint 4.2 describes how the raw material is divided into parts, which is always fixed at the first level. The constraint ensures that the amount of raw material on first level is the same as the input. The data  $part_{xi}$  keeps the values for the division. The decision variable,  $level1_{t,i}$ , represents how much extra raw material needs to be bought from vendor at each time, that is added to the model in first level.

### Constraints for second level

$$\sum_j y_{t,i,j} = x_{t,i} \quad \forall_{t,i} \quad (4.3)$$

$$y_{t,i,j} = 0 \quad \forall_{i,j} \notin IJ \quad (4.4)$$

Constraint 4.3 is the input balance for the product through the second level in the model. The constraint ensures that the amount of raw material in second level is the same as in first level. Parts from first level can only go a certain way through to second level and constraint 4.4 ensures that it goes the right way through to the next level.

### Constraints for third level

$$\sum_k z_{t,j,k} = \sum_i y_{t,i,j} \quad \forall_{t,j} \quad (4.5)$$

$$z_{t,j,k} = 0 \quad \forall_{j,k} \notin JK \quad (4.6)$$

$$z_{t,j,k} \leq upperlimit_{j,k} \cdot \sum_i y_{t,i,j} \quad \forall_{t,j,k} \quad (4.7)$$

$$z_{t,j,k} \geq lowerlimit_{j,k} \cdot \sum_i y_{t,i,j} \quad \forall_{t,j,k} \quad (4.8)$$

Constraint 4.5 is the input balance for the product through to the third level in the model. It ensures that the amount of raw material in third level is the same as in second level. Constraint 4.6 describes the flow from the second level to the third level, i.e. how raw

material from second level can be allocated into products on third level. Constraint 4.7 refers to the upper limit of production for each product in third level and constraint 4.8 to the lower limit. The lower- and upper limit constraints control how the raw materials from second level go through to third level. The lower limits are only for products like Cut off, Lard, Loin speck and Trash. Raw material that goes into these products cannot be utilized into other pork products. This is described further in chapter 4.6.2.

### Other constraints

$$prod_{t,k} = \sum_j z_{t,j,k} + level3_{t,k} \quad \forall_{t,k} \quad (4.9)$$

$$inv_{t,k} = prod_{t,k} - demand_{t,k} + beginv_k \quad \forall_{t=1,k} \quad (4.10)$$

$$inv_{t,k} = inv_{t-1,k} + prod_{t,k} - demand_{t,k} \quad \forall_{t>1,k} \quad (4.11)$$

Constraint 4.9 represents the production level for each product. The decision variable,  $level3_{t,k}$ , represents how much extra raw material needs to be bought from vendor at each time, that is added to the model in third level. Constraint 4.10 represents the necessary inventory level at the beginning of the time period and constraint 4.11 represents the inventory level in all other time periods.

## 4.5 Mathematical model verification

When developing a mathematical model; constraints, variables and objective function needs to be verified and ensured that they are working as it is supposed to. The process of testing and improving a model to increase its validity is commonly referred to as model validation [10]. A model validation was done in this research by developing a simplified version of the model. Then we move toward a more extensive model that reflects the complexity of the real problem.

The simplified version of the model was verified by applying a test case to the model. The test case is for a given data; the input is a certain amount of carcasses that flows through the model, product value, lower- and upper limit for production and fixed division of the carcass. The model consisted of three levels with four products, as can be seen in Figure 4.2. The first node is the amount of carcasses that is supposed to be processed, which



is divided into two parts by a fixed proportion. At every step, all constraints, variables and objective function were calculated by hand to ensure that the model is giving right results and behaving as it is supposed to. In this type of model, we are not producing up to demand. We assume all products produced will be sold. This model does not take time into account.

Figure 4.2 shows the simplified version of the model. It shows how a 10 kg. of carcass flows through the model. It divides into two parts in first level; Side and Leg. The Side flows through to the next level undivided. Leg divides into Ham and Bayon ham as shown in Figure 4.2. The raw material Side is then allocated into the products; Side, Ham I, Cut off and Lard. The upper limit represents how much of the raw material Side can be allocated into each product. The lower limit represents the minimum amount of raw material that needs to be allocated into each product. As can be seen, not all products have lower limit. The reason for this is because products like Cut off or Lard is a part of the pork that cannot be utilized in other products. The raw material Leg can be allocated into the products; Cut off, Bayon ham, Pesto ham and Lard with a given upper- and lower limit. Then the Bayon leg can be allocated into the products; Cut off, Bayon ham and Lard. The maximized revenue from the simplified version of the model was 1.367 ISK. The upper- and lower limit is characterized as UL and LL in Figure 4.2.

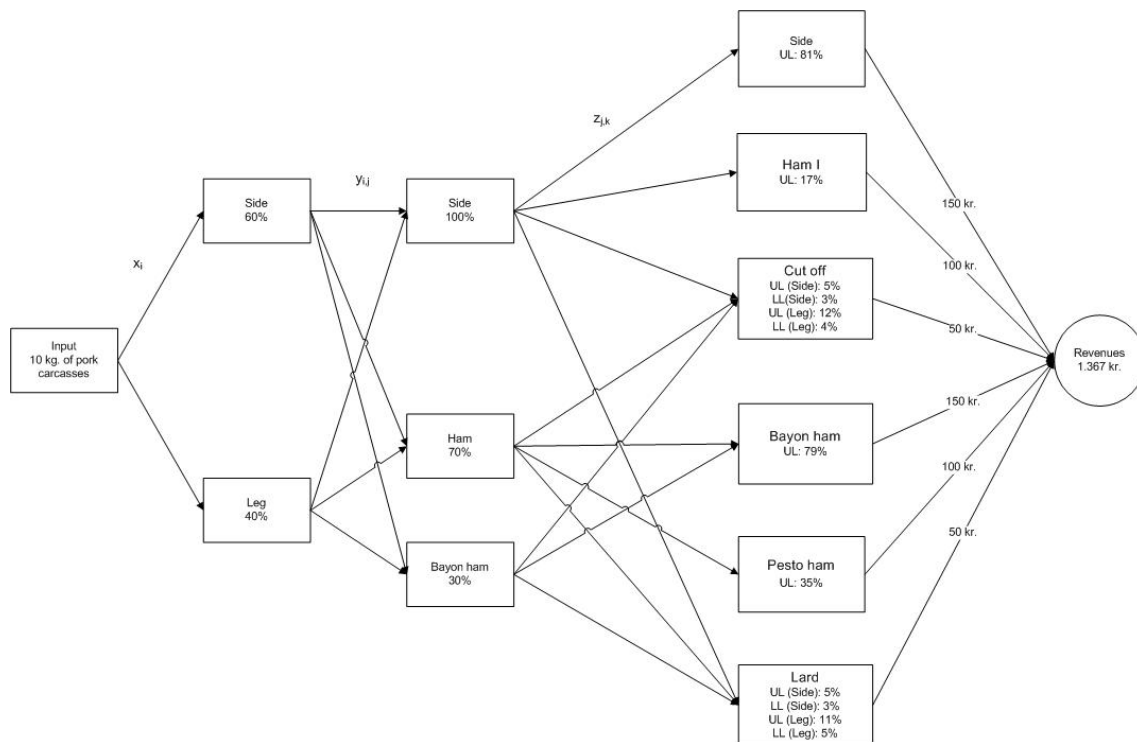


Figure 4.2: Illustration of the model using simple data set.

One possible way to assess the closeness with which the final version of the model represents reality is to compare model production with actual production over the planning horizon to which the data used to test the model applies. Results from this comparison can be seen in chapter 6.1.

## **4.6 Model limitations and simplifications**

### **4.6.1 Limitations**

There are a few limitations in the data set available from the meat processing plant under study, which certainly affect the final results. The first limitation is that the production cost for each product is not taken into account in the model. The main reason is a lack of information from the meat processing plant under study. The meat plant does not keep track of this cost. It would however be interesting to add this variable to the model to get more realistic value for each product and in the end a more realistic result of revenues. A valuable product can have a high production cost, which might lead to the model to allocate the raw material into other products. If production cost would be taken into account, it would be subtracted from the value of each product. The variable production cost would only be taken into account. Fixed cost does not affect results.

The second limitation is that the production capacity was not taken into account in the model, such as employees, machines and available raw material. It would be interesting to add these constraints in the model.

Third limitation is the inventory level at the meat plant was not available from the business ERP system. Therefore the inventory level that the model proposes cannot be compared to actual inventory level.

The fourth and the last limitation is the data discrepancy in the data set. The data discrepancy is described in chapter 5.2.

### **4.6.2 Simplifications**

Several simplifications have been done in the model. The first was the fixed division of carcasses in the first level in the model. Weight and type of the carcasses are different between carcasses, so these numbers are not always the same. On average, the division is as provided in the model.

Secondly, the lower limits of production on third level in the model, were too restrictive and not giving the right picture of how it is in reality. These limits were then critically reviewed by researcher and most of the lower limits were removed. By doing this, the model has more flexibility to allocate raw materials into products. After the review, only Cut-off, Lard, Loin speck and Trash have a lower limit of production. The reason why these products still have lower limit is because it is not possible to allocate these products into other pork products. It is possible to use, e.g. Ham I, which is the most valuable ham, in Ham II or Ham III products, though it is not as valuable. It is however possible.

The third and the last simplification is the input in the model, which is the sum of the demand, for each time period. In reality, it would be the amount available at each time, i.e, what the meat plant is supplied from the slaughterhouse.



# Chapter 5

## Data

To attain accurate and useful results the model requires accurate data. The data applied to the model was gathered from the business ERP system at the meat plant. Our main contact for this research in the meat processing plant in Hvolsvöllur, is Ágúst Guðjónsson, head of the planning department. Our data consists of the following variables.

### **Demand**

The demand used in the model is actually the amount processed in each time period, either for use right away within the meat plant or to keep as an inventory for high peak seasons like summer, Christmas or Easter. The demand data describes the demand that is within the meat plant, not the demand for final products that goes to retailers.

If the mathematical model will be used to make a production plan for a given planning horizon in the nearest future, then the model requires that the meat plant makes a forecasted demand plan for the given planning horizon. The model is based on this forecasted demand, so it is fundamental that the forecast is done in the most accurate way as possible. Forecasted demand can be done in many different ways, but it is important that historical data are available. The forecasted demand needs to reflect all seasonal fluctuation for the planning horizon.

### **Product value**

The products in this research are not final products that go to retailers. They are pork products used in further production mix within the meat processing plant. Pork products are used in different kind of final products. For example is Ham I, Ham II and Ham III,

used in different kind of ham in slices of meat, mixed with other raw materials. Ham I is the most valuable ham, and goes therefore into luxury ham, as honey glazed ham. All kind of sausages and force meat produced in the meat plant are a product mix of pork, beef and lamb products, along with other raw materials. Other products are used for example in chops, marinated barbeque products, smoked rack of pork or in readymade meals.

The product value is primarily based on the price that the meat plant buys the carcass from the slaughterhouse. The market price is also estimated and the fat proportion. The material is worth more if it has a low fat proportion. The products available from processing pork carcass can be seen in Table 5.1 sorted from the lowest value to the highest.

<i><b>Product</b></i>
Trash
Blood meat
Lard
Loin speck
Cut off
Shank
Side
Spare ribs
Fore saddle
Ham III
Bacon ham
Shoulder
Bayon ham
Loin without crackling
Christmas ham
Ham II
Pesto ham
Saddle
Ham I
Fillet
Loin boneless in chop
Loin boneless in slices of meat

Table 5.1: Pork products sorted from the lowest value to the highest.

## Input

The input in the model is the sum of total demand for each month in the planning horizon. If the mathematical model will be used to make a production plan for a given planning horizon in the nearest future, then the input would be what the meat plant in Hvolsvöllur is supplied from the slaughterhouse at each time.

The model allocates first the whole carcass into parts. Then the parts are divided into raw material and finally into pork products after forecasted demand in the most optimal way possible. If the input is more than forecasted demand, the model suggests what products should produce and store as inventory. The input might not be enough to meet the demand, so the meat plant is not able to produce to meet forecasted demand. Then the model allocates the raw material into products, first according to the minimum production constraint, which is the lower limit constraint for each raw material. Then the rest is allocated into products with the goal to reach the demand constraint in the most valuable way.

### **Division of the carcass**

The division of the carcass into parts in the first level is fixed. The division was determined by our contact at the meat plant. The division is based on how it was on average in the year of 2010.

### **Lower- and upper limit of production in third level**

In the abstract model, every node from second level enters every single node at third level. Our data restrict these conditions with lower- and upper limit on each product in third level. The lower- and upper limit for how the raw material flows from second level to the third level was determined by our contact in the meat plant. These limits are mostly based on how it was on average in the year 2010 and partly based on experience. The lower limits were too restricted as was mentioned earlier. These limits were therefore critically reviewed and products like Cut off, Lard, Loin speck and Trash were the only products left that still have a lower limit of production. There is always certain percentage of the carcass that goes to the Trash. Same applies to Cut off, Lard and Loin speck, there is always certain percentage of the carcass that goes to these products and it is not possible to allocate these products into other products. Lower limit of production can be seen in Table B.1. Upper limit of production can be seen in Tables B.2 and B.3.

The lower limits represent a minimum production level of the product. For example is minimum production of the product Lard from the raw material Bayon ham 5.0% as can be seen in Table B.1. The upper limits represent a maximum production level of each product. For example is maximum production of the product Spare rib from the raw material Side 14.0% as can be seen in Table B.2. If a product has no lower or upper limit from a certain raw material, it means that this product cannot be produced from that certain raw material.

### **Extra raw material bought from vendor**

Hot dogs and all kind of sausages along with dry sausages, like Salami and Pepperoni are a product mix of many different kinds of products and one of them is Lard. The Lard processed in the meat plant is often not enough to reach demand, so the meat plant is supplied with extra Lard from vendor. The extra Lard bought from vendor is added to the model in third level.

The meat plant processes the part Side for vendor and they return it back as bacon. The Side is added to the model in first level.

### **Cost variables**

Inventory cost was not available from the business ERP system at the meat plant. The inventory cost for the slaughterhouse was however available. It was estimated to be 20% of production cost of slaughtering, which is 8 ISK per kilogram.

Product deterioration for inventories at the end of the planning horizon is estimated to be 30% of product value. This is to prevent that the model collects unnecessary inventory over the planning horizon.

Cost of buying extra Lard from vendor is 65 ISK per kilogram and Side is 385 ISK per kilogram.

## **5.1 Division of pork carcass**

According to our data, the network diagram, as can be seen in A.1, describes how a carcass of pork is divided into different parts on first level, then further into different raw material on second level and at last in pork products on third level. The division of the pork can be seen graphically in Figure 5.1. The division is also listed along with proportion of each part of the total carcass, which is fixed, in Table 5.2.

From the first level, Loin and Leg divides further to the second level, into different kind of raw material. Loin divides into three raw materials and Leg divides into two raw materials as can be seen in table 5.2. Other parts from first level go through to the second level undivided. Then all raw materials from second level are allocated into products according to product value, forecasted demand and lower- and upper limit on production capacity to the third level. The products in third level in the model can be seen in Table 5.1 and are sorted from the lowest value to the highest.



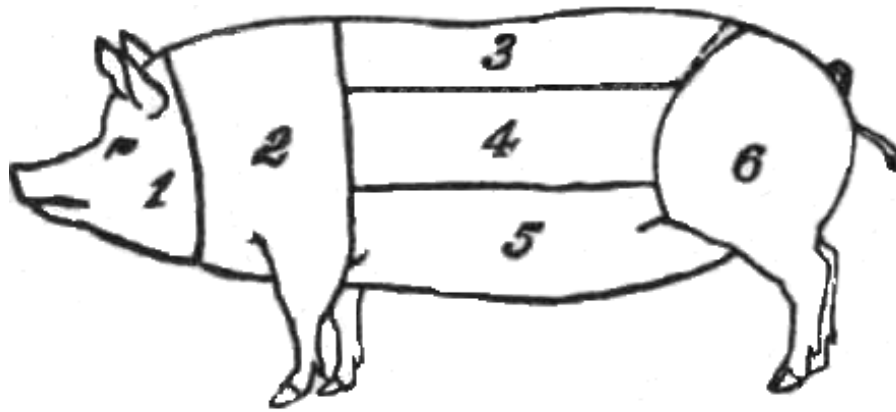


Figure 5.1: Diagram of how pork is divided into parts [9].

<i>Nr.</i>	<i>Part/ Raw material</i>	<i>Proportion</i>
1.	Saddle	15,4%
1.	Fore saddle	2,5%
2.	Shoulder	16,6%
3.	Fillet	1,8%
4.	Loin	15,5%
	Loin in slices of meat	
	Loin	
	Loin chop	
5.	Side	15,4%
6.	Leg	32,8%
	Leg	
	Bayon ham	

Table 5.2: The division of pork carcass into parts and raw material.

## 5.2 Data discrepancy

A part of the data set includes some discrepancy which makes it impossible to run the model with the constraint that describes the upper limit of production on third level, unless a decision variable is added to the model that describes the necessary inventory level at the beginning of the planning horizon.

The discrepancy is characterized by the upper limit is in some cases not high enough. The production of Bacon ham, for example, in January 2010, was 3.214 kg. actually. Bacon ham is only made from the Shoulder part of the pork, which is 16.6% of the total input. Available raw material from Shoulder part is 8.291 kg. in January 2010. The upper limit of production of Bacon ham is 15% of this 8.291 kg. So, it is possible to produce 1.243,5 kg. of Bacon ham in January 2010, according to our data. Therefore, there is a lack of

1.970,5 kg. to reach as it was actually. The variable that describes the necessary inventory level at the beginning of the planning horizon is to cover when this happens.

# Chapter 6

## Results

The goal with the model is to make a mathematical model that maximizes the product value of pork, by giving a suggestion of how to allocate the input into products according to product value and demand, as previously mentioned. In this chapter the results from the model will be summarized. The results will be compared with the existing production plan for pork products. A sensitivity analysis will be done for first and third level of the model and the shadow prices interpreted. The solution time of the linear model is less than 5 seconds. The model consists of 4.462 variables and 7.284 constraints.

### 6.1 Comparison between actuality and model

In this section a comparison of the results from the model with how the production actually was in the year 2010. We used historical data to reconstruct the past to determine how well the model and the resulting solution would have performed if they had been used. Comparing the effectiveness of this hypothetical performance with what actually happened indicates whether using the model tends to yield significant improvements [10].

As have been mentioned before, a decision variable was added to the model that describes the beginning inventory level at the meat plant. The model suggests that the meat plant should begin with the inventory level shown in Table 6.1. Products that have beginning inventory are products that can't be produced to meet demand in the beginning of the planning horizon, because of the discrepancies in the upper limit constraint of production.

<i><b>Product</b></i>	<i><b>Beginning inventory (Kg.)</b></i>
Side	12.466,4
Cut off	14.187,6
Bacon ham	1.970,5
Fillet	45,0

Table 6.1: Necessary initial inventory at the beginning of the time period.

This type of model reflects the actual production plan for 2010 as closely as possible. Production for most products are rather similar between actual and what the model proposes. In some instances the model produces more than in actual and keeps what is in excess as an inventory. The model also has beginning inventory level, because of the discrepancy in the dataset. Revenues are obtained by multiplying the value of each product with the production quantity. Inventory- and production cost is not taken into account in actuality, as have been mentioned before. However, inventory cost is included in results from the model.

Following is more detailed results for products showing interesting comparison between how it was actually and what the model proposed. As have been mentioned before, the inventory level cannot be compared, because of lack of information about how the inventory level is actually at the meat plant. So the following graphs show production and inventory level as the model proposes and production level for how it was actually. First a comparison will be shown where the model is producing exactly like actually. Then where the model is producing

### 6.1.1 Production same in model and actually

Figure 6.1 shows an example of a product where the model was producing exactly the same as actually. There was one product in addition to Shank, that were producing exactly the same as actually, which was Christmas ham. There were also nine products that were producing over the year the same as actually, but in some months the model was producing more and kept the excess of demand in stock. Figure 6.2 shows an example of product that collects in stock in some months. As can be seen in Figure 6.2 the demand for this product, Lard, is rather steady over the year. The production increases however during the summertime. The reason is preferably that Lard is processed in all kinds of hot dogs which are very popular products over the summertime. SS hot dogs are very popular in Iceland and the market share has been rather high over the past decades. In 2010 the amount of produced hot dogs at the meat plant was 750 tons [6].

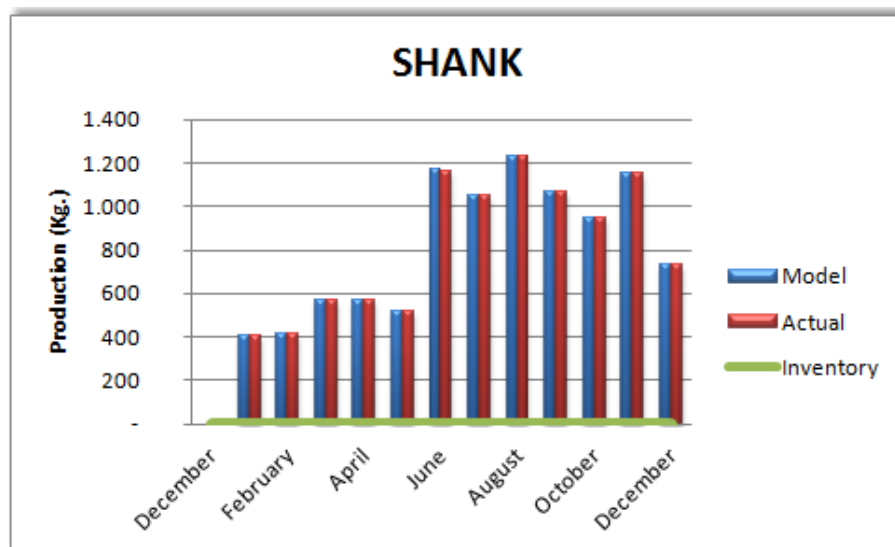


Figure 6.1: Comparison of production level for Shank from January until December 2010 and inventory level from the end of December 2009 until December 2010.

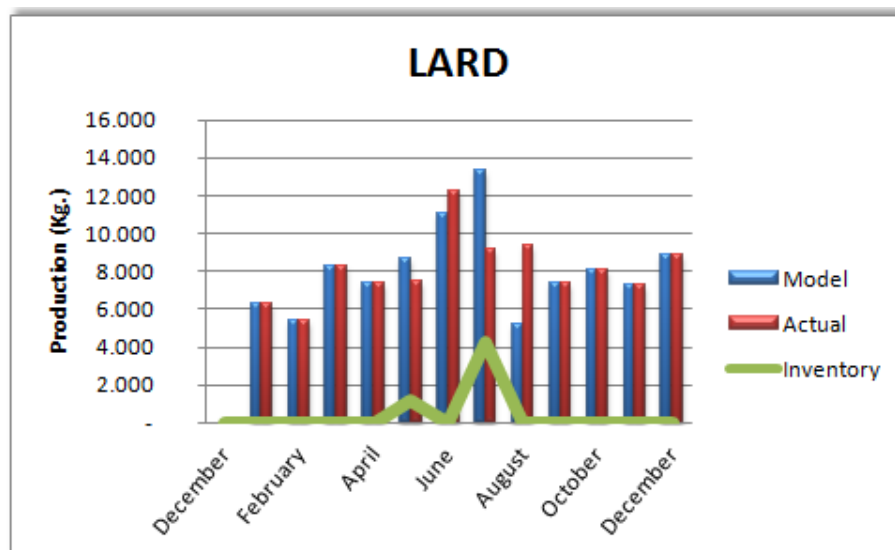


Figure 6.2: Comparison of production level for Lard from January until December 2010 and inventory level from the end of December 2009 until December 2010.

### 6.1.2 Model producing more than actually

Figure 6.3 shows a product where the demand is rather high in December compared to other months. The reason for this high demand in December is because Loin without crackling is processed in smoked rack of pork, which is very popular in December over the Christmas time. The meat plant collects these loins over the year, where they choose the best loins every time and place them in the freezer.

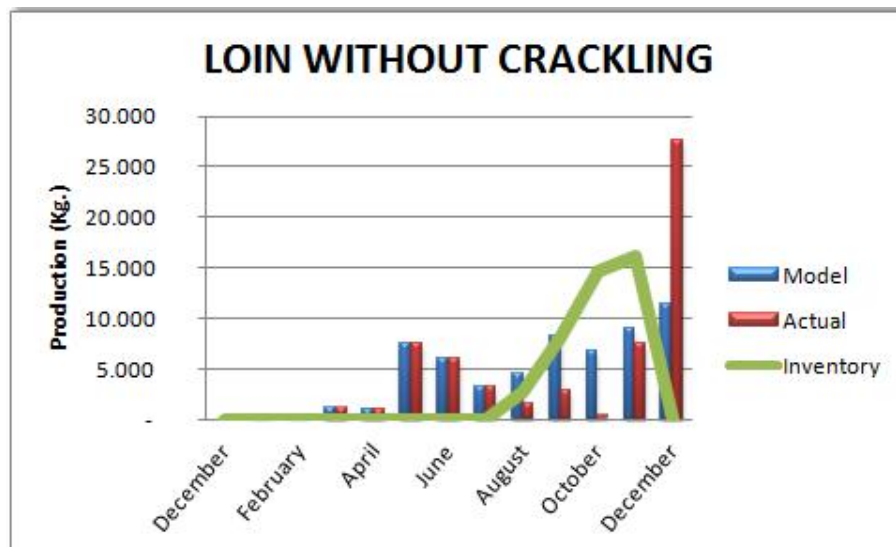


Figure 6.3: Comparison of production level for Loin without crackling from January until December 2010 and inventory level from the end of December 2009 until December 2010.

In December the model has over 16.000 kg. of raw material to allocate to other products. The model allocated most of it into Ham I and Fillet. Figure 6.4 shows how the model allocates almost 9.000 kg. more than needed of Ham I in December. This can indicate what products SS should concentrate on in a marketing point of view.

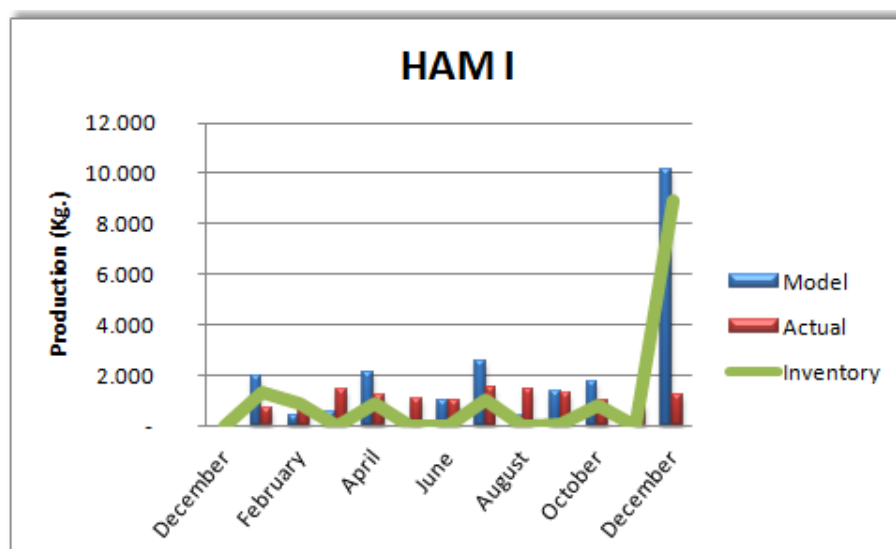


Figure 6.4: Comparison of production level for Ham I from January until December 2010 and inventory level from the end of December 2009 until December 2010.

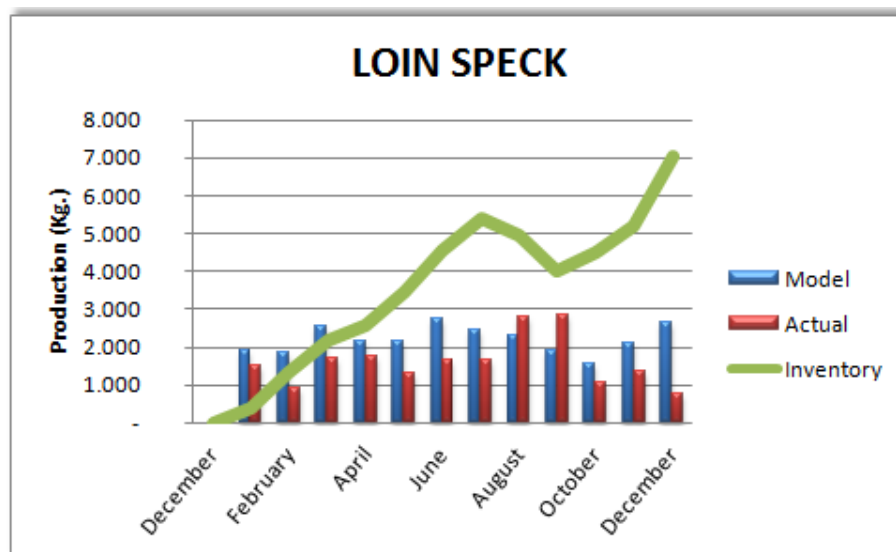


Figure 6.5: Comparison of production level for Loin speck from January until December 2010 and inventory level from the end of December 2009 until December 2010.

### 6.1.3 Limitations in lower limit constraint

The lower limit constraint of production was in some cases not providing an accurate picture of how it was actually. Loin speck is one of the products that still has a lower limit of production in the model and an example of a product where the lower limit constraint is forcing the model to produce more than needed in almost every month, as can be seen in Figure 6.5. At the end of the planning horizon the inventory level is over 7,000 kg. There were four additional products where this occurs, such as Side, as can be seen in Figure 6.6.

### 6.1.4 Limitations in upper limit constraint

The upper limit constraint was not in all cases providing an accurate picture of how it was actually. It can be seen if we look at the product Cut off. The Cut off is utilized in a lot of products in the meat plant, like hot dogs, schnitzel, meat puddings and meat balls. The model suggests that the inventory level at the beginning of the planning horizon of Cut off should be over 14,000 kg. This is because, though the model is producing maximum amount possible according to the upper limit constraint, it doesn't manage to produce enough to meet the demand. This happens in June until August, as can be seen in Figure 6.7. At the end of the year the inventory level is zero. It is possible to add a constraint that forces the model to end up with inventory level at the end of the planning horizon, to cover the lack of Cut off the first months.

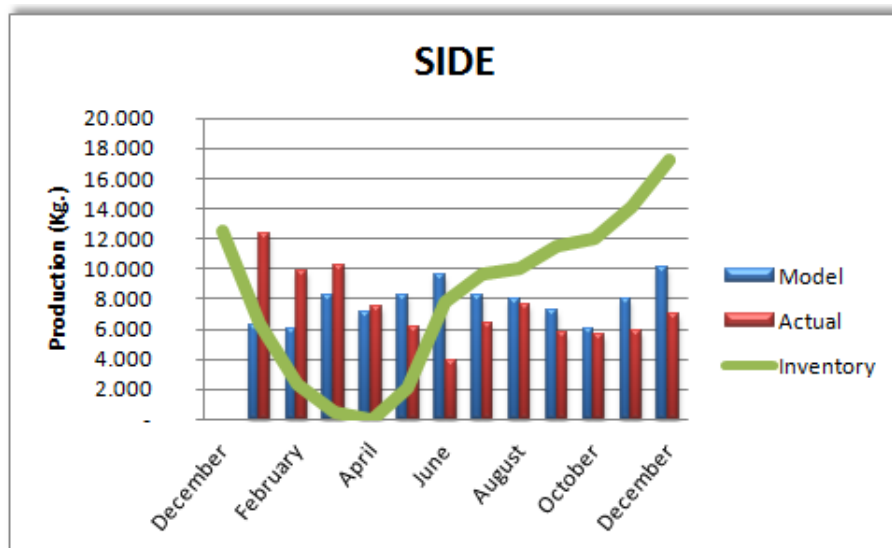


Figure 6.6: Comparison of production level for Side from January until December 2010 and inventory level from the end of December 2009 until December 2010.

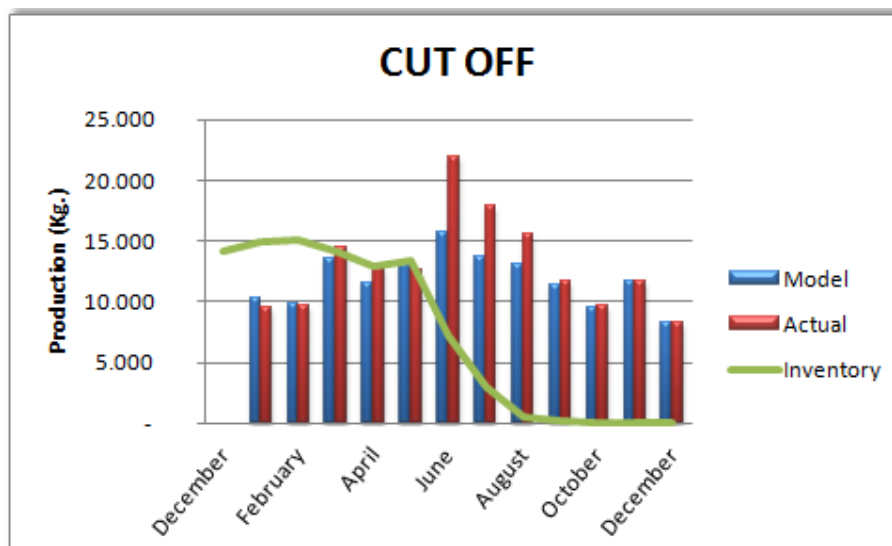


Figure 6.7: Comparison of production level for Cut off from January until December 2010 and inventory level from the end of December 2009 until December 2010.



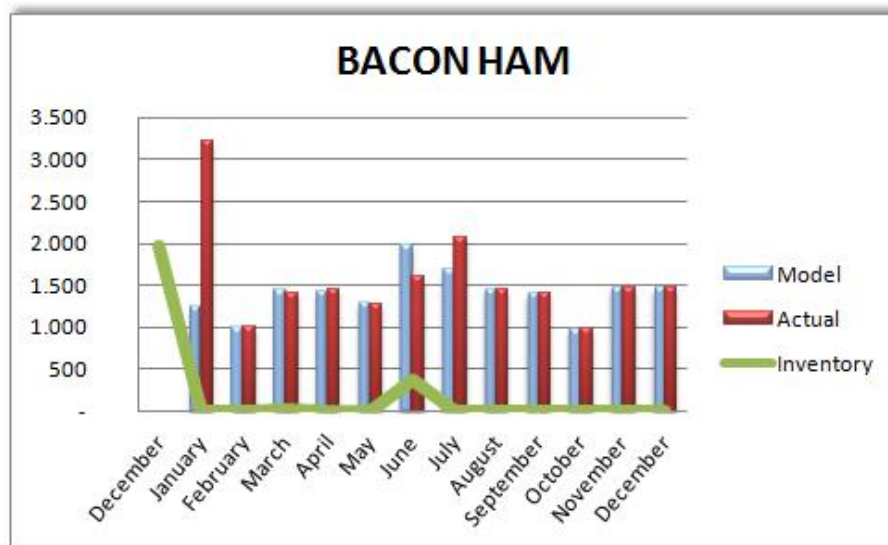


Figure 6.8: Comparison of production level for Bacon ham from January until December 2010 and inventory level from the end of December 2009 until December 2010.

The same applies to the product Bacon ham. The model suggests that the inventory level at the beginning of the planning horizon should be 1.970 kg, as can be seen in Figure 6.8. This is because though the model is producing maximum amount possible according to the upper limit constraint, it does not reach to meet demand. This only happens in January. The production as the model proposes and in actual are very similar the rest of the planning horizon.

### 6.1.5 Comparison of revenues

If we compare revenues from the model with as it was actually, then gross income is 10.15% lower in actual then from the production that the model proposes. The model is however producing 4.17% more than actual production was, in the form of beginning inventory and buying from vendor. When cost is taken into account the income is 5.3% lower in actuality. The cost in the model is much higher than in actual, both because it includes inventory cost and also because we assume that the inventory level at the end of the planning horizon costs 30% of product value. The model however assumes that all products produced sells.

## 6.2 Sensitivity analysis

Sensitivity analysis on a linear programming model provides information on the sensitivity of the optimal solution when data values are changed. When formulating a linear program model, data is often estimated based on experience and assumptions and can therefore be inaccurate. Sensitivity analysis in linear programming offers extensive capability to demonstrate how the optimal solution changes when the data is changed. Questions such as "what happens if there is higher demand at time  $t$  for product  $k$ " or "is it worth it to produce product  $k$ ", can be answered by applying sensitivity analysis. Are there new opportunities out there which the processing plant should respond to? Has demand changed for certain products? Is it too expensive to produce certain range of products? For each of the constraints a shadow price is given, which is the rate at which the objective function changes. A sensitivity analysis will be done for the first and third level in the model. The constraint in second level is only showing how parts from first level go through to next level. Parts from first level go in almost all cases undivided to next level. Only Loin and Leg divides further and they can go 100% in each raw material they divide into. At last a sensitivity analysis will be done for the demand constraint.

### 6.2.1 Level 1

Since the limits are fixed in first level in this model, constraints cannot be changed in order to find better solution. If we look at the results given in MPL, regarding shadow prices for level 1, the order of importance can be seen, for the parts of the carcass. Fillet is the most expensive part of the carcass, then Side and the least expensive part is Fore saddle. The order of importance can be seen in Figure 6.9.

The shadow prices for this level represents how much the objective function would increase if one extra kilogram of carcass could go through each part. The objective function would clearly increase most if higher proportion of the carcass could be allocated into Fillet.

### 6.2.2 Level 3

The highest shadow prices for third level will be examined and looked at the possibility of changing the constraints in order to obtain better solutions. The prior constraint for this level is representing the input balance. Shadow prices for that constraint will not be examined. This constraint is only ensuring that the amount of carcasses in third level

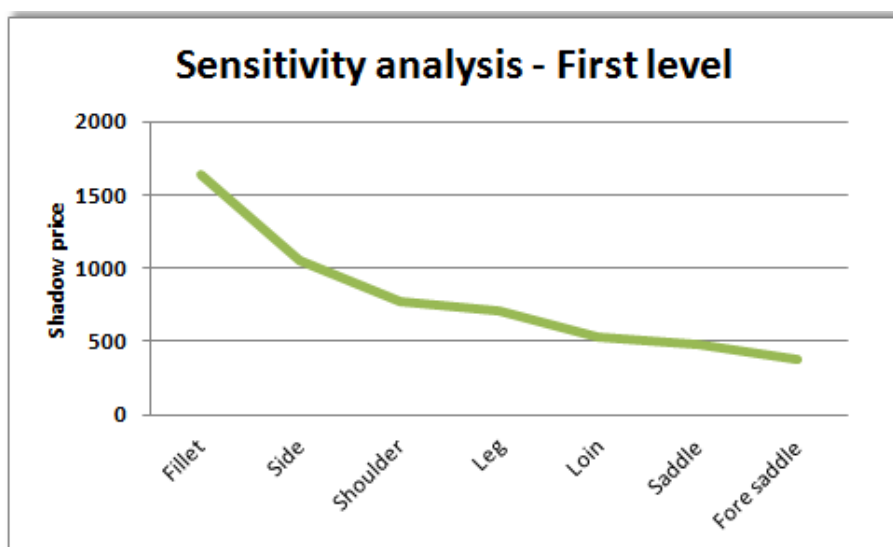


Figure 6.9: The parts of the carcass in order of importance

is the same as in second level. Constraints that represent the lower- and upper limit of production will however be examined here. Shadow prices for these constraints for each raw material at second level in the model can be seen here below.

### Side

The raw material Side can be allocated into four products; Ham III, Spare ribs, Side and Cut off. The production of the product Side could not meet demand in the first four months as previously mentioned, so there is no slack in production in these months. The shadow prices that show up in these months are for products that cannot be produced from the raw material Side, as Bacon ham and Fillet. Shadow prices can be seen in Table 6.6. The shadow price of Bacon ham in January represents how much the objective function would increase if it would be possible to produce Bacon ham from the raw material Side. Bacon ham can't meet demand at that time because of the discrepancy in the upper limit constraint as have been mentioned before.

As can be seen in Figure 6.6, production of the product Side from January until April was always lower than production was actually. Then in May until December the production is in all instances higher than production was actually. This is because the upper limit constraint forces the model to always produce maximum amount of Side, because it is already allocating maximum amount in other products that is allowed to allocate from the raw material Side. This explains why there are so many products with rather high shadow prices in the period from May until November. It would clearly be more value adding if

<i><b>Product</b></i>	<i><b>Shadow price (ISK)</b></i>	<i><b>Period</b></i>
<i><b>Upper limit</b></i>		
Ham I	546,0	May - Nov.
Ham II	554,0	
Ham III	554,0	
Lard	554,0	
Cut off	965,0	
Christmas ham	546,0	
Pesto ham	554,0	
Bayon ham	565,7	
Bacon ham	554,0	
Loin boneless in slices of meat	546,0	
Loin boneless in chop	535,5	
Loin without crackling	432,0	
Shoulder	554,0	
Saddle	237,0	
Bacon ham	236,0	Jan.
Fillet	552,7	Feb. - Apr.
Fillet	1.804,0	May - Oct.
<i><b>Lower limit</b></i>		
Cut off	-83,0	Jan. - Apr.
Trash	-1.110,0	Jan. - Apr.

Table 6.2: Shadow prices for products from the raw material Side.

the raw material Side could be allocated into these products listed in Table 6.2 in May until November.

The lower limit constraint shows shadow prices that describes how much the objective function would decrease if the model would allocate more raw materials into a certain products. Products that show shadow prices from the raw material Side are Cut off and Trash. The shadow prices can be seen in Table 6.6. Both products have negative values of shadow prices which would lead to the objective function decreasing of the amount given in Table 6.6 if the lower limit would increase according to one extra kilogram.

### **Loin in slices of meat**

The raw material Loin in slices of meat can be allocated into five products; Ham I, Ham III, Loin speck, Cut off and Loin in slices of meat. In January the highest shadow prices is for the products; Spare ribs, Side, Bacon Ham and Fillet. The shadow prices can be seen in Table 6.3. The reason for this high shadow price of Side and Bacon ham in January is because the model is not able to produce to meet demand of these products at that

<i>Product</i>	<i>Shadow price (ISK)</i>	<i>Period</i>
<b><i>Upper limit</i></b>		
Spare ribs	668,7	Jan. - Apr.
Side	668,7	Jan. - Apr.
Bacon ham	904,7	Jan.
Loin in slices of meat	167,8	Jan. - Dec.
Loin boneless in chop	111,0	Jan. - Dec.
Fillet	552,7	Jan. - Oct.
<b><i>Lower limit</i></b>		
Loin speck	-640,0	Jan. - Apr.
Fore saddle	-485,0	Mar.
Trash	-610,0	Jan. - Mar.

Table 6.3: Shadow prices for products from the raw material Loin in slices of meat.

time. The reason is because of the discrepancies in the upper limit constraint. In other months after January, the highest shadow price is for Fillet. Then in every month are shadow prices for Loin in slices of meat and Loin boneless in chop. In average they are as interpreted in Table 6.3. Shadow prices for all products mentioned above, except Loin in slices of meat represent what the objective function would increase if these products could be produced from the raw material Loin in slices of meat. In reality these products cannot be processed from that raw material. The shadow price for Loin in slices of meat represent what the objective function would increase if the upper limit constraint would allow one extra kilogram of production of this product.

Concerning shadow prices for the lower limit constraints, products like Loin speck, Fore saddle and Trash show shadow prices from the raw material Loin in slices of meat. As can be seen in Table 6.3, the shadow prices have all negative values for the lower limit constraint. This would lead to the objective function decreasing of the amount given in Table 6.3 if the lower limit would increase according to one extra kilogram of raw material. It is interesting to see that the shadow price for Loin speck has higher negative value than for Trash, as can be seen in Table 6.3. Meaning that the objective function would decrease more if the lower limit would allow one extra kilo of raw material into Loin speck rather than into Trash. This is mainly because Loin speck comes from the raw materials; Loin in slices of meat, Loin, Loin chop and Saddle. These raw materials are allocated into rather expensive products. While Trash comes from all possible raw materials. So it would have more influence on available input to allocate it into raw materials that is allocated into Loin speck rather than into other raw material that can be allocated into less expensive products.

## **Loin and Loin chop**

If we look at the upper limit constraint, the highest shadow prices from January until April is for Spare ribs, Side, Cut off and Fillet. The shadow prices can be seen in Table 6.4. As can be seen, it is all products that can't be produced from this type of raw material actually, except Cut off. So, these shadow prices represent what the objective function would increase if it would be possible to produce these products from Loin or Loin chop.

The reason for the high shadow price of Side and Bacon ham is the same as have been mentioned before, the discrepancies in the upper limit constraint.

The high shadow price of Cut off from January until October is because the model cannot, in a few instances, produce to meet demand of Cut off because of the discrepancies in the upper limit constraint. The model suggests that inventory level in the beginning of the planning horizon should be over 14.000 kg. of Cut off to cover the lack of this product. It is more expensive to keep the product as an inventory over the whole planning horizon than producing the product constantly over the planning horizon. That's the reason for why the shadow price of Cut off is more than double the value of the product.

Fillet has generally very high shadow price because of its high product value. The shadow prices for Fillet decreases vastly in November and December. The reason is because the model is forced to produce more than demand in these months. The raw material Fillet is allocated into the products Cut off and Fillet. The model produces minimum amount of Cut off and the rest goes to Fillet which is more than demand in November and December. So it is not value adding for the objective function of producing more of the product in these months.

The model does not produce maximum amount of the products Loin without crackling or Loin boneless in chop through the planning horizon, so there is always slack in the upper limit constraint for this product. This leads to a zero value in the shadow price, meaning that the objective function would not increase if the upper limit constraint would allow more production of this product.

Concerning shadow prices for the lower limit constraint, products like Loin speck and Trash show shadow prices from the raw material Loin and Loin chop. As can be seen in Table 6.4, the shadow prices have all negative values for the lower limit constraint. This would lead to the objective function decreasing of the amount given in Table 6.4 if the lower limit would increase according to one extra kilogram. As before, the shadow price of Loin speck gives higher negative value than for Trash. The reason is the same as for Loin in slices of meat.

<i>Product</i>	<i>Shadow price (ISK)</i>		<i>Period</i>
	<i>Loin</i>	<i>Loin chop</i>	
<b><i>Upper limit</i></b>			
Spare ribs	608,5	509,8	Jan. - Apr.
Side	608,5	509,8	Jan. - Apr.
Cut off	532,1	428,2	Jan. - Oct.
Bacon ham	841,1	745,1	Jan.
Fillet	1.362,8	1.265,8	Jan. - Oct.
<b><i>Lower limit</i></b>			
Loin speck	-535,2	-633,3	Jan. - Mar.
Trash	-505,2	-603,3	Jan. - Mar.

Table 6.4: Shadow prices for products from the raw material Loin and Loin chop.

### Saddle and Fore saddle

If we look at the upper limit constraint, the highest shadow prices are very similar as for the Loin parts; Loin in slices of meat, Loin and Loin chop. Here we have high shadow prices for Side from January until April. The reason is the same as was previously mentioned. The model cannot produce to meet demand because of the discrepancies in the upper limit constraint. Shadow price for Bacon ham is high in January as for all other parts so far for the same reason as before mentioned. Cut off and Fillet have high shadow prices from January until October as can be seen in table 6.5. The reason is the same as was mentioned earlier.

The model does not produce maximum amount of the products Saddle or Fore saddle through the planning horizon, so there is always slack in the upper limit constraint for this product. This leads to a zero value in the shadow price, meaning that the objective function would not increase if the upper limit constraint would allow more production of this product.

Again we have higher negative value for shadow price of Loin speck than for Trash, for the lower limit constraint. The reason is the same as for Loin in slices of meat.

### Shoulder, Ham and Bayon ham

The highest shadow prices for the raw materials; Shoulder, Ham and Bayon ham are for the products; Spare ribs, Side, Cut off, Bacon ham and Fillet. Shadow prices can be seen in Table 6.6. As was mentioned earlier, the reason for the high shadow price of Side, Cut off and Bacon ham is because of the discrepancies in the upper limit constraint. As

<i>Product</i>	<i>Shadow price (ISK)</i>		<i>Period</i>
	<i>Saddle</i>	<i>Fore saddle</i>	
<b>Upper limit</b>			
Spare ribs	811,0	987,0	Jan. - Apr.
Side	811,0	987,0	Jan. - Apr.
Cut off	728,0	904,0	Jan. - Oct.
Bacon ham	1.047,0	1.223,0	Jan.
Fillet	1.567,0	1.743,0	Jan. - Oct.
<b>Lower limit</b>			
Loin speck	-331,0	0,0	Jan. - Mar.
Trash	-301,0	-125,0	Jan. - Mar.

Table 6.5: Shadow prices for products from the raw material Saddle and Fore saddle.

was mentioned in the section for Loin and Loin chop, the high shadow price for Fillet is because of the high product value. Then in November and December the shadow price decreases vastly for the same reason as have been mentioned before.

The shadow prices for Cut off and Bacon ham describes what the objective function would increase if it would be possible to increase the upper limit constraint according to one extra kilogram of raw material. Other shadow prices represents what the objective function would increase if it would be possible to process these products from the raw materials; Shoulder, Ham or Bayon ham.

The lower limit constraint only shows shadow prices for Trash, as can be seen in Table 6.6. These shadow prices represents what the objective function would decrease if the lower limit would increase according to one extra kilogram allocated into this product from the raw materials Shoulder, Ham or Bayon ham.

### Fillet

The raw material Fillet can only be allocated into the products Cut off and Fillet. The highest shadow price here is for the product Fillet, with shadow price of 839 ISK, in January, February, April and July. In other months the shadow price is zero.

## 6.2.3 Demand

A sensitivity analysis was done for the demand constraint in the model, in order to find what products are important to concentrate on. The demand was increased by 10% for ev-



<i>Product</i>	<i>Shadow price (ISK)</i>			<i>Period</i>
	<i>Shoulder</i>	<i>Ham</i>	<i>Bayon ham</i>	
<b><i>Upper limit</i></b>				
Spare ribs	466,0	466,0	458,1	Jan. - Apr.
Side	466,0	466,0	458,1	Jan. - Apr.
Cut off	411,0	411,0	399,3	Jan. - Oct.
Bacon ham	690,0	690,0	682,2	Jan.
Loin in slices of meat	213,0	213,0	213,0	Dec.
Loin in chop	111,0	111,0	111,0	Dec.
Loin without crackling	93,8	93,8	93,8	Dec.
Fillet	1.250,0	1.250,0	1.246,6	Jan. - Oct.
<b><i>Lower limit</i></b>				
Trash	-650,0	-650,0	-665,9	Jan. - Mar.

Table 6.6: Shadow prices for products from the raw materials Shoulder, Ham and Bayon ham.

ery product. The model was run for each product individually to find if revenues would increase or decrease after increasing the demand. The results can be seen in Table 6.7.

By concentrating on increasing demand for the products that increase revenues when increasing demand, SS might increase its profit considerably according to the sensitivity analysis. The products that show increase in revenues are the most valuable products, except for Loin speck. Loin speck is the fourth least valuable product, as can be seen in Table 5.1. The reason for why the revenues would increase if demand for Loin speck would increase is because the model is in almost every month, in the planning horizon, producing more than needed because of the discrepancies in the lower limit constraint. So, instead of collecting the product into stock, it would be more value adding if the demand would increase and the products could be used in further product mix within the meat plant.

Revenues would decrease if demand for Fillet would increase. Fillet is however the third most valuable product, as can be seen in Table 5.1. This is interesting in the light of results of sensitivity analysis in the first level. The most important part of the carcass was the Fillet, according to the sensitivity analysis. The sensitivity analysis on first level only described what the objective function would increase much if it would be possible to add one extra kilo of each raw material through the model. It is clearly not profitable when the amount of Fillet is tens of thousands of kilos and has clearly more affect on other products.

<i>Product</i>	<i>Difference in revenues (%)</i>
<b><i>Increased revenues</i></b>	
Christmas ham	0,01
Loin boneless in chop	0,02
Pesto ham	0,04
Fore saddle	0,08
Loin speck	0,09
Ham I	0,19
Ham II	0,20
Loin boneless in slices of meat	0,33
Loin without crackling	0,36
Saddle	0,40
<b><i>Decreased revenues</i></b>	
Bayon ham	-0,01
Shoulder	-0,01
Spare ribs	-0,14
Shank	-0,14
Bacon ham	-0,19
Fillet	-0,26
Side	-0,94
Ham III	-1,79
Lard	-2,71
Cut off	-4,93

Table 6.7: Sensitivity analysis for demand.

## 6.3 Summary

The discrepancies in the data set is affecting results in a major way. The upper limit constraint is obviously not representing enough for how it is actually. Same applies to the lower limit constraint for production. If the lower- and upper limit constraints were representative of how it is in reality, the model would have more flexibility to produce products to meet demand rather than to produce often huge amount of products that go directly to the stock.

## Chapter 7

### Conclusion

Linear programming seems to be a very effective way to transform data into valuable information to support the daily decision making regarding production planning. In this research, a linear programming model has been applied to the determination of optimal production plan for pork products by maximizing the product value. The model was developed in a way that it can be implemented on any kind of carcass and even in any kind of food processing plant, only by applying different data to the model. This is not tested in this research, it would however be interesting to do so as a next step.

The model was designed to be used as a decision tool for the meat processing plant in SS. The model is considered to be used on a monthly or yearly basis and it should take into account the latest available information. In that way, an up to date production plan can be made. The model gives timing of when to produce each product, what to keep as inventory and the production level for a given planning horizon. The model also generates a shadow prices that can give helpful information's for the meat plant. Such as what products to concentrate on to try to get higher demand in order to maximize revenues.

A comparison between what the model proposes and from a data set from the meat plant under study was done. Results from the comparison indicated that a linear programming model fits very well for the production planning problem.

If the mathematical model would be implemented for use at the meat plant, it would be necessary to develop a graphical user interface. The system would have to allow the production planner to add necessary information into the system, as forecasted demand and available input along with their requests and constraints.

In a further research it would be interesting to take production cost and production capacity into account. It would give more realistic results. The value of the model depends on

the quality of the input data. In this research the data set included some major discrepancies that affected the final results. It would be interesting to apply another data set to verify the model.

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# **Appendix A**

## **A.1 Network diagram**

A network diagram that shows how the input of carcasses flows through the model can be seen in Figure A.1. The first level shows how the carcass divides into different parts. Second level shows how these parts divide further into raw materials. Then these raw materials divide into products, which are shown in third level.

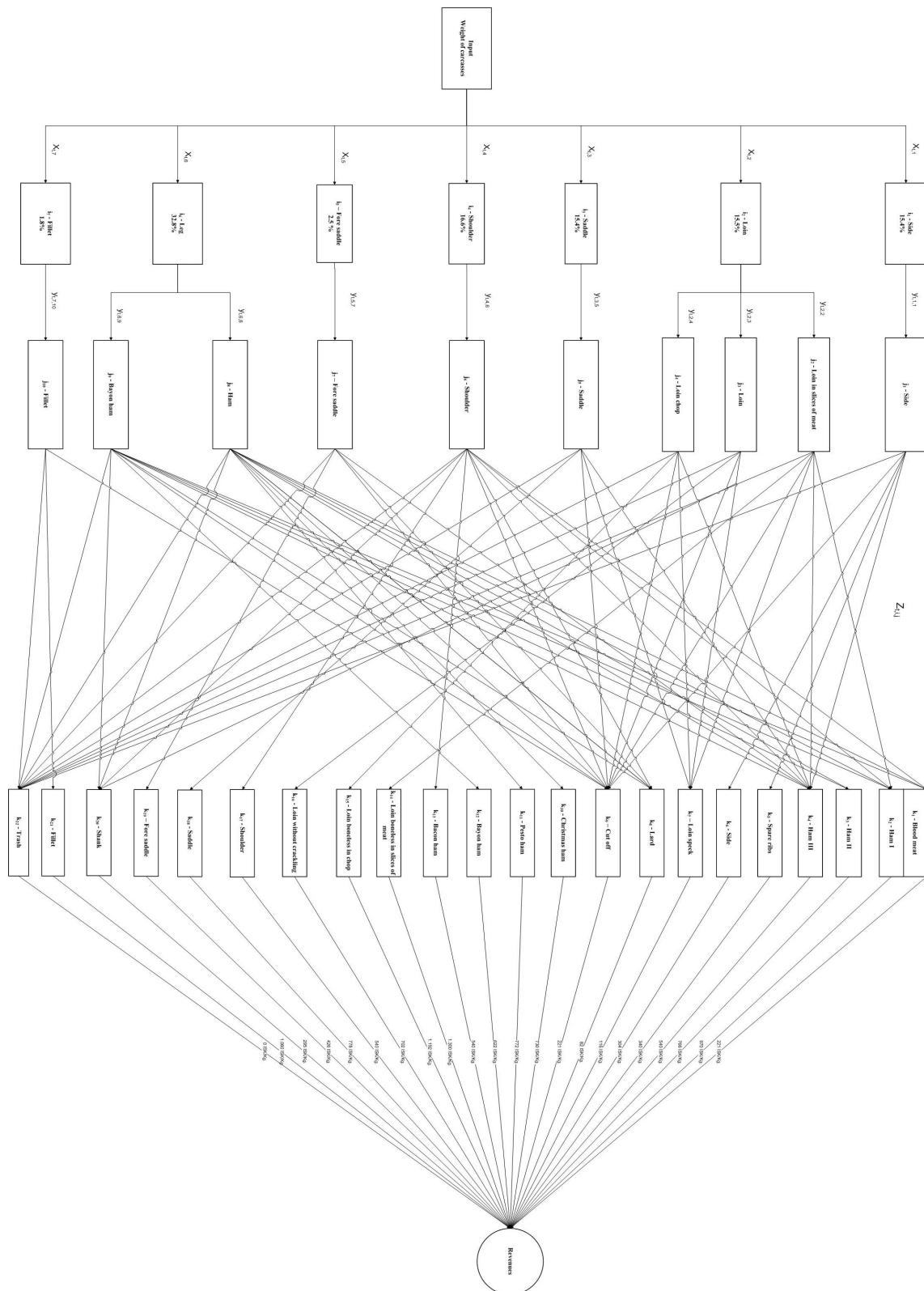


Figure A.1: Network diagram of the flow of the carcass through the model. It shows how the carcass divides into parts and further into raw material and finally into products.

## **Appendix B**

### **B.1 Lower- and upper limit of production**

<i>Raw material</i>	<i>Product</i>	<i>Lower limit (%)</i>
Side	Cut off	0,5
	Trash	5,0
Loin in slices of meat	Loin speck	15,0
	Cut off	8,0
	Trash	20,0
Loin	Cut off	10,3
	Trash	1,0
Loin chop	Loin speck	10,0
	Cut off	6,4
	Trash	23,0
Saddle	Loin speck	10,2
	Cut off	13,1
	Trash	13,0
Shoulder	Lard	10,0
	Cut off	10,0
	Trash	15,0
Fore saddle	Lard	2,0
	Cut off	10,0
	Trash	8,0
Ham	Lard	10,0
	Cut off	10,0
	Trash	8,9
Bayon ham	Lard	5,0
	Cut off	5,0
	Trash	10,0
Fillet	Cut off	26,0

Table B.1: Lower limit of production.

<i>Raw material</i>	<i>Product</i>	<i>Upper limit (%)</i>
Side	Ham III	0,3
	Spare ribs	14,0
	Side	81,0
	Cut off	2,0
	Trash	7,0
Loin in slices of meat	Ham I	42,0
	Ham III	9,0
	Loin speck	25,0
	Cut off	12,0
	Loin boneless in slices of meat	38,0
	Trash	24,0
Loin	Loin speck	16,3
	Cut off	0,3
	Loin without crackling	89,0
	Trash	3,0
Loin chop	Ham III	2,0
	Loin speck	20,0
	Cut off	10,4
	Loin boneless in chop	55,0
	Trash	27,0
Saddle	Ham III	16,1
	Loin speck	42,6
	Cut off	19,1
	Saddle	42,6
	Trash	17,0
Shoulder	Blood meat	3,0
	Ham I	20,0
	Ham III	26,0
	Lard	25,0
	Cut off	30,0
	Bacon ham	15,0
	Shoulder	10,0
	Shank	10,0
	Trash	19,0
Fore saddle	Ham III	8,0
	Lard	6,0
	Cut off	16,0
	Fore saddle	69,0
	Trash	12,0

Table B.2: Upper limit of production.

<i>Raw material</i>	<i>Product</i>	<i>Upper limit (%)</i>
Ham	Blood meat	3,0
	Ham I	30,0
	Ham II	30,0
	Ham III	20,0
	Lard	30,0
	Cut off	30,0
	Christmas ham	30,0
	Pesto ham	20,0
	Shank	6,0
	Trash	12,9
Bayon ham	Blood meat	3,0
	Ham I	30,0
	Ham II	30,0
	Ham III	20,0
	Lard	30,0
	Cut off	30,0
	Bayon ham	30,0
	Shank	6,0
	Trash	14,0
Fillet	Cut off	32,0
	Fillet	74,0

Table B.3: Upper limit of production.

# Appendix C

## C.1 The code written in MPL

### *INDEX*

```
i = DATAFILE(i.dat);
j = DATAFILE(j.dat);
k = DATAFILE(k.dat);
t = DATAFILE(t.dat);
```

### *DATA*

```
input[t] := SPARSEFILE(input.dat);
v[k] := DATAFILE(price.dat);
part[i] := DATAFILE(const.dat);
demand[t,k] := SPARSEFILE(Demand.dat);
rawmaterial[i,j] := SPARSEFILE(upperry.dat);
lowerlimit[j,k] := SPARSEFILE(lowerlimitz.dat);
upperlimit[j,k] := SPARSEFILE(upperlimitz.dat);
cl := DATAFILE(lardcost);
cs := DATAFILE(sidecost);
invcost := DATAFILE(invcost.dat);
```

### *DECISION VARIABLES*

```
x[t,i];
y[t,i,j];
z[t,j,k];
prod[t,k];
inv[t,k];
```

$side[t,i];$   
 $lard[t,k];$   
 $beginv[k];$

### **MACRO**

$prodvalue := sum(t,k: v[k]*prod[t,k]);$   
 $invcost := sum(t,k: inv[t,k] * invcost);$   
 $lardcost := sum(t,k: lard[t,k] * cl);$   
 $sidecost := sum(side[t,i] * cs);$   
 $beginvcost := sum(beginv[k] * invcost);$   
 $invcost2 := sum(k < 22: inv[12,k]*v[k]*0.30);$   
 $revenues := prodvalue - invcost - lardcost - sidecost - 100000*beginvcost - invcost2;$

### **MODEL**

$MAX\ PROFIT = revenues;$

### **SUBJECT TO**

#### **Level 1**

$Constant1[t,i]: x[t,i] = part[i]*input[t] + side[t,1];$

---

#### **Level 2**

$Balance2[t,i]: sum(j: y[t,i,j]) = x[t,i];$

$Upperlimit2[t,i,j]: y[t,i,j] \leq rawmaterial[i,j]*10000000;$

---

#### **Level 3**

$Balance3[t,j]: sum(k: z[t,j,k]) = sum(i: y[t,i,j]);$

$Upperlimit3[t,j,k]: z[t,j,k] \leq upperlimit[j,k]*sum(i: y[t,i,j]);$

$Lowerlimit3[t,j,k]: z[t,j,k] \geq lowerlimit[j,k]*sum(i: y[t,i,j]);$



*Production* $z[t,k]$ :  $prod[t,k] = sum(j: z[t,j,k]) + lard[t,k]$ ;

*BeginningInventory* $[k]$ :  $inv[1,k] = prod[1,k] - demand[1,k] + beginv[k]$ ;

*Inventory* $z[t \geq 2,k]$ :  $inv[t,k] = inv[t-1,k] + prod[t,k] - demand[t,k]$ ;







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