LATERAL OPTIMIZATION OF AIRCRAFT TRACKS IN REYKJAVIK AIR TRAFFIC CONTROL AREA

May 2012

Einar Ingvi Andrésson

Master of Science in Decision Engineering

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by

Einar Ingvi Andrésson

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at Reykjavík University in partial fulfillment of
the requirements for the degree of
Master of Science in **Decision Engineering**

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The undersigned hereby certify that they recommend to the School of Science and Engineering at Reykjavík University for acceptance this research thesis entitled **Lateral Optimization of Aircraft Tracks In Reykjavik Air Traffic Control Area** submitted by **Einar Ingvi Andrésson** in partial fulfillment of the requirements for the degree of **Master of Science** in **Decision Engineering**.

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Abstract

The requirement of minimizing fuel oil burn and greenhouse gas emissions in aviation is an ever increasing factor in air traffic management. The approach used in this project is to minimize fuel burn and emissions by optimizing flight tracks with respect to wind. This is based on the assumption that by minimizing the flight time between two points, the amount of fuel is also minimized. The optimization is done with the Dijkstra search algorithm, which is used to calculate the shortest path in terms of time for aircrafts in cruise phase. A geographical grid is constructed using flight plans from Icelandair, with the initial flight plan as a reference point to calculate the shortest path through. The starting coordinates and last known coordinates for cruise phase during the flights were used as the starting and end points for the algorithm. Two flights originating in Iceland were examined in detail and both showed reduction in the time spent in cruise phase from their respective flight plans. These improvements were also used to estimate the fuel saved per kilograms. The reduction in fuel burn was subsequently used to estimate the reduction of CO_2 emissions during the cruise phase. The results show that using wind optimization to reduce cruise time with a shortest-path search algorithm can likely offer opportunities for improved fuel efficiency and merits further research. Further research of this method would be extended to airlines that transit through the Reykjavik control area in cruise phase without landing or take-off from Keflavik, giving a better picture of traffic within the Reykjavik control area.

Bestun Flugferla í Láréttu Plani á Flugstjórnarsvæði Íslands

Einar Ingvi Andrésson

Maí 2012

Útdráttur

Kröfurnar um að minnka eldsneytisbrennslu og útblástur gróðurhúsloftte- gunda í flugumferð er sífellt að öðlast meira vægi í flugumstjórn. Aðferðin til lágmarka eldsneytisbrennslu og útblástur í þessu verkefni er með bestun á flugferlum með tilliti til vinda, bæði vindhraða og vindáttar. Bestunin var framkvæmd með Dijkstra leitaralgríminn, sem finnur þá flugleið fyrir flugvélar í flughæð sem lágmarkar flugtímann. Hnitakerfi (net) var smíðað utan um áætlaða flugleið frá flugplani véla Icelandair, þar sem fyrsti þekkti punktur í flughæð var upphafspunktur og til síðustu þekktra hnita í flughæð. Tvö flug voru reiknuð og bæði flugin sýndu fram á betri niðurstöður heldur en áætlaða flugáætlunin gerði ráð fyrir. Þessi bæting var svo notuð til að áætla magn af eldsneyti sparað, sem einnig gefur áætlað magn af CO_2 útblæstri sem er sparað í flughæð. Niðurstöðurnar sýndu að bestun með tilliti til vinda til að lágmarka tíma í flughæð með leitaralgrími sé nokkuð sem vert er að skoða frekar. Í framtíðarrannsóknum á þessari aðferð væri nauðsynlegt að fá gögn frá flugfélögum sem fljúga í gegnum íslenska flugstjórnarsvæðið án þess að lenda eða taka á loft frá Keflavík. Bein flug í flughæð í gegnum flugstjórnarsvæðið gæfu betri mynd af flugumferð innan íslenska svæðisins.

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Isavia is also thanked for supporting this project financially and providing the data necessary for me to fulfil this assignment. Hjalti Pálsson, senior engineer at Isavia receives special mention for his input in the thesis and prior research under the AIRE project.

Furthermore I would like to thank Icelandair and Captain Jóhann Engilbertsson, for providing real world flight data and assisting me in decoding data collected from the aircraft's. Belgingur the Meteorological Research Institute and Ólafur Rögnvaldsson its director receive special thanks for providing weather data and insight into high altitude weather. Thanks are due to Tern Systems and Birgir Már Þorgeirsson for providing me with the BADA fuel burn model and Kristbjörn Helgason for his advices in R programming.

Thanks to all of you

Einar Ingvi Andrésson, B.Sc. Reykjavik, June 2012

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Chapter 1

Introduction

This thesis was carried out with the support and in collaboration with Isavia and it's subsidiary Tern Systems. Isavia has for some time been active in projects addressing fuel burn reduction and the reduction of greenhouse gas (GHG) emissions by air traffic in the North Atlantic Region. In part this is due to the fact that air traffic management service providers have for some time been under increasing pressure to take measures aimed at lessening fuel consumption within their areas. The introduction of carbon charges by the European Union is making emissions of GHG's a new cost item for the airlines. According to a sample of 45 major airlines researched by IATA, the International Air Transport Association [1]. The largest single cost item in 2008 for the global airline industry was fuel costs. Fuel represented 32.3 percent of the total operating cost of airlines, with current and past trend in fossil fuel prices indicating that aviation fuel costs will continue to increase in the foreseeable future. These factors are the motivation behind this optimization study to reduce the amount of fuel burned and consequently reduce emissions. As an example the estimated economic benefit of 1% fuel saving by all air traffic within the Reykjavik Air Traffic Control Area, would result in approximately 6000 metric tonne savings annually based on 2011 traffic figures. The price for metric tonne of jet fuel in May 2012 was $969, using this price the economic savings would amount to almost $6 million by aircraft within the Reykjavik Control Area (CTA).

The average time spent within the Reykjavik air traffic control area is 100 minutes. The majority of these aircrafts transit through the CTA without landing or taking off from Keflavik or other airports within the area. These aircrafts are in the cruise phase portion of their flights. Cruise phase is typically the longest segment of a flight, after the top of climb has been reached. This phase terminates when the top of descent point is reached. Aircraft typically cruise long distances at the same altitude or with occasional increase in altitude. In this thesis the aim is to minimize the flight time in cruise phase for the flights that were studied by applying optimization methods to take advantage of wind speed and wind direction. The North Atlantic jet stream is a primary example of wind currents that affect aircraft track consisting of winds blowing predominantly from west to east. Airlines flying from North America to Europe attempt to utilize the tail wind created by the jet stream as much as possible to increase their ground speed. Flying from Europe to North America the airlines try to minimize the impact of headwind during the flight. The North Atlantic jet stream is described further in chapter 3. This study utilized real performance data from Icelandair for entire flights i.e. in climb, cruise phase and descent in the optimization study which is however limited to the cruise phase. The data from Icelandair included critical data such as the take off weight, landing weight and fuel burned within the Reykjavik Air Traffic Control Area (BIRD). This data was incorporated with data received from Isavia for flights within BIRD. The data from these two sources combined to provide a view of the flight tracks studied in the thesis both within BIRD and outside of BIRD.

The flights studied in this thesis were optimized only in cruise phase with special attention to performance within BIRD. The purpose of the study is to evaluate the feasibility and the potential benefit of optimizing aircraft flight paths with respect to wind speed and wind direction rather than choosing for example optimizing the shortest distance between point of departure and destination. This would result in shorter time needed to complete the cruise phase and consequently less fuel burned during the cruise phase. The flights chosen for optimization were flights ICE615 from Keflavik to New York on 14th of July 2011 and ICE204 from Keflavik to Copenhagen on the same date. The reason for selecting the former is because of the strong North Atlantic jet stream effect. The jet stream provides variability in wind speed and wind direction to optimize the flight track on-the-fly from Keflavik to New York. The reason for choosing flight ICE204, is the relatively long time spent in cruise within BIRD boundaries.

Extensive research and optimization has already been done in flight optimization by a large number of research organizations. However the aviation industry is a competitive industry, making it hard to obtain research material and real world data from multiple sources to analyse. Fortunately, in addition to data provided by Isavia and Tern Systems, Icelandair and Belgingur Institute of Meteorological Research (IMR) cooperated and provided vital information and data during the course of the project. Other optimization methods Air traffic management providers and airlines allocate resources to shortest path optimization

The remainder of this thesis describes the background and technical aspects in chapter 2, the thesis moves then on to the technical approach. After the necessary parameters and numbers are known for the optimization model, the wind optimization model is detailed in chapter 4. The wind optimization results and graphical figures of optimized flights are shown in chapter 5. Conclusions are then discussed in chapter 6.

Chapter 2

Background and technical aspects

Global aviation is currently at a turning point influenced by coinciding external factors such as, the rise in fossil fuel prices and ever increasing demand for transportation capacity. At the same time there is increased pressure for reduction in the environmental footprint of aviation. Competition in the air transport industry also makes it a low margin industry which is constantly looking for means of reducing its operating costs. In order to create optimization models that can be used in this effort a great deal of data from multiple sources is needed. In addition to data, newer technologies such as the Automatic Dependent Surveillance-Broadcast (ADS-B) system are needed for supporting optimized operation of aircraft in the future. The ADS-B system is a new technology used for tracking aircraft with a network of ground stations. Figure 2.1 shows the proposed oceanic corridor in the North Atlantic, where ground stations in Iceland, Greenland and the Faroe Islands provide considerable oceanic coverage. Within this corridor Isavia will be able to provide better surveillance coverage for oceanic flights and potentially better weather data. Having the best available weather data at hand is important in wind optimization for aircrafts in cruise phase which is the subject of this thesis. In order for aircraft to have the best available weather data in the future. The aircraft might need methods possibly based on the ADS-B technology. In this optimization study, post processed weather data was used to optimize the time of flight for aircraft in the cruise phases.

2.1 Isavia projects and linked projects

Isavia is currently establishing a contiguous Trans-Atlantic ADS-B surveillance corridor in cooperation with Naviair, delivering more safety and economic benefits by being able to reduce separation of aircraft in the area. Naviair is the Danish equivalent to Isavia.

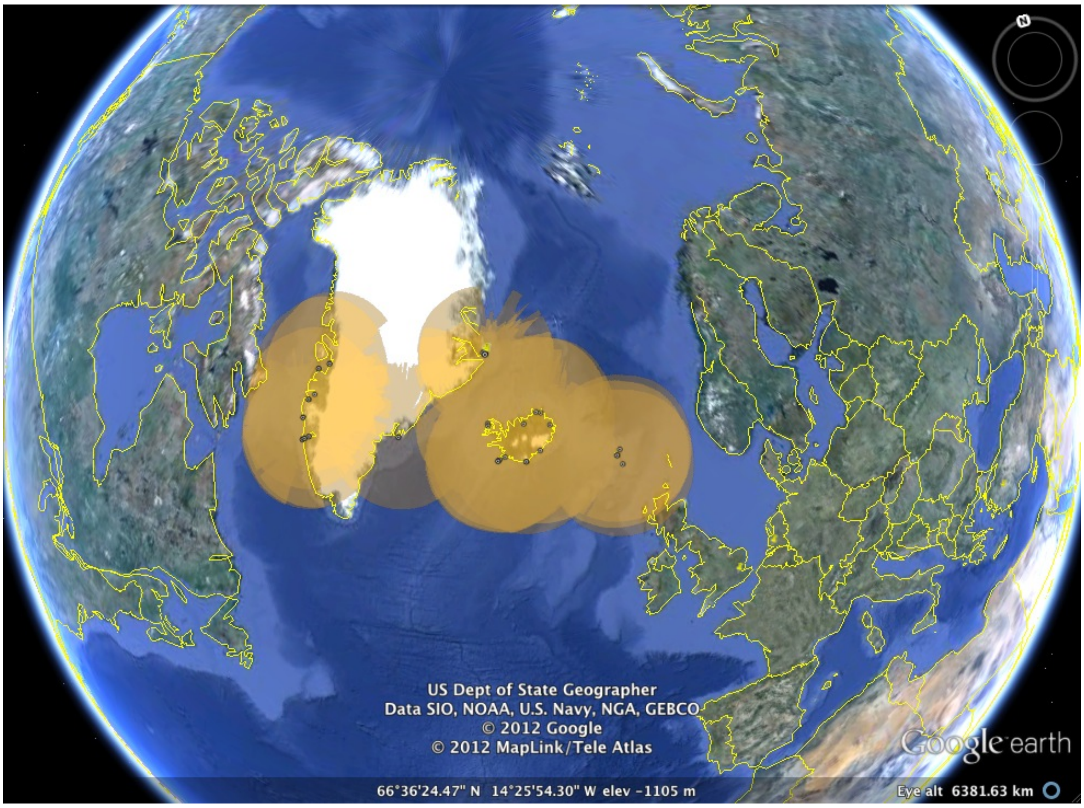

Figure 2.1: Proposed ADS-B Oceanic Corridor in the North Atlantic

Isavia has been engaged in projects regarding flight optimization with respect to different factors. The AIRE project is of particular interest in this context. This project strove to minimize emissions in the North Atlantic by optimizing flight profiles, i.e. the altitude and speed for entire flights between the origin and destination airports. Results of the analysis showed considerable improvements. Icelandair ran 48 flight trials on the route between Keflavik and Seattle from 17. October 2009 to 15. January 2010 traversing the area that will fall within the North Atlantic surveillance corridor. According to these trials installation of ADS-B receiver stations at key locations in Greenland will increase the surveillance duration of the aircraft track along this route from 55% to 85% of the total distance. Estimated annual benefits of the optimization were 1.240 tonnes of fuel savings, $870.000 annual savings and 3.912.200 kg CO_2 emission savings for the air traffic transiting through the area by installing ADS-B receiver stations at strategic locations [2].

Project DORIS is a collaboration project between the airlines Air Europa and Iberia with consultancy from NAV Portugal, INECO, AESA and Senasa for oceanic "Gate-to-Gate" flight operations, regarding emissions in oceanic flights. DORIS stands for Dynamic Optimization of the Route In flight at Santa Maria. This project was performed during the year 2011 and its main objectives were optimizing flight paths with in flight dynamic optimization. In flight dynamic optimization is when aircraft perform in flight course alterations, deviating from the flight plan. For example to reduce the effect of headwind. The project's aim was to minimize greenhouse gas emissions such as, CO_2 in oceanic domain. Air Europa and Iberia operate several long haul flights from Europe to South America. In the DORIS project Air Europa's flights are optimized with respect to fuel savings while Iberia's flights are optimized w.r.t. a combination of fuel and time savings.

Preliminary results from DORIS show that in transoceanic flights fuel consumption is between 10 and 20 kg/nm on the average. Meaning a 1 nautical mile in deviation from the shortest distance path for a city pair can result in between 10 - 20 kg extra fuel burn [3].

Dynamic Airborne Reroute Procedures (DARP) is an in-flight optimization to minimize fuel burn, taking advantage of updated weather reports. DARP is part of the Asia and Pacific Initiative to Reduce Emissions (ASPIRE) program. ASPIRE is a cooperation between the Federal Aviation Administration (FAA), Japan Civil Aviation Bureau, New Zealand Airways, Airservices Australia and the Civil Aviation Authority of Singapore for reducing greenhouse gas (GHG) emissions in aviation [4]. DARP can provide significant savings in fuel and therefore emissions. A recent Air New Zealand analysis concluded that 58 percent of all flights from Auckland to North America assessed during the analysis achieved in an average fuel burn reduction of 453kg per flight Gate-to-Gate, or roughly 1431kg of CO_2 emissions. Certain factors make it easier to obtain these results in Pacific airspace when compared with North Atlantic flight, such as longer distances and significantly less traffic. This makes separation between aircraft's less of an issue and there are fewer restrictions on airspace in Pacific flights, both with respect to altitude changes and lateral changes. This gives increased flexibility for optimizing flight profiles, to lower fuel burn and emissions. ADS-B equipped aircraft meeting certain requirement within ASPIRE'S ADS-B coverage can be separated safely by 30 nm in lateral and longitudinal direction of the aircraft tracks instead of the traditional 60 nm lateral and longitudinal separation on the North Atlantic [4].

2.2 Reykjavik Control Area

The North Atlantic airspace delegated for control by Isavia is known as the Reykjavik Control Area (Reykjavik CTA). However it is also know by its International Civil Aviation Organization (ICAO) identifiers as BIRD CTA. Its commonly referred to as BIRD. This control area is among the largest oceanic control areas in the world totalling about 5.4

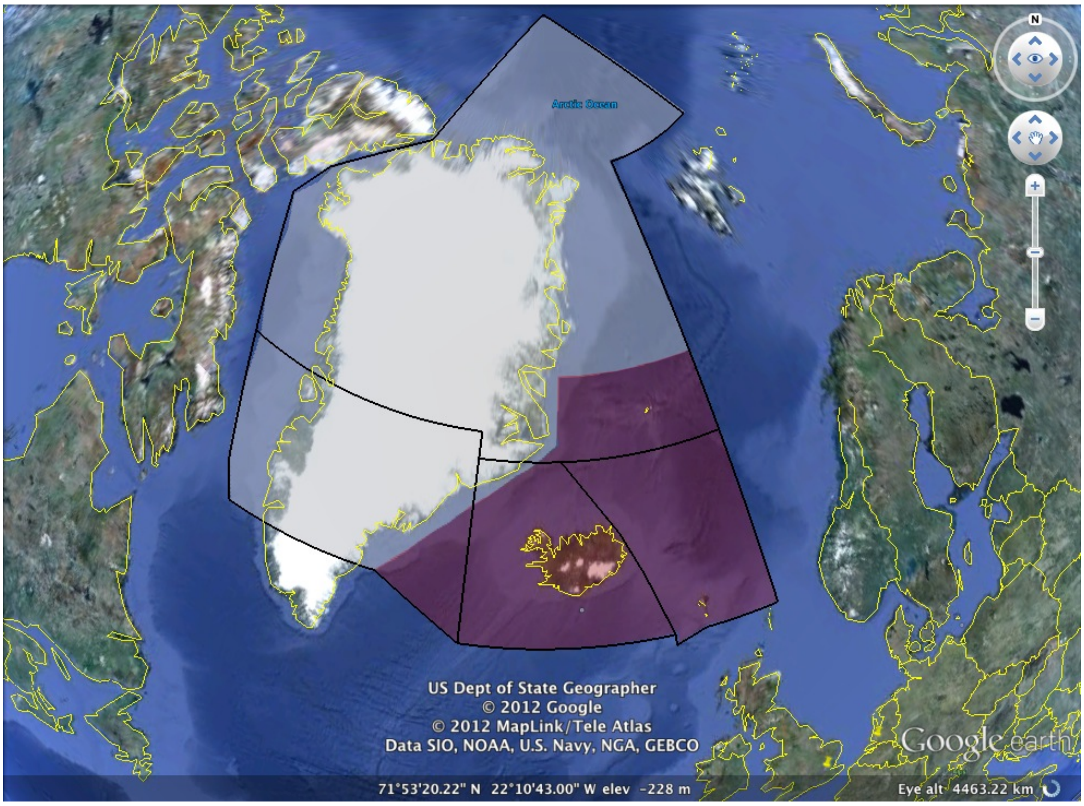

Figure 2.2: Reykjavik ATC Area and it's Four Sectors

million sq. km, where over 110.000 aircraft spend on average 100 minutes within its boundaries. [1] Figure 2.1 shows the Reykjavik Control Area and its four sectors. The area shown in the magenta color identifies the indigenous Flight Information Region (FIR) of Iceland, Reykjavik FIR. The other part of the area that covers a large part of Greenland and extends to the North Pole is a part of the Sondrestrom FIR. Air traffic control above 19,500 ft is provided by the Reykjavik Area Control Center (ACC) under an agreement between Denmark and Iceland.

2.3 Great Circle Distance

The shortest path between any two points on the surface of a sphere lies on the Great Circle Distance (GCD), also known as geodesic distance. To find the great circle (geodesic) distance between two points located at specified latitudes δ and longitudes λ, where point 1 is (δ_1, λ_1) and point 2 is (δ_2, λ_2) on a sphere of radius a. The polar radius is 6371 km while the equatorial radius is 6378 km. In the wind optimization model developed in this study the radius used was mean radius of earth a $\approx$ 6371 km. Furthermore it is necessary

[1] http://www.isavia.is/English/

to be able to convert from spherical coordinates to Cartesian coordinates with equation 2.1

$$\mathbf{r}_i = \begin{bmatrix} \cos\lambda_i \cos\delta_i \\ \sin\lambda_i \cos\delta_i \\ \sin\delta_i \end{bmatrix} \tag{2.1}$$

where δ denotes latitude and λ longitude. Furthermore latitude is related to colatitude [2] where the angle ϕ of spherical coordinates is related to the latitude $\delta = 90^\circ - \phi$, the conversion to Cartesian coordinates replaces sin ϕ and cos λ by cos δ and sin δ. To find the angle α between $\mathbf{r}_1$ and $\mathbf{r}_2$ in equation 2.2 that converts spherical coordinates into Cartesian coordinates dot product of the two vectors is used:

$$\begin{aligned} \cos\alpha &= \mathbf{r}_1 \cdot \mathbf{r}_2 \\ &= \cos\delta_1 \cos\delta_2 (\sin\lambda_1 \sin\lambda_2 + \cos\lambda_1 \cos\lambda_2) + \sin\delta_1 \sin\delta_2 \\ &= \cos\delta_1 \cos\delta_2 \cos(\lambda_1 - \lambda_2) + \sin\delta_1 \sin\delta_2 \end{aligned} \tag{2.2}$$

thus the GCD is represented by equation 2.3 where *d* is the GCD

$$d = R_m(acos^{-1}[\cos\delta_1 \cos\delta_2 \cos(\lambda_1 - \lambda_2) + \sin\delta_1 \sin\delta_2]) \tag{2.3}$$

Where R_m is the mean radius of the earth. [3]

The GCD for any point chosen, is computed using Matlab. Matlab has built-in functions for GCD calculations. Matlab also contains a Mapping Toolbox which contains specialized functions for navigational problems and computations. These functions were used in the study to find the GCD for city pairs. In Icelandair's Flight Plans, the coordinates of both take-off and landing were provided, these coordinates were put into a function in Matlab's Mapping Toolbox. In this study the city pairs considered were Keflavik - Seattle, Keflavik - New York, Keflavik - Copenhagen and Keflavik - London. Icelandair's real data obtained from aircraft allowed a comparison between the real flight paths flown between city pairs and the GCD shortest paths between the city pairs. The GCD was calculated by creating a Matlab script that read in all waypoints during the flights and calculated the cumulative GCD flown between them, also calculating the aircrafts great circle course for each leg. Leg is the distance from current node to the adjacent nodes available in the right

[2] http://mathworld.wolfram.com/Colatitude.html
[3] http://mathworld.wolfram.com/GreatCircle.html

direction, legs are described further later on. The great circle course is the corresponding course for the great circle distance between any two points. Table 2.1 shows comparison between the GCD between city pairs and distance flown between them in accordance with to Icelandairs flight plans for the city pairs. Finally the table shows the deviation from GCD during the respective flights according to flight plan. The small deviation indicates that the flight plans on this day were close to the great circle paths between the point of origin and destination.

City	GCD in kilometers	Distance Flown (km)	Deviation (km)
Seattle	5875.8	5961.2	85.4
New York	4182.5	4245.5	63
Copenhagen	2136.2	2189.8	53.6
London	1888.8	2034.6	145.8

Table 2.1: Comparison of Distance Flown on 14/7/2011 and GCD from Keflavik.

2.4 Flight paths chosen

Icelandair's scheduled flight network consists exclusively of flights to and from Europe and North America. All flights land and take off from Keflavik. Therefore the two flights studied in this thesis took off from Keflavik. Hence the fuel spent reaching top of climb is not taken into consideration. Flights inbound to Keflavik were also under consideration. The reason for choosing flights originating in Keflavik was that fuel spent during the climb was known with greater accuracy than when the flights had been airborne for longer period. To obtain actual performance data for non-stop flights in cruise phase through BIRD it would have been desirable to establish a cooperation with an airline, that flies non-stop flights through BIRD on a regular basis. This would provide data for continuous cruise through BIRD airspace for processing. To depict Icelandair's network and for getting an idea of where an aircraft would enter BIRD, four tracks were chosen initially, two tracks to Europe and two to North America. Figure 2.2 shows the tracks of the four flights. It should be kept in mind that this is a three dimensional picture from Google Earth. The gnomonic projection obscures the GCD tracks flown as the figure is projected from Google Earth to a two dimensional figure. The European tracks were from Keflavik to Copenhagen and to London. The North America tracks were from Keflavik to New York and Seattle. In this study only two flights were examined. The reasons for going to two tracks was due to the amount of effort required manually processing the weather data. However these two flight tracks were examined thoroughly with the emphasis on

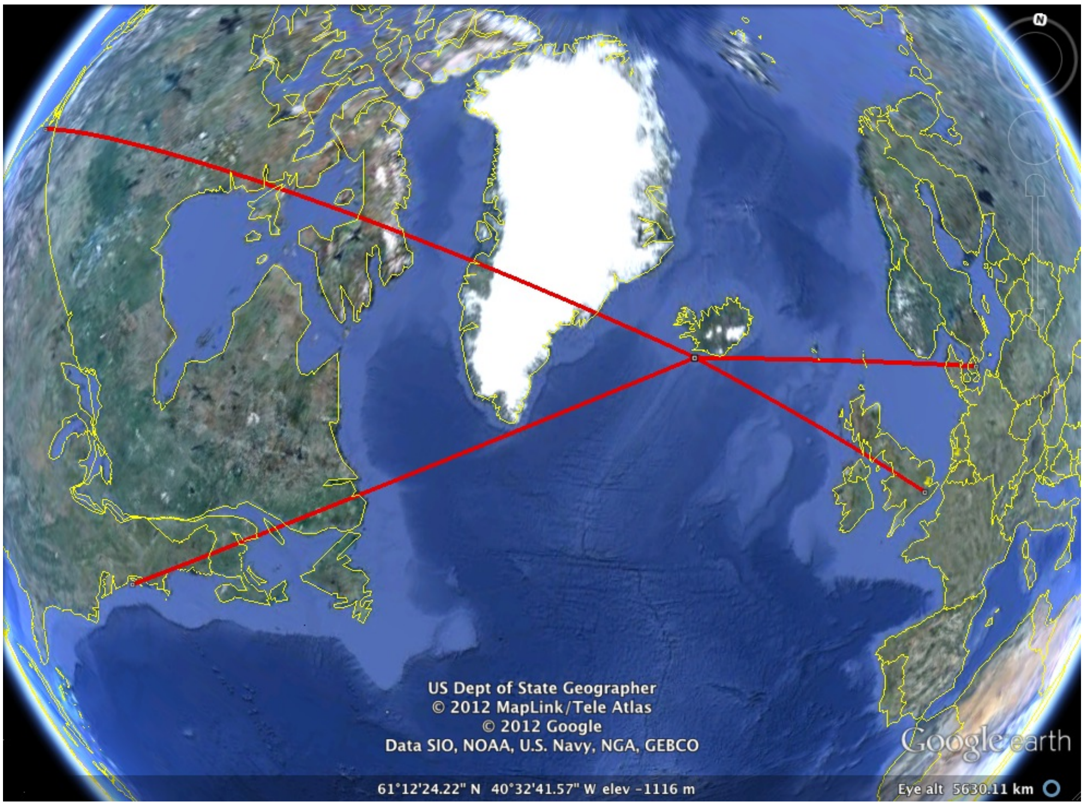

Figure 2.3: Flight Paths from Keflavik

demonstrating the feasibility of the method employed for the optimization. The sample examined consisted of one long and one short flight having a different fuel burn. The fuel load changes w.r.t. time during the flight. Flights to and from Copenhagen are relatively short. Therefore the aircraft take off with light fuel loads whereas the New York flight has a much higher fuel load. As aircraft burn fuel their mass decreases, usually resulting in less fuel consumption per minute. Furthermore specific factors such as the North Atlantic jet stream in case ICE615's and the time spent within BIRD in ICE204's case while in cruise phase were also taken into consideration when choosing flight tracks.

2.5 Lateral trajectory optimization

Various parameters come into play when dealing with aircraft in flight, these are somewhat simplified in this study by the fact that this thesis deals only with wind optimization in horizontal plan namely at constant cruise altitude. However to generate a wind optimal route for an object at constant altitude h over earth's spherical surface we need optimal aircraft heading with respect to wind, and in order to find optimal aircraft heading for wind optimal route over a spherical surface. We need equations 2.4 to 2.6. Equation

2.4 depicts the longitude component of the optimal heading and Equation 2.5 depicts the latitude component of the optimal heading.

$$\phi = \frac{Vcos(\psi) + u(\phi, \theta, h)}{Rcos(\theta)} \tag{2.4}$$

$$\theta = \frac{Vsin(\psi) + v(\phi, \theta, h)}{R} \tag{2.5}$$

$$\psi = \frac{-[F_w(\psi, \phi, \theta, u, v) + F_c(\psi, \phi, \theta, u, v, K)]}{Rcos(\theta)(Ct + K(\phi, \theta, h)} \tag{2.6}$$

Equation 2.6 depicts the optimal heading angle as a function both latitude and longitude. Where $F_w(\psi, \phi, \theta, u, v)$ and $Fc(\psi, \phi, \theta, u, v, K)$ are aircraft heading dynamics in response to wind speed, wind direction and temperature, respectively. The flight path angle is zero and ϕ is longitude and θ is latitude, V is airspeed, ψ is heading angle and R is Earth's radius. The east-component of the wind velocity is *u*(ϕ,θ,h), and the north-component of the wind velocity is *v*(ϕ,θ,h) [8].

The lateral trajectory is optimized by determining the shortest leg. To determine the heading angle that minimizes a cost function, great circle course, and GCD were needed. The cost function contains wind direction, wind speed, temperature and GCD for each edge between adjacent nodes in the grid. During the wind optimization it was decided to ignore the affect of vertical wind components on the aircraft, it would have increased the already large wind data set and further complicated matters. It was not possible to examine winter flights because of there was not available data from all sources needed for cross-referencing. Of course in future research it would be preferable to have better data. For instance with actual performance data for the entire flight from airlines, for each waypoint. The data should include exact time at waypoint, cumulated fuel burn and weather conditions. Due to difficulties in obtaining detailed data, it was only possible to examine two flights in this thesis. Because of the effort needed to manipulate data before it could be worked with and the short time frame for the thesis. It has to be take into consideration the complexity of the problem at hand. For instance in *Cross Polar Aircraft Trajectory Optimization and the Potential Climate Impact* [8] research paper done by NASA Ames Research Center, there is focused on two flights.

Chapter 3

Technical approach

The purpose of the wind optimization model is to adjust the course of the aircraft during cruise by changing the aircraft heading. In *Cross Polar Aircraft Trajectory Optimization and the Potential Climate Impact* [8] research paper done by NASA Ames Research Center the focus is on reducing emissions while flying cross-polar flights. The method used in the this research project was to deviate from the fixed routes defined for flights across the North Pole using a dynamic programming algorithm to generate minimum time wind-optimal routes. In this study the same method is used in principal. An optimization algorithm is used to select the course of an aircraft at each waypoint in order to to minimize the time needed for flying the track between specified initial and final points of the route segment. Thus the flight track deviates from the track defined by the flight plan during cruise. In order to achieve this a wind optimization model was constructed.

Normally the full equations of motion would be required in order to generate the aircraft trajectory in addition to the wind velocity. However in the case of cruising flight over the North Atlantic the true airspeed (TAS) has already been chosen by the crew and/or the air traffic control center. The only variable to be chosen for lateral optimization is the heading of the aircraft that determines the course of the aircraft path. To make the wind optimization model as accurate as possible, real weather data is needed, real aircraft performance data and data on real flight profiles. The wind optimization model is dependent on wind speed, wind direction and temperature as inputs. These weather elements are used to calculate the distance flown by an aircraft with respect to the ground, i.e. the ground speed. The determination of ground speed is defined and explained in chapter 4.4. As mentioned before three sources of data were available. More data sources would have been preferable, especially from other air traffic contol centers and airlines. Data was provided by Isavia, Icelandair and IMR. The flight track between Keflavik and New

York is affected with the North Atlantic jet stream making it an interesting flight track to examine in detail. Jet streams are relatively strong winds concentrated within a current of air in the atmosphere. Jet streams are considered to be present wherever it is determined that wind speeds exceed 50 knots. [1]

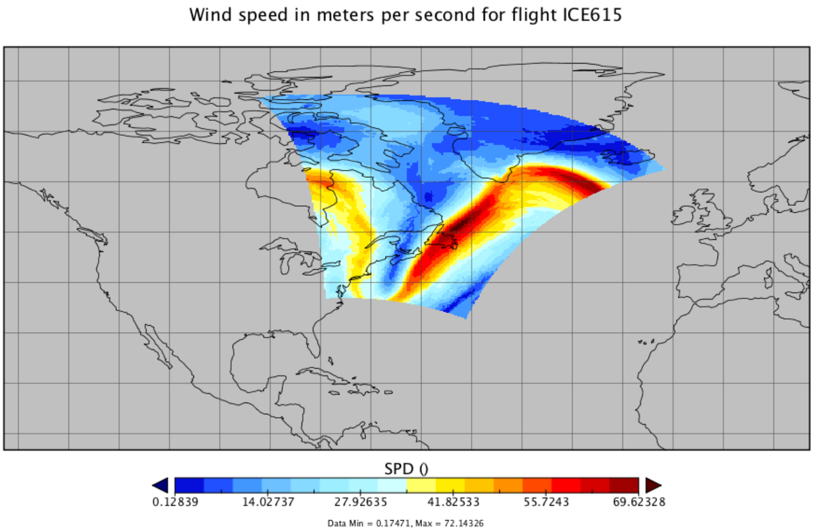

Figure 3.1: Weather Map showing Wind speed for the North Atlantic Jet Stream at 19:00 GMT on 14/7/2011

The North Atlantic jet stream changes rapidly and presents both opportunities and problems to air traffic traversing it. Flights from North America to Europe utilize the jet stream to increase their ground speed (GS) by altering their course depending on the wind direction within the jet stream. Conversely flights travelling from Europe to the Northern parth of the Eastern seaboard of North America try to minimize the effect of headwinds resulting from the jet stream. Figure 3.1 shows how the jet stream affects flight tracks between North America and Europe. The red line on the figure parallel to the Eastern seabord of North America and Greenland represents the jet stream. This red line indicates the jet stream is present at 35,000 ft during ICE615's flight on the 14th of July 2011. The wind speed within the jet stream is from 55 meters per second to 69 meters per second, during ICE615's cruise phase on that date.

The jet stream falls within the IMR's Weather data sets that is used by the wind optimization (WO) model. The model reads in wind direction and wind speed values from the weather grid. The values are used to construct a matrix, utilized during the WO. The jet stream has a high variance in wind direction, consequently having an influence on the wind optimization in flight ICE615's case. The jet stream also affects the generation of flight plans, because of its high variance. However there is considerable deviation in the

[1] http://amsglossary.allenpress.com/glossary/search?id=jet-stream1

flight plans, both in fuel burn and flight duration. One factor that could improve them is less variance between weather reports and the actual weather faced during the flight, by having better and more frequently updated weather reports.

3.1 Weather Data

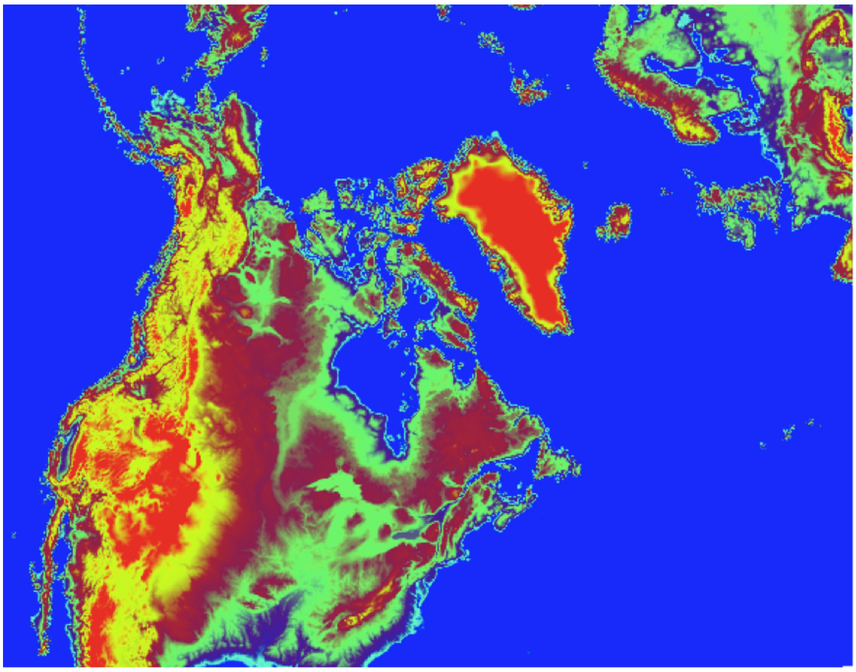

Figure 3.2: Geographical Area covered by the IMR weather data sets

IMR provided high resolution weather data, i.e. wind and temperature data, for four days. Figure 3.2 shows the area covered by this data. Two days during the winter and two days during the summer were generated. The dates chosen were the 13th and 14th of February and then the 13th and 14th of July 2011. The data was received from IMR in Network Common Data Form format (netCDF). The weather data provided by IMR was actual weather that was collected and post processed, in February of 2012. These data sets were defined on a grid with a 9 kilometre resolution in the lateral plane for the altitudes most commonly used by commercial jet aircraft i.e. between 30.000 ft to 42.000 ft. For sake of minimizing the data sets, altitudes chosen for the model were from 33.000 ft to 39.000 ft resulting in seven horizontal planes evenly distributed with 1000 ft altitude increments. This weather data provided an in depth view of the conditions faced by the flights selected during those days, with temperature readings in Celsius, wind direction in degrees and wind speed in meters per second [m/s]. The 9 km spacing between data points was much finer resolution than needed. In order to cover the four flight routes chosen the resulting grid covers much of the North Atlantic, North America and Northern Europe.

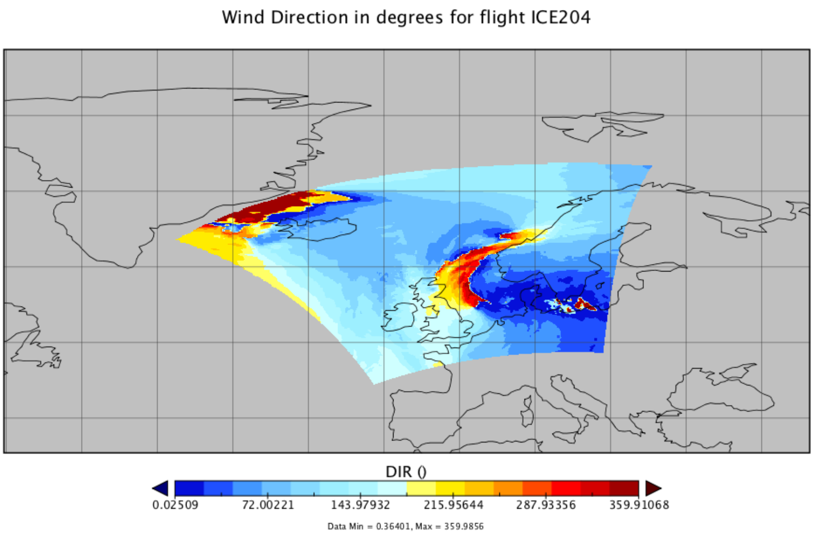

Figure 3.3: Map showing Wind Direction in degrees for flight ICE204 at cruise altitude

Using the software Panoply netCDF Viewer created by NASA it is possible to view the netCDF files, export single variables as csv files to Matlab and calculate mean and standard deviation of wind speed encountered in cruise phase for a single flight.

The weather data is fairly large, each hour of weather data requiring close to a gigabyte, containing: wind speed, wind direction, temperature and ground surface altitudes for given coordinates in latitude and longitude. The time span for each set of the weather data was twenty six hours spanning from 23:00 hours GMT on the 13th of July 2011 to 00:00 on the 15th of July 2011, totalling about 24 gigabytes of weather data.

Early on in the project it was realized that given the current hardware and time, it would be necessary to decrease the amount of data for computation. Therefore the original data set was divided into subsets surrounding the flight profiles. Four different grids were extracted with only the relevant grids for the flight profiles. Figures 3.3 and 3.4 show the wind direction and wind speed affecting flight ICE204 from Keflavik to Copenhagen during its cruise phase.

3.2 Aircraft Flight data and Air Traffic Control data

Isavia provided pre take-off flight plans and detailed flight profile data for all flights within its area. Thus giving a precise information about these flights within its area. All data

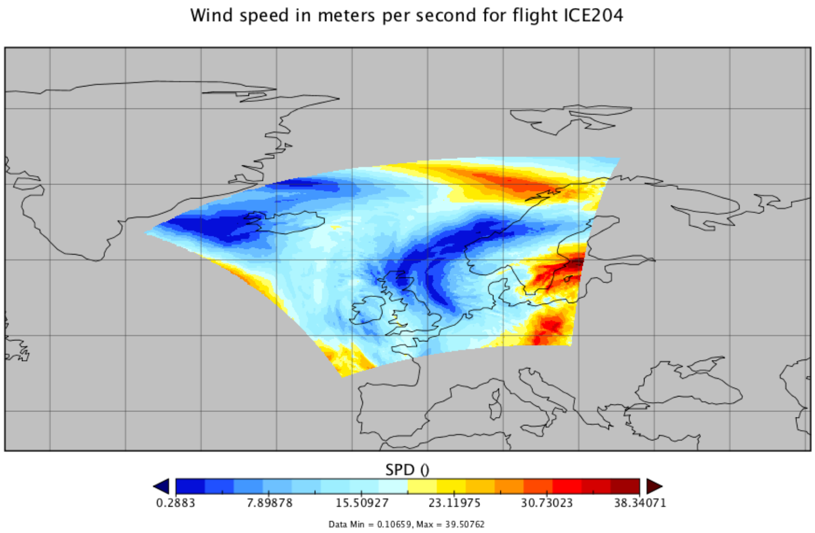

Figure 3.4: Wind speed in m/s during ICE204's cruise phase on 14/7/2011 at cruise altitude

provided by Isavia was in Comma-Separative Values (CSV) format, which is readily accessible.

The data included ADS-B positioning data sent directly from aircraft while flying within BIRD making it possible to calculate the great circle distance flown by the aircraft in order to compare the actual flight path with the planned flight route within BIRD.

Icelandair provided detailed pre take-off flight plans for the entire flights, making it possible to cross reference flight paths and performance for each leg. Furthermore Icelandair provided actual post flight data downloaded from the aircrafts after landing, making it possible to analyse each aircraft performance of each flight both within BIRD and for the entire flight. Data provided by Icelandair was mainly in the Hist file format. Figure 3.5 shows the actual ADS-B position transmissions within BIRD for flight ICE615 from New York to Keflavik, making it possible to plot the actual flight path within BIRD. Figures showing ICE615's and ICE204's tracks for the days 13th and 14th of July 2011 can be found under Appendix A.

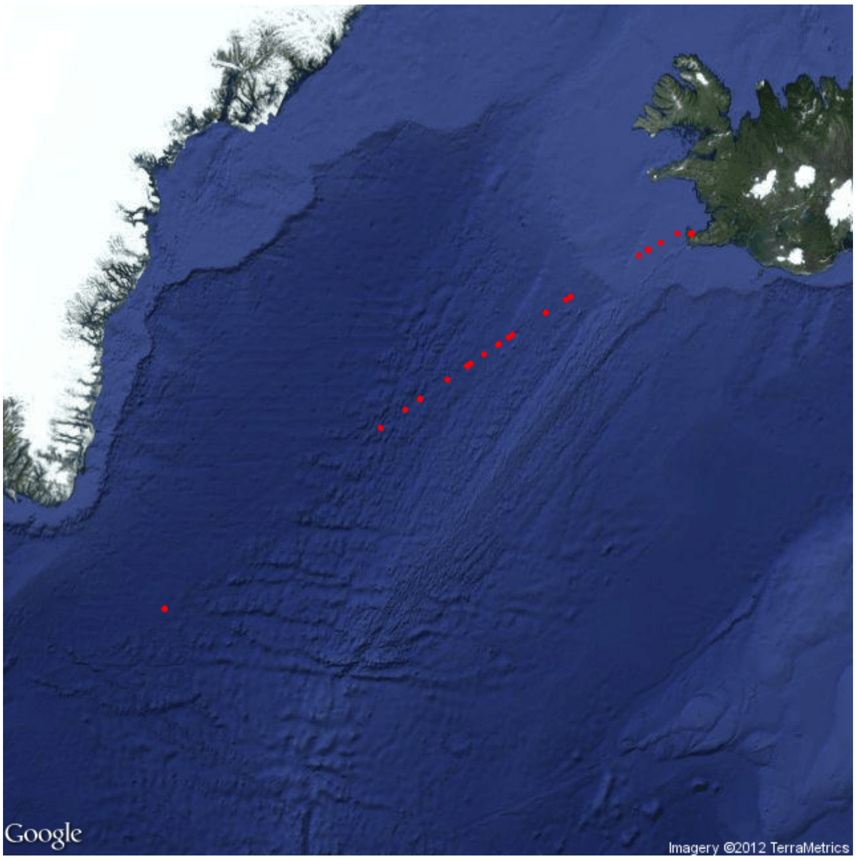

Figure 3.5: ADS-B Data from Isavia showing ICE615's Keflavik - New York Flight Track on 14/7/2011

Chapter 4

The Wind Optimization model

In order to construct an optimization model, solid mathematical foundations are needed. Algorithm and model designers have developed several fundamental algorithm design techniques such as dynamic programming, depth-first search and manipulation of data at their disposal. The wind optimization model takes data from various sources and utilizes it to solve the problem, by converting messy applications through modelling into clean mathematical problems suitable for application of an optimization algorithm [7].

There are number of suitable search algorithms applicable for use in this optimization. The algorithm chosen for the wind optimization is called Dijkstra's algorithm. The A^* algorithm was initially considered during the course of the project. A^* is in fact based on Dijkstra's algorithm and is a best first search algorithm. It generally has a faster runtime than Dijkstra. However A^* would not have significantly decreased the run time of this particular optimization model. The fact that in this optimization problem the direction of the network is already defined, i.e. the heading direction of the aircraft, effectively makes Dijkstra's run time the same as A^*'s. Furthermore A^* is more complicated to construct and to use. Dijkstra's algorithm is used as route planning software such as directions on Google Maps and GPS receivers which provide shortest path from a start point to end point.

All calculations in this thesis were performed by 2.4 GHz Intel Core 2 Duo, 4 GB 1067 MHz DDR Macbook Pro laptop. For graphical presentation the RStudio GUI of the R programming language was used and for optimization and algorithm running the numerical computing environment of Matlab was used. The Matlab versions used were 7.11.0 (R2010b) and the (R2012a). For netCDF viewing Panoply viewer edition 3.1.2. provided by NASA was used.

4.1 Model limitations

The number of parameters and constraints in a real world scenario are too numerous to take all of them into consideration in this model. The question is where to stop and how much benefit would they add to the model. The question is also how much increase in runtime would be required to incorporate additional parameters and constraints. Constraints can be in the form of flight track alterations i.e. when traffic makes it impossible to fly the optimal flight track. Another constraint would be denied use of optimal flight track by ATM service providers. However there are moving parts that are necessary to take into consideration when modelling this kind of problem created by practical constraints. This is the case when dealing with fuel burn optimization with changing variables like this model. The moving parts are the flight paths flown by the aircraft, wind direction, wind speed and altitude in cruise phase. Certain factors however are difficult to estimate or calculate with the desired accuracy. Reasons vary from capabilities of hardware used to data being unobtainable and/or assumptions that had to be made. These are discussed further in the following sections.

4.1.1 Weather data limitations

The weather data used in this model was not available before or during the flights in question, as this data is gathered retroactively. During the study the detailed pre flight weather forecasts for ICE615 and ICE204 and in flight weather forecasts were not analysed. In Icelandair's flight plans for the flights studied, basic weather parameters was available. However this data was not used as it would have added little value to the study. Icelandair did not provide the detailed weather data which they had available at the time of preparation of ICE615's and ICE204's flight plans on 14th of July 2011, or the weather forecasts for these flights in their cruise phases. It would have been useful to compare Icelandairs weather data to the IMR's weather data used in this optimization study in order to assess the difference.

4.1.2 Computational limitations

While constructing the grid representing nodes available for the aircraft during cruise phase, it became clear that the 9km weather data resolution was too high.

For convenience and for keeping the grid structure simple it was decided to construct a cylinder around the flight track, as defined by Icelandair's pre take-off flight plans.

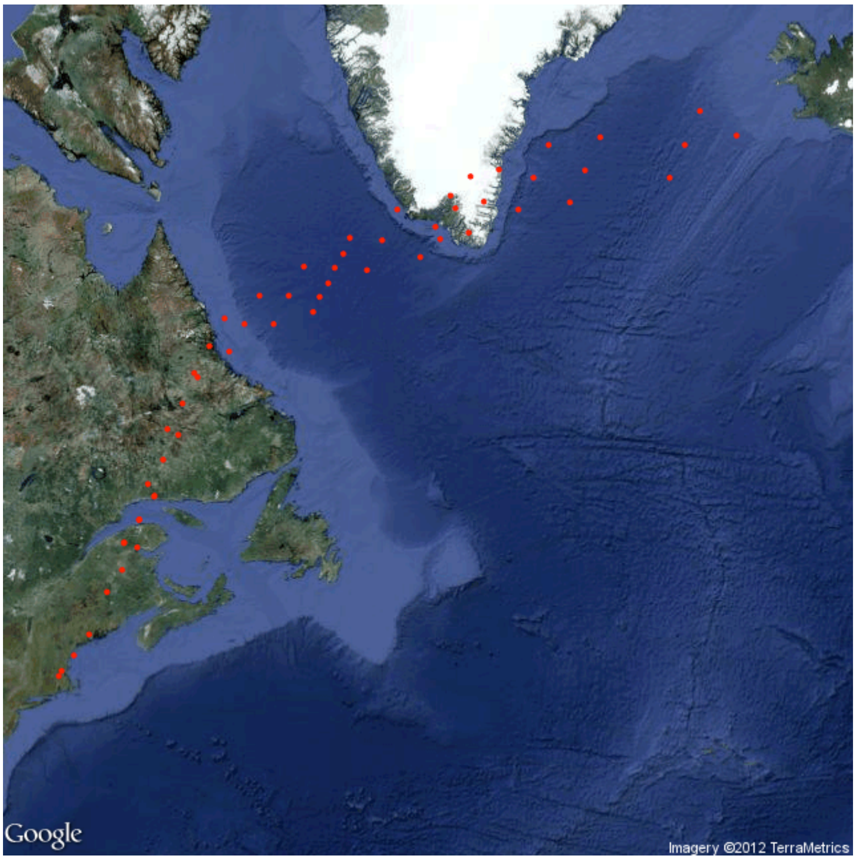

Figure 4.1: Coordinate Grid for ICE615

Figure 4.1 shows the grid used for the wind optimization with the Dijkstra algorithm over ICE615's cruise phase. The cylinder defined around the flight track has a +/- 1 degree off-set in latitude and longitude from the nodes of the grid. The reason for defining the grid in this way apart from convenience was that this study serves as a proof of concept. There are multiple ways to define the grids. With better computational capabilities larger grids with smaller separation between nodes could be defined. By increasing the number of nodes with shorter separation and/or laterally extending the grids more accuracy could be obtained. The grid is explained and described further in the following sections.

4.2 Grid for Lateral trajectory optimization

To optimize the lateral trajectory for an aircraft it was decided to construct a grid around the flight track specified by the flight plan. As described in chapter 4.1.2. The grids are constructed around Icelandair's flight plans for flights ICE615 and ICE204. The distance between nodes is called a leg. For instance when the aircraft is in node 1 in the grid the algorithm has three legs to choose from. Leg 1 is the GCD between node 1 and node 2.

Leg 2 is the GCD between node 1 and node 3 etc.Clearly the GCD between coordinates is not a fixed distance on a spherical surface. As latitude increases the distance between grid poles also decreases due to the fact that the distance between meridians is reduced. Thus the GCD and course between nodes in the grid varies, meaning that length of each leg in the grid has to be calculated independently. Furthermore true airspeed at each waypoint in Icelandairs flight plans for ICE615 and ICE204, was used at corresponding nodes in the grids. The fuel burn savings obtained by chasing more favourable winds decrease as you go further away from the great circle path. The deviation however is dependent on wind speed and wind direction. If both are favourable then considerable deviation from the great circle path can pay off. Therefore an off-set of +/- 1 degree in latitude and longitude from the planned route was considered adequate.

4.3 Two scenarios for calculating cost per leg

The optimization was initially done with respect to time. When the time saved during the wind optimization was known it was easy to plug into the BADA model to get an estimate of the fuel saved with wind optimization. Isavia and Tern Systems provided Eurocontrol's Base of Aircraft Data Revision 3.9 (BADA) fuel burn model [9]. The model incorporates performance data from aircraft manufacturers such as Boeing and Airbus, allowing the computation of the fuel saved when the time saved by the optimization was known. The other way is to optimize w.r.t. fuel burn directly. To do so with certainty involves a significantly more complex method than is required for time based optimization. However a rough estimate can be obtained from Icelandair's real fuel burn data, given the time saved, the initial take-off weight, fuel burn within BIRD and the landing weight is known. Therefore its possible to interpolate the aircraft's mass decrease during it's cruise phase and thereby calculate the fuel burn during this phase. To get an estimate of fuel saved with wind optimization, known parameters were inserted in to BADA which uses the input to estimate the fuel consumption rate in [kg/min] as a function of cruise altitude for the specified aircraft type.

4.4 Variables affecting time based wind optimization

As mentioned above wind speed and wind direction affect aircraft velocity in flight as it moves with the air mass. However temperature and air density also affect the performance of aircraft at altitude. At cruise altitude the air is less dense and the temperature generally decreases with increasing altitude. Therefore temperature and air pressure are important variables when airspeed is calculated from Mach number or indicated airspeed. The ISA, International Standard Atmosphere model is used in the airspeed calculations to provide the relationship of these variables. [1]

Aircraft performance dictates true airspeed (TAS). Airspeed must provide enough lift for the aircraft to match its weight. In this WO model we focus on True Airspeed (TAS) and Ground Speed (GS). Thus TAS is either specified directly or indirectly in terms of Mach number, where the following relationship exists:

$$TAS = a_0 M \sqrt{\frac{T}{T_0}} \tag{4.1}$$

When TAS is given as a Mach number conversion into knots is necessary in order to calculate GS. Equation 4.4 provides the relationship between Mach number where a_0 is the speed of sound at sea level, M is the current Mach number typically, 0.78 to 0.82, T is static temperature in Kelvin degrees and T_0 is temperature at sea level under ISA conditions 288.15 Kelvin degrees. Equation 4.4, was used to calculate TAS of an aircraft, with a known Mach number M. In Icelandairs flight plans for the studied flights each waypoint had a TAS value defined directly either in knots or a Mach number. [9].

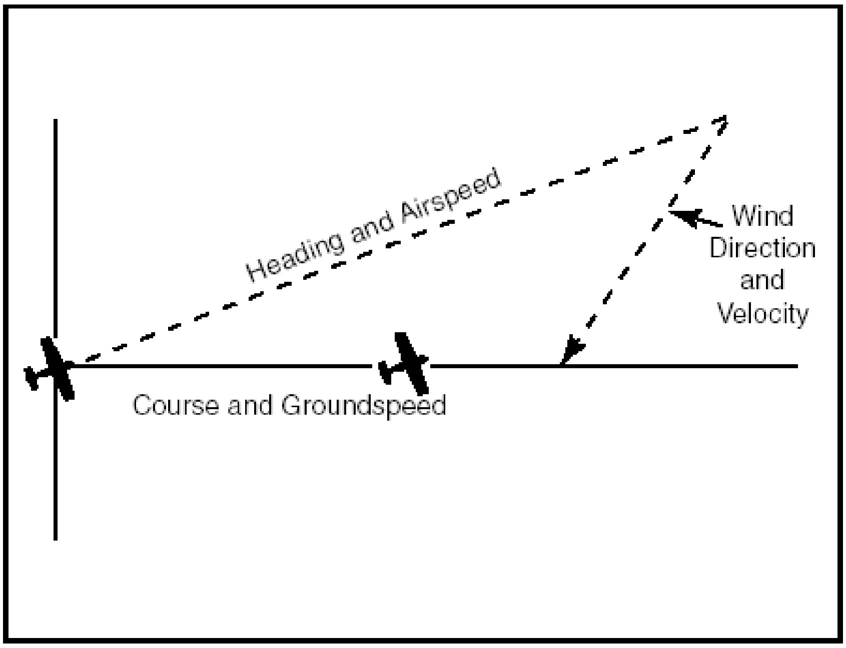

Figure 4.2: The Wind Triangle showing Wind Speed and Wind Direction affect on aircrafts

[1] http://wahiduddin.net/calc/refs/PDBSPEC2_-1.pdf

GS is the magnitude of the velocity vector that the aircraft is moving at relative to the ground when all factors are taken into considerations: TAS, heading, wind speed and wind direction. Therefore ground velocity is calculated as the vector sum of an aircraft's TAS, heading and the wind speed and wind direction. This variable is imperative for the optimization, as its average dictates the time it takes to cover each leg within the grid. Each leg in the grid is unique and requires separate calculations to find the time it takes to cover it's respective distance. [2] Figure 4.2 shows the relations between the TAS vector, wind direction and wind speed and their affect on aircraft ground speed and course. Furthermore Equation 4.5 shows how the relations between the vectors and the angles between them. The equation shows that when wind direction is at an angle relative to the aircraft path, the aircraft is forced to adjust it's heading i.e. to partly head into the wind to compensate for wind drift. [3]

GS clearly is determined as the TAS times the cosine of the wind correction angle plus or minus the wind speed vector, along the ground track of the aircraft.

$$GS = TAS \cdot cos(Drift) + Windspeed \cdot cos(Drift\,Correction\,Angle) \tag{4.2}$$

where wind velocity is expressed by the wind speed and direction at the aircraft cruising altitude.

4.5 BADA fuel burn model

After the flight time per leg is found, it is possible to calculate the amount of fuel saved. Calculating this quantity is done by importing the time saved into the BADA, fuel-burn model which provides an assessment of aircraft fuel burn by type of aircraft. A detailed Matlab file is provided for each aircraft type to calculate fuel burn in [kg/min] when taking predefined specific inputs into consideration, i.e. aircraft mass in kilograms, altitude, temperature and speed in terms of Mach number. BADA relies on performance data provided by aircraft manufacturers and consequently can be compared to look-up tables. In this project it is used to determine Boeing 757-200 performance at altitude under International Standard Atmosphere model (ISA) conditions. This is provided in terms of fuel consumption rate as shown in Figure 4.3 as an example which demonstrates the variation as a function of altitude. The nominal aircraft mass in this case is 95,000 kg [9].

[2] http://www.free-online-private-pilot-ground-school.com/navigation-principles.html
[3] http://delphiforfun.org/programs/Math_Topics/WindTriangle.htm

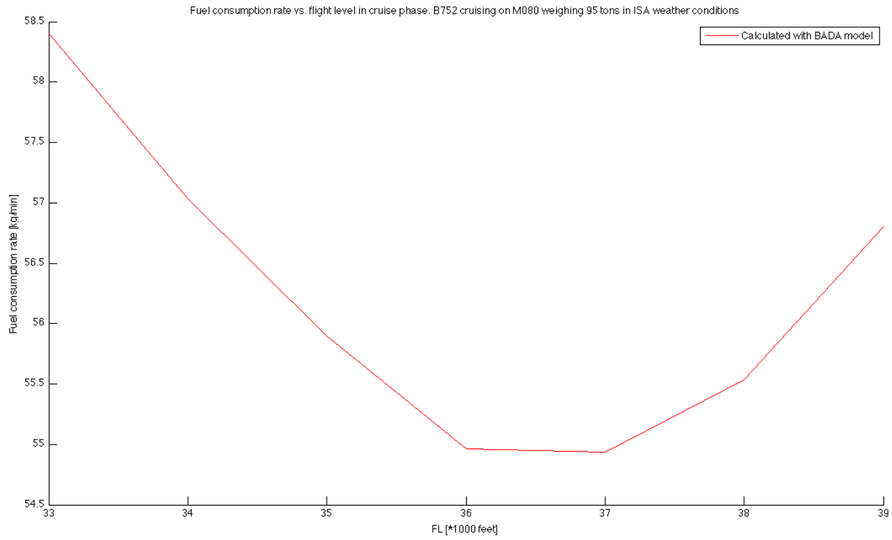

Figure 4.3: Boeing 757-200 Fuel consumption rate as a function of Altitude at ISA Atmospheric Conditions

ISA conditions however are rare in the flight paths chosen in this thesis. ISA conditions for instance assume a standard atmospheric pressure of 1013 millibars at sea level and 15 °C at sea level. [4] Therefore the WO paths with respect to fuel burn in [kg/min] is an approximation. As the temperature at sea level was not included in IMR's weather data, the exact temperature at sea level was not known for relevant coordinates and during the time span. Therefore the ISA conditions were used. However as the date chosen for optimization was during the summer, this is not expected to have a major effect. It must be kept in mind that the fuel savings are primarily dependent on the estimated reduction in flight time.

4.6 Methodology and Computations for ICE615

In order to determine the WO flight, the following steps were performed.

1. Matrix A is defined as a grid of waypoints surrounding the flight path actually flown, the grid was constructed from Icelandair's flight plan, where the initial point represented the first point in the cruise phase known with exact latitude and longitude coordinates. The first point in the grid can only be used to move forward in the grid. This is important because it means that the grid is a directed graph. When constructing the matrix it was decided to add and subtract 1 degree from the latitude

[4] http://www.flywestwind.com/wtc/pprograms/isa.htm

and longitude of each known position of the flight plan, creating a 20x3 matrix representing the nodes. The matrix represents the 20 known coordinates during cruise specified by the flight plan. The middle column is the planned flight track, whereas columns 1 and 3 contain the coordinates added for the purpose of optimization. The reason for adopting this method was that even though the weather data has far greater resolution an airliner would not alter its course very rapidly. One degree in latitude equals about 60 nautical miles or about 111.3 km. However, one degree in longitude varies, in terms of distance with the cosine of latitude. Hence the distance between nodes varies slightly within the grid. (The node matrix is shown in Appendix C)

2. When the nodes have been defined, the next step was to calculate the edges linking the nodes. This had to be carried out in a number of steps, the first step being to calculate the great circle distance (GCD) linking the nodes. This means that GCD had to be calculated for instance, from node 1 to all it's adjacent nodes. These calculations created a 19x7 matrix with GCD distance between all nodes. Where the middle column represented the flight path according to flight plan, other columns represented edges needed to cover all possible paths through the grid. (The GCD matrix is shown in Appendix C)

3. The "cost" of each leg is the time it takes to travel that leg. Thus the time, T_i, for leg i needed for each leg is:

$$T_i = \frac{GCD_i}{GS_i} \tag{4.3}$$

   Equation 4.6 represents the time for each leg that is dependent on the GCD of the leg and the GS of the aircraft while flying this leg. The GCD matrix is known, making the GS the next variable to find. In order to calculate GS wind speed, wind direction, TAS and track angle must be known.

4. After creating a matrix for leg cost, an adjacency matrix is designed. The adjacency matrix specifies which nodes are connected to each other.

5. At this point when the matrices have been constructed it is possible to run the Dijkstra algorithm on the grid to find the optimal shortest path w.r.t time. The algorithm starts at node 1 and works it's way to the final known coordinates of the cruise phase according to Icelandair's flight plan, node 56. Output from the algorithm consists of the total time needed to traverse the grid and the flight path through the grid.

6. To get an estimate of fuel burn per minute during the cruise, the BADA model is used. Aircraft mass decreases with flight time. The difference between Boeing's

757-200 take off weight and landing weight can be anywhere from 10 to 25 tonnes depending on the length of flight. As the aircraft mass decreases the fuel burn rate decreases, calling for mass interpolation to determine the cruise weight of the aircraft.

7. As the estimate of fuel burn per minute from BADA is known, that estimate is multiplied with the fuel burn bias factor to get the closest estimate possible of the fuel burn rate. The fuel burn bias accounts for the real burn of the specific aircraft. In the BADA model engine performance data is provided by aircraft producers. The performances of engines can vary with maintenance level and service life. However in this study the actual fuel burn bias is provided in the data from Icelandair. Therefore the Boeing 757-200 aircraft used in flight ICE615 on 14th of July 2011 had a fuel burn bias of 1.02. In other words there is an aircraft specific increase of 2% in fuel burn over the nominal aircraft performance.

These steps were also used to optimize flight ICE204.

4.7 Dijkstra Algorithm

A *path* is a sequence of edges connecting two vertices. Often there are multiple ways to traverse a path within a graph. Dijkstra's algorithm (DA) is the method of choice for finding a shortest-path in an edge and/or vertex weighted graph if available [10]. DA solves the single-source shortest-paths problems on weighted directed graphs G = (V, E). To do so there cannot be any non-negative weights on edges. We assume that $w(u,v) \geq 0$ for each edge $(u,v) \geq$ **E**. This means that the Dijkstra algorithm works only properly on graphs without negative-cost edges, which is not an issue in this instance, because fuel burn cannot be negative nor negative time per leg in the graphs.

Dijkstra's algorithm maintains a set of S vertices whose final shortest-path weights from the source s have already been determined. The algorithm repeatedly selects the vertex $u \in$ V - S with the minimum shortest-path estimate, adds u to S, and relaxes all edges leaving u. [7]

As shown in Algorithm 1, the pseudocode starts at line 1 by assigning values to unknown distances from the start node to neighbouring nodes, by having all distances from the start node as infinity to other nodes and the distance zero for the initial starting node. The algorithm does this by initializing d and π. Line 2 initializes the set S to the empty set, which is the set of unvisited nodes. Then the algorithm maintains the invariant that Q = V - S at the start of each iteration during the **while** loop for lines 4-8, the loop searches

Algorithm 1 Dijkstra(G,*w*,*s*)

1 INITIALIZE-SINGLE-SOURCE(G,s)
2 S $\leftarrow \emptyset$
3 Q $\leftarrow$ V[G]
4 **while** Q $\neq \emptyset$
5 **do** $u \leftarrow$ EXTRACT-MIN(Q)
6 S $\leftarrow$ S $\cup$ u
7 **for** each vertex $v \in$ Adj[u]
8 **do** RELAX(u,v,w)

for the adjacent nodes from the current node going in the right direction. Line 3 is the initialization of the min-priority queue Q to contain all the vertices in V; since S = $\emptyset$ at that point in time, the invariant is true after line 3. For every iteration done by the **while** loop of lines 4-8, a vertex u is extracted from Q = V - S and added to set S, therefore maintaining the invariant. The algorithm checks the leg lengths from the current node to the adjacent nodes and overwrites the beginning assigned values. During the first run through the loop, u = s. Vertex u. therefore, has the smallest shortest-path estimate of any vertex in V - S. Then lines 7-8 relax each edge (u,v) leaving u, thus updating the estimate $d[v]$ and the predecessor $\pi[v]$ if the shortest path to v can be improved by going through u. The **while** loop of lines 4-8 extracts each vertex from Q and adds to S exactly once therefore the loop iterates exactly |V| times, the while loop overwrites values for legs if multiple ways are optional to go through the graph, until the shortest path is found as shown in Introduction to Algorithms [7].

Chapter 5

Optimization Results

Flights operated on the 14th of July 2011 were chosen because of data availability. Data from all participants i.e. Isavia, Icelandair and the Institute for Meteorological Research had to be available for these flights: comprising weather data for the flight tracks from IMR, flight profile from Isavia, detailed pre take-off flight plans from Icelandair and post flight analysis data, also from Icelandair. The surveillance data from Isavia shows the exact flight profiles for the flights within BIRD, which can then be compared to Icelandair's flight plans and examined for flight plan variance.

The flight plan time variance shows the accuracy of the flight plan. This can be done by comparing predicted flight duration according to the flight plan with the actual flight duration. The mean deviation for four flights on the 14th of July 2011 according to the data available was 3.6% for the total flight duration, Table 5.1 shows the deviation per flight and difference between predicted flight time and actual flight time. However there can be multiple reasons for delays during the actual flights, such as traffic congestions at airports which may force aircraft into holding patterns. The data from Icelandair for the actual flight did not include any explanations of delays. Reasons for delay could be bad weather during take-off or landing or constrictions from air traffic control. For this reason it was decided to compare the performance using the data obtained from IMR and the original flight plans, instead of actual flight performance. However the actual flight performance was examined and shown in tables as well.

Table 5.2 shows the total fuel burn and predicted burn in kilograms from the flight plan, with mean deviation in total burn of a few percent. The deviation for flight ICE204 from Keflavik to Copenhagen is uncharacteristically high compared to other flights examined during the same day. This is one of the reasons for examining this flight further in this study. Also it is worth mentioning that according to table 5.2 Icelandair appears to con-

Flight	From	Dest	Flight time	Predicted FT	Difference	Deviation
ICE614	JFK	KEF	4:48	4:49	0:01	0.35%
ICE615	KEF	JFK	5:43	5:35	0:08	2.33%
ICE204	KEF	CPH	2:59	2:41	0:18	11.18%
ICE680	SEA	KEF	6:52	6:54	0:02	0.49%

Table 5.1: Total flight time prediction Accuracy for Flights on 14/7/2011

sistently overestimate the fuel burn, by 7.06 %. Reducing the excess fuel load with better statistical analysis can further improve the fuel burn performance of the aircraft with lower take-off weight.

Flight	Total Burn	Predicted Burn	Difference	Deviation
ICE614	16693	17629	936	5.61%
ICE615	19197	19954	757	3.94%
ICE204	9325	10927	1602	17.18%
ICE680	23950	24311	361	1.51%

Table 5.2: Fuel burn Prediction accuracy for Flights 14/7/2011

The optimization is done over the entire cruise phase of the flights chosen, the reason for doing so is to get longer distance to optimize over. It is also important to take into account the portion of the flight flown within BIRD. Table 5.3 shows the respective fuel burned and time spent within BIRD. It is clear that the flight track between Keflavik and New York spends a relatively short amount of time within BIRD, however the length of the cruise phase and the variance in wind direction and wind speed make it an intriguing example to study. Which is why the flight track between Keflavik and New York was studied.

Flight	Time in BIRD	% of FT	Burn within Bird	% of Total Burn
ICE614	42	14.53%	1996	11.96%
ICE615	59	17.61%	8709	2.33%
ICE204	1:32	51.4%	6096	65.37%
ICE680	2:13	32.13%	6278	26.21%

Table 5.3: Time and fuel burn within BIRD compared to total flight burn on 14/7/2011

Hence the results shown in table 5.3 the flight ICE204 was chosen for further examination, as this flight burned 65.37% of it's fuel within BIRD and spent 51.4% of it's time within BIRD during the flight.

5.1 Flight ICE615

The main reason for choosing ICE615 other than availability of data from all participants, is that the flight ICE615 traverses the North Atlantic jet stream making it an intriguing case to study. As this flight is flying westbound against the North Atlantic Jet stream, the optimization consists of minimizing the effect of headwind on the aircraft during ICE615's cruise phase, taking advantage of wind speed and wind direction variance which is significantly higher than on the average. Table 5.4 shows ICE615's cruise phase portion of fuel burn, flight track and flight time. The North Atlantic jet stream makes it interesting

Flight	Portion of Burn	Portion of Flight track	Portion of FT
ICE615	74.75%	88.05%	89.25%

Table 5.4: Flight ICE615 cruise phase's portion of Fuel burn, Flight track and Flight time on 14/7/2011

to see the flight path suggested by the algorithm for ICE615's cruise phase, compared to the initial flight plan from Icelandair. Figure 5.1 shows the grid constructed for ICE615's cruise phase, using Icelandairs initial flight path as a reference point.

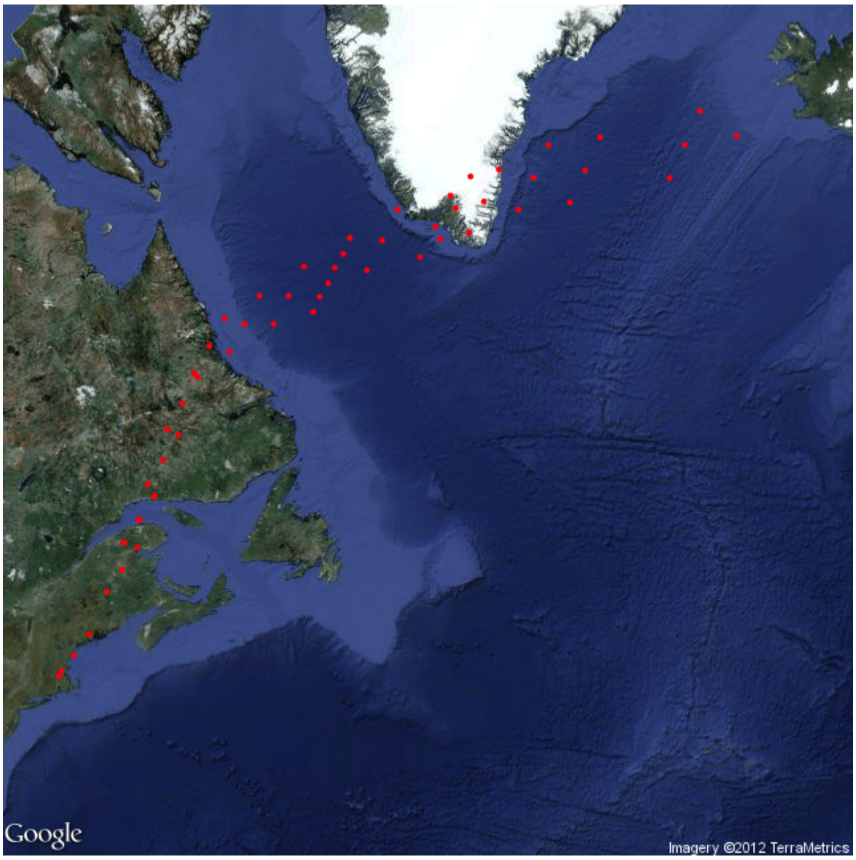

Figure 5.1: The grid for for flight ICE615 from Keflavik to New York on 14/7/2011

Figure 5.2 shows the deviation from Icelandairs flight plan for ICE615 during the cruise phase. The algorithm performs frequent course adjustments to minimize the headwind effect during the cruise phase. The predicted flight time for the cruise phase by the flight plan was used as a reference point for the calculations, which the algorithm tried to improve. In the case of flight ICE615 on 14th of July 2011, the flight plan calculated the time needed to traverse the cruise phase as 299 minutes with the weather data available at the time.

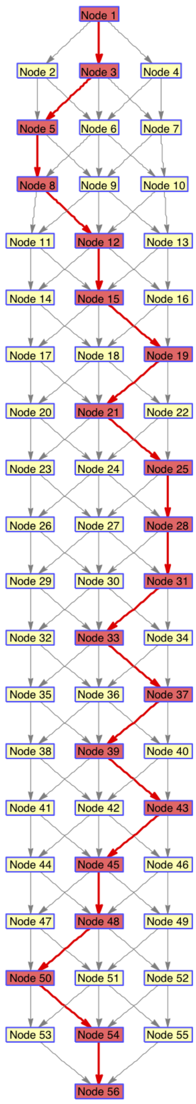

Figure 5.2: The shortest path for ICE615's grid representing cruise phase

To prevent difference in weather data accuracy between the weather data available at the time and the post processed weather data obtained from IMR the route suggested by Icelandairs flight plan shown in Figure 5.3, was also calculated with IMR's weather data. Using IMR's weather data, the calculations showed that the cruise phase should have been 297 minutes, when calculating the route from Icelandairs flight plan. The new route calculated by the algorithm needs 294.7 minutes to traverse the cruise phase, with the data available from IMR. According to this the algorithm calculated a route that took 4.3 minutes less to complete than the flight plan predicted with the data available at the time. The improvement on the original flight plan results, is due in part to more data available for the algorithm to work with. According to the data from IMR, the time improvement calculated by the algorithm for cruise phase was 297 - 294.7 = 2.3 minutes. The algorithm achieved this by deviating from the flight track, defined by the flight plan. The deviations are noticeable in Figure 5.4.

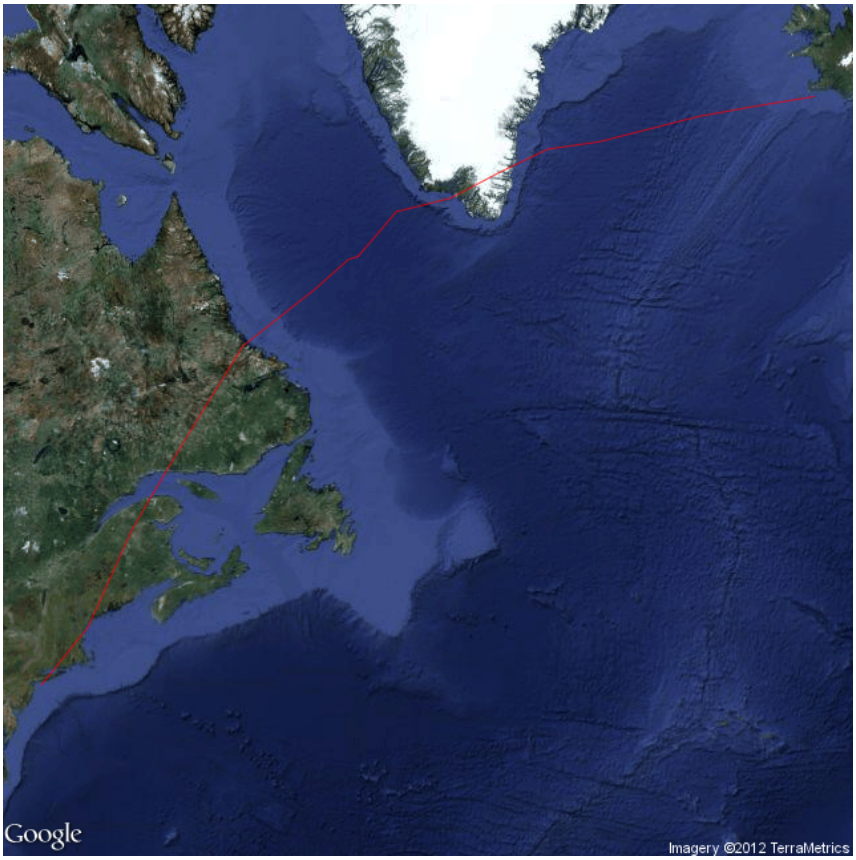

Figure 5.3: The initial flight plan for ICE615

The planned route was 2050.1 nm in length but the route suggested by the algorithm is 2124.8 nm, meaning it's 74.7 nm longer than the planned route. This is a deviation of 3.6% from the planned route in nautical miles. Albeit the route computed by the algorithm takes **2.3** minutes less time to travel it according to the algorithm and the weather data at hand.

To estimate the fuel consumption during the cruise phase, certain parameters are needed such as the cruise altitude. During ICE615's cruise phase the cruise altitude was 35,000 ft, and the mean mass of the aircraft during cruise phase was 93,536 kg, it's cruise speed

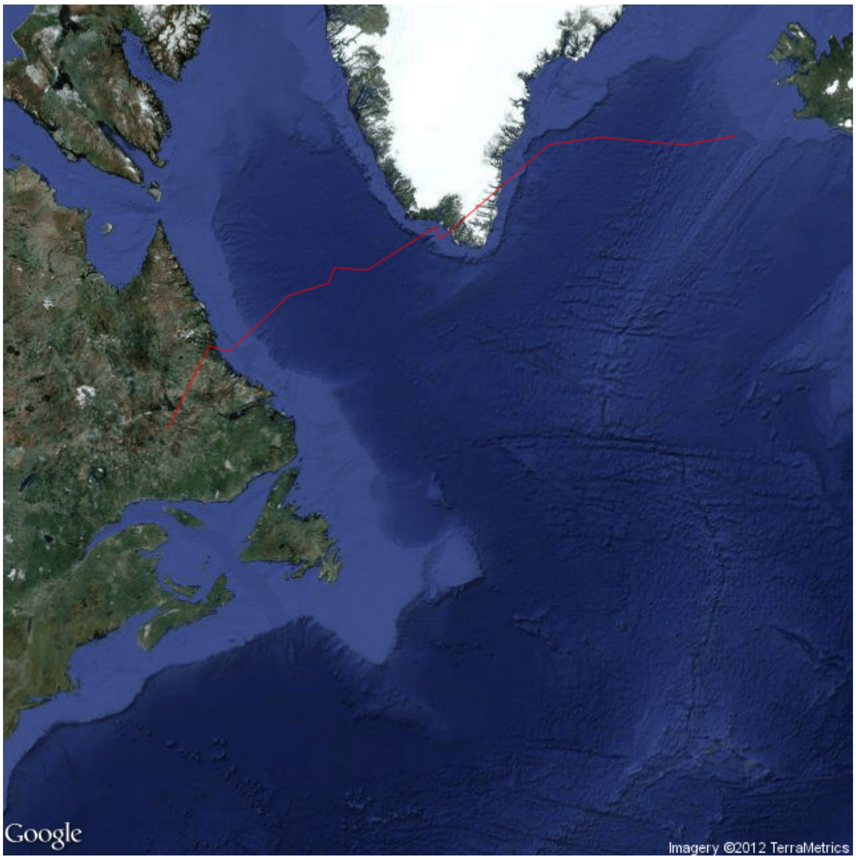

Figure 5.4: ICE615

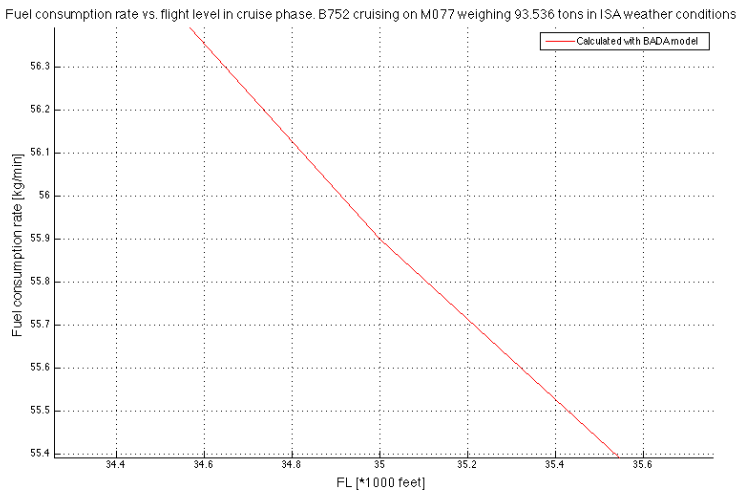

Figure 5.5: BADA fuel consumption estimate in kg/min for flight ICE615 from Keflavik to New York during cruise phase as a function of altitude

was Mach 0.77. These parameters were put into BADA. Calculated fuel consumption rate by BADA according to these parameters was 55.9 kg/min at 35,000 ft and ISA weather conditions. Figure 5.5 shows the fuel consumption rate for ICE615's Boeing's 757-200 as a function of altitude. Based on this fuel consumption the saved fuel during cruise phase was 129 kg. Taking the fuel bias into consideration from Icelandairs flight plan for flight ICE615, where the rated fuel bias for both of the aircraft engines was 1.02, the expected fuel saved was:

$$\mathbb{E}[Fuel\,Saved] = 129kg \cdot \textbf{bias} = 132kg \tag{5.1}$$

The improvements on cruise performance therefore would be 0.9% fuel burn savings w.r.t the planned burn for cruise phase and 0.8% reduction of time, when comparing the results of the original flight plan and the optimized route using the data from IMR. Table 5.5 summarizes ICE615's results when using IMR's weather data.

Flight	**Dijkstra Path in min**	**Flight plan in min**	**Fuel saved in kg**
ICE615	294.7	297	132

Table 5.5: Summary of Flight ICE615 cruise phase's optimization results Fuel burn, Flight track and Flight time on 14/7/2011

These results show that the fuel saved within BIRD would be 23.45 kg, according the estimate done with BADA. It is necessary to take into consideration that this flight was chosen for optimization because of the North Atlantic jet stream, not because of fuel burn within BIRD.

5.2 Flight ICE204

The reason for choosing flight ICE204 from Keflavik to Copenhagen on 14th of July 2011 was that 65.37% of its fuel burn was within BIRD according to Icelandairs data available for the actual flight. The initial flight plan predicted the time in cruise phase within BIRD as 70 minutes, which represents 55.5% of the predicted total cruise phase time of 126 minutes, according to Icelandairs flight plan. Table 5.6 shows ICE204's cruise phase portion of fuel burn, flight track and flight time. The portion of fuel burn is lower when compared to ICE615 this is the results of a shorter cruise phase and less headwind during the flight than ICE615 faced.

Flight	Portion of Burn	Portion of Flight track	Portion of FT
ICE204	62.95%	74.65%	70.4%

Table 5.6: Flight ICE204 cruise phase's portion of Fuel burn, Flight track and Flight time on 14/7/2011

The same method as in ICE615's case was used to construct a grid for ICE204, the grid for ICE204 is shown here in Figure 5.6.

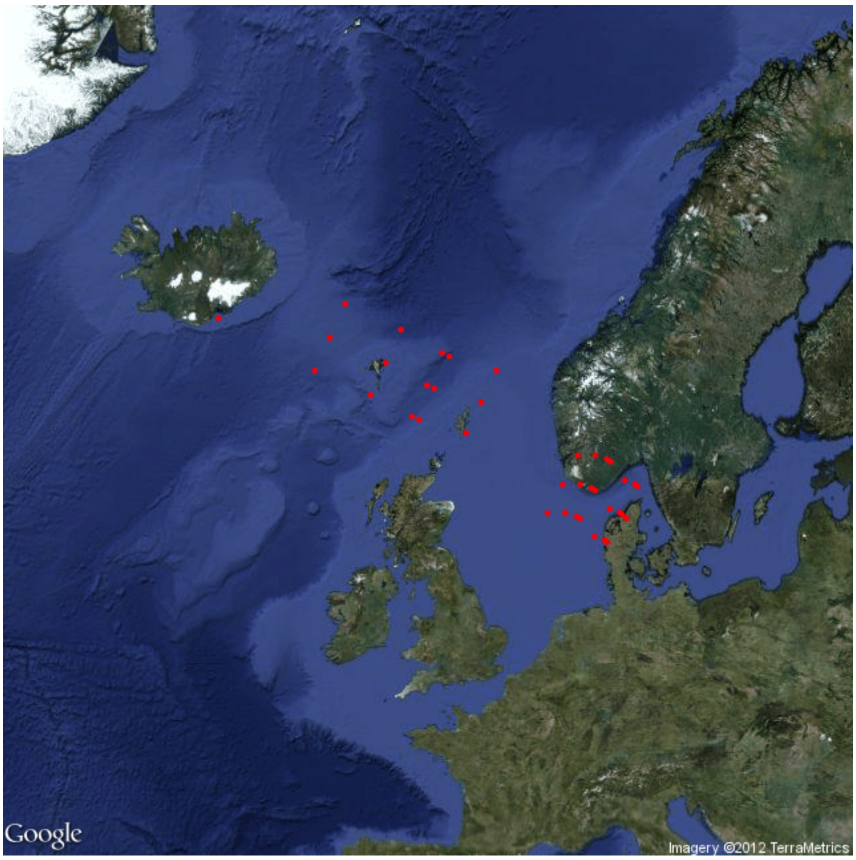

Figure 5.6: Coordinate grid for ICE204 cruise phase grid used for track optimization

Figure 5.7 shows the deviation from Icelandairs flight plan for ICE204 during the cruise phase. The algorithm does not perform as many course adjustments from the initial flight plan as in ICE615's case. The reason is the low wind speed faced during the cruise phase. The low wind speed reduces the possibility of using the wind to increase ground speed with WO, even though wind direction changes during the flight are considerable. Therefore the low wind speed values makes it more difficult to increase performance during the cruise phase. When using the data from IMR to calculate the route suggested by Ice-

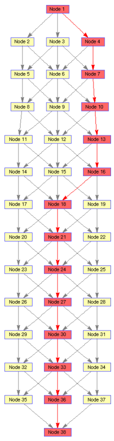

Figure 5.7: The shortest path for ICE204's cruise phase grid

landairs flight plan as shown in Figure 5.8. The calculations showed that the cruise phase should have been 124.5 minutes instead of 126 minutes. The algorithm calculated the shortest path for cruise phase as 123.3 minutes for ICE204, which is 2.7 minutes less than predicted by the original flight plan, with the data available at the time. When looking at the algorithms results with the weather data from IMR, the time improvement calculated by the algorithm for the cruise phase was 124.5 - 123.3 = 1.2 minutes. Figure 5.9 shows the deviation from flight track defined by the flight plan. By using the Faroe Islands and the Shetlands as reference points the difference in the flight path's can be observed in Figures 5.8 and 5.9. The difference is the deviation suggested by the optimization algorithm from the flight plan, shown in Figure 5.7.

The planned route was 904.1 nm in length but the route suggested by the algorithm was 900.2 nm. Therefore the algorithm comes up with a better solution both for distance flown and time to traverse the grid representing ICE204's cruise phase improving the flight plan by 3.8 nm and **1.2** minutes. The short cruise phase makes it even more difficult to increase the performance during the cruise phase.

The cruise altitude in ICE204's case is 35,000 ft, and the mean mass of the aircraft during cruise phase was 87,900 kg and with a cruising speed of Mach 0.76. Unlike ICE615 this was a Boeing 757-300 instead of Boeing 757-200 needing appropriate adjustments when

Figure 5.8: The planned flight track for ICE204's cruise phase on 14/7/2011

dealing with the BADA model. Calculated fuel consumption by BADA and according to the input parameters was 54.96 kg/min at 35,000 ft and ISA weather conditions. Based on this fuel consumption the saved fuel during cruise phase was 66 kg. The rated fuel bias for this aircraft was 1.012, hence expected fuel fuel saved was:

$$\mathbb{E}[Fuel\,Saved] = 66kg \cdot \textbf{bias} = 66.8kg \tag{5.2}$$

These improvements in cruise performance therefore are estimated to be 0.96% fuel burn savings w.r.t the planned burn for cruise phase and 0.95% w.r.t time and with 55.56% of the cruise phase within BIRD the fuel saved within BIRD ought to be 36.7 kg, according to the estimate done with BADA.

Flight	**Dijkstra Path in min**	**Flight plan in min**	**Fuel saved in kg**
ICE204	123.3	124.5	66.8

Table 5.7: Summary of Flight ICE204 cruise phase's optimization results Fuel burn, Flight track and Flight time on 14/7/2011

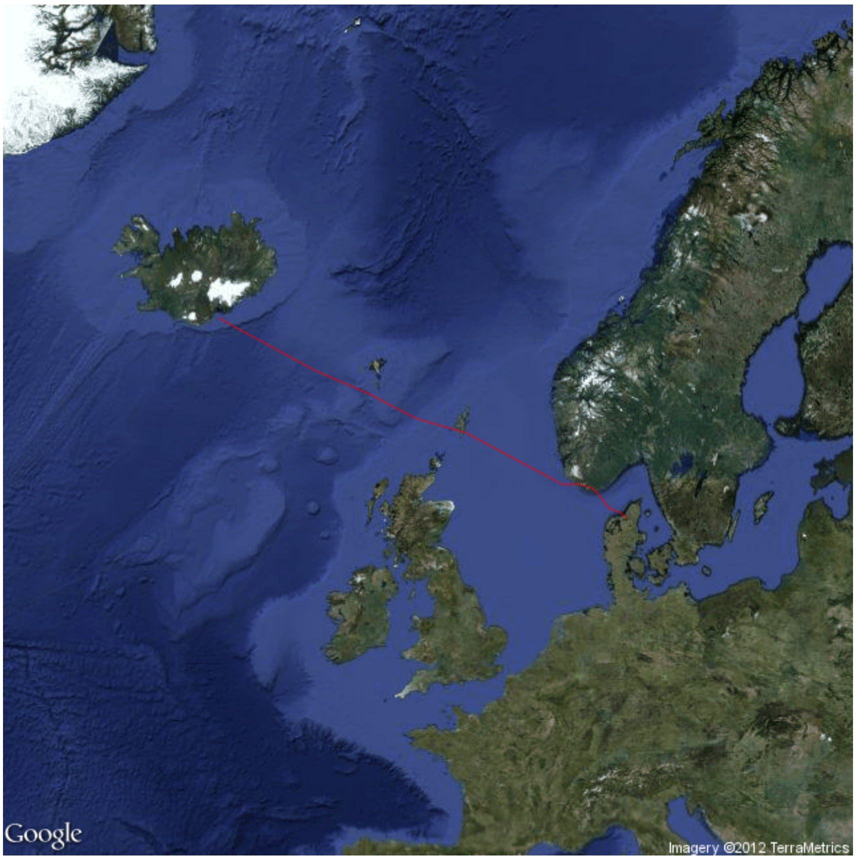

Figure 5.9: The optimized flight track for ICE204's cruise phase on 14/7/2011

Its worth mentioning that the actual total flight time was 18 minutes longer than predicted by the flight plan. When looking at the wind speed faced by ICE204 during cruise phase, as shown in Figure 3.4, it is highly unlikely that this delay was caused by the winds en-route. As Figure 3.4 shows that at ICE204 cruise altitude the wind speed ranged from 2 to 34 knots during the flight, which is rather low.

5.3 Greenhouse gas emissions

GHG emissions are part and parcel of flying as this mode of transport is heavily dependent on the use of hydrocarbon fuels. Therefore every effort taken to reduce fuel burn has the by-product of also reducing the environmental effects of aviation. Small carbon emission savings in each flight add up to a considerable amount when the scale of today's aviation industry is taken into consideration. Hence successes in fuel burn reduction are a step in making aviation companies more environmentally friendly. Calculations of how much CO_2 would be saved as a result of the fuel saved during flight, must be assessed because of anticipated future GHG restrictions and levies imposed by states. In order to convert aviation fuel into CO_2 a conversion factor of 3.155 applies. This means that the 1 kg of consumed fuel produces 3.155 kg of CO_2.[1] Accordingly, the optimized shortest path could potentially have saved 416.5 kg CO_2 during ICE615's cruise phase. For ICE204's cruise phase the CO_2 emission savings could have been 217.7 kg.

[1] http://www.epa.gov/cleanenergy/energy-resources/calculator.html

Chapter 6

Conclusions and Recommendations

6.1 Summary and Conclusions

The effect of wind speed and wind direction can be observed when driving long distances on rural roads in strong winds. The fuel consumption of an automobile increases when facing headwind and similarly it decreases with tailwind. This study focuses on the aircraft's capability to deviate in the horizontal plane from the flight track to either minimize the impact of headwind or take advantage of tailwind by optimizing the aircraft heading. The results of the wind optimization show that aircraft flight path optimization is a field of study with potential benefits as fossil fuel prices continue to increase and carbon charges create strong incentives for airlines to improve their fuel usage. The growing pressure for reduced fuel consumption also creates service opportunities for Air Traffic Service providers like Isavia, that have a desire to support their customers in this endeavour. This is achieved by optimization of aircraft paths in the lateral plane by minimizing flight time resulting in reduced fuel burn. However, this process is dependant on the quality and accuracy of the wind data when the optimization is performed. It is also dependent on the variability of the winds that tend to be much less during the summer than in winter.

Therefore flight plan prediction should be more accurate during the summer, with less deviation between actual flight time and predicted flight time. The deviation in flight time for 14 July 2011 as shown in table 5.1 shows that there was room for improvements on that day. The improvements in flight time calculated in this study for flights ICE615 and ICE204 by application of the Dijkstra algorithm and accurate weather data strengthen the claim that improvements could be made. The cruise phase is by far the largest portion of North Atlantic flight profiles and is exposed to the effect of strong winds. Therefore it is logical to optimize this phase, also because there is more flexibility to optimize in cruise phase as the climb phase and the descent phase are more restricted. Optimized flight routes for ICE615 and ICE204 show that with more accurate weather data further improvements could be possible during cruise. The question is how much room there is for improvements with wind optimization during cruise. The study achieves an approximately one percent improvement on a day where wind speed was low, when compared to other days.

The results of the study indicate that when the conditions are favourable aircraft flight profile optimization with respect to known wind conditions can improve performance more than achieved in this study. However research involving multiple flight tracks examined over different parts of the year is needed to verify how significant this benefit is. Seasonal fluctuations should be analysed in order to give a better estimate of year round fuel savings. According to *Cross Polar Aircraft Trajectory Optimization and the Potential Climate Impact* [8] Wind-optimal routes reduced average fuel burn of flight plan routes by 4.4% on December 4, 2010. In order to minimize the difference in results during the estimation of the fuel burn savings IMR's data was used to estimate the cruise time for the path calculated by optimization and the path defined by the flight plan. The fuel consumption rate was obtained from the BADA model to determine the corresponding fuel savings.

6.2 Recommendations for future research

Future research could focus on the cost of constructing and implementing a search algorithm, analysing the cost of WO versus fuel saved. Obviously such a project would need the cooperation of air navigation service providers such as Isavia, airlines, meteorological institutions and IT companies to be possible. Preferably such a project would receive data from a number of airlines and weather sources, compiling as much weather and performance data as possible for data mining. The quality and the data format are of great importance, as well as the definition of the relevant datasets to minimize data size. Despite increasing data storage and processing capabilities, it is preferable to keep the run time of such programs to a minimum. During the study considerable manual effort was needed which can be automated to increase the efficiency of the wind optimization. A refinement of data mining techniques and more data compatibility is needed and further access to data stored aboard the aircrafts. The data aboard the aircrafts contains values of wind speed, wind direction and temperature faced in each waypoint. In order to further increase the accuracy of the fuel savings computations, more accurate data for the aircrafts mass in each waypoint is needed and temperature at sea level to plug in to the BADA model. In general the most accurate weather data has to be available, also to know the variance in weather forecasts and finally better validation of weather data is needed.

Bibliography

[1] CUI Deguang Rui, CHENG Peng.*Dynamic Air-Route Adjustments Model, Algorithm, and sensitivity Analysis*. Tsinghua National Laboratory for Information Science and Technology (TNList). Department of Automation, Tsinghua University, Bejing 10084, China, volume 14 edition, February 2009

[2] Hjalti Pálsson. *Reduction of Emissions on the North Atlantic by the Implementation of ADS-B*. Isavia, Reykjavíkurflugvöllur 101 Reykjavík Iceland, April 2010

[3] Unknown. *Performance of Flight Trials and Demonstrations Validating Solutions For the Reduction of CO_2 Emissions*. EUROCONTROL, November 2011.

[4] ASPIRE *Asia and Pacific Initiative to Reduce Emissions (ASPIRE) Strategic Plan*, March 2011

[5] Edward A. Lester and R. John Hansman. *Benefits and Incentives for ADS-B Equipage in the National Airspace System*. MIT International Center for Air Transportation, Department of Aeronautics Astronautics Massachusetts Institute of Technology Cambridge, MA 02139USA, August 2007

[6] Jonathan A. Lovegren and R. John Hansman. *ESTIMATION OF POTENTIAL AIRCRAFT FUEL BURN REDUCTION IN CRUISE VIA SPEED AND ALTITUDE OPTIMIZATION STRATEGIES*. MIT International Center for Air Transportation, Department of Aeronautics Astronautics Massachusetts Institute of Technology Cambridge, MA 02139 USA, February 2011

[7] Ronald L. Rivest, Clifford Stein, Thomas H. Cormen, Charles E. Leiserson.*Introduction to Algorithms*. Deparment of Electrical Engineering and Computer Science, Department of Electrical Engineering and Computer Science Massachusetts Institute of Technology Cambridge, MA 02139USA, second edition, 2001.

[8] Shon Grabbe Neil Chen Hok K. Ng, Banavar Sridhar.*CROSS-POLAR AIRCRAFT TRAJECTORY OPTIMIZATION AND THE POTENTIAL CLIMATE IMPACT*. NASA Ames Research Center, NASA Ames Research Center, Moffett Field, CA 94035.

[9] EUROCONTROL EXPERIMENTAL CENTRE. *BASE OF AIRCRAFT DATA (BADA) AIRCRAFT PERFORMANCE MODELLING REPORT*, 2008-2009.

[10] Steven S. Skiena.*The Algorithm Design Manual*. Department of Computer Science State University of New York, University of New York at Stony Book New York, USA, second edition, 2008

Appendix

Appendix A

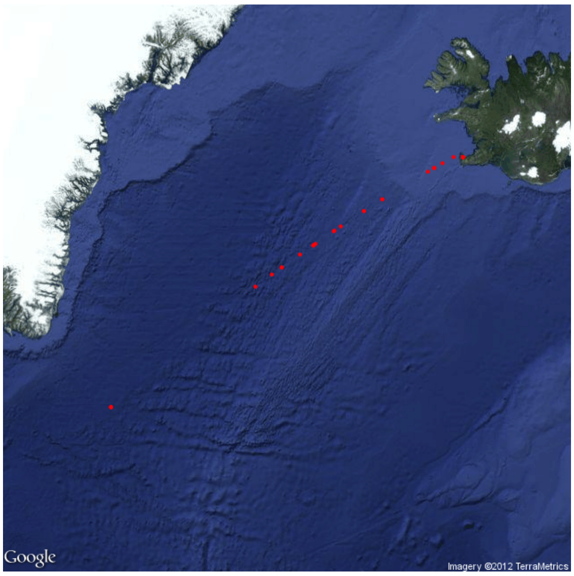

Figure 6.1: ICE615 actual Flight within BIRD 13/7/2011

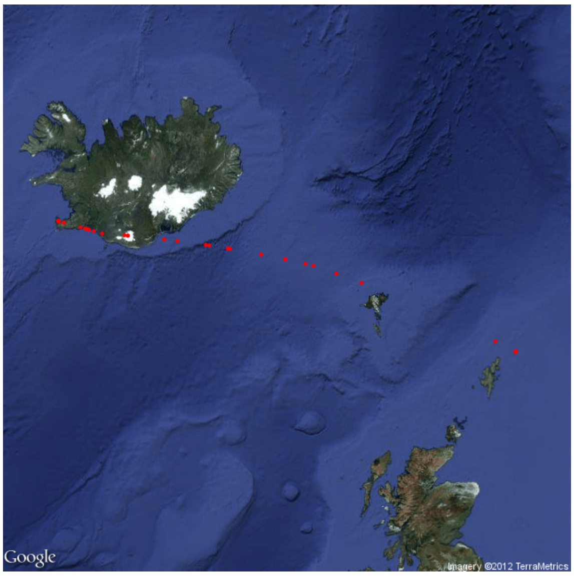

Figure 6.2: ICE204 actual Flight within BIRD 14/7/2011

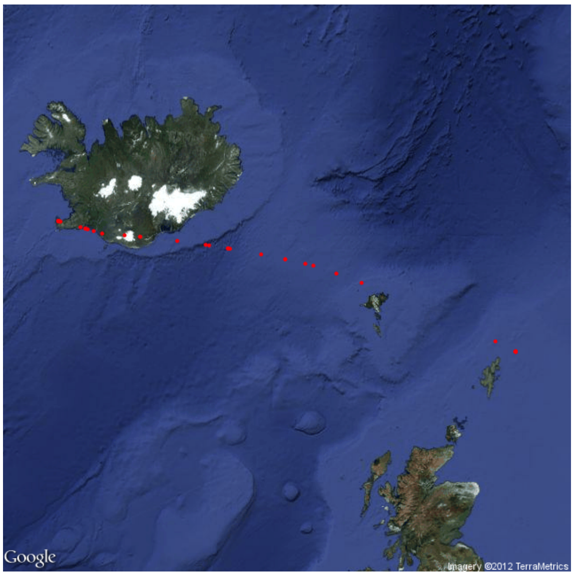

Figure 6.3: ICE204 actual Flight within BIRD 13/7/2011

Appendix B

Matlab Code for flight ICE204

```
%% ICE204 14/7/2011 Boeing 757-300

tic

alt = 36.000;
bias = 1.012;
K = 273.15;
T = K-51.26; % average temperature in Kelvin at cruise altitude
Mach_number = 0.77;

%% TAS in knots

calc_TAS = 38.975*Mach_number*sqrt(T)

airspeed = [ 441.2336   441.2336   447.0393   441.2336   441.2336
441.2336 434   435   436   436   437   437   438   ];

% mean(TAS) = 438.872

%% Coordinates for grid
```

```
grid = [NaN    NaN    63.58   -17.36  NaN    NaN
64      -9      63      -10     62      -11
63.24   -5.31   62.24   -6.31   61.24   -7.31
62.54   -2.58   61.54   -3.58   60.54   -4.58
62.44   -2.11   61.44   -3.11   60.44   -4.11
62      1       61      0       60      -1
59.25   6.38    58.25   5.38    57.25   4.38
59.26   7.55    58.26   6.55    57.26   5.55
59.13   8.3     58.13   7.3     57.13   6.3
59.05   8.54    58.05   7.54    57.05   6.54
58.4    9.48    57.4    8.48    56.4    7.48
58.27   10.16   57.27   9.16    56.27   8.16
58.19   10.32   57.19   9.32    56.19   8.32
NaN     NaN     57.06   9.59    NaN     NaN];


%% Create GCD matrix all distances in NM

% plus 1 direct path
lat_1 = grid(:,1);
lon_1 = grid(:,2);

[coursegc_1,distgc_1] = legs(lat_1,lon_1,'gc');

% Original flight plan
lat = grid(:,3);
lon = grid(:,4);

[coursegc,distgc] = legs(lat,lon,'gc');

% Minus 1 direct path
lat_minus_1 = grid(:,5);
lon_minus_1 = grid(:,6);

[coursegc_minus_1,distgc_minus_1] = legs(lat_minus_1,lon_minus_1,'gc');
```

```
% Columns 1, 4 and 7 in GCD matrix
straight_edges = [distgc_1 , distgc , distgc_minus_1];


% Find angled edges

lat_1_to_lat = [64    62.24    63.24    61.54    62.54    61.44
62.44    61    62    58.25    59.25    58.26    59.26    58.13    59.13
58.05  59.05    57.4    58.4    57.27    58.27    57.19    58.19
57.06  ];
lon_1_to_lon = [ −9    −6.31    −5.31    −3.58    −2.58    −3.11
−2.11            0     1     5.380    6.38     6.55     7.55     7.3
8.3     7.54  8.54     8.48     9.48     9.16    10.16    9.32    10.32
9.59  ];


[coursegc_lat_1_to_lat ,distgc_lat_1_to_lat] = legs(lat_1_to_lat ,lon_1_to


lat_to_lat_1 = [ 63.5800    64    63    63.24    62.24    62.54
61.54    62.44    61.44    62    61    59.25    58.25    59.26    58.26
59.13  58.13    59.05    58.05    58.4    57.4    58.27    57.27
58.19  ];
lon_to_lon_1 = [  −17.3600    −9   −10    −5.31    −6.31    −2.58
−3.58   −2.11    −3.11     1            0     6.38     5.38     7.55
6.55     8.3  7.3     8.54     7.54     9.48     8.48    10.16     9.16
10.32  ];

[coursegc_lat_to_lat_1 ,distgc_lat_to_lat_1] = legs(lat_to_lat_1 ,lon_to_l


lat_minus_1_to_lat = [ 62    62.24    61.24    61.54    60.54
61.44    60.44    61    60    58.25    57.25    58.26    57.26    58.13
57.13    58.05  57.05    57.4    56.4    57.27    56.27    57.19
56.19    57.06  ];
lon_minus_1_to_lon = [ −11    −6.31    −7.31    −3.58    −4.58
−3.11    −4.11            0    −1     5.38     4.38     6.55     5.55
```

```
7.3      6.3      7.54  6.54      8.48      7.48      9.16      8.16      9.32
8.32      9.59  ];

[coursegc_lat_minus_1_to_lat,distgc_lat_minus_1_to_lat] = legs(lat_minu


lat_to_lat_minus_1 = [  63.58     62     63     61.24     62.24     60.54
61.54     60.44     61.44     60     61     57.25     58.25     57.26     58.26
57.13  58.13     57.05     58.05     56.4     57.4     56.27     57.27
56.19  ];
lon_to_lon_minus_1 = [  −17.36   −11   −10     −7.31     −6.31     −4.58
−3.58     −4.11     −3.11     −1          0     4.38     5.38     5.55
6.55     6.3  7.3     6.54     7.54     7.48     8.48     8.16     9.16
8.32  ];
%G=lat_to_lat_minus_1(1:2:end);
%H=lon_to_lon_minus_1(1:2:end);

[coursegc_lat_to_lat_minus_1,distgc_lat_to_lat_minus_1] = legs(lat_to_la

% columns 2,3,5 and 6 in GCD matrix
angled_edges = [  distgc_lat_1_to_lat ,  distgc_lat_to_lat_1 ,
distgc_lat_minus_1_to_lat,distgc_lat_to_lat_minus_1     ];

angled_edges_2 = [  angled_edges(1:2:end,1),  angled_edges(1:2:end,2),  an

angled_course = [  coursegc_lat_1_to_lat ,  coursegc_lat_to_lat_1 ,
coursegc_lat_minus_1_to_lat,coursegc_lat_to_lat_minus_1    ];

angled_course_2 = [  angled_course(1:2:end,1),  angled_course(1:2:end,2),

%angled_edges_3 = [  angled_edges_2(1:12,1),  angled_edges_2(1:11,2),  ang



GCD = [  NaN       NaN       222.9529702       201.5334902       NaN       198.589
108.4799723       128.4231688       128.0969082       111.6185507       132.433
85.6964332       112.8414339       105.3259782       87.91454153       108.724
```

```
14.35076403     67.71354088     68.09891935     14.74682073     68.9311
90.93960238     105.2382619     121.628048      93.68950104     125.260
228.6984381     260.3535088     217.6913798     232.1121129     222.762
35.91626558     59.67531316     90.79043517     36.96432434     92.2388
24.34577398     68.29238703     75.55357838     24.98275592     76.6872
8.823978131     69.06573852     67.50023619     9.004790148     68.1402
48.79979243     99.0849038      64.83297011     49.30711518     66.4613
22.80937208     68.61215504     74.90905902     23.37706839     75.9784
6.975205914     70.21101095     66.58528747     7.078677161     67.1614
NaN     71.78909112     NaN     11.76229094     66.99074827     NaN

%% Find GS for the GCD matrix

mean_wd = 112.84; % mean wind direction in degrees at 35.000 ft
std_wd = 71.158; % standard deviation of wind direction in degrees at 3

mean_ws = 14.6939; %in meters/second
std_ws = 7.162;% in m/s

c = 1.9438; % convert m/s to knots

mean_ws_knots = mean_ws*c; % 28.562
std_ws_knots = std_ws*c; % 13.9215

wind_direction = normrnd(112.84,71.158, [1 13])';
windspeed = normrnd(28.562,13.9215, [1 13])';

windfrom = [ 156.1923 104.9931  132.5833  119.0340  125.1925
145.1166   69.8949   56.1254  138.5031  246.9794   91.0072
40.7766   95.8656  ];

windspeed = [  21.8238   24.0049   35.1819   26.7521   20.2877
22.3841   10.1504   16.5962    8.8909   21.3787   13.4558
18.7173   24.3625  ];
```

```
% Need the course as well

%course_matrix = [coursegc_1, angled_course(1:2:end,1), angled_course(1

%course_matrix_2 = [coursegc_1, coursegc, coursegc_minus_1];

course_matrix_3 = [ NaN NaN      79.77037524     96.64261175     NaN
113.2089546     144.14854       81.45427048     112.4792454     81.6852
118.1453968     153.9809811     78.50540478     117.346437      78.8312
114.5235987     192.9840998     36.84306711     113.8187058     37.7419
105.5059896     144.2979097     72.15214186     105.0091975     72.6463
133.8107817     147.8926988     116.0472316     132.9639672     115.360
88.53941619     174.8373864     47.17386963     88.57187084     47.9859
108.3767512     186.6645377     45.51980811     107.8864259     46.3327
122.8764821     200.4649457     34.55757353     122.1341439     35.3222
142.6995732     181.1225409     70.26517469     141.925719      70.7544
109.7209592     188.7087658     45.08234642     109.2182181     45.8695
133.4525706     202.9071175     33.45882204     132.663126      34.1876
NaN     199.3901068     NaN     131.4602209     38.23709108     NaN


course_lat_1= course_matrix_3(:,1)';
course_lat_1_to_lat = course_matrix_3(:,2)';
course_lat_to_lat_1 = course_matrix_3(:,3)';
course = course_matrix_3(:,4)';
course_lat_minus_1_to_lat = course_matrix_3(:,5)';
course_lat_to_lat_minus_1 = course_matrix_3(:,6)';
course_lat_minus_1=course_matrix_3(:,7)';


% Calculate GS matrix

% column 1
[heading_lat_1,groundspeed_lat_1,windcorrangle_lat_1] = driftcorr(cours

% column 2
[heading_lat_1_to_lat ,groundspeed_lat_1_to_lat ,windcorrangle_lat_1_to
```

```
% column 3
[heading_lat_to_lat_1 ,groundspeed_lat_to_lat_1 ,windcorrangle_lat_to_la

% column 4
[heading,groundspeed,windcorrangle] = driftcorr(course,airspeed,windfrom
417.4222   412.9985   414.5855   422.1362   419.3260   424.3720
424.5325   427.4623   441.0635   424.1979   437.2156   417.9599

% column 5
[heading_lat_minus_1_to_lat,groundspeed_lat_minus_1_to_lat,windcorrangl

% column 6
[heading_lat_to_lat_minus_1,groundspeed_lat_to_lat_minus_1,windcorrangl

% column 7
[heading_lat_minus_1,groundspeed_lat_minus_1,windcorrangle_lat_minus_1]

% in knots
GS = [groundspeed_lat_1', groundspeed_lat_1_to_lat', groundspeed_lat_to

%% Calculata cost per leg in time

time_cost = GCD ./ GS; % time per leg in hours

adj_matrix = [ 0   1   1   1   0   0   0
0   0   0   0   1   1   0   0   0
0   0   0   0   1   1   1   0   0
0   0   0   0   0   1   1   0   0
0   0   0   0   0   0   0   1   1
0   0   0   0   0   0   0   1   1
```

```
0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
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0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
];

t_adj_matrix = [ 0 0.511 0.4689 0.4679 0 0 0
0 0 0 0 0.2599 0.304 0 0 0
```

0	0	0	0	0.3056	0.2673	0.3076	0	0
0	0	0	0	0	0.316	0.2749	0	0
0	0	0	0	0	0	0	0.2076	0.2724
0	0	0	0	0	0	0	0.2475	0.2128
0	0	0	0	0	0	0	0	0.2556
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

```
%% Shortest Path

FMG = sparse(t_adj_matrix)

h = view(biograph(FMG,[],'ShowWeights','on'))

[dist,path] = graphshortestpath(FMG,1,38)

set(h.Nodes(path),'Color',[1 0.4 0.4])
edges = getedgesbynodeid(h,get(h.Nodes(path),'ID'));
set(edges,'LineColor',[1 0 0])
set(edges,'LineWidth',1.5)


% dist = 2.1017 hours

time_in_cruise_phase=dist*60; % 126.102 min

%% Dijkstra


[dist,path, pred] = graphshortestpath(FMG,1,38,'directed',true,'method'

% Total time in cruise according to Dijkstra

% the same as in shortest path 126.102 min.

set(h.Nodes(path),'Color',[1 0.4 0.4])
edges = getedgesbynodeid(h,get(h.Nodes(path),'ID'));
set(edges,'LineColor',[1 0 0])
set(edges,'LineWidth',1.5)

% distance in NM flown according to Dijkstra
```

```
lat_dijkstra = [ 63.5800    62.0000    61.2400    60.5400    60.4400
60.0000    58.2500    58.2600    58.1300    58.0500    57.4000
57.2700    57.1900    57.0600 ];
lon_dijkstra = [ -17.3600   -11.0000    -7.3100    -4.5800    -4.1100
-1.0000     5.3800     6.5500     7.3000     7.5400     8.4800
9.1600     9.3200     9.5900 ];
[coursegc_dijkstra,distgc_dijkstra] = legs(lat_dijkstra,lon_dijkstra,'g
dist_dijkstra_NM = sum(distgc_dijkstra) % 900.2649 NM

%dijkstra_cruise_phase_portion_of_total_flight=dist_dijkstra_NM/sum(dis


%% Comparison with Flight plan:

% Cruise distance of original flight plan: 904.092 Nautical miles and s

t_flight_plan = sum(time_cost(:,4)); % 128.3098 minutes  128.3098-126.1

improvement_over_flight_plan = (t_flight_plan/dist)-1; %  1.75%

% The flight plan

lat_fp=[ 63.59 63.42 63.5800    63.0000    62.2400    61.5400
61.4400    61.0000    58.2500    58.2600    58.1300    58.0500
57.4000    57.2700    57.1900    57.0600 55.53 55.51 55.37 ];
lon_fp=[ -22.36 -20.36 -17.3600   -10.0000    -6.3100    -3.5800
-3.1100          0     5.3800     6.5500     7.3000     7.5400
8.4800     9.1600     9.3200     9.5900 10.31 10.55 12.39 ];

[coursegc_fp,distgc_fp] = legs(lat_fp,lon_fp,'gc'); % entire flight 120

%% Comparison with real number

TOW = 94129;
LDGWGHT = 84804;
GWT_BDRY=88033;
```

```
initial_cruise_weight=TOW-2790; % 91339
end_cruise_weight=TOW-9669; % 84460

%mean_cruise_weight= 87900

Burn = 54.96; % kg/min

Fuel_saved=(Burn*2.2078)/bias % 119.9019 kg

toc
```

Code for Overlay on Google Earth images in R

```
# Flight ICE680 From Seattle to KEF 13 july 2011 Boeing 757−200
# Time schedule according to flight plan: dep: 23:39 lan: 4:15
# Actual time according to ADS−B data from Isavia: dep: 23:48 lan: 6:32
# Alt = 37.000 ft, M = 0.80
lat <− data[472672:472717,c('loc_lat')]
lon <− data[472672:472717,c('loc_lon')]
#distance flown calculated with
library(geosphere)
xy <− rbind(c(lat), c(lon))
coordinates <− t(xy)
# distm(coordinates, fun = distHaversine)


map_center <− c( max(lat), min(lat), mean(lat), c(max(lon), min(lon), m
map_center
# map_center values: 69.05000 63.98500 65.86872 −22.60500 −61.94100 −
bb <− qbbox(c(69.05,63.985,65.86872),c(−61.941,−22.605,−33.65496), TYPE
MyMap <− GetMap.bbox(bb$lonR, bb$latR,destfile = "North_America_SEA_KEF
mymarkers <− cbind.data.frame(lat = lat, lon = lon, col = 'red')
```

```
bb <- qbbox(lat = mymarkers[,"lat"], lon = mymarkers[,"lon"])

# MyMap <- GetMap.bbox(lonR=bb$lonR, latR=bb$latR, destfile = "North_At

zoom <- min(MaxZoom(latrange=bb$latR,lonrange=bb$lonR));

png("OverlayTest_SEA_KEF_13_7.png",640,640);


plot.new()


PlotOnStaticMap(MyMap, lat = lat, lon = lon, pch =20, col='red')

dev.off()
```

GCD calculations in R

```
## Calculate great circle distance for a path with rdist.earth


# rdist.earth(x1, x2, miles = FALSE, R = NULL)

# have to have the x1 and x2 matrix as lon/lat

data <- read.csv("Icelandair_NY_FLIGHT_lat_lon.csv", header = TRUE)

lat <- data$lat1

lon <- data$lon1

# combine the vectors into a 22x2 lon/lat matrix

matrix <- cbind(lon,lat)

# calculate the great circle distance matrix

dist <- rdist.earth(matrix, miles = FALSE, R = NULL)

# sum of the matrix

sum(dist) # 662548.6 km

#for (i in matrix) print(i)

# for (j in matrix) print(j)


for (i in lon) print(i)

for (j in lat) print(j)

for (i,j) rdist.earth((i,j), miles = FALSE, R = TRUE)
```

Appendix C

NY grid

64.2800 -25.5800 63.2800 -26.5800 62.2800 -27.5800 64.0000 -29.0000 63.0000 -30.0000 62.0000 -31.0000 63.2300 -35.5900 62.2300 -36.5900 61.2300 -37.5900 63.0000 -39.0000 62.0000 -40.0000 61.0000 -41.0000 62.2500 -42.2800 61.2500 -43.2800 60.2500 -44.2800 62.0400 -44.1600 61.0400 -45.1600 60.0400 -46.1600 61.4400 -45.4800 60.4400 -46.4800 59.4400 -47.4800 61.0000 -49.0000 60.0000 -50.0000 59.0000 -51.0000 60.0800 -52.1300 59.0800 -53.1300 58.0800 -54.1300 59.5500 -52.5600 58.5500 -53.5600 57.5500 -54.5600 59.1200 -55.1700 58.1200 -56.1700 57.1200 -57.1700 58.1200 -58.1000 57.1200 -59.1000 56.1200 -60.1000 57.3200 -60.4100 56.3200 -61.4100 55.3200 -62.4100 55.1600 -62.1900 54.1600 -63.1900 53.1600 -64.1900 52.9300 -63.4700 51.9300 -64.4700 50.9300 -65.4700 50.4400 -65.0400 49.4400 -66.0400 48.4400 -67.0400 50.4100 -65.0600 49.4100 -66.0600 48.4100 -67.0600 48.2200 -66.1800 47.2200 -67.1800 46.2200 -68.1800 44.2500 -69.3600 43.2500 -70.3600 42.2500 -71.3600 43.5000 -70.1800 42.5000 -71.1800 41.5000 -72.1800

NY GCD

0 0 0 94.2733 0 0 0 181.7185 81.0291 77.6502 187.6811 157.5284 144.1102 193.6022 93.5954 159.5513 231.6611 96.7346 145.8943 157.5284 99.8470 101.1149 53.6595 81.0291 103.8442 67.7842 145.8943 106.5645 54.2238 117.1173 159.5513 55.9114 111.6215 67.7842 57.5855 52.0157 81.2719 53.6595 52.8971 25.8403 111.1755 53.7827 105.1093 169.1281 117.1173 108.2264 161.8112 26.0662 111.3181 107.6572 33.1860 81.2719 110.1062 65.9620 161.8112 112.5484 34.3670 141.2231 169.1281 34.5153 102.3288 65.9620 34.6656 83.9825 61.1980 31.1430 86.2125 61.7988 102.3288 88.4242 109.5111 152.4691 118.0853 111.7009 175.5286 61.7988 113.8840 88.2735 74.4685 61.1980 89.9880 45.3494 175.5286 91.6959 142.6276 210.2698 152.4691 143.2765 212.7523 45.3494 143.9309 141.2832 91.9954 74.4685 141.6315 74.5539 212.7523 141.9825 160.5076 73.3389 210.2698 160.9789 231.6299 74.5539 161.4521 1.9569 71.6055 91.9954 1.9633 69.9016 231.6299 1.9696 138.5992 345.8916 73.3389 138.8811 209.7147 69.9016 139.1637 272.4427 74.0655 338.0820 273.6083 201.0846 209.7147 274.7694 0 0 0 57.7012 0 0 0

NY Course Matrix

NaN NaN 304.9053 261.2577 NaN 239.7604 NaN 258.2371 246.1410 246.1410 258.6885 157.5284 277.4945 259.1058 263.0398 305.8504 305.8504 263.3032 145.8943 241.5531 263.5465 245.0223 230.7359 230.7359 245.7552 67.7842 283.7895 246.4427 257.3869 332.5060 332.5060 257.7922 111.6215 230.4576 258.1693 226.7512 215.9234 215.9234

227.6552 25.8403 337.2720 228.5207 256.9926 295.5311 295.5311 257.4054 161.8112 239.6804 257.7899 240.5050 228.8525 228.8525 241.2498 65.9620 275.0885 241.9537 202.3779 31.5084 335.1575 202.9716 102.3288 206.7576 203.5539 253.2235 234.1225 224.4887 253.6897 61.7988 304.3049 254.1267 238.0143 270.8195 270.8195 238.7338 175.5286 228.5135 239.4170 238.0181 226.2746 226.2746 238.7121 45.3494 285.8980 239.3718 205.3485 201.0564 201.0564 205.9003 212.7523 208.0675 206.4401 199.1448 203.6773 203.6773 199.5517 74.5539 187.9931 199.9511 201.9700 193.7067 193.7067 202.3864 231.6299 206.2394 202.7941 203.0199 212.9104 212.9104 203.4505 69.9016 33.2086 203.8721 198.8643 183.8442 204.8494 199.2036 209.7147 204.8413 199.5360 210.1492 211.9317 207.9133 210.5670 201.0846 212.3482 210.9732 NaN 35.4983 NaN 218.9827 216.7580 NaN NaN

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