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in Economics

Harvesting Policy under Uncertainty
The Case of the Deep Beaked Redfish in the Irminger Sea

Arnaldur Smári Stefánsson

Supervisor: Ragnar Árnason
Faculty of Economics
October 2012

HÁSKÓLI ÍSLANDS

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School of Social Sciences
University of Iceland
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Abstract

The Deep Stock of Beaked redfish (*Sebastes Mentella*) in the Irminger Sea is a commercial deep-sea species that mainly inhabits depths between 600 and 900 meters. As is the case for some other deep-sea-species, biological information on the stock is incomplete. While annual catch data is available, only five biomass assessments have been carried out. The stocks mortality is unknown and no firm estimates have been put forth for the carrying capacity. The economic data is as well incomplete. No separate financial statistics exist for the redfish fleet as such. Data on the Icelandic fleet as whole is the most precise, including annual data on revenue. However, there is no data on cost solely due to harvesting of Beaked redfish nor information on effort. A model is presented that takes these data shortcomings into account by adding distributions to number of variables including mortality, carrying capacity, output elasticity of effort and the ratio between the catchability coefficient and marginal variable cost. Given that the model is appropriate it might be used as a gauge for fishery harvesting policies that have a high degree of uncertainty. This holds in spite of the controversial parameters of the model, which are given in distributions instead of values. Finally, the model indicates that the present value of profits yielded by an optimal path of harvest would be between almost the same to € 116.5 higher than from an optimal path to the MSY equilibrium. The model further indicates that total allowable catch set by the Northeast Atlantic Fisheries Commission for the next years should be lower if the goal is to maximise the economic performance of the fishery.

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1 Introduction

Beaked redfish (*Sebastes mentella*) is primarily a deep-sea commercial species harvested in the Atlantic. The main fishing grounds can be found in the Barents Sea, the continental shelf and the slope west of Norway, the Atlantic waters in the Norwegian Sea and Irminger Sea and adjacent waters. Scientists differentiate between two stocks of Beaked redfish in the Irminger Sea by separating them into the Shallow Stock which can be found above depths of 500 meters, and the Deep Stock which is most abundant at depths of 600-900 meters (Sigurdsson et al., 2006).

The Irminger Sea consists of an area an area both stretching inside national exclusive economic zones (EEZs) as well as extending into international waters. The fishery is managed by the Northeast Atlantic Fisheries Commission (NEAFC) while the International Council of the Exploration of the Sea (ICES) holds an advisory role. In 2009, most of the nations taking part in the fishery agreed to ban fishing from the Shallow Stock, but the annual total allowable catch (TAC) for the Deep Stock was set at 30-40 thousand tonnes in 2011 and 2012 (Jakobsdottir and Kristinsson, 2010).

In this essay a bioeconomic model for the Deep Stock fishery in the Irminger Sea is developed. The model used is an aggregate biomass model, where no attempt is made to model individual fish cohorts. As is the case for some other deep-water species, information on the stock is incomplete. Although several stock assessments have taken place, the year-to-year variations in stock size are not well documented. The natural mortality has not been determined, and no firm estimates have been put forth for the carrying capacity. Annual catch data does however exist and information on revenue for the Icelandic fleet as a whole is available but, there is no data available on costs that are solely due to harvesting of Pelagic redfish nor on effort. The Icelandic fleet is responsible for the biggest share of catches from the beginning of the fishery, or 47% of total recorded landings (see table A.4). The model developed takes these data shortcomings into account. First a simple model is derived and then expanded by allowing for uncertainty in mortality, carrying capacity, discount value, price, output elasticity of effort and costs. An analytic solution of the traditional uncertain optimal path will not be presented because of various reasons. First of all the aim of this essay is not to propose a specific path of harvest but rather to address the distribution of possible optimal paths. Secondly, to address one path would not necessarily be very practical because of the uncertainty. Hence, if one possible optimal path would be presented it might be far from the true optimal path.

The optimal path of harvests is defined as the path that yields the maximum attainable level of the present value of profits. The corresponding equilibrium, if it exists, is called the optimal equilibrium (OE). An attempt will be made to quantify a possible range of the differences between present values from optimal paths to the maximum sustainable yield (MSY) equilibrium and the OE equilibrium. Finally, NEAFC recommendations about TAC will be compared with the optimal paths of the model and an attempt made to examine whether similar bioeconomic models may be used as a gauge for harvesting policies in fisheries with a high degree of uncertainty, such as the Deep Pelagic redfish stock and many other deep-sea fisheries.

This paper is organised in the following manner. Section 2 describes the fishery and the available data. In the next sections the traditional model without uncertainty is introduced and its analytical solution. In section 4 the traditional model is extended by adding uncertainty and parametrisation of the model are discussed. Results are presented in section 5 and section 6 is the conclusion.

2 The Fishery

2.1 The Stock

Redfish is a name commonly applied to members of the deep-sea genus *Sebastes* which contains hundreds of species. Most of the species are found in the Northern Pacific Ocean, but four species reside in the North Atlantic, i.e. Golden redfish (*Sebastes marinus*), Beaked redfish (*Sebastes mentella*), Norway redfish (*Sebastes viviparus*) and Acadian redfish (*Sebastes fasciatus*).

Beaked redfish consists of various stocks. A demersal stock is found on the Icelandic continental shelf and slope. A Northeast Atlantic stock inhabits the Norwegian and Barents Seas. The third component is the Pelagic redfish stock (Jakobsdottir and Kristinsson, 2010). The International Council for the Exploration of the Sea (ICES) has defined Pelagic redfish in the Irminger Sea and adjacent waters, see figure 2.1, as two different biological stocks, a Deep Pelagic stock and a Shallow Pelagic stock (Sigurdsson et al., 2006). The two stocks will further be referred to as the Deep and Shallow stocks, respectively for the sake of simplicity and together as Pelagic redfish. The Shallow adult stock is found at a depth range above 500 meters, while the Deep adult stock is most abundant between 600 and 900 meters, but can be found at depths of up to 1000 meters (Jakobsdottir and Kristinsson, 2010). From April to May the stocks in Irminger Sea concentrate because of larval hatching and from June to July due to the feeding period. Concentrations may sometimes be found during the mating period, from August to September. During the rest of the year, no dense aggregations of the stock are found (Rikhter, 1996).

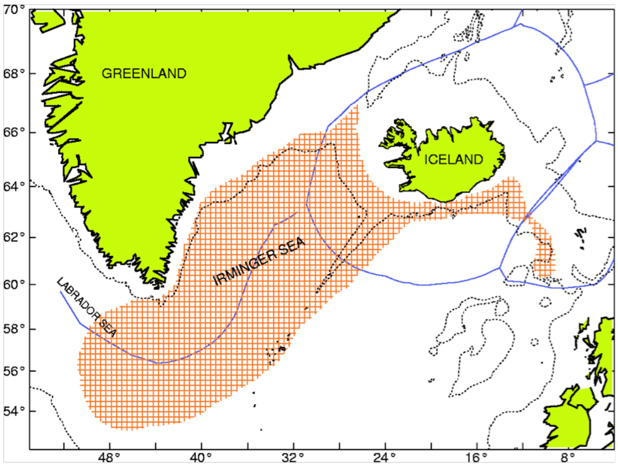


Figure 2.1: Geographical distribution of *Sebastes mentella* in Irminger Sea and adjacent waters. Source: (Sigurdsson et al., 2006)

Beaked redfish is a long lived, late-maturing species. It has been estimated to reach ages of up to 75 years, but the maximum age for redfish in the Irminger Sea is uncertain (Campana et al., 1990). The observed maximum length of individuals from the Deep Pelagic stock is 56 cm, while the average length for females is 35 cm and the average for males is 29-33.5 cm at 50% maturity. Age of recruitment to the fishery of the stock in unknown, but is believed to be near maturity. Recruitment fluctuates greatly, with long periods of very low recruitment. The causes for this variability are unknown. The natural mortality is unknown but thought to be 0.05-0.1 for both stocks (Jakobsdottir and Kristinsson, 2010).

Biomass assessments for the Deep stock in the Irminger Sea and adjacent waters, carried out by Icelandic and German groups, are shown in figure 2.2. Five comparable assessments have been made, in 1999, 2001, 2003, 2009 and 2011. In the first and last two assessments, in 1999, 2009 and 2011, the biomass was estimated to be close to 500 thousand tonnes. The biomass estimate in 2001 was more than double the two year earlier estimate. The biomass in 1999 was estimated to be 497 thousand tonnes while it was estimated to be 1057 thousand tonnes in 2001. Between 2001 and 2003 the biomass was estimated to have dropped again by almost 40%, or to 678 thousand tonnes.

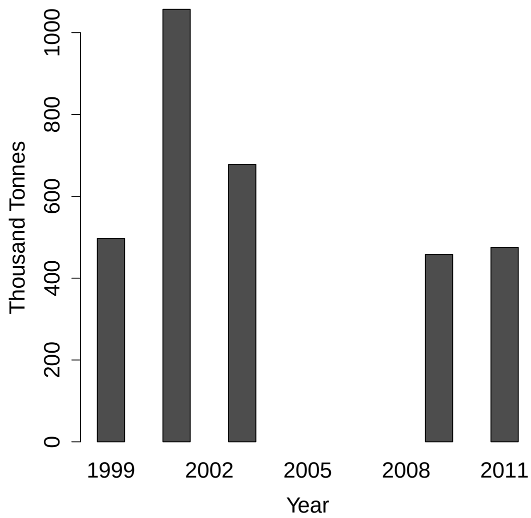


Figure 2.2: Biomass assessments for the Deep Pelagic redfish stock in the Irminger Sea and adjacent waters.

Source: Marine Research Institute of Iceland

2.2 The Fishing

The Pelagic redfish fishery in the Irminger Sea and adjacent waters takes place both inside the EEZs of Iceland and Greenland, as well as in the regulatory areas of the North East Atlantic Fisheries Commission (NEAFC). NEAFC is responsible for the management of the species, and sets the management policies for both stocks, with ICES having an advisory role. Catches from the Deep Stock are mainly from three ICES areas, Va, XII and XIV, see figure 2.3. In the early 1990's catches were mainly from areas XII and XIV but since 1997 almost all catches come from areas Va and XIV. Until 1999 almost all catches from the Shallow Stock were caught in ICES areas XII and XIV. However, since 2000 catches have been divided between ICES area XII and NAFO ares 1F and 2J (Jakobsdottir and Kristinsson, 2010). To minimise mixed-stock catches ICES recommended separate management units based on geographic proxies. The Deep Pelagic Management Unit is in the northeast Irminger Sea, while the Shallow Pelagic Management Unit is in NAFO areas 1 and 2, and ICES areas Vb, XII and XIV (ICES, 2009).

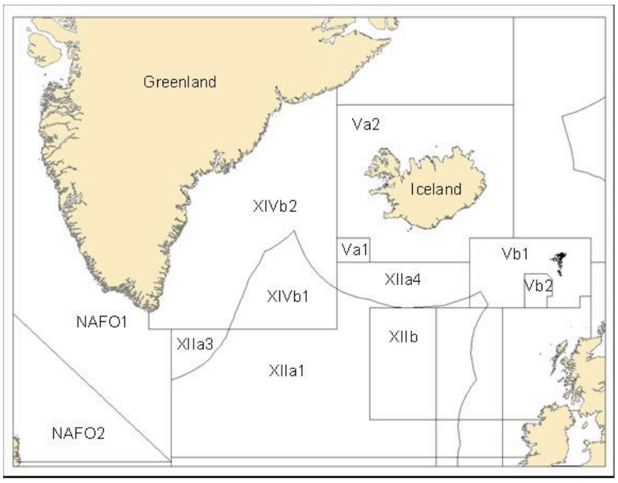


Figure 2.3: Partition of ICES and NAFO areas in Irminger Sea and adjacent waters.

Source: (Jakobsdottir and Kristinsson, 2010)

The management is based on setting a TAC for each stock, but technical measures (minimum mesh size) are also used. Additional measures may be introduced by individual nations. Icelandic vessels are, for instance, subject to the system of individual transferable quotas. Management in recent years has been severely hampered by the inability of nations taking part in the fishery to agree on total quota and allocation keys. Russia is not a party to an agreement made by the EU, Faroe Islands, Greenland, Iceland and Norway in 2009 which bans fishing from the Shallow Stock and sets a TAC for the Deep Stock of 38 thousand tonnes for 2011 and 32 thousand tonnes for 2012. Instead Russia has set a unilateral quota of 29.5 thousand tonnes for both stocks in 2011, stating that the stocks are not separated and that their status is not as depleted as otherwise believed (Jakobsdottir and Kristinsson, 2010).

Catches from the Deep Pelagic redfish stock in the Irminger Sea and adjacent waters are shown in figure 2.4. First reported catches were in 1989, but catches did not reach a thousand tonnes until 1992. Catches increased slowly at first, before jumping from 15 thousand tonnes in 1993 to almost 140 thousand tonnes in 1996. Catches then declined but remained between 84 and 104 thousand tonnes in the ensuing years. Since 2005, annual catches have been between 30 and 67 thousand tonnes. However, catches are thought to have been underestimated due to incomplete reporting.

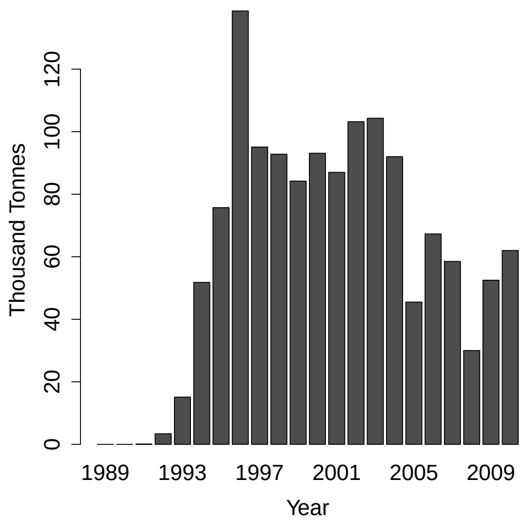


Figure 2.4: Reported catches from the Deep Pelagic redfish stock in the Irminger Sea and adjacent waters.

Source: Marine Research Institute of Iceland

Icelandic trawlers began exploiting the Deep Pelagic stocks in the early 1990s, with other nations not registering any substantial catches until 1995, see table A.4. The share of Icelandic catches has since steadily declined and is now down to 20-30% of total annual catches. The fishery is mainly carried out during the concentration of the stock from April to July (Jakobsdottir and Kristinsson, 2010).

2.3 The Sample

Data on the economic performance of the fishery is limited because no separate financial statistics exist for the redfish fleet and there is no data available on cost that is solely due to harvesting of

Pelagic redfish (IoES, 2010). However, a sample with operating accounts of 429 Icelandic units, fishing for various species.during the years 1992 to 2010 is available. Each unit consists of one or more ships a year. Thus if a vessel is participating in the fishery for more than one year it is more than one unit. Among these, 184 units were harvesting Pelagic redfish. The reduced 184 unit sample contains 52% of the Icelandic Pelagic redfish catches from 1992 to 2010. There is no information on the partition of catches from the Deep and the Shallow Stock. However, in 1992 to 2010, almost 86% of Icelandic Pelagic redfish catches are believed to have been from the Deep stock (see table A.3).

In 1992 to 2010 almost 92% of Icelandic Beaked redfish catches were frozen at sea (see table A.1) and over 98% of catches were harvested by vessels that had Pelagic Trawl (see table A.2)[1]. Pelagic trawls are towed through the water above the sea floor but not along or close to the sea floor as bottom trawls. While bottom trawling can cause destruction on the ocean bottom, pelagic trawling usually does not. Thus, concerning the sea floor, pelagic trawls can be assumed to produce less externalities than bottom trawls. Landings are not sold on local market for direct consumption and only fresh landings are sold on local markets for processing. Exported landings are mainly to markets in Germany, Japan and Russia (Jakobsdottir and Kristinsson, 2010).

Pelagic redfish is the most harvested species in the reduced sample, about 26.7% of total catch and it's value is 19.5% of the total catch value. The share of Pelagic redfish catches has though decreased. Between 1992 and 2008 Pelagic redfish catches varied from 20% to 40% but since then they have been lower, or 9% of total catches in 2008 to 2010.

A histogram of the proportion between each unit's total annual operating expenses ($C_{i,t}$) and catch value ($R_{i,t}$) in the years the biomass was estimated is shown in figure 2.5 and summarised in table A.7. The median value is 0.93 while the mean is 1.15. This is simply because for some units expenses are way higher than the value of catches. The 25th percentile is 0.88 and the 75th percentile is 1.06.

[1]If the data from Statistics Iceland and Marine Research Institute is compared it seems like what Statistic Iceland defines as Oceanic redfish is Shallow and Deep Pelagic redfish.

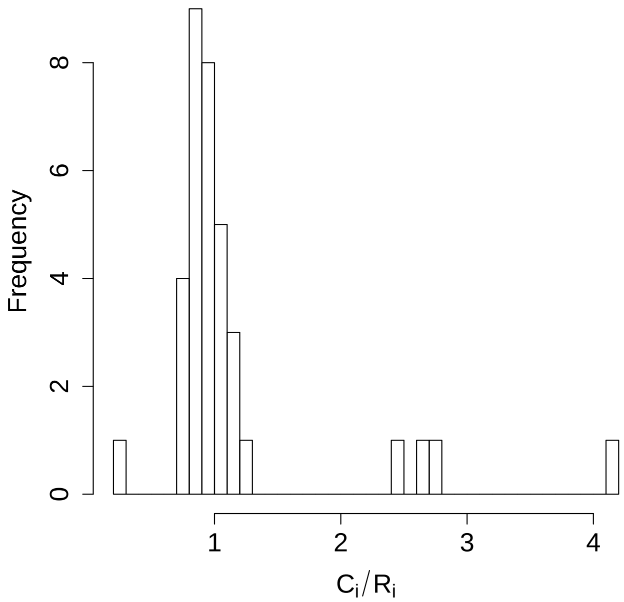
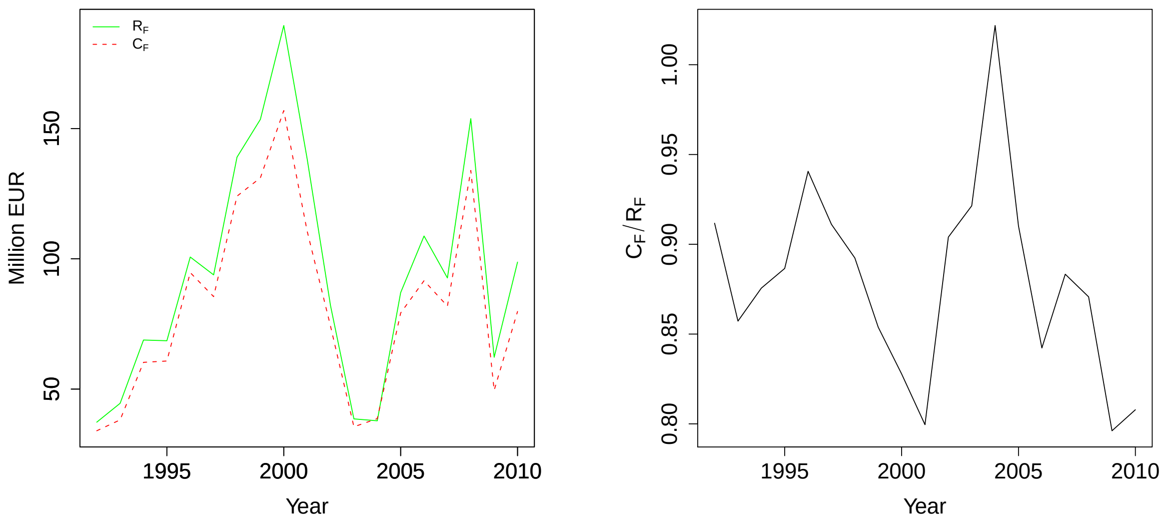


Figure 2.5: Histogram of $C_{i,t}/R_{i,t}$, where $C_{i,t}$ and $R_{i,t}$ are each unit's total annual operating expenses and catch values.

Source: Statistics Iceland

The annual proportion between the fishery's total operating expenses, $C_{F,t}$, and total catch values, $R_{F,t}$, is shown in figure 2.6a and $C_{F,t}/R_{F,t}$ is in figure 2.6b. The fishery's total operating expenses and it's total catch values is in the range between € 34 million and € 190 million. Catch values are higher than expenses in all years except 2004 and $C_{F,t}/R_{F,t}$ is in the range between 0.80 and 1.02.



(a) (b)

Figure 2.6: The fishery's total annual operating expenses, $C_{F,t}$, catch values, $R_{F,t}$, and $C_{F,t}/R_{F,t}$.

Source: Statistics Iceland

3 Traditional modelling

3.1 The Traditional Model

The model defined is an aggregate bioeconomic model[2]. Let X_t denote biomass at the time t. The biomass is the aggregate weight of the stock. The natural growth is defined as the change in the biomass that is not due to harvest. For each period the natural growth is thus determined by recruitment, weight gain or loss and mortality that is caused by other factors than fishing. Further, the natural growth of biomass is described by the Fox function[3](Fox, 1970) where growth is determined by biomass

$$G(x_t) \equiv a \cdot X_t - b \cdot X_t \cdot \log(X_t). \tag{3.1}$$

Jensen (2005) refined the Fox function as

$$G(X_t) = r_{\max} \cdot X_t\Big(\log(K) - \log(X_t)\Big) - Q_t \cdot X_t,$$

where $r_{\max}$ is the intrinsic growth rate, K is the carrying capacity and $Q_t \cdot X_t$ is meant to capture environmental fluctuations. In equation (3.1) environmental fluctuations are not considered and Jensen's version may thus be refined as

$$G(X_t) = r_{\max} \cdot X_t\Big(\log(K) - \log(X_t)\Big). \tag{3.2}$$

[2]The bioeconomic model simply integrates biological and economic systems. It is said to be aggregate because the fleet is treated as one individual.

[3]The following deraviation leading to equation (3.3) shows that if natural mortality (M) has been predetermined only one variable, the carrying capacity (K), needs to be parameterised. Other growth functions were considered, for instance the Logistic growth function. Parametrisation of other considered growth functions yielded values for K that were outside it's expected range. Therefore, the Fox function was chosen and K parameterised for predetermined values of M taken from it's expected range.

If natural factors change the carrying capacity can change and thus a function of time. However, K is assumed to be a constant and thus the maximum level of biomass, so

$$0 \leq X \leq K.$$

When the biomass is at its maximum level the natural growth is zero, thus, $G(K) = 0$. Comparing equations (3.1) and (3.2) gives the following equalities, $a = r_{\max} \cdot \log(K)$ and $b = r_{\max}$. Furthermore, as shown in appendix A.1, in (3.1) $r_{\max}$ is equal to fishing mortality at maximum sustainable harvest, F_{MSY}. Finally a proxy for F_{MSY} is natural mortality, M (Fox, 1970). Equation (3.1) may thus be refined as

$$G(X_t) = M \cdot X_t \cdot \Big(\log(K) - \log(X_t) \Big). \tag{3.3}$$

Harvest may be described by the Cobb-Douglas production function (Cobb and Douglas, 1928)

$$Y(E_t, X_t)_t \equiv q \cdot E_t^{\alpha} \cdot X_t^{\beta}, \tag{3.4}$$

where E is fishing effort. The parameter $q > 0$ is referred to as the catchability coefficient (Arnason et al., 2009) and $\alpha > 0$ and $\beta > 0$ are the output elasticities of effort and biomass respectively (Cobb and Douglas, 1928). The output elasticity of biomass, β, indicates the degree of schooling behaviour by the stock (Arnason et al., 2009).

There is only one value of Y for each value of X that keeps X unchanged between $t + 1$ and t. That is the value oft Y that is equal to the value of G. The biomass is said to be in a steady state if it is unchanged between periods. Thus, the maximum sustainable yield (MSY) is equal to the maximum level of natural growth.

Let $\dot{X}_t$ denote the change in biomass between t and $t + 1$. The change in biomass between period t and $t + 1$ is the difference between natural growth and harvest at time t

$$\dot{X}_t \equiv G_t - Y_t. \tag{3.5}$$

Price, p, is assumed to be an exogenous variable. This will hold if catches from the concerned stock do not affect the market price. For each period t to $t + 1$, the revenue of the fishery from catches is the multiple of price and total harvest

$$R(Y_t) \equiv p \cdot Y_t. \tag{3.6}$$

For simplicity, fixed costs is not included and the cost of harvesting is assumed to be a linear function of effort plus crew shares

$$C(E_t, Y_t) \equiv c_0 \cdot E_t + s \cdot p \cdot Y_t, \tag{3.7}$$

where $c_0 > 0$ is marginal variable cost excluding crew shares, and s is the proportion of revenues dedicated to the crew. Isolating the effort in the definition of harvest in equation (3.4) gives the following

$$E = q^{-1/\alpha} \cdot X^{-\beta/\alpha} \cdot Y^{1/\alpha}.$$

The cost (3.7) may thus be refined as

$$C(X_t, Y_t) = c \cdot X^{-\beta/\alpha} \cdot Y^{1/\alpha} + s \cdot p \cdot Y_t, \tag{3.8}$$

where $c = c_0 \cdot q^{-1/\alpha}$. Now, $c > 0$ since $c_0, q > 0$. The profit from fishing in the period between t

and $t + 1$, is the difference between revenue and cost, $\Pi_t \equiv R_t - C_t$, or

$$\Pi(X_t, Y_t) = (1 - s) \cdot p \cdot Y_t - c \cdot X^{-\beta/\alpha} \cdot Y^{1/\alpha}. \tag{3.9}$$

3.2 The Objective

The objective must be defined before making an attempt to identify the optimal path of harvest. The price of harvest is assumed to measure the marginal social benefits from consumption, Y. The social benefit from harvests in each period of time is expressed as

$$B(Y) = \int_0^Y p(y)dy. \tag{3.10}$$

This is consistent with Clark and Munro (1975). There are not assumed to be any externalities from harvesting, or at least they are negligible. The consumer cost from harvesting is therefor zero and the social cost equals the producers cost from harvesting, C(X,Y). The societies dynamic utility function from harvesting may thus be addressed as

$$J = \int_0^\infty \Big(B(Y) - C(X, Y)\Big) \cdot \exp(-r \cdot t)dt. \tag{3.11}$$

If demand for fish is perfectly elastic the social benefit from harvest can be refined as

$$\begin{aligned} B(Y) &= \int_0^Y pdy \\ &= p \cdot Y, \end{aligned}$$

and society's utility function becomes

$$
\begin{aligned}
J &= \int_0^\infty \Big(p \cdot Y - C(X, Y)\Big) \cdot \exp(-r \cdot t) dt \\
&= \int_0^\infty \Pi(X, Y) \cdot \exp(-r \cdot t) dt,
\end{aligned}
\tag{3.12}
$$

where $\Pi(X, Y)$ is the instantaneous profit from harvesting. This is consistent with the Gordon (1954), Scott (1955) and Clark and Munro (1975). The aim is to find optimal steady state solutions and optimal paths to that solutions. The equilibrium solution of an optimal control problem with the objective functional J in equation (3.12) is referred to as the optimal equilibrium (OE).

Arnason, Roy, and Schrank (2008) emphasise that fisheries and other natural resources can easily constitute base industries. The contribution of base industries to GDP play a greater role in overall economic activity than measured by their value added to GDP because of multiplier effects. As a result profitable fisheries are a stimulus for higher economic growth. So even though the demand for fish is not perfectly elastic, societies and producers interest may perfectly coincide (Arnason et al., 2008).

3.3 The Analytical Solution

The task is to find a path of harvest, Y, that leads to a steady state that maximises the present value of profits. Functions that describe the optimal dynamic solution can be found analytically with optimal control theory. The case when the harvest, Y, is the control variable and the biomass, X, is the state variable is analysed. The objective is to maximise the present value of profits, formally (Kamien and Schwartz, 1991)

$$
\max_{\{Y_t\}} J = \int_0^\infty \Pi(Y_t, X_t) \cdot \exp(-r \cdot t) dt, \tag{3.13a}
$$

with the constraints

$$\dot{X} = G - Y, , \tag{3.13b}$$

$$Y_t \in [0, \bar{Y}_t]. \tag{3.13c}$$

Here, $\bar{Y}_t$, is an upper constraint on the harvest, either X_t or maximum capacity of the fishery. The current value Hamiltonian (Kamien and Schwartz, 1991) of the problem is

$$\mathcal{H}(Y_t, X_t, t) = \lambda_0 \cdot \Pi(Y_t, X_t) + \lambda_t \cdot (G(X_t) - Y_t),$$

where λ_0 and λ_t are Lagrange multipliers. Further λ_t represents the shadow value of biomass at time t, $\lambda_t = \partial J / \partial X_t$.

The Pontryagin Maximum Principle (Pontryagin et al., 1962) gives conditions that a path of the control and state variable of an optimal control problem must satisfy to be optimal. Among these necessary conditions are (Kamien and Schwartz, 1991):

(i) λ_0 is a constant, 0 or 1 for all t, λ is a continuous function of time and λ_0 and λ are not both 0, $(\lambda_0, \lambda) \neq 0$.

(ii) Y maximises $\mathcal{H}$ for allt t.

(iii) $\dot{\lambda} - r \cdot \lambda = -\mathcal{H}_X$.

According to condition (ii),

$$Y_t = \begin{cases} 0, & \text{if } \mathcal{H}_Y < 0 \text{ for all } Y_t \in [0, \bar{Y}_t] \\ \bar{Y}_t, & \text{if } \mathcal{H}_Y > 0 \text{ for all } Y_t \in [0, \bar{Y}_t] \end{cases} \tag{3.14}$$

If neither of the conditions in (3.14) hold then Y_t is determined by $\mathcal{H}_Y = 0$. Now, if $\mathcal{H}_Y = 0$ for some $Y \in [0, \bar{Y}_t]$ then

$$\lambda_0 \cdot \Pi_Y = \lambda.$$

If $\lambda_0 = 0$ then it must hold that $\lambda = 0$. Thus, by condition (i) $\lambda_0 = 1$ and

$$\lambda = \Pi_Y. \tag{3.15}$$

By combining neccesary condition (iii) and equations (3.15) and (3.14) an optimal adjustment path is achieved

$$\begin{aligned}
&Y_t = 0, \text{ if } G_X + \tfrac{\Pi_X}{\Pi_Y} + \tfrac{\dot{\lambda}}{\lambda} < r \text{ for all } Y_t \in [0, \bar{Y}_t], \\
&Y_t = \bar{Y}_t, \text{ if } G_X + \tfrac{\Pi_X}{\Pi_Y} + \tfrac{\dot{\lambda}}{\lambda} > r \text{ for all } Y_t \in [0, \bar{Y}_t], \\
&G_X + \frac{\Pi_X}{\Pi_Y} + \frac{\dot{\lambda}}{\lambda} = r, \text{ else }.
\end{aligned}$$

The steady state when $\dot{\lambda} = 0$, is perfectly described by the following

$$\begin{aligned}
&Y = 0, \text{ if } G_X + \tfrac{\Pi_X}{\Pi_Y} < r \text{ all } Y \in [0, K] \text{ and } G = 0, \\
&Y = \bar{Y}, \text{ if } G_X + \tfrac{\Pi_X}{\Pi_Y} > r \text{ for all } Y \in [0, K] \text{ and } G = 0, \\
&Y = G \text{ and } G_X + \frac{\Pi_X}{\Pi_Y} = r, \text{ else.}
\end{aligned}$$

The last two equalities

$$G_X + \frac{\Pi_X}{\Pi_Y} = r, \tag{3.16a}$$

$$Y = G, \tag{3.16b}$$

can be analysed further. Equation (3.16b) simply says that harvest should be equal to the natural growth. Equation (3.16a) can be thought of as a modification of the golden-rule equilibrium equation. Clark and Munro (1980) called the left side of equation (3.16a) the resources own rate of interest. Equation (3.16a) thus states that at the optimum, the owners rate of return of the resource is equal to the rate of discount. The resources own rate of interest quantifies the owners rate of return of the resource at steady state. The first component, G_X, is the instantanous marginal product of the stock. The other component of the resources own rate of interest, Π_X/Π_Y is called the marginal stock effect (Clark and Munro, 1975). The marginal stock effect quantifies the marginal value of biomass relatively to the marginal value of harvest. The rate of discount is opportunity cost of managing the resource (Baumol, 1968; Arrow, 1999). The owner should thus harvest at a rate such that the rate of return from the resource is equal to the next best investment alternative. If equation (3.16a) is applied to the model defined in section 3.1 then

$$\Pi_X = c \cdot \beta \cdot \alpha^{-1} \cdot X^{-\beta/\alpha-1} \cdot Y^{1/\alpha}, \tag{3.17}$$

$$\Pi_Y = (1 - s) \cdot p - c \cdot \alpha^{-1} \cdot X^{-\beta/\alpha} \cdot Y^{1/\alpha-1}, \tag{3.18}$$

$$G_X = M \cdot (\log(K) - \log(X) - 1). \tag{3.19}$$

Existence and sufficiency are discussed in appendixes A.2 and A.3. Given the assumptions in section 3.1 and that $X > 0$ and $\alpha \leq 1$ so that the optimal path exists, along with if the steady state is uniquely determined by 3.16, an approximated optimal path from the initial state to the optimal equilibrium will be an approximation of the optimal path

4 Modelling with uncertainty

4.1 The Model Extendend

Many components of the model are uncertain. Thus, some parameters will be given distributions instead of specific values. These parameters are, $\tilde{M}$, $\tilde{K}$, $\tilde{\alpha}$, $\tilde{c}$, $\tilde{p}$ and $\tilde{r}$, where the tilde indicates that the parameters now follows a distribution instead of taking one particular value. The distributions of price, $\tilde{p}$, and discount rate, $\tilde{r}$, are assumed to be independent. However, the distribution of $\tilde{\alpha}$ is used in the estimation of $\tilde{c}$ and the distribution of $\tilde{M}$ is used when $\tilde{K}$ is generated. These distributions are thus not independently distributed. Therefore, for each i, the pares ($\tilde{\alpha}_i$, $\tilde{c}_i$) and ($\tilde{M}_i$, $\tilde{K}_i$) are stored. If the parameters that are given predetermined distributions are believed to lie in a specific range, but no value in that range believed to be the most probable, they will be given an uniform distribution

The path of harvest that maximises the expected present value from the fishery will not be found. If the goal is to find that path, the distributions of the variables discussed later in section 4.2 would need to be analysed properly since the distributions will probably have a major effect on the path that optimises the expected value. However, if the parameters are thought to be in the range of the distributions discussed in section 4.2, even though the shape of the distribution is not sufficiently precise, given the other model assumptions, the true optimal path may lie within the range of the 1000 paths.

There is large uncertainty concerning the biology of the system. Neither the carrying capacity, K, or the mortality, M, of the specific stock has been estimated. The growth function is made uncertain by adding distributions to K and M. Equation (3.3) is thus refined as

$$\tilde{G}(X_t) = \tilde{M} \cdot X_t \cdot \Big(\log(\tilde{K}) - \log(X_t) \Big). \tag{4.1}$$

The statistical properties of $\tilde{M}$ and $\tilde{K}$ will be discussed in section 4.2.4. By adding predetermined uniform distribution to $\tilde{M}$ the distribution of $\tilde{K}$ will be generated with numerical minimisation.

Since data on effort is lacking, biomass estimates are scarce and harvests are misreported, it is difficult to estimate the production function. The production function (3.4) becomes uncertain through the output elasticity, $\tilde{\alpha}$

$$\tilde{Y}(E_t, X_t) = q \cdot E_t^{\tilde{\alpha}} \cdot X_t^{\beta}. \tag{4.2}$$

The uncertain output elasticity $\tilde{\alpha}$ is assumed to follow uniform distribution, further discussed and justified in section 4.2.3. By combining equations (4.1) and (4.2), the development of the biomass defined by equation (3.5) may be refined as

$$\dot{\tilde{X}}_t = \tilde{G}_t - \tilde{Y}_t. \tag{4.3}$$

The same limitations plus deficiencies on cost data make estimations of the cost equation (3.8) difficult to define. This is taken into account by adding distributions to the parameter, c, in the cost equation (3.8). The distribution of $\tilde{c}$ will then be generated by estimations in section 4.2.3 and price becomes uncertain, $\tilde{p}$, by giving it normal distribution in section 4.2.1 The profit in equation (3.9) may then be refined as

$$\tilde{\Pi}(X_t, \tilde{Y}_t) = (1 - s) \cdot \tilde{p} \cdot \tilde{Y}_t - \tilde{c} \cdot X^{-\beta/\tilde{\alpha}} \cdot \tilde{Y}^{1/\tilde{\alpha}}. \tag{4.4}$$

Finally, society's discount value, r, is not clear and may change. The discount value both affects the optimal equilibrium and the optimal path to the equilibrium. The distribution of the uncertain discount rate $\tilde{r}$ will be formed with the justification of a number of references in section 4.2.2. The formal problem (3.13) can now be restated. As mentioned above, the expected present value will not be optimised. However, for each set of parameters $(\tilde{K}_i, \tilde{M}_i, \tilde{\alpha}_i, \tilde{c}_i, \tilde{p}_i, \tilde{r}_i)$ the present value of the fishery is maximised

$$\max_{\{Y_t\}} J = \int_0^{\infty} \tilde{\Pi}_i(X_t, \tilde{Y}_t) \cdot \exp(-\tilde{r}_i \cdot t)dt, \tag{4.5a}$$

where $\tilde{\Pi}_i(X_t, \tilde{Y}_t) = (1 - s) \cdot \tilde{p}_i \cdot \tilde{Y}_{i,t} - \tilde{c}_i \cdot X^{-\beta/\tilde{\alpha}_i} \cdot \tilde{Y}_{i,t}^{1/\tilde{\alpha}_i}$, with the constraints

$$\dot{\tilde{X}} = \tilde{G}_i - \tilde{Y}_i, \tag{4.5b}$$

$$\tilde{Y}_t \in [0, \bar{Y}_t], \tag{4.5c}$$

$$\tilde{X}_0 = \bar{X}. \tag{4.5d}$$

where $\bar{X}$ is some fixed, current value of biomass.

4.2 Parametrisation

4.2.1 Price

Figure 4.1 shows Icelandic vessels average price of Pelagic redfish catches in 1992 to 2010 and the demersal catch index in 1997 to 2010.

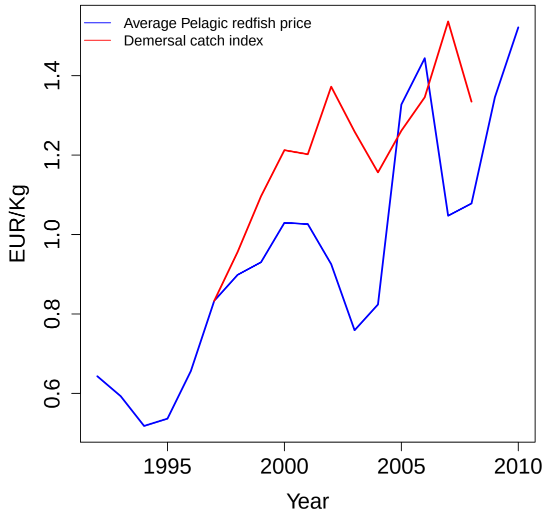


Figure 4.1: Average price of Pelagic redfish catches and demersal catch index.

Source: Statistics Iceland

There seems to been an upward trend in both prices of Pelagic redfish as well as in overall demersal catches. Since 1992 price has almost tripled. In 1992 price was slightly over €0.6 per kg but in 2010 the price was approximately €1.5 per kg. What matters here is how price is relative to cost. To estimate a trend for cost and price is outside the scope of this text. Price in the model will be normally distributed with a mean of €1.50 per kg and a standard deviation of €0.35 per kg, formally

$$\tilde{p} \sim N(\text{€}1.50 \text{ per kg}, \text{€}0.35 \text{ per kg}). \tag{4.6}$$

4.2.2 Discount Rate

When choosing the optimal path of harvest one has to compare the benefits and costs of actions that takes place at different times. This comparison may be implemented by the use of a discount rate. Baumol (1965) argued that the use of an incorrect estimate of the discount value might lead to very serious misallocation of resources. Further, Koopmans (1965) described how he found the problem of optimal growth so complex that one should not make a priori choice of discount rate before knowing the effects of alternative choices.

In the economic literature the appropriate discount rate for public projects is argued to be the one which measures the social opportunity cost (Baumol, 1968; Arrow, 1999). However, to measure the social opportunity cost is neither an easy nor undisputed task. Baumol (1968) argued that what should be considered, is the welfare loss from not having these instantaneous benefits in the form of consumption or reinvestment and a premium due to risk. According to Arrow (1999) the discount rate quantifies societies concerns about intergenerational equity, expectations about growth of income.

The Ramsey equation, derived in appendix A.4, describes the rate of return on capital in welfare optimum, formally

$$r = \delta + \eta \cdot g, \tag{4.7}$$

where r is the equilibrium real return on capital, δ represents pure time preference, η quantifies elasticity of the marginal utility of consumption and g is the growth rate of consumption per generation (Norhaus, 2007; Stern, 2007; Weitzman, 2007). All the variables in equation (4.7) may vary between, time, individuals and nations. The pure time preference (δ) and the marginal elasticity (η) describe controversial preferences while g may be easier to measure with empirical observations (Weitzman, 2007). Further, η measures at what rate the marginal utility from consumption declines with increasing consumption (Arrow et al., 1996) coupled with risk aversion (Weitzman, 2007). Thus, the more important equity between generations, the higher the value of η will be (Arrow et al., 1996)

Ramsey was the first economist to address the idea of the pure time preference in his classic paper *A Mathematical Theory of Saving* published in 1928. In his paper, Ramsey argues that in a social context, by moral reasons, the pure time preference should be zero. This was the general view in texts published the following years and decades.(Pigou, 1946; Harrod, 1948; Solow, 1974). Further research on the topic led to arguments for positive values. Both Koopmans (1965) and Mirrlees (1967), claimed that no time preference would lead to very low levels of consumption. With the same emphasis, Arrow (1999) has explained that by weighting all generations welfare equally would imply very high savings on the present generation. He states that the pure rate of time preference, with prejudice, should be 1%, which is the same value Stern (2007) used. However, Stern's argument for using positive value is because of the probability of extinction. Other suggestions have been in the range 0% to 2% (Weitzman, 2007). The distribution of δ in the following simulations will be uniformly distributed between 0% and 2%, or

$$\delta \sim U(0\%, 2\%). \tag{4.8}$$

Attempts to estimate η indicate a range of values. In his controversial paper about climate change, Stern (2007) used the value $\eta = 1$. Arrow et al. (1996) point out that there seems to be an harmony about η being in the range of 1 and 2. This is consistent with Fellner (1967) and Scott (1989) who estimated η to be about 1.5. Wietzman (2007) suggest using higher values, in the range of 2 to 3 while others suggest using even lower values. Estimates for the United Kingdom indicated that it should be in the range between 0.8 and 0.9 (Pearce and Ulph, 1995). This is not far from Brent's (1994) suggestion of using η between 0 and 1 with 0.5 as a benchmark. With this in mind a uniform distribution is chosen for η in the range of 0.5 to 2.5, that is

$$\eta \sim U(0.5, 2.5). \tag{4.9}$$

The growth rate of consumption is generally based on empirical observations of actual consumption growth. Many studies have suggested a value of 2% per year (Arrow, 1999; Baumol, 1968; Weitzman, 2007; Prescott, 2002). Stern (2007) was more pessimistic than his colleagues using $g = 1.3\%$. Here, g will be normally distributed with a mean of $g = 2\%$ per year and a standard deviation of $\sigma_g = 0.25\%$ per year, formally

$$g \sim N(2\% \text{ per year}, 0.25\% \text{ per year}). \tag{4.10}$$

The distribution of the discount rate is generated by taking 1000 combinations ($\tilde{\delta}_i$, $\tilde{\eta}_i$, $\tilde{g}_i$) from the distributions described in equations (4.8), (4.9) and (4.10) and from each combination forming the relevant discount rate described in equation (4.7). The histogram of $\tilde{r}$ is shown in figure 4.2 and summarised in table 4.1. The median value of $\tilde{r}$ is 3.93% while its mean is 4%. Further, 25% of $\tilde{r}$ are lower than 2.96% and other 25% are greater than 4.99%.

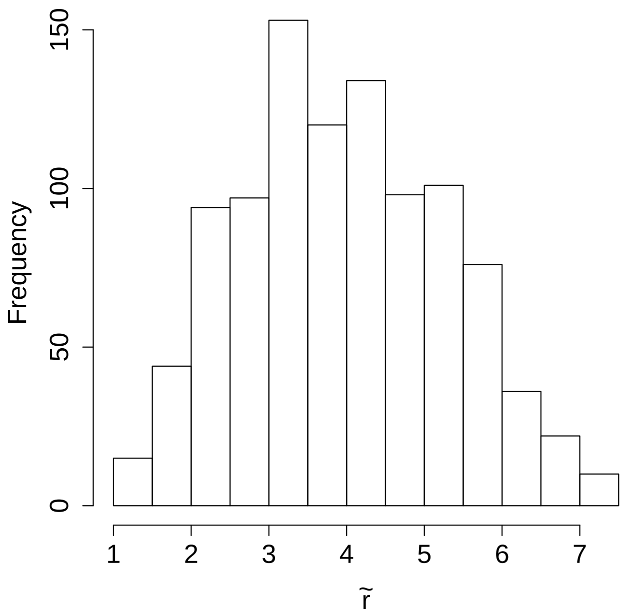


Figure 4.2: Histogram of $\tilde{r}$.

Table 4.1: Summary of the uncertain appropriate discount rate, $\tilde{r}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1.15	3	3.88	3.96	4.96	7.45

4.2.3 The Cost Function

The cost function depends on both the output elasticity of effort (α) and output elasticity of biomass (β). Empirical studies indicate that the value of output elasticity of biomass is close to 0.5 in for miscellaneous pelagic fishes (Arnason et al., 2009). The value of β is thus fixed as 0.5, implying declining returns to stock size. Studies further indicate that the output elasticity of effort is often close to unity. Hence, $\tilde{\alpha}$ will be uniformly distributed between 0.8 and 1.0

$$\tilde{\alpha} \sim U(0.8, 1.0). \tag{4.11}$$

The median value of shares as proportion of total catch value in the sample is 37% while the mean value is 40% (see table 4.2). The proportion of shares is fixed as, $s = 0.4$.

Table 4.2: Summary of crew share, s.

Source: Statistics Iceland

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0	0.34	0.37	0.4	0.4	2

Let the proportion between each unit's annual pelagic redfish catch, $Y_{i,t}$, and total catch, $Y_{i,t}^T$, be denoted as

$$\kappa \equiv \frac{Y_{i,t}}{Y_{i,t}^{T}} \tag{4.12}$$

In an attempt to quantify the solely cost of pelagic redfish harvesting the following equation is estimated

$$\frac{C_{i,t}}{R_{i,t}} = \frac{C_{F,t}}{R_{F,t}} \cdot \exp(\theta_t \cdot \kappa). \tag{4.13}$$

Here $C_{i,t}$ and $R_{i,t}$ represent each unit's total annual cost and revenue, while $C_{F,t}$ and $R_{F,t}$ are the fisherie's total total annual cost and revenue. Note that if $\kappa = 1$, then

$$C_t = R_t \cdot \frac{C_{F,t}}{R_{F,t}} \cdot \exp(\theta_t),$$

where C_t and R_t are the cost and revenues from harvesting pelagic redfish. The least squares estimation of (4.13) is summarised in table 4.3. All p-values are relatively high indicating that the estimates are not statistically significant.

Table 4.3: Summary of $\hat{\theta}_t$.

	Year	$\hat{\theta}_t$	$\hat{\sigma}_{\theta_t}$	p-value
1	1999	0.10	0.07	0.21
2	2001	−0.02	0.11	0.86
3	2003	−0.02	0.10	0.81
4	2009	−0.01	0.04	0.89

Now, for each $j = 1, \ldots, 1000$, a pseudo value for cost of pelagic redfish harvesting is generated by

$$\tilde{C}_{j,t} = R_t \cdot \frac{C_{F,t}}{R_{F,t}} \cdot \exp(\tilde{\theta}_{j,t} \cdot \kappa),$$
$$\tilde{\theta}_t \sim N(\hat{\theta}_t, \hat{\sigma}_{\theta_t}),$$

For each value of $\tilde{C}_{j,t}$ a value for $\tilde{c}_j$ is generated by the ordinary least squares (OLS) estimation of

$$\tilde{C}_j - s \cdot R_t = \tilde{c}_j \cdot \hat{X}^{-\beta/\alpha_j} \cdot Y^{1/\alpha_j}, \tag{4.14}$$

The distribution of $\tilde{c}$ is show in figure 4.3. The median value of $\tilde{c}$ is 11.33 and it's first and third quartiles are 10.58 and 12.11.

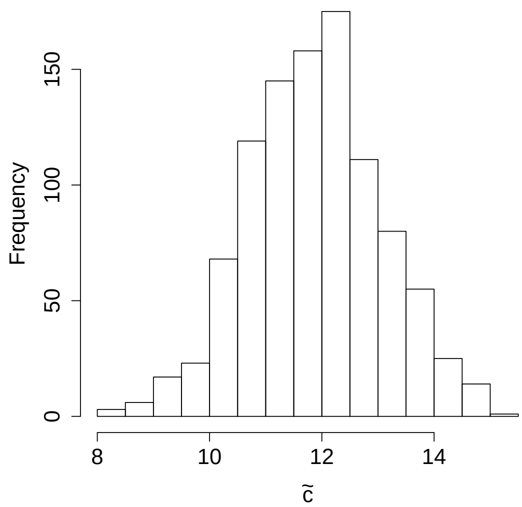
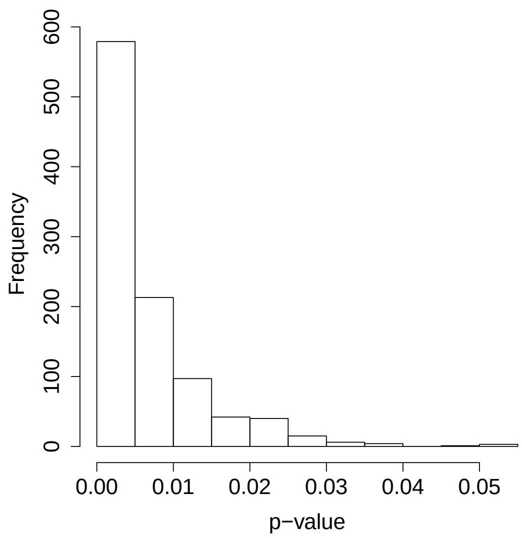


Figure 4.3: Histogram of $\tilde{c}$, the uncertain parameter in the cost function.

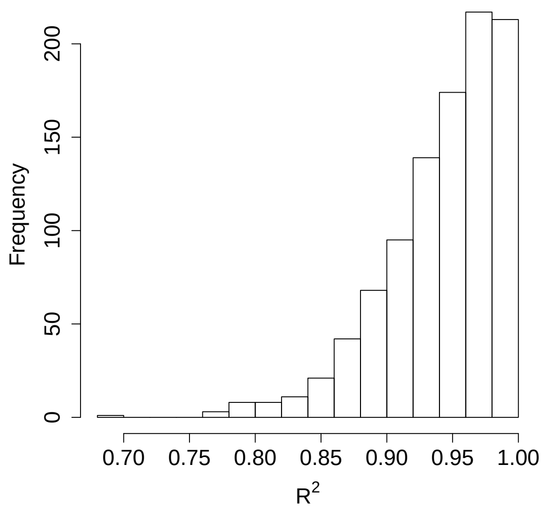
Table 4.4: Summary of $\tilde{c}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
8.18	10.58	11.33	11.37	12.11	14.91

The distribution of the p-values from the OLS estimation of (4.14) is shown in figure 4.4a and listed in table A.13. The maximum value of the p-values is 0.052. Thus, all estimates of $\tilde{c}$ are statistically significant at the a significance level of 0.052. The corresponding distribution of the coefficient of determination, R^2, is shown in figure 4.4b and listed in table A.14. The first quartile is 0.93 while the third is 0.98. However, the values regressed, $\tilde{C}$ were generated. Thus, low p-values and high values of R^2 do not necessarily say anything about the real harvesting cost of the Deep Stock as a function of biomass and harvest.



(a)



(b)

Figure 4.4: Histogram of p-values and R^2 from the estimation of $\tilde{c}$.

4.2.4 The Growth Function

The growth function, (4.1) consists of one variable and two parameters. The variable is the relevant stock size and the parameters are the carrying capacity ($\tilde{K}$) and the natural mortality ($\tilde{M}$). The natural mortality is unknown but probably between 0.05 and 0.1(Jakobsdottir and Kristinsson, 2010). Thus, $\tilde{M}$ will be uniformly distributed in the interval [0.05,0.10], formally

$$\tilde{M} \sim U(0.05, 0.10). \tag{4.15}$$

To determine the distribution of $\tilde{K}$, a thousand simulations of the fishery are carried out, with the corresponding values of $\tilde{M}$ determined by (4.15). For each case, i, initial biomass is $\tilde{K}_i$. In each simulation the value of $\tilde{K}_i$ is determined by minimising the sum of squares between the simulated biomass X_t and the estimated values of biomass, $\hat{X}_t$, written as

$$\min_{\tilde{K}_i} \sum_t (\hat{X}_t - X_t) \text{ for all i and relevant t.} \tag{4.16}$$

Simulations are carried out for three different cases. In case 1 all the biomass assessments are used. In case 2 the biomass estimate in 1999 is assumed to be underestimated and consequently left out, whereas in case 3 the biomass estimate in 2001 is assumed to be overestimated. The distribution of $\tilde{K}$ is then determined by taking 334 values from case 1 and 333 from case 2 and 3. A histogram of $\tilde{K}$ is shown in figure 4.5 and summarised in table 4.5.

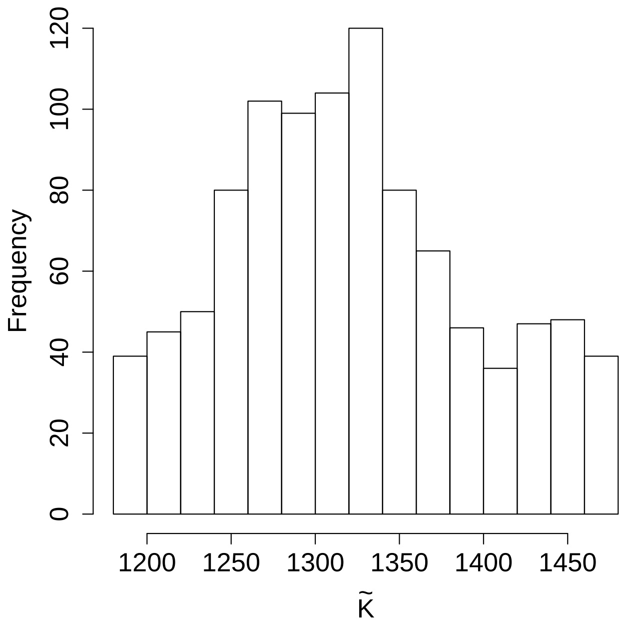


Figure 4.5: Histogram of $\tilde{K}$, the uncertain carrying capacity.

Table 4.5: Summary of the uncertain carrying capacity, $\tilde{K}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1180	1267	1317	1321	1368	1480

If the smallest and largest 25% of $\tilde{K}$ are excluded then the rest is distributed in the range between 1267 and 1368 thousand tonnes. Further, the median value of $\tilde{K}$ is 1317 thousand tonnes. By the definition of the model (see equation (3.5)), each year the carrying capacity should be equal to the sum of the current biomass and the natural growth since harvesting began minus the sum of harvest from the beginning of the fishery. If the initial year is denoted as $t = 0$ and the current year as $t = \kappa$ then

$$K = x_\kappa + \sum_{t=0}^{\kappa-1} (y_t - G_t).$$

Thus, given that $G_t > 0$ for all t then

$$K < x_\kappa + \sum_{t=0}^{\kappa-1} y_t. \tag{4.17}$$

Hence, the carrying capacity should be less than the sum of the current level of biomass and catches from the beginning of the fishery. The sum on the right side of $<$ in equation (4.17) is for all years the biomass was estimated greater than the values in the distribution of $\tilde{K}$, except for 1999. The specific sums are 969.6, 1706.9, 1518.1, 1695.7 and 1827.2 thousand tonnes respectively for the years estimated.

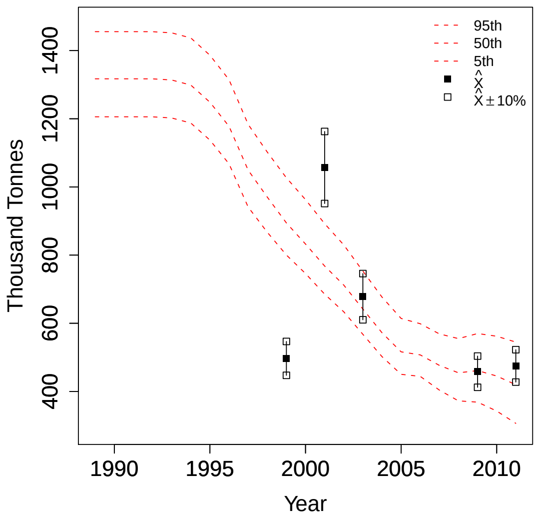


Figure 4.6: Simulation of biomass with the parametrised natural mortality, $\tilde{M}$, and carrying capacity, $\tilde{K}$.

Simulations of the biomass with the corresponding distributions of $\tilde{M}$ and $\tilde{K}$ are shown in figure 4.6. The red dotted lines represent the 5th, 50th and 95th percentiles of biomass value. The interval between the 5th and the 95th percentile is referred to as the 90% interval. The black squares are the point estimates of biomass and the white squares represent the 20% interval (± 10%) around the point estimate. The point estimates of the biomass estimates in 2003, 2009 and 2011 are included in the 90% interval of the simulations. However, neither the point estimates or their 20% intervals are included in the 90% interval of the simulations in 1999 and 2001.

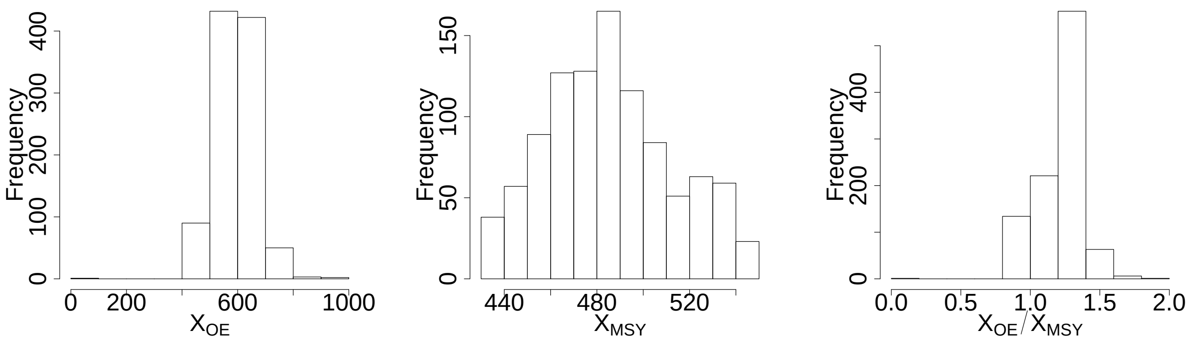
5 Harvesting Policy

5.1 Equilibrium Solutions

From the distributions of the six parameters ($\tilde{p}$, $\tilde{r}$, $\tilde{\alpha}$, $\tilde{c}$, $\tilde{M}$ and $\tilde{K}$) that were parametrised in section 4.2, 1000 random combinations are picked and the MSY and OE steady state solutions for each combination calculated. The interval between the 5th and 95th percentile is called the 50% range while the one between the 25th and 75th percentile is called the 90% range.

Histograms of the optimal biomass equilibrium, X_{OE}, and the biomass equilibrium corresponding to MSY, X_{MSY}, are shown in figures 5.1a and 5.1b, and a histogram of X_{OE}/X_{MSY} is in figure 5.1c. The optimal biomass equilibrium, X_{OE}, ranges from being almost the same size to 1.43 times as large as X_{MSY} (see table 5.1). Therefore, a harvesting policy leading to the OE equilibrium would be more effective than one leading to the MSY equilibrium, in avoiding risk of collapse of the stock or prolonged depression.

The 90% and 50% ranges for X_{OE} are [480, 702] thousand tonnes and [562, 636] thousand tonnes respectively, while X_{MSY}'s 90% and 50% ranges are [444, 535] thousand tonnes and [466, 503] thousand tonnes. The biomass estimate in 2011, 475 thousand tonnes, is inside X_{MSY}'s 50% range and X_{OE}'s 90% range, but the median value of X_{MSY} is 485 thousand tonnes, making it 10 thousand tonnes larger than the 2011 estimate.



(a) (b) (c)

Figure 5.1: Histograms of the optimal equilibrium of biomass (X_{OE}), biomass equilibrium corresponding to MSY (X_{MSY}) and their proportion (X_{OE}/X_{MSY}).

Table 5.1: Equilibrium solutions for biomass in thousand tonnes.

X	OE	MSY	OE/MSY
5%	480	444	1.00
25%	562	466	1.17
50%	598	485	1.23
75%	636	503	1.30
95%	702	535	1.43

Considering harvesting policy it is useful to realise how the distributions of the corresponding equilibrium values of harvest, Y_{OE} and Y_{MSY}, look like. Their histograms are shown in figures 5.2a and 5.2b, as well as a histogram of Y_{OE}/MSY in figure 5.2c. The optimal equilibrium harvest, Y_{OE}, ranges from being almost the same size to about 92% of MSY. The 90% range for Y_{OE} is [26, 44] while MSY's 90% range is [27, 45] thousand tonnes. However, the 50% range for Y_{OE} spans only 9 thousand tonnes, [30, 39] thousand tonnes, while MSY's range is a thousand tonnes bigger, or [31, 41] thousand tonnes (see table 5.2). Reported catches in 2010 were 62 thousand tonnes, more 1.7 times bigger than the median values of Y_{OE}, 36 thousand tonnes, and MSY, 34 thousand tonnes.

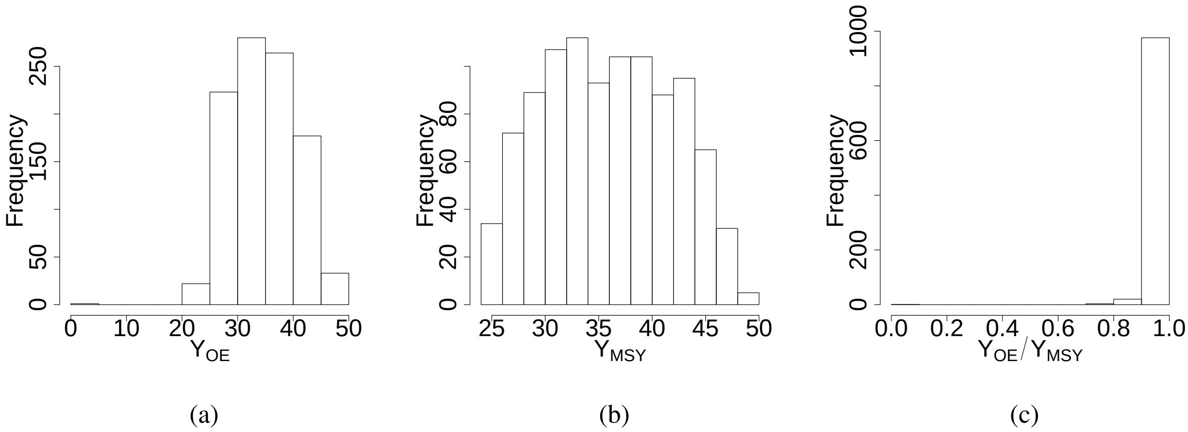


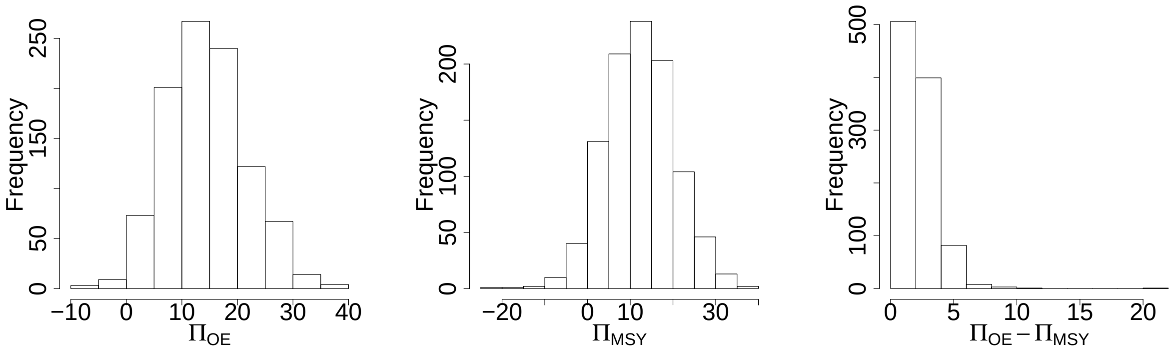
Figure 5.2: Histograms of the optimal equilibrium of harvest (Y_{OE}), maximum sustainable yield,MSY, and and their proportion (Y_{OE}/MSY).

Table 5.2: Equilibrium solutions for harvest in thousand tonnes.

Y	OE	MSY	OE/MSY
5%	26	27	0.92
25%	30	31	0.96
50%	34	36	0.97
75%	39	41	0.99
95%	44	45	1.00

The instantaneous profit, Π, in the optimal equilibrium, Π_{OE}, is in all instances higher than instantaneous profits in the MSY equilibrium, Π_{MSY} even though it is in some cases almost the same. In addition, the median value of the difference between Π_{OE} and Π_{MSY} is €2.0 million (see table 5.3). Part of the MSY equilibrium yields negative profits, with 5% yielding €-0.2 million or less. However, the 50% range for Π_{MSY} is [6.8, 17.4] €million, while Π_{OE}'s 50%

range is [9.2, 19.0] € million.



(a) (b) (c)

Figure 5.3: Histograms of the optimal equilibrium of instantaneous profits (Π_{OE}), equilibrium of instantaneous profits corresponding to MSY (Π_{MSY}) and their difference ($\Pi_{OE} - \Pi_{MSY}$).

Table 5.3: Equilibrium solutions for instantaneous profits in EURmillion.

Π	OE	MSY	$OE - MSY$
5%	3.6	−0.2	0.00
25%	9.2	6.8	1.20
50%	13.9	12.1	1.97
75%	19.0	17.4	2.85
95%	27.0	25.5	4.58

5.2 Optimal Paths

The main concern is not the instantaneous profit, Π, in equilibrium but the present value, PV, that the path to relevant equilibrium yields. For the sake of simplicity the optimal path to the OE equilibrium is called the optimal path while the optimal path to the MSY equilibrium is called the MSY path. The objective is to find the optimal path of harvest for each set of parameters and interpret the corresponding distributions. For each case i, the value of harvest is defined by the feedback-rule

$$Y = \tilde{\omega}_{1,i} \cdot X + \tilde{\omega}_{2,i} \cdot X^2, \tag{5.1}$$

and the goal is to optimise numerically

$$\max_{\tilde{\omega}_{1,i},\tilde{\omega}_{2,i}} J = \int_0^T \Pi(X_t; \tilde{\omega}_{i,1}, \tilde{\omega}_{i,2}) \cdot \exp(-r_i \cdot t)dt, \tag{5.2a}$$

with the constraints

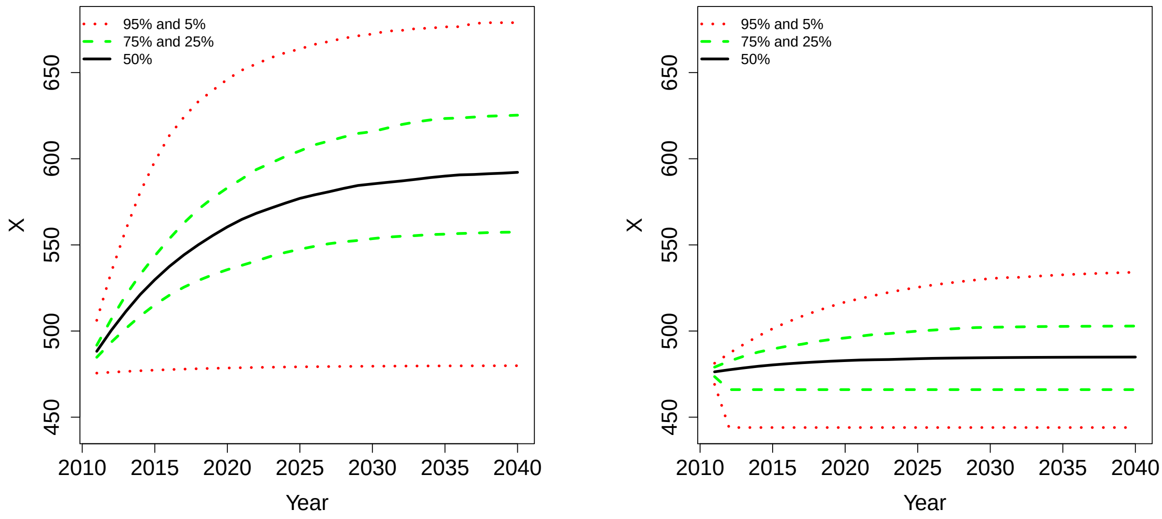
$$\dot{\tilde{X}} = \tilde{G} - Y, \tag{5.2b}$$

$$Y_t \in [0, \bar{Y}_t], \tag{5.2c}$$

$$Y^* = \tilde{\omega}_{1,i} \cdot X^* + \tilde{\omega}_{2,i} \cdot X^{*2}, \tag{5.2d}$$

where Y^* and X^* represent the relevant equilibrium values, OE or MSY, of harvest and biomass. Since the objective function is optimised numerically, ∞ is replaced with some terminal time T. By choosing sufficiently large T this should not affect the outcome. The cases where $\Pi_{MSY} < 0$ are excluded for two reasons. First, it is odd to maximise the present value of the fishery to a steady state that yields negative instantaneous profit. Second, some might argue that it is highly unlikely that the MSY equilibrium yields negative instantaneous profit. Since the newest numbers on harvest in the dataset are from 2010, the initial year is chosen to be 2011.

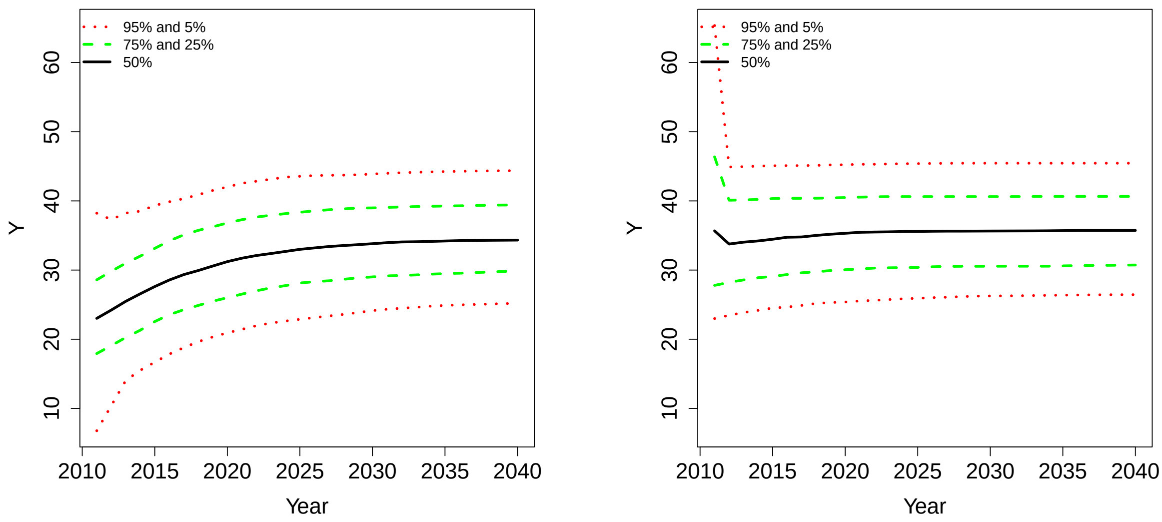
The optimal time paths of biomass to the OE and MSY equilibrium are shown in figure 5.4. The red dotted lines represent the 5th and 95th percentiles, while the green broken lines represent the 25th and 75 percentiles and the black lines the median values. Since the current value of biomass is most likely closer to the MSY value than the OE value, the MSY paths are flatter. In some cases the current value of biomass is even greater than the MSY value and the decreases (see figure 5.4b). The optimal path of biomass to the OE equilibrium is in more than 95% of the cases increasing, but the steepness depends on the corresponding OE equilibrium values.



(a) (b)

Figure 5.4: Optimal paths of biomass to the optimal equilibrim (OE) and the equilibrium corresponding to the maximum sustainable yield (MSY).

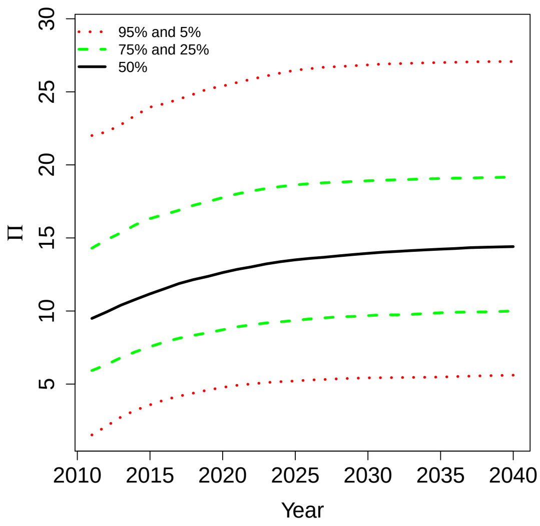
The paths of harvest, the control variable, to the two equilibrium are shown in figure 5.5. The optimal paths are shown in figure 5.5a. In all instances the optimal path to the OE equilibrium is increasing. The optimal paths of harvest to the MSY, are as in the case of biomass, flatter than the optimal paths. Leaving some paths increasing while others are decreasing (see figure 5.5b).



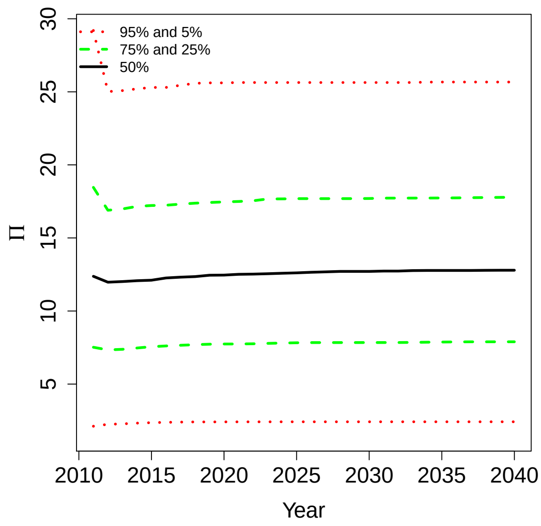
(a) (b)

Figure 5.5: Optimal paths of harvest to the optimal equilibrim (OE) and maximum sustainable yield (MSY).

As shown in figure 5.6a the optimal path of harvest yields relatively low instantaneous profits to begin with. The time paths of instantaneous profits, Π, to the MSY equilibrium are relatively flatter than those to the OE equilibrium since the current value of biomass is most likely closer to the MSY equilibrium value (see figure 5.6b). In some cases the path of Π to MSY is even decreasing. Equilibrium in the fishery is defined to be present when $|\Pi^* - \Pi_t| <$ €0.05 million, where Π^* is the relevant equilibrium value of instantaneous profit. In all cases it takes longer time (T^*) to reach the OE equilibrium with median value of the difference of 19 years (see table A.15). This is simply because the fishery is believed to be further away from the OE equilibrium than the MSY equilibrium. In 50% of the cases it takes longer than 32 years to reach the OE equilibrium while only 14 years to reach the MSY equilibrium.



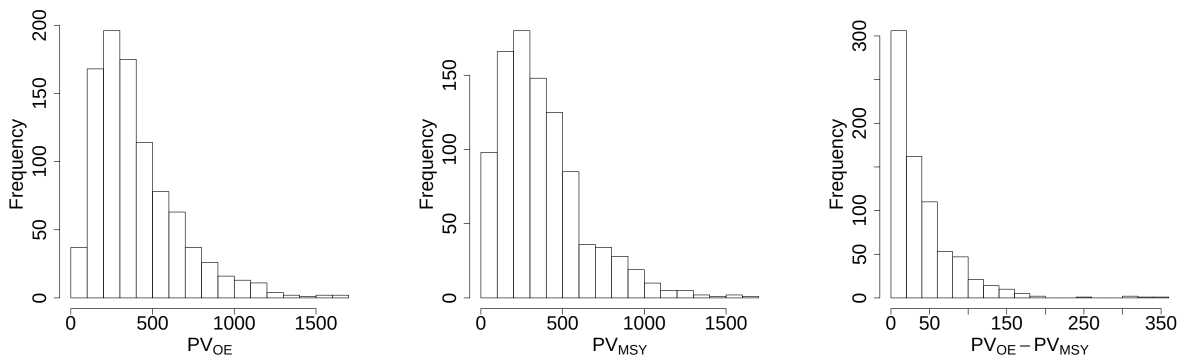
(a)



(b)

Figure 5.6: Optimal paths of instantaneous profit to the optimal equilibrim (OE) and the equilibrium corresponding to the maximum sustainable yield (MSY).

Histograms of the present values from the paths to the two equilibriums and their difference is shown in figure 5.7. The present value from the optimal paths, PV_{OE}, are in all cases higher than from the MSY paths, PV_{MSY}, with a median value difference of €26.4 million. The 90% range for the difference is [0.0, 116.5] € million and the 50% range is [5.4, 54.8] € million. Therefore, in 75% of the cases the optimal path yields a present value of profits that is at least €5.4 million greater than the present value of profits from the MSY path. Further, the 50% range for PV_{OE} and PV_{MSY} are [218.3, 515.8] € million and [183.1, 489.0] € million respectively.



(a) (b) (c)

Figure 5.7: Histograms of present value of profits yielding from the optimal paths,PV_{OE}, present value of profits yielding from the optimal paths to the MSY equilibrium, PV_{MSY} and their difference $PV_{OE} - PV_{MSY}$.

Table 5.4: Solutions for PV

PV	OE	MSY	$OE - MSY$
5%	110.9	60.2	0.0
25%	218.3	183.1	5.4
50%	341.2	318.2	26.4
75%	515.8	489.0	54.8
95%	915.0	876.4	116.5

5.3 Comparison and Practical Value

In 2011 the NEAFC decided the total annual catch from the stock till 2014 (Sigurdsson and Magnusson, 2011). The total annual catch in 2011 should sum up to 38 thousand tonnes, but then decrease by 6 thousand tonnes per year till 2014 when total annual catches will be 20 thousand tonnes. These values of TAC can be compared with the values from feedback-rule promoted in section 5.2. Table 5.5 shows the 5th, 25th, 50th, 75th and 95th percentiles of the optimal harvest from 2011 to 2014 as well as the NEAFC recommendation. The path of the TAC recommended by the NEAFC is decreasing while the optimal path is in over 95& of the cases increasing. Further, the NEAFC recommendation for catches in 2011 equates 95th percentile of the optimal path, and is 1.6 times greater than the median value of the optimal path (18 thousand tonnes). In 2012 the NEAFC recommendation is less than the 75th percentile of the optimal path, but is still more than 50% higher than the median value of 24 thousand tonnes. In 2013 the NEAFC recommendation is thousand tonnes less than the 50th percentile of the optimal path, and thousand tonnes lower than the 25th percentile in 2014.

Table 5.5: Distribution of feedback rule in 2011 to 2014 and NEAFC recommendation in thousand tonnes.

Year	5%	25%	50%	75%	95%	*NEAFC*
2011	7	18	23	29	38	38
2012	10	19	24	30	37	32
2013	14	20	25	31	38	26
2014	16	21	27	32	39	20

As mentioned above, the NEAFC recommendations for the TAC is greater than the 75th percentile values of the optimal paths till 2013. This means that if harvests will be as recommended by the NEAFC through 2013, the value of biomass in 2014 will most likely be lower than it would be if the TAC was decided each year by the median value of the optimal paths. The optimal path is an increasing function of biomass, so even though the NEAFC recommendation in 2014 is lower than the 25 percentile of the optimal path, in the same year, the true optimal value may be larger than the true optimal value.

The annual 90% and 50% ranges of harvest by the feed-back rule are shown in table 5.6. The 90% range is decreasing from year to year varying from being 23 to 35 thousand tonnes while the 50% range is 11 thousand tonnes in all years, from 2011 to 2014.

Table 5.6: Annual range of the feedback rules in 2011 to 2014 in thousand tonnes.

Year	5% to 95%	25% to 75%
2011	31	11
2012	27	11
2013	24	11
2014	23	11

The ranges in table 5.6 can be put into context by comparing them with their median values. Table 5.7 shows the ranges from table 5.6 divided by the corresponding median values. Both proportions are decreasing from year to year. The 90% ranges vary from being 85% of the median value to 1.52 times greater. However, the 50% ranges are in all cases smaller than the median values. Varying from being 41% to 49% of the median values.

Table 5.7: Range of feedback rule range divided by median value.

Year	5% to 95%	25% to 75%
2011	1.35	0.48
2012	1.11	0.45
2013	0.94	0.43
2014	0.87	0.41

6 Conclusions

The present values of profits from the optimal paths are in all cases greater than present values from the MSY paths. The difference has a median value of €26.4 million and its 50% range is [5.4, 54.8] € million. The optimal path to the MSY equilibrium should yield higher present values of profits than all other paths to the MSY equilibrium. Thus, an even greater difference might be expected if the present values of profits from the optimal OE paths would be compared with present values of profits from catch rules aiming for the MSY equilibrium. Further, X_{OE} is in all cases greater than X_{MSY}. Therefore, a harvesting policy leading to the OE equilibrium would be more effective than one leading to the MSY equilibrium, in avoiding risk of collapse of the stock or prolonged depression.

Various components of the model might be refined. All parameters distributions and values could be analysed further. The carrying capacity, K, and the natural mortality, M, both affect the OE and the MEY equilibrium. Thus, even though their value were under- or overestimated the effect might have consistent effects on both equilibrium. Other parameters only affect the OE equilibrium. The parameter, $\tilde{c}$, in the cost function of pelagic redfish harvesting could have been estimated differently. One way would be to estimate a cost function were total cost would be a function of total, but separated, harvests and the corresponding biomass. Further, the types of cost and growth functions were chosen with simplicity in mind. There are certainly some fixed costs in the fishery, and harvesting costs might not linear in effort. As well, the growth might be better described by some other function than the Fox function. However, even though parameters or functions do not perfectly reflect the fishery in reality, the model may be used to map the relative outcomes of different harvesting policies.

Reaching an desirable equilibrium will not happen over a night. The worse the condition of the stock and the lower the natural mortality, the longer time it takes to reach both the OE and MSY equilibrium. In the model presented in this essay the time it takes to reach the OE equilibrium with the optimal paths ranges from being 9 to 50 years. Thus, if the stock will be further researched, conditions for harvesting policies might change on the way to the equilibrium. Therefore, even though uncertainty concerning the stock may be considered high, it is important do define the objective of managing the stock.

The NEAFC recommended TAC path is quite different from the optimal path. The optimal path of harvest increases with biomass. The biomass assessment in 2011 and the model indicate that biomass in 2011 was lower than the optimal equilibrium value. Thus, if the aim is to achieve the optimal equilibrium, the optimal path of harvest in 2011 to 2014 should increase from year to year. However, the TAC recommended by the NEAFC for the same period is decreasing from year to year. Between 2011 and 2014 the TAC will change from 36 to 20 thousand tonnes. If the model used with its distributions of parameters is believed to describe the fishery then the discussion in section 5.2 indicates that the TAC should initially be lower and increasing from year to year. However, since there is no catch rule in use and catches only recommended until 2014 it is very difficult, or even impossible, to quantify how badly the NEAFC recommendations will perform compared to the optimal path in terms of present value of profits.

The model might be used as a gauge for harvesting policies for fisheries with a high degree of uncertainty. This is argued because of two reasons. The first being that at least in one year NEAFC recommendations equates the 95th percentile of optimal harvest. Therefore, if some of the 1000 combinations of parameters used in our model can be used to describe the real fishery, it demonstrates that the recommended TAC in 2011 is likely to high. The second being that the addressed model addressed strongly indicates that the optimal path of harvests is increasing but not decreasing as the NEAFC recommendation proposes.

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Appendix

A.1 Intrinsic growth rate

Note that

$$G_X = r_{max} \cdot (\log(K) - \log(X) - 1).$$

Thus, at MSY when $G_X = 0$, the following holds

$$G_X = r_{max} \cdot (\log(K) - \log(X_{MSY})).$$

Multiplying both sides with X_{MSY} gives

$$r_{max} \cdot X_{MSY} = r_{max} \cdot X_{MSY} \cdot (\log(K) - \log(X_{MSY})) = G(X_{MSY})$$

Therefore,

$$r_{max} = \frac{G(X_{MSY})}{X_{MSY}}. \tag{A.1}$$

Fishing mortality, F, is defined by

$$Y = F \cdot X.$$

Since at MSY, $Y = G(MSY)$, this means that, $G(MSY) = F_{MSY} \cdot X_{MSY}$. Therefore

$$F_{MSY} = \frac{G(X_{MSY})}{X_{MSY}}. \tag{A.2}$$

Combining equations (A.1) and (A.2) shows that

$$r_{max} = F_{MSY}.$$

A.2 Existence

The Pontryagin Maximum Principle only gives necessary conditions for the optimal path. It is possible that there is no optimal path. However, if the objective function, $\Pi \cdot \exp(-r \cdot t)$ is concave in Y, the state equation, $\dot{X}$ is linear in Y and both are continous and bounded with bounded derivatives then the optimal path exists (Kamien and Schwartz, 1991; Cesari, 1966). Now, $\dot{X}$ is linear in Y (see equation (3.5)). Both $\dot{X}$ and $\Pi \cdot \exp(-r \cdot t)$ are continuous and bounded, given that $X > 0$ (see equation (3.9)). Further, $\dot{X}_X = G_X$ and $\dot{X}_Y = -1$. Thus, as long as $X > 0$, the derivatives of $\Pi \cdot \exp(-r \cdot t)$ and Y are bounded (see equations (3.17), (3.18) and (3.19)). At last

$$\Pi_{YY} = c \cdot \alpha^{-1} \cdot (1 - \alpha^{-1}) \cdot X^{-\beta/\alpha} \cdot Y^{1/\alpha - 2}.$$

First, since $X, Y \geq 0$ then $X^{-\beta/\alpha} \cdot Y^{1/\alpha-2} \geq 0$. Second, $c > 0$ so assuming that $0 < \alpha \leq 1$ provides that $c \cdot \alpha^{-1} \cdot (1 - \alpha^{-1}) \leq 0$ and $\Pi_{YY} \leq 0$. Thus, given the previous assumptions, $\Pi \cdot \exp(-r \cdot t)$ is concave in Y assuming that $\alpha \leq 1$. Given the following arguments, the optimal path exists as long as $X > 0$ and $\alpha \leq 1$ in addition to previous assumptions in section 3.1.

A.3 Sufficiency

There are two famous theorems about sufficiency of Pontryagin's theorem. Mangasarian (1966) proved that if in (3.13) the set of the control variable is convex and if the Hamiltonian of the problem is concave in the state and control variables then the necessary conditions in Pontryagin's theorem are sufficient. Arrow (1968) extended Mangasarian theorem and proved that if the maximised Hamiltonian[4] is a concave function of the state variable then the necessary conditions in Pontryagin's theorem are as well sufficient (Kamien and Schwartz, 1991). Now, if an optimal path exists, then its equilibrium state will be described by equations (3.16). Because of the complexities of the model the optimal path of harvest will first be derived analytically. Then the the optimal equilibrium values of biomass and harvest will be found by equations (3.16). Last, the optimal path to that equilibrium is numerically approximated. However, if given the assumptions in section 3.1 and that $X > 0$ and $\alpha \leq 1$ so that the optimal path exists, along with if the steady state is uniquely determined by 3.16, the approximated optimal path from the initial state to the optimal equilibrium will be an approximation of the optimal path.

A.4 Ramsey Equation

The aim is to maximise society's utility with consumption, ψ, as a control variable and capital, k, as state variable. The output of capital is denoted by $f(k_t)$. The change in capital is the difference between it's output and consumption,

$$\dot{k}_t = f(k_t) - \psi_t.$$

Society's instantaneous utility from consumption is $U(\psi_t)$. The utility is assumed to be and increasing and concave function of consumption, $U_\psi > 0$, $U_{\psi\psi} \leq 0$. Social welfare is represented by the intertemporal sum of the instantaneous utility. The objective is thus formally (Kamien and Schwartz, 1991)

$$\max_{\{\psi_t\}} J = \int_0^\infty U(\psi_t) \cdot \exp(-\delta \cdot t) dt, \tag{A.3a}$$

[4] The maximised Hamiltonian is the value of the Hamiltonian evaluated at the optimal path of the control variable.

with the constraint

$$\dot{k} = f(k_t) - \psi_t, , \tag{A.3b}$$

$$\tag{A.3c}$$

where δ is the pure time preference. The current value Hamiltonian (Kamien and Schwartz, 1991) of the problem is

$$\mathcal{F}(\psi_t, t) = \mu_0 \cdot U(\psi_t) + \mu_t \cdot (f(k_t) - \psi_t),$$

The Pontryagin Maximum Principle (Pontryagin et al., 1962) gives conditions that a path of the control and state variable of an optimal control problem must satisfy to be optimal. Among these necessary conditions are (Kamien and Schwartz, 1991):

(i) μ_0 is a constant, 0 or 1 for all t, μ is a continuous function of time and μ_0 and μ are not both 0, $(\mu_0, \mu) \neq 0$.

(ii) ψ maximises $\mathcal{F}$ for allt t.

(iii) $\dot{\mu} - \delta \cdot \mu = -\mathcal{F}_k$.

According to condition ii, $\mathcal{F}_\psi = 0$. Thus

$$\mu_0 \cdot U_c = \mu.$$

If $\mu_0 = 0$ then it must hold that $\mu = 0$. Thus, by condition (i) $\mu_0 = 1$ and

$$\mu = U_\psi.$$

Now, $\dot{\mu} = U_{\psi\psi} \cdot \dot{c}$. Combining this with condition (iii) gives

$$U_{\psi\psi} \cdot \dot{c} - \delta U_{\psi} = -U_{\psi} \cdot f_k.$$

By defining the social marginal product of capital as $r \equiv f_k$ and rearranging

$$r = \delta - \frac{U_{\psi\psi}}{U_{\psi}} \cdot \dot{c}.$$

Consumption growth is $g \equiv \dot{c}/c$ and the elasticity of the marginal utility of consumption is $\eta \equiv -U_{\psi\psi}/U_{\psi} \cdot c$. Thus, the rate of return on capital in social optimum is

$$r = \delta + \eta \cdot g.$$

A.5 Figures

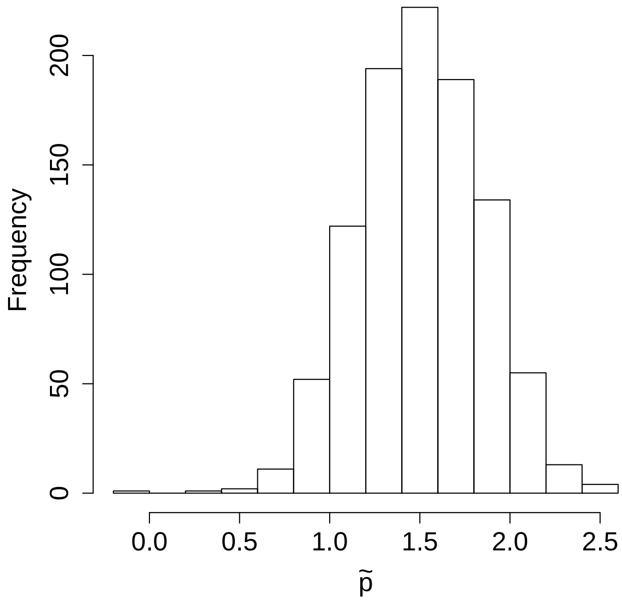
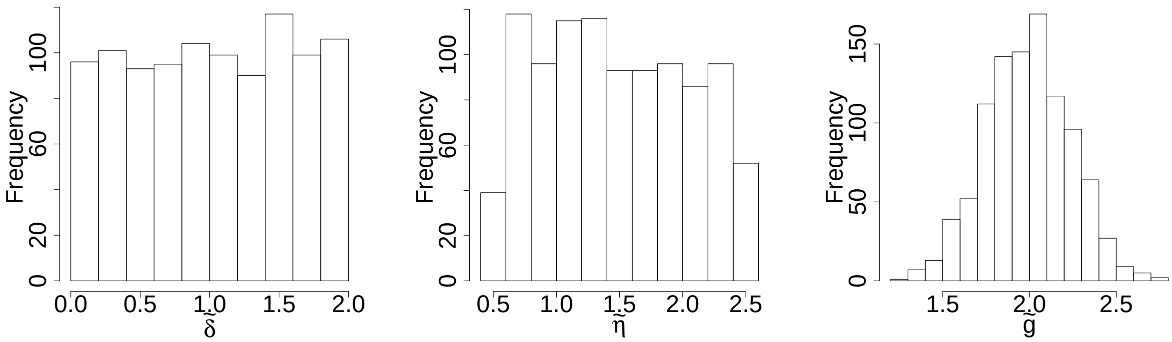


Figure A.1: Histogram of the uncertain price, $\tilde{p}$.



(a) (b) (c)

Figure A.2: Histogram of the uncertain pure time preference, $\tilde{\delta}$, elasticity of marginal utility of consumption, $\tilde{\eta}$, and growth rate of consumption, $\tilde{g}$.

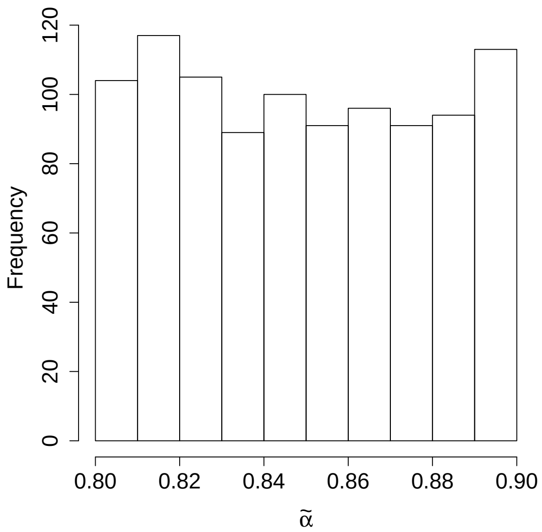


Figure A.3: Histogram of the uncertain output elasticity of effort, $\tilde{\alpha}$.

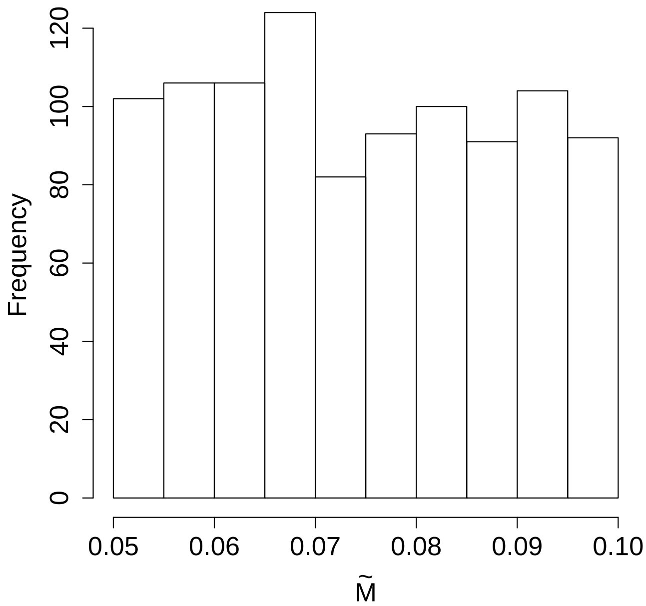
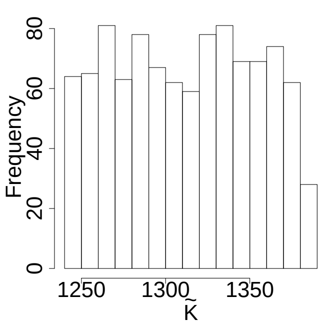
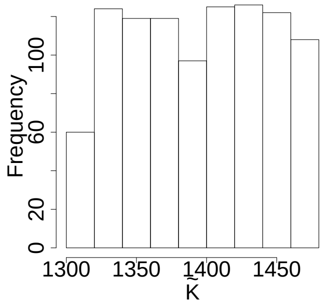


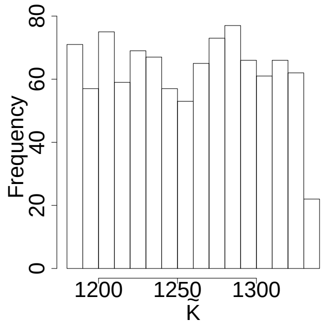
Figure A.4: Histogram of the uncertain natural mortality, $\tilde{M}$.



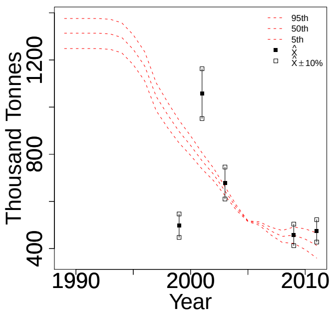
(a) Histogram of $\tilde{K}$ in case 1.



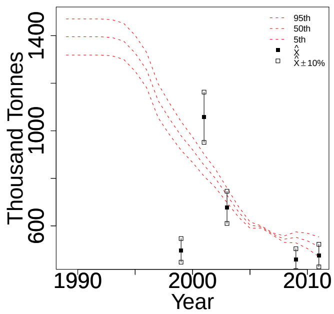
(b) Histogram $\tilde{K}$ in case 2.



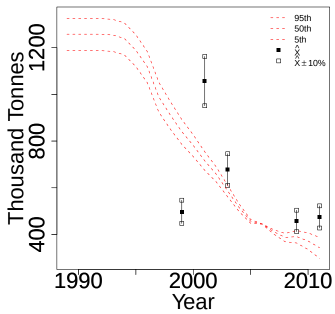
(c) Histogram of $\tilde{K}$ in case 3.



(d) Simulations of X in case 1.



(e) Simulations of X in case 2.



(f) Simulations of X in case 3.

Figure A.5: Histograms of the uncertain carrying capacities, $\tilde{K}$, and simulations of biomass (X) in cases 1,2 and 3.

A.6 Tables

Table A.1: Icelandic pelagic redfish cathces by type of processing in thousand tonnes

Year	Land Freezing	Frozen at Sea	Freshly Iced Fish Exported by Air	Reduction	Freshly Iced Fish Landed Abroad	Freshly Iced Fish Exported in Containers	Domestic Consumption
1992		13.85					
1993	0.08	19.66					
1994	5.89	40.95	0.00			0.25	
1995	2.45	25.47		1.35		0.01	
1996	8.40	43.21	0.12	0.49		0.77	
1997	2.07	34.03	0.07	1.06	0.12	1.26	
1998	2.94	40.57	0.37	0.12	1.69	1.42	0
1999	1.36	38.79	0.18		0.65	2.02	
2000	1.14	40.50	0.61		0.59	2.38	
2001	1.27	38.92	0.46			1.79	
2002	1.34	42.59	0.02		0.50	0.05	
2003	1.20	45.96	0.06		0.76	0.43	
2004	0.04	36.44			0.26	0.08	
2005	0.02	15.81				0.17	
2006	0.00	24.64					
2007		19.92					
2008		6.79		0.00			
2009	0.00	15.14				0.39	
2010	0.05	14.41	0.02			0.32	
Sum	28.26	557.65	1.91	3.02	4.57	11.34	0

Table A.2: Icelandic Pelagic redfish cathces by gear in thousand tonnes

Source: Statistics Iceland

	Year	Bottom Longline	Bottom Trawl	Pelagic Trawl
	1992			13.8
	1993			19.7
	1994		0.0	47.1
	1995	0.0		29.2
	1996	0.5	0.0	52.5
	1997			38.6
	1998		0.3	46.8
	1999			43.0
	2000		1.5	43.7
	2001			42.4
	2002		4.3	40.2
	2003		1.4	47.0
	2004		0.5	36.3
	2005			16.0
	2006		0.0	24.6
	2007		0.0	19.9
	2008		0.0	6.8
	2009		1.3	14.2
	2010			14.8
Sum		0.5	9.5	596.7

Table A.3: Shallow and Deep Pelagic redfish catches.

Source: Marine Research Institute of Iceland

	Year	Icelandic Shallow	Total Shallow	Icelandic Deep	Total Deep	Sample
	1992	12.1	62.6	3.4	3.4	10.8
	1993	10.2	100.8	12.7	15.1	9.9
	1994	5.9	96.9	47.4	51.8	29.9
	1995	8.7	100.1	25.9	75.7	14.3
	1996	5.8	41.8	57.1	138.6	38.6
	1997	4.4	27.7	36.8	95.1	25.2
	1998	2.0	24.2	46.5	92.8	28.7
	1999	3.7	25.5	40.3	84.2	30.9
	2000	3.8	33.2	41.5	93.1	22.8
	2001	14.7	41.8	27.7	87.0	20.4
	2002	5.2	43.2	39.3	103.2	19.9
	2003	4.3	56.7	44.6	104.3	12.7
	2004	5.7	33.9	31.1	92.0	10.8
	2005	3.1	28.2	12.9	45.5	12.2
	2006	1.3	15.7	20.9	67.3	15.4
	2007	0.1	6.1	18.1	58.5	10.9
	2008	0.1	2.0	6.7	30.0	5.5
	2009	0.4	2.7	15.1	52.5	6.6
	2010	0.2	2.4	14.6	62.0	6.3
Sum		91.7	745.5	542.6	1352.1	331.8

Table A.4: Catches of Deep Pelagic redfish by country.

Source: (Jakobsdottir and Kristinsson, 2010)

Year	BG	CA	EE	FO	FR	DE	GL	IS	JP	LV	LT	NL	NO	PL	PT	RU	ES	UK	UA	Total
1991								43												43
1992								2615												2615
1993				310		1135		13354					878							15678
1994						2019		47421					523		377	1465				51805
1995	1140	181	5056	1572	68	8271	1579	26197	396	1501	6868	4	3169		2955	15868	2620		956	78399
1996	1654	307	3351	3748		15549	1671	57616	196	512	5031		5161		1903	36400	5558	123	245	139025
1997		9	315	435		11200		36915	3				2849		3307	33237	6895			95164
1998			76	4484		8368	302	46524	1		34		438		4073	25748	2758			92805
1999			53	3466		8218	3271	40223					3337		4240	11419	9885	5		84115
2000			7733	2367		6827	3327	41753			0		3108		3694	14851	9740			93399
2001			878	3377		5914	2360	28901			7515		4275		2488	23810	8649			88166
2002			15	3664		7858	3442	39289			9771		4197		2208	25309	7402			103155
2003				3938		7028	3403	44588					5185		2109	28638	9374			104263
2004				4670		2251	2419	31112					6277	1889	2286	31067	9996			91968
2005				1800		1836	1431	12919			1027		3950	1240	1088	16323	3871			45485
2006				3498		1830	744	20948			1294		5968	1356	1313	23670	6673			67294
2007				2902		1110	1961	18091		575	1394		4628	636	2067	21337	3810			58511
2008				2653			1170	6721			749		571	219	1733	15106	1179			30100

BG=Bulgaria, CA=Canada, EE=Estonia, FO=Faroe Islands, FR=France, DE=Germany, GL=Greenland, IS=Iceland, JP=Japan, LV=Latvia, LT=Lithuania, NL=Netherlands, NO=Norway, PL=Poland, PT=Portugal, RU=Russia, ES=Spain, UK=United Kingdom, UA=Ukraine

Table A.5: Distribution of catches in the sample in thousand tonnes.

Source: Statistics Iceland

Cod	Haddock	Saithe	G. Halibut	P. Redfish	Other	Total
281.7	73.5	118	79.4	332	358.9	1243.5

Table A.6: Distribution of catch values in the sample in million euros.

Source: Statistics Iceland

Cod	Haddock	Saithe	G. Halibut	P. Redfish	Other	Total
463.2	112	108.6	192.6	295.3	344.8	1516.5

Table A.7: Summary of the proportion between each unit's total operating costs and catch values, $C_{i,t}/R_{i,t}$.

Source: Statistics Iceland

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.27	0.88	0.93	1.15	1.06	4.13

Table A.8: Summary of the uncertain price, $\tilde{p}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
−0.2	1.25	1.51	1.5	1.74	2.53

Table A.9: Summary of the uncertain pure time preference, $\tilde{\delta}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0	0.53	1.01	1.02	1.52	2

Table A.10: Summary of the uncertain elasticity of marginal utility of consumption, $\tilde{\eta}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.5	1	1.43	1.48	1.95	2.5

Table A.11: Summary of the uncertain consumption growth, $\tilde{g}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1.2	1.82	1.99	1.99	2.16	2.73

Table A.12: Summary of $\tilde{c}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
8.18	10.58	11.33	11.37	12.11	14.91

Table A.13: Summary of p-value in estimations of $\tilde{c}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
$2.776e-06$	0.001295	0.003751	0.006372	0.008651	0.05156

Table A.14: Summary of R^2 in estimations of $\tilde{c}$.

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.77	0.93	0.96	0.95	0.98	1

Table A.15: Solutions for T

T	OE	MSY	$OE - MSY$
5%	9	2	0
25%	27	2	12
50%	32	14	19
75%	41	24	26
95%	50	36	34