

**OPTIMIZATION OF THE
OPERATION OF A NETWORK OF
LOW TEMPERATURE
GEOTHERMAL RESERVOIRS**

January 2013

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Master of Science in Decision Engineering



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Optimization of the operation of a network of low temperature geothermal reservoirs

by

Hrannar Már Sigrúnarson

Research thesis submitted to the School of Science and Engineering
at Reykjavík University in partial fulfillment of
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Master of Science in Decision Engineering

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Jan 2013

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Date

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Abstract

In 1930 the first low temperature geothermal reservoir was harvested in Reykjavík. In the decades that followed more reservoirs were harvested and in 1990 Nejsavellir power plant began operation and supplied hot water to Reykjavík. Today all hot water used in Reykjavík has its source from low temperature geothermal reservoirs or power plants that heat up cold ground water with geothermal heat. Around Reykjavík there are four low temperature reservoirs and two power plants that produce water for district heating. Optimization model has been made for one of the low temperature reservoir to optimise the production from, Laugarnes. Lumped parameter modeling was used to fit the parameters for the reservoir and mixed integer linear programming for the optimization. In this project the production optimization model is extended to take into account more than one low temperature reservoir. The model was tested and four experiments conducted. They showed good results though the model could only run over 28 weeks due to computational time. The computational time was extended from previous model because the optimization model was non linear instead of being linear and that proves to need much more processing power.

Bestunarlíkan fyrir rekstrarumhverfi af neti af lághitasvæðum

Hrannar Már Sigrúnarson

Jan 2013

Útdráttur

1930 var fyrsta laghitasvæðið nýtt á Íslandi. Áratugina sem að fylgdu þar á eftir voru fleirri lághitasvæði nýtt og árið 1990 byrjaði framleiðsla á heitu vatni á Nesjavöllum. Í dag kemur allt heitt vatn sem að notað er í Reykjavík frá lághitasvæðum eða virkjunum sem að nota jarðvarma til að hita upp kalt grunnvatn. Í kringum Reykjavík eru fjögur laghitasvæði og tvær virkjanir sem að sinna þörfum Reykjavíku fyrir heitu vatni. Bestunarlíkan hefur verið gert til að besta rekstur eins af lághitasvæðunum, Laugarnes. Punktalíkna var notað til þess að herma eftir lághitasvæðinu en línuleg blönduð heiltölubestun til að besta reksturinn. Í þessu verkefni er bestunarlíkaninu breytt á þann veg að hægt sé að taka tillit til fleirri en eins laghitasvæðis. Líkanið var prófað og fjögur tilfelli rannsökuð. Tilfellin sýndu góðar niðurstöður en niðurstöðurnar voru takmarkaðar þar sem að aðeins var hægt að athuga vinnslu yfir 28 vikur vegna keyrslubunga líkansins. Keyrslubunginn var að hluta til vegna þess að líkanið er ólínulegt og þarf það meiri tíma til útreikninga en línulegt.

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Contents

List of Figures	viii
List of Tables	x
1 Introduction	1
2 Methods	5
2.1 Lumped Parameter Reservoir Model	5
2.2 Optimization model	7
3 Experiments and results	13
3.1 Data fit	13
3.2 Experiments	14
3.2.1 Experiment 1, setup	14
3.2.2 Experiment 1, results	15
3.2.3 Experiment 2, setup	17
3.2.4 Experiment 2, result	18
3.2.5 Experiment 3, setup	18
3.2.6 Experiment 3, result	19
3.2.7 Experiment 4, setup	20
3.2.8 Experiment 4, results	21
3.3 Size of models and solution time	22
4 Conclusions and future work	25
Bibliography	
Appendix	

List of Figures

1.1	Overview of suppliers for Reykjavík	2
1.2	Location of warm water reservoirs and power plants around Reykjavík . .	3
2.1	Three tank system for a single reservoir for a single reservoir in a lumped parameter model[10]	5
2.2	The reservoir systems serving a single demand	6
3.1	Drawdown in reservoirs in experiment 1	15
3.2	Production from each reservoir in experiment 1	16
3.3	Drawdown in reservoirs in experiment 2	17
3.4	Production from each reservoir in experiment 2	18
3.5	Drawdown in reservoirs in experiment 3	19
3.6	Production from each reservoir in experiment 3	20
3.7	Drawdown in reservoirs in experiment 4	21
3.8	Production from each reservoirs in experiment 4	22

List of Tables

3.1	Parameters for reservoir 1 which where attained in Sigurdardottir et al work for Laugarnes[10]	14
3.2	Parameters for reservoir 2, these parameter are fictional and is to reflect a smaller reservoir than reservoir 1	14
3.3	Parameters for experiment 4	20
3.4	Running time and constraints of each experiment	23

Chapter 1

Introduction

There are a number of low temperature reservoirs in and around Reykjavík. In 1930 the first low temperature reservoir was used as a source of hot water for district heating. It was Laugarnes that was first used and it was used to heat up the newly built Austurbæjarskóli elementary school. Since it was an experiment a boiler room was included in Austubæjarskóli as it was custom to heat up water with coal. Soon other large buildings followed and in 1931 the state hospital, Reykjavik swimming pool in 1934 and Laugarnesskóli elementary school in 1935. The harvestation of low temperature geothermal water started to grow rapidly during the second world war and the 1970's oil crisis. Until 1990, when Nesjavellir power plant was built, the demand for hot water grew and it had started to impact the yield of the reservoirs. But after Nesjavellir power plant started supplying hot water the pressure on the low temperature geothermal reservoir lowered and they grew to full potential in a few years. Today Reykjavík and neighbouring communities are supplied with hot water from two power plants and four low temperature geothermal reservoirs. The supply has grown from 15 kg/sec when Laugarnes was the only supplier to being able to supply upto 5000 kg/sec today. This is known as the largest supplier in the world of warm water that comes from geothermal heat [1].

Nesjavellir and Hellisheiði are suppliers of hot water. The hot water is produced by extracting cold ground water and heated up with geothermal fluid. After the cold water has cooled the turbines the water has reached 60°C. The water is heated to around 85°C and hydrogen sulfide is pumped into the water to remove oxygen from the water before it is transported to Reykjavík. There is no need for modelling for Nesjavellir and Hellisheiði as the cold water source is almost infinite with respect to the demand. Nesjavellir power plant was started in 1990 and has since produced warm water and electricity as a by-product with excessive steam. It produces around 1640 kg/sec of 83°C water or equivalent

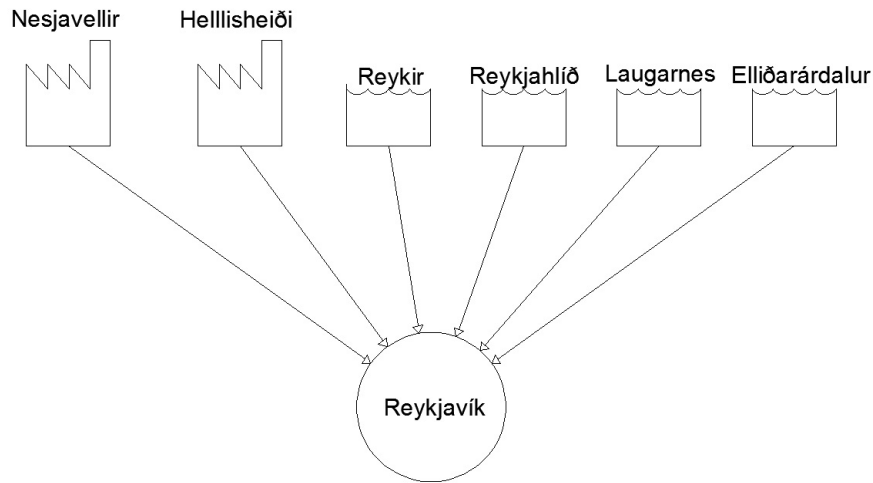


Figure 1.1: Overview of suppliers for Reykjavík

to 300MW of warm water. Nesjavellir also produces 120MW of electricity but it is considered as secondary production as it's Nesjavellir main objective is to supply hot water. Hellisheiði main focus is production of electricity not hot water production for district heating. Hellisheiði power plant produces much smaller amount of hot water, between 200-400 kg/sec of 85°C warm water is sent from Hellisheiði when needed [2].

Reykir and Reykjahlíð are located in Mosfellsbær and are very close to each other. No research have been done to examine if the reservoirs are connected and if extraction from one has any influence on the other that the author is aware of. But due to the proximity it is more likely than not that they are connected.

Elliðaárdalur is located in Reykjavík and is one of Reykjavík recreational areas. Hot water has been extracted for several decades in Elliðaárdalur but has reduced significantly in recent years, mostly due to the introduction of the Nesjavellir power plant. Location of the reservoirs around Reykjavík are illustrated in figure 1.2

Laugarnes is located in middle of Reykjavík and as Elliðaárdalur is a recreational area. Despite being the oldest source, Laugarnes is still being harvested for hot water, since Nesjavellir was built the extraction of water has significantly decreased. The hot water from the reservoir is used in domestic use. The water from Nesjavellir and Hellisheiði are pre warmed cold ground water and is injected with hydrogen sulphide to remove excessive oxygen to prevent corrosion in pipelines. Due to unknown reasons the water from the power plant and the water from Reykjahlíð and Reykir can't be used together, that the author is aware of. If blended the pipelines will corrode quickly.



Figure 1.2: Location of warm water reservoirs and power plants around Reykjavík

Geothermal energy can be considered a renewable source as removal of energy from the source is replaced in a similar timescale. It is not to be confused with sustainable energy. Sustainability refers to a exploitation to a resource over a long period of time while a production system is able to use it's energy [3,4]. It is important to use geothermal areas in effective and in a renewable way. Along with renewable production considerations need to be done to the limitations and constraints in the operational environment. Therefore is the operation of controlling the production in a network of low temperature geothermal reservoirs both a complicated and an important problem. Because of the complicity and size of the problem it is interesting to use systematic mathematical methods to optimise the operation. Several studies have been done in assessing geothermal systems with different methods. Volumetric methods, detailed mathematics and lumped parameter modeling are the methods that are mostly used. Volumetric and detailed mathematics are powerful ways to study geothermal system, but both of them are very complicated and studies made with these method have been used to study what if scenarios in simulation. It is interesting to use optimization to find the best way to operate low temperature geological reservoirs as it is a optimisation method, not a what if simulation . For optimization to be possible the model needs to be fast and computationally efficient but also accurate. For the model to be fast and efficient it is considered that lumped parameter modling is best suited. The gain of using lumped parameter modeling is that it doesn't require much processing power and it uses parameter that are common and easily available. The benefits of using a lumped parameter model instead of using for example volumetric or numerical medeling model is that it is commonly available and few parameters, it does not need much processing

power and it has acceptable accuracy in pressure changes when modeling isothermal low temperature geothermal reservoirs [5,6].

Around the world lumped parameter modeling has been applied successfully to assess geothermal systems, for example Iceland, Turkey, P.R. of China and in Central America [5,7,8,9]

One of the geothermal reservoirs that has been studied with help of a lumped parameter model is Laugarnes. Sigurdardottir et.al[10] developed a optimization model that optimizes the operation side for a low temperature geothermal reservoir and connects that to the lumped parameter model for Laugarnes. The objective of this paper is to extend Sigurdardóttir et.al[10] model to consider a network of low temperature geological reservoirs. The objective is to construct a model that considers more than one low temperature geothermal reservoir and maximises the profit of the operation. After that has been done four scenarios will be examined to see the effect of having more than one reservoir will have on the drawdown of the watertable on the reservoirs.

Chapter 2

Methods

2.1 Lumped Parameter Reservoir Model

The low temperature reservoir modeling is based on lumped parameter description of a liquid phase hydrothermal reservoir in this study. It is based on a three tank system where the reservoir is divided into three connected areas. The smallest tank is where the extraction well is placed. The other two tanks are the near regions where drawdown or pressure changes can be measured through inspection wells. The tanks are connected to each other so fluid can flow between them, also fluid can flow from the tank nearest to the extraction well to the surface. Figure 2.1 shows the relation between the tanks where K represents the mass of the tank, σ the flow resistance, h the drawdown and m the extraction from tank 1. Usually lumped parameter models for low temperature reservoirs are based on pressure changes but since the data in this project is based on drawdown in meters, the model will take this into count.[5,6]

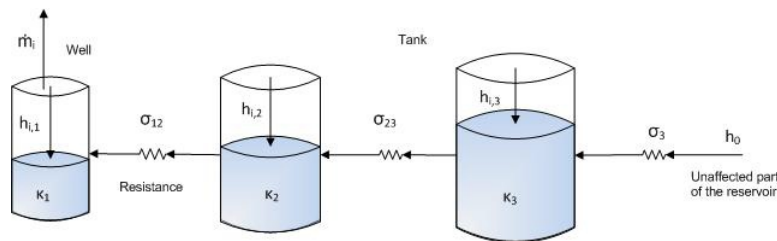


Figure 2.1: Three tank system for a single reservoir for a single reservoir in a lumped parameter model[10]

Sigurdardottir et al [10] has in her work developed a model to fit the parameters to historical data needed for the optimization model. In her work she has assumed that there was only one reservoir but in this paper the optimization model will be extended to take into account more than one reservoir.

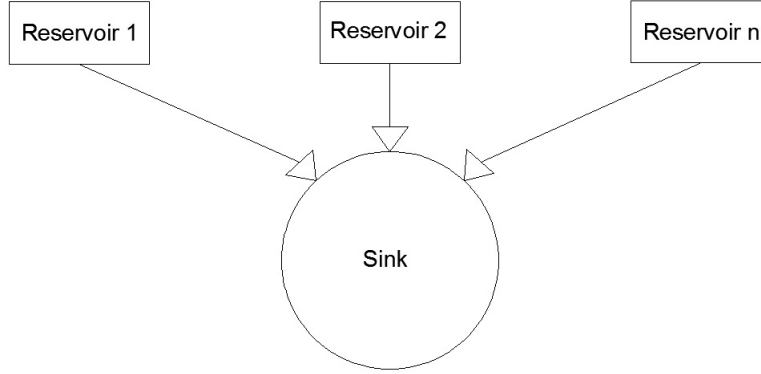


Figure 2.2: The reservoir systems serving a single demand

The mass flow from the reservoirs will be denoted as $m_{i,f}$ which will be the flow at time i from reservoir f . The capacity of each tank will be denoted as k and the resistance between tanks will be denoted as σ . The relationship between the tanks in a single reservoir is written in the following three differential equations.

$$k_1 \frac{dh_1}{dt} = \sigma_{12}(h_2 - h_1) + m/\rho g \quad (2.1)$$

$$k_2 \frac{dh_2}{dt} = \sigma_{12}(h_1 - h_2) + \sigma_{23}(h_3 - h_2) \quad (2.2)$$

$$k_3 \frac{dh_3}{dt} = \sigma_{23}(h_2 - h_3) + \sigma_3(h_0 - h_3) \quad (2.3)$$

Where k represents the storage coefficient of the storage tanks, σ represents the conductivity between the tanks and h the drawdown in the tanks. And in more compact in matrix form and with respect of more than one reservoir.

$$K_f \frac{\partial}{\partial t} h_f = S_f h_f + u_f \quad (2.4)$$

where

$$K_f = \begin{bmatrix} k_{1,f} & 0 & 0 \\ 0 & k_{2,f} & 0 \\ 0 & 0 & k_{3,f} \end{bmatrix} \quad h_f = \begin{bmatrix} h_{1,f} \\ h_{2,f} \\ h_{3,f} \end{bmatrix}$$

$$u_f = \begin{bmatrix} m_f / \rho g \\ 0 \\ \sigma_{3,f} \cdot h_{0,f} \end{bmatrix} \quad S_f = \begin{bmatrix} -\sigma_{12,f} & \sigma_{12,f} & 0 \\ \sigma_{12,f} & -\sigma_{12,f} - \sigma_{23,f} & \sigma_{23,f} \\ 0 & \sigma_{23,f} & -\sigma_{23,f} - \sigma_{3,f} \end{bmatrix}$$

$$\forall f \in \{1, 2, \dots, F\}$$

There are number of ways to solve the system both analytically with differential calculations and numerical. Sigurdardottir et.al [10] uses modified Eulers method which is a first order numerical method to solve the system, mainly due to the available data and it works without limit on the time step is chosen. Modified Euler is considers the function both at the beginning and the end of the timestep and takes the average of the two. Solving the equation for h_{i+1} gives equation 2.9.. The model then estimates the parameters by minimising the squares of the difference between predicted drawdown and empirical data. This returns the values for k , h_1 and σ .

2.2 Optimization model

The goal of the optimization model will be to maximise the profit over time. The only income is the sale of hot water. The costs are two the production and pump cost. The production cost is the cost of extracting water from the wells while the pump cost is the cost of installing a new pump. There are many other cost to consider, like maintenance, but for simplifications only these two are considered.

Objective Function

$$PV_{Income} = \sum_i \sum_f \frac{\Delta t \cdot m_{f,i} \cdot C_{water}}{(1+r)^i} \quad (2.5)$$

$$PV_{Production} = \sum_i \sum_f \frac{(m_{f,i}^{mean} \cdot h_{1,f,i})) \Delta t \cdot C_{elect} \cdot g}{(1+r)^i} \quad (2.6)$$

$$PV_{Pump} = \sum_i \sum_f \frac{y_{i,f} \cdot C_{pump}}{(1+r)^i} \quad (2.7)$$

$$PV_{Profit} = PV_{Income} - PV_{Production} - PV_{Pump} \quad (2.8)$$

Formulas 2.5 to 2.7 are from Sigurdardottir et al project[10] The income, production cost and pump cost will be the summation over all areas and time. In the end the objective will be to maximize the difference between Income and production cost. C_{water} is the fixed price on which the water will be sold at in $\$/m^3$. C_{elect} is the price of electricity at $\$/kWh$, the electricity is needed to pump up the water from the aquifers. C_{pump} is the price of a new pump if there is a need for installing a new pump. $y_{i,f}$ is a integer that is more than one if a pump need to be installed at time i in reservoir f . The constraints will be as follows.

Constraint: Mass balance equation on matrix form

This constrain describes the dynamics of the three tank system and how drawdown in the tanks is affected by the production. The relationship is carried out with a discrete approximation of the lumped parameter model to make the model more easily solvable as the lumped parameter model contains differential equations. It was used implicit approach to solve the differential equation witch results in a stable model but requires some simple matrix operations in each time step. [11].

$$h_{j,f,i+1} = (K_f - \frac{\Delta t}{2} S_f)^{-1} ((K_f + \frac{\Delta t}{2} S_f) h_{j,f,i} + \frac{\Delta t}{2} (u_{f,i+1} + u_{f,i})) \quad (2.9)$$

$$\forall i \in N \text{ and } j \in \{1, 2, 3\}, f \in \{1, 2, \dots, F\}$$

where

$$u_{i,f} = \begin{bmatrix} m_{f,i+1}/g\rho \\ 0 \\ \sigma_{3,f} \cdot H_{0,f} \end{bmatrix} \quad (2.10)$$

$$\forall i \in N \text{ and } f \in \{1, 2, \dots, F\}$$

Constraint: Demand

This constraint states that total production from the reservoirs f in time i can't be more than the total demand for period i .

$$\sum_f m_{f,i} \leq md_i \quad (2.11)$$

$$\forall i \in \{1, 2, \dots, N\} \text{ and } f \in \{1, 2, \dots, F\}$$

Constraint: Max drawdown

There is power required to extract the water from the reservoir and each unit extracted of water contains exergy. $P_{Well,f}$, in equation 2.12, is the power needed for extracting water from the well and X_{Water} , in equation 2.13, is the water exergy.

$$P_{Well,f} = m_{j,f}^{mean} gh_{j,f} \quad (2.12)$$

$$X_{Water} = m_f^{mean} e_{x,f} \quad (2.13)$$

If P_{Well} is required to be equal or less than X_{Water} for a long period of time sustainability criteria is obtained.

$$P_{Well} \leq \delta X_{Water} \quad (2.14)$$

Under Carnot conditions δ would be equal to 1, but that is not possible in the case of district heating. Here δ is 10% which means that there is only 10% of the energy contained in the water utilised.

C is the heat capacity of the water and if 15°C is the temperature of the water then $c(J \cdot kg^{-1}K^{-1})$ is the heat capacity. In this example temperature is measured in Celsius not Kelvin for simplification and it doesn't matter as the following formula uses heat difference. $T_h(K)$ is the heat from the source and $T_0(K)$ is the heat of the water after it has been used. Exergy per unit can be approximated as following:

$$e_x = c((T_h - T_0) - T_0 \ln \frac{T_h}{T_0}) \quad (2.15)$$

Using these assumptions maximum drawdown is determined by the following equation.

$$h_{1,f,1}^{max} = \frac{e_x \delta}{g} \quad (2.16)$$

Due to renewability the drawdown is not allowed to exceed the maximum,

$$h_{i,f,1} \leq h_{1,f,1}^{max} \quad (2.17)$$

$$\forall f \in \{1, 2, \dots, F\} \text{ and } i \in \{1, 2, \dots, I\}$$

Constraint: Production Capacity

In the Max drawdown constraint it was described how power is needed to pump up water from the reservoirs. The power demand needed at each timestep for pumping is calculated as following.

$$P_{f,i} = gh_{1,f,i} \cdot m_{f,i}^{\text{mean}} \leq (P_{\text{pump},f} \cdot \sum_{k=1}^i y_{f,k}) \quad (2.18)$$

$$\forall f \in \{1, 2, \dots, F\} \text{ and } i \in \{1, 2, \dots, I\}$$

$P_{\text{pump},f}$ is the power consumption of the pump in reservoir f and $y_{f,k}$ a integer that represents how many pumps need to be added at reservoir f at time i . There are many

brand and types of pumps to choose from. In this work we use 250kW pumps as (Silja Sigurdardottir) used in her research.

The optimization model is therefore:

$$\max_{h_{i,f}, m_{i,f}, y_{i,f}} PV_{\text{Profit}} = PV_{\text{Income}} - PV_{\text{Production}} - PV_{\text{Pump}}$$

s.t

$$h_{j,f,i+1} = (K_f - \frac{\Delta t}{2} S_f)^{-1} ((K_f + \frac{\Delta t}{2} S_f) h_{j,f,i} + \frac{\Delta t}{2} (u_{f,i+1} + u_{f,i}))$$

$$m_{f,i} \leq m_{d,f,i}$$

$$h_{i,f,1} \leq h_{1,f,1}^{\max}$$

$$g(h_{1,f,i} \cdot m_{f,i}^{\text{mean}}) \leq (P_{\text{pump},f} \cdot \sum_{k=1}^i y_{f,k})$$

$$m_{f,i} \geq 0$$

$$\forall j \in \{1, 2, 3\}, f \in \{1, 2, \dots, F\} \text{ and } i \in \{1, 2, \dots, I\}$$

Chapter 3

Experiments and results

There are number of things to explore and investigate regarding the reservoirs near Reykjavík. Among those things are mixing of chemicals in pipelines, over exploration of reservoirs and the impact of power plants. The experiments will try to illustrate similar circumstances as are in Reykjavík, where there are many reservoirs with different attributes and chemical mixture. The model is programmed in matlab with Tomlab optimization environment and it is a mixed integer non linear programming. It will be solved with a Gurobi solver and constraints will be added to reflect circumstances.

3.1 Data fit

As the only real data in this project is production history from Laugarnes and all other data uses Laugarnes as reference point all results will be hypothetical. It was made with another reservoir that resembles Laugarnes but only smaller, all parameters where done smaller so it should contain less water and the resistance between tank is less, see tables 3.1 and 3.2. The parameters in table 3.1 are the parameters that Sigurdardottir et al[10] to Laugarnes in her research paper.

Parameters	Parameters value	Unit
k_1	50	ms^2
k_2	1712	ms^2
k_3	10,426,317	ms^2
σ_{12}	0.093	ms
σ_{23}	0.00026	ms
σ_3	0.0098	ms
$h_{1,i=1}$	131.99	m
$h_{2,i=1}$	130.63	m
$h_{3,i=1}$	39.57	m
h_0	6.35	m
Water heat	71	C

Table 3.1: Parameters for reservoir 1 which where attained in Sigurdardottir et al work for Laugarnes[10]

Parameters	Parameters value	Unit
k_1	49	ms^2
k_2	1611	ms^2
k_3	10,126,317	ms^2
σ_{12}	0.090	ms
σ_{23}	0.00022	ms
σ_3	0.0093	ms
$h_{1,i=1}$	121.99	m
$h_{2,i=1}$	120.63	m
$h_{3,i=1}$	29.57	m
h_0	3.35	m
Water heat	69	C

Table 3.2: Parameters for reservoir 2, these parameter are fictional and is to reflect a smaller reservoir than reservoir 1

3.2 Experiments

3.2.1 Experiment 1, setup

In this scenario it is assumed that there are two reservoirs where one is Laugarnes, reservoir 1, and the other is slightly smaller, reservoir 2. Maximum drawdown was added as a constraint in this experiment. As the model only runs for 28 weeks the maximum drawdown constraints where modified so that the model would be forced to use them.

Maximum drawdown for Laugarnes was set to be 140 meters and for reservoir 2 it was set to be 130 meters. This means that the heat from the sources are 71°C for Laugarnes and 69°C for reservoir 2. In both cases 15°C was assumed to be the dead state temperature, or when the water has no more heat than the surrounding. In reality the maximum drawdown is greater than this, closer to 500 meters as the water is a lot hotter, but this is hypothetical and only to show the function of the model. As mentioned before the model only ran through 28 weeks and is that due to computational time.

- Parameters for experiment 1
 - Historical demand is 520 kg/sec
 - Maximum drawdown for reservoir 1 is 140 meters
 - Maximum drawdown for reservoir 2 is 130 meters
 - One pump is installed in the first week for each reservoir

3.2.2 Experiment 1, results

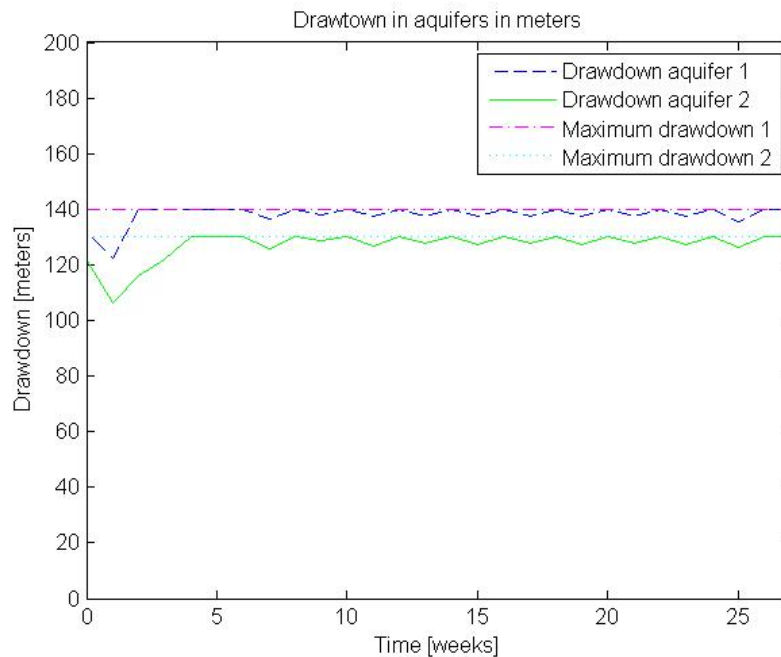


Figure 3.1: Drawdown in reservoirs in experiment 1 shows that the reservoirs hit the constraint every second week.

With the demand of 520 kg/sec it only took a little more than one week for the reservoirs to hit the maximum drawdown. As they hit the maximum drawdown constraint production dropped the following week. The following week, when the reservoirs have

had some time to gain it's potential, they can again supply upto demand. This pattern continued through the experiment. The fluctuation in production is most likely due to that the profit is calculated as a net present value. Because of that it aims to produce as much as possible early. The experiment illustrates a example of a over exploitation of the reservoirs as it was in Reykjavík before the power plants where built and helped to keep up with increasing demand of warm water. New pumps where installed for reservoir 1 at time 2.

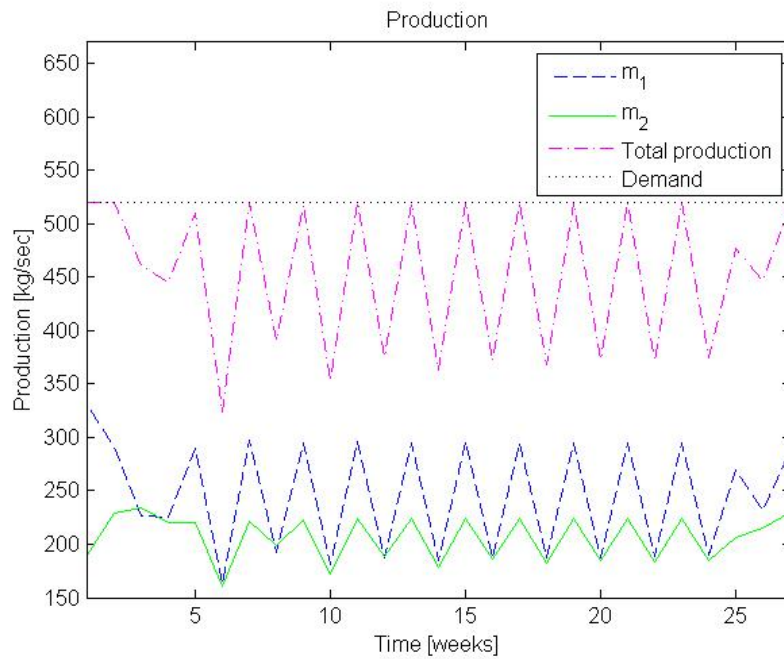


Figure 3.2: Production from each reservoir in experiment 1 show that only every second week the reservoirs can supply the historical demand

3.2.3 Experiment 2, setup

In this experiment it was assumed that a power plant is producing hot water to help with the demand from experiment 1. The reservoir will have to supply a demand of 320 kg/sec instead of 520 kg/sec as the power plant is a constant supplier of 200 kg/sec of warm water. This will show what influence a power plant has on the reservoirs. The parameters for reservoir 1 and 2 are the same as in the previous experiment.

- Other parameters for the experiment
 - Historical demand is 320 kg/sec
 - Maximum drawdown for reservoir 1 is 140 meters
 - Maximum drawdown for reservoir 2 is 130 meters
 - One pump is installed in the first week for each reservoir
 - Power plant supplies 200 kg/sec

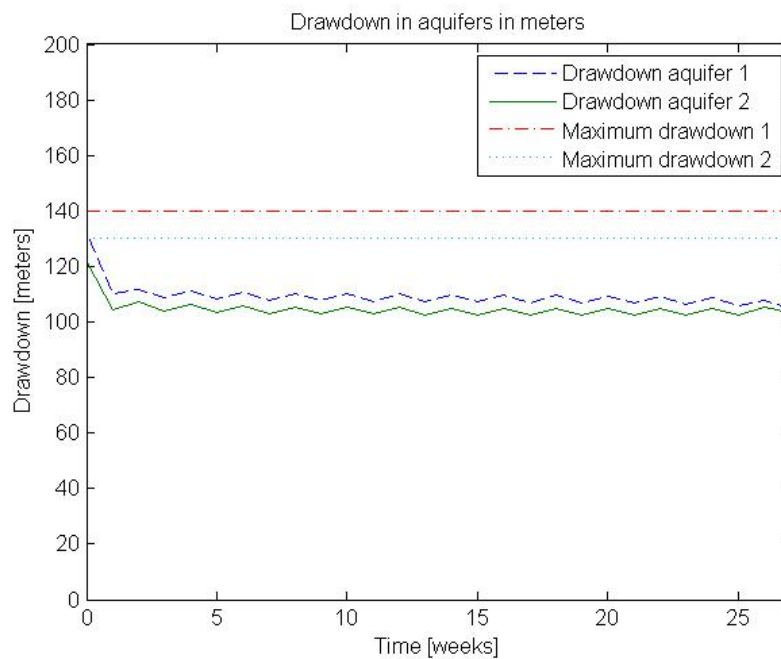


Figure 3.3: Drawdown in reservoirs in experiment 2 shows that the maximum drawdown constraint is never hit. It seems as if the height of the water table in the reservoirs is increasing in time

3.2.4 Experiment 2, result

It is clear from figure 3.3 that it has significant results on the drawdown to have a power plant, like Nejsavellir or Hellisheiði, to help with the demand. Neither of the reservoirs hit the maximum drawdown constraint and can meet supply for demand. The difference in drawdown can be explained by the difference mass of the water in reservoir 1 and reservoir 2. Water in reservoir 1 weights 0,977kg/L and in reservoir 2 0,978kg/L due to temperature difference. It shows a clear example on how much impact Nesjavellir power plant had on the reservoirs when it started in 1990 and the reservoirs had time to gain their attributes and over exploration on the reservoirs stopped.

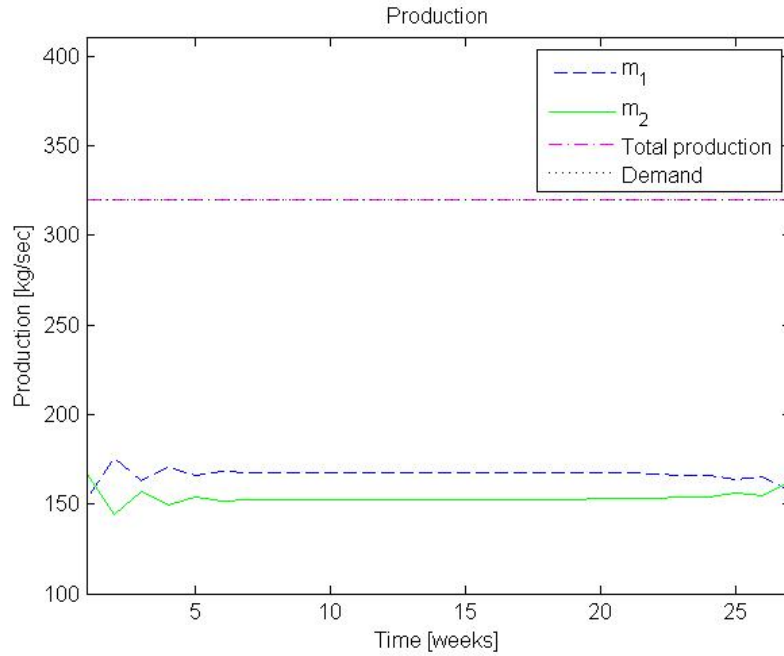


Figure 3.4: Production from each reservoir in experiment 2 shows that the supply keeps up with the historical demand. There is no line to show the total production as it followed the historical demand and wasn't visible

3.2.5 Experiment 3, setup

In experiments 1 and 2 it was assumed that the demand was constant. That is not the case in reality as the demand fluctuates from season to season. In this experiment it is assumed that the demand fluctuates as follows 320->420->520->420 and this pattern then repeats itself and all values are in kg/sec.

- Other parameters for the experiment

- Maximum drawdown for reservoir 1 is 140 meters
- Maximum drawdown for reservoir 2 is 130 meters
- One pump is installed in the first week for each reservoir

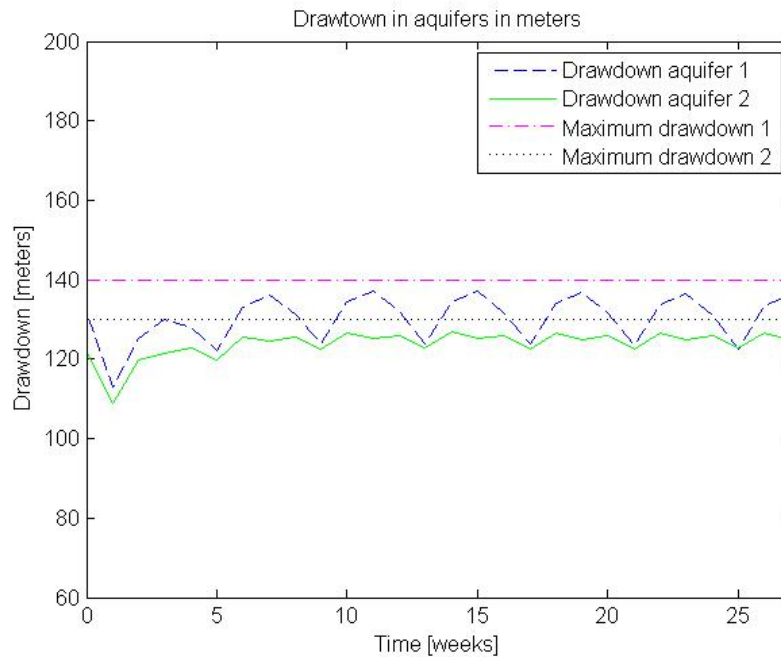


Figure 3.5: Drawdown in reservoirs in experiment 3 shows that the maximum drawdown constraint is never hit. There is more fluctuation in the drawdown in reservoir 1 and that directly related to the production.

3.2.6 Experiment 3, result

Figure 3.5 shows the drawdown in the reservoirs. Neither of the reservoirs go up to the maximum drawdown constraint. It is as the model seeks to have the production from reservoir 2 with less fluctuation than reservoir 1, as seen in figure 3.6, that is probably due to the pump constrain. When the demand is 420 kg/sec equal amount is produced in the reservoirs. When the demand is 520 kg/sec reservoir 1 produced around 300 kg/sec while production in reservoir 2 drops. When the demand is 320 kg/sec reservoir 2 produces the greater part of the demand while production in reservoir 1 drops to around 150 kg/sec. It looks like as the time increases that the production from reservoir 2 stabilises and get more evenly distributed. New pump was needed for reservoir 1 at time 3.

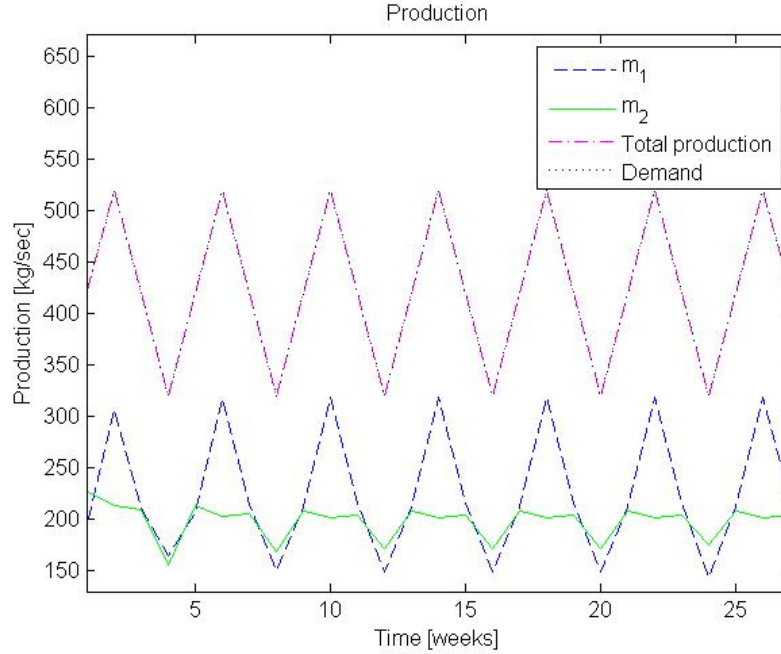


Figure 3.6: Production from each reservoir in experiment 3 shows that the supply keeps up with the demand. The two lines are in sync and it seems like it's only one line in the graph. The two reservoirs both at some point produce more than the other. Reservoir 1 produces more when the demand is 520 kg/sec and reservoir 2 when the demand is 320 kg/sec. As time progresses fluctuation in the production from reservoir 1 becomes smaller.

3.2.7 Experiment 4, setup

As mentioned earlier in the paper there can be scenarios where water from two reservoirs can't be mixed together due to mixing of chemicals. It is for example the case of Reykjahlíð and Nesjavellir power plant. In this experiment it will be considered as a chemical can not exceed or go under a certain threshold. It can be done to assure that enough H₂S is in the water to prevent corrosion. There is also another constraint added here that says that production from reservoir 2 must be greater or equal to the production of reservoir 1. It is because production is necessary in Reykjahlíð at certain times to prevent hot spring forming on the surface. Otherwise the same constraints where in this experiment as in experiment 1.

Parameter	Value	Units
Concentration in reservoir 1	1.0	con/m ³
Concentration in reservoir 2	1.2	con/m ³
Total concentration threshold	$1.1 \leq \text{concentration} \leq 1.5$	con/m ³

Table 3.3: Parameters for experiment 4

Constraint for chemical mixing: $1.1 \leq \frac{m_1 \cdot \text{concentration}_{res1} + m_2 \cdot \text{concentration}_{res2}}{m_1 + m_2} \leq 1.2$

3.2.8 Experiment 4, results

The substance was stable at 1.1 throughout the inspection time.

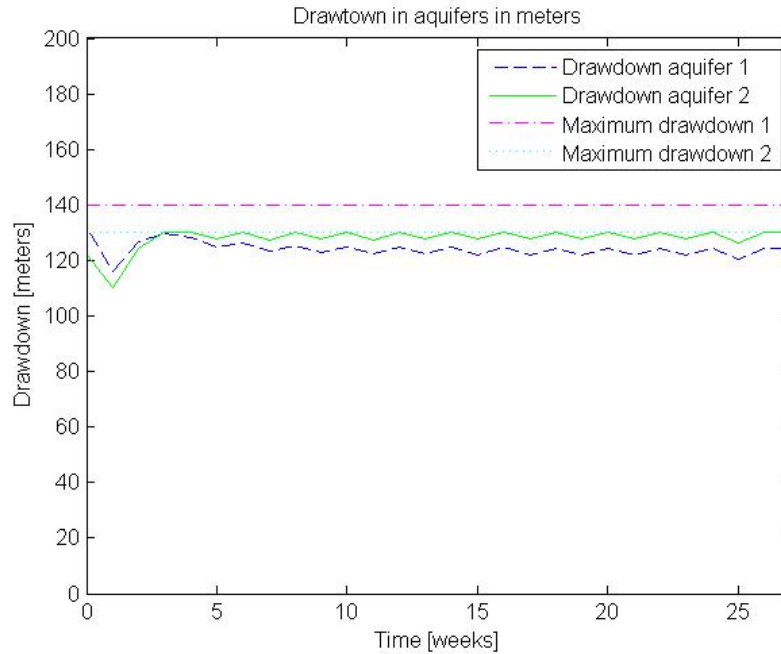


Figure 3.7: Drawdown in reservoirs in experiment 4 fluctuate almost in sink. The drawdown in reservoir 2 hits the maximum drawdown constraint quickly and after that it controls the total production from the reservoirs.

There is not much fluctuation in production or the drawdown in the reservoirs. The drawdown in reservoir 2 soon hits its constraint. After it hits the constraint it controls the total production. As the production from reservoir 2 must at least be as equal as the production from reservoir 1 and the constraint for the chemical mixing must be at least 1.1 the production from reservoir 1 can't pick up the needed production. As a result every second timestep reservoir 2 hits its drawdown constraint while the drawdown in reservoir 2 declines as seen in figure 3.5. as seen in figure 3.6 the production fluctuates in sink with the drawdown. The production from reservoir 1 and reservoir 2 are equal throughout the inspection time. New pumps were installed for reservoir 1 and 2 at time 2.

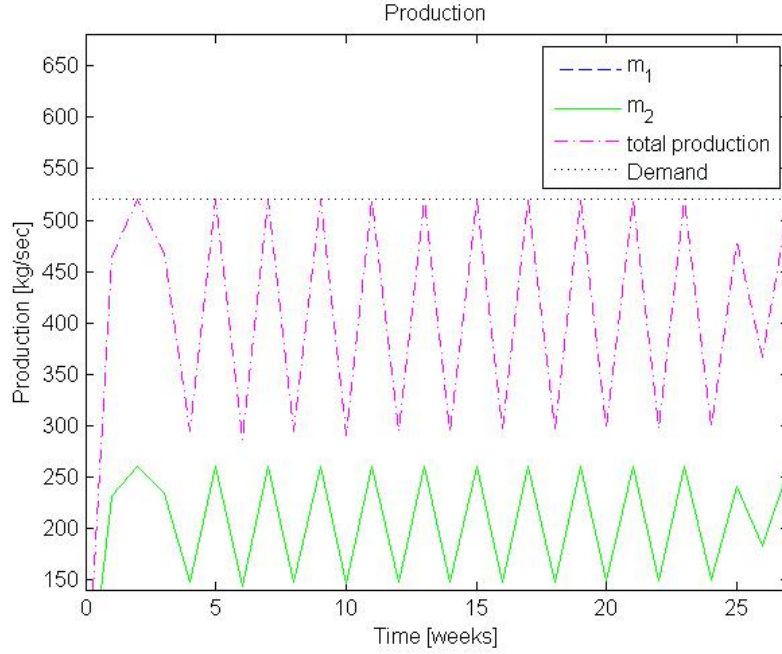


Figure 3.8: Production from the reservoirs in experiment 4 are always equal. That is due to the constraint that says production from reservoir 1 can't exceed production in reservoir 2. Production can't fulfil demand in every week and is that due to constraints in maximum drawdown.

3.3 Size of models and solution time

The running time seemed to correlate with how many constraints it had to take into account when calculating the drawdown and production, see table 3.4. Experiment 3 took the longest time to run and it is likely due to that it where many constraints on each time interval it had to take into account. Experiment 1 was a slightly faster through the calculations but nonetheless it took 4 hours. All the experiments where run on a Dell Studio XPS 1640 laptop. The computer has a 2,53GHz CPU, 4GB ram and a 64bit Windows 7 operating system. Experiment 2 was only 30 minutes to find solution and that must be because it doesn't need to consider the maximum drawdown constraints. Experiment 2 was run over longer period than experiments 1 and 2 but as soon as the time went over 35 weeks the running time of the model increased and was finally stopped after more than 24 hour running. It is likely due to that at each time interval 13 new constraints are added to the model in experiments 1 and 2 and 16 more for experiment 3. The running time should decrease if the model was made linear but as it is now it is non linear. Linear model should

Experiment	Number of constraints	Solution time
1	364	4 hours
2	364	30 minutes
3	364	4 hours
4	448	5 hours

Table 3.4: Running time and constraints of each experiment

require less computational time and the model would be able to run over a longer period of time without increasing the computers power. The computations should also be easier if the timesteps were in months not weeks. Then it could be considered as tank one is empty because it is so small and therefore it would be a two tank system instead of a three tank system. The downside of that is that the model would not be as detailed.

Chapter 4

Conclusions and future work

A optimization model has been made that maximizes profit for the operation of a network of low temperature geothermal reservoirs. The model has been used to optimize a system with two reservoirs and four scenarios where analysed. Due to high computational time inspection time was limited to 28 weeks. That meant results where limited and could not show what effects long time extraction has on the reservoirs.

The downside of the model is that it is a non linear model and due to that it needs much processing power to find solution. If the model is changed to a linear model, computation would be easier and less processing power should be needed and most likely it could be run over a longer period of time. The linearisation is however not trivial and outside the scope of this thesis. The timesteps could also be changed from weeks to months but that would result in less accuracy in the model.

It is fairly easy to run experiments and what if scenarios in the model and to put other constraints and reservoirs in it. But if more constraints or reservoirs where added the model would become bigger and therefore longer to find a optimal solution. It would be interesting to see the results if the model would run for inspection time of 20-50 years.

Though the inspection time was limited to 28 weeks the optimisation model and constraints seem to work fine. It would be better to have a more powerful computer to run the model and one that is only dedicated to solve the model. It is hard to say at this point with full certainty if the model works in practise as there where only parameter for one reservoir. The model needs to be tested with real parameters from two or more reservoirs to fully test it's potential. So the conclusion is that the model works but has limitations due to processing power needed for optimisation.

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Appendix

Indices

t_i	Discrete time index, $t_1 \leq t_i \leq t_N$, for all $i \in \{1, 2, \dots, N\}$, (s)
i	$t_i = i$ for simplification

Decision Variables

$m_{i,f}$	Extraction from tank 1 in reservoir f at time i , (kg/s)
Y_f	Pumps needed in reservoir f

State Variables

$h_{i,j,f}$	Drawdown at time i in tank j in reservoir f , for all $j \in \{1, 2, 3\}$, $i \in \{1, 2, \dots, N\}$ and $f \in \{1, 2, \dots, F\}$, (m)
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Parameters

$\sigma_{12,f}$	The conductivity between tanks 1 and 2 in reservoir f , ($\text{m} \cdot \text{s}$)
$\sigma_{23,f}$	The conductivity between tanks 2 and 3 in reservoir f , ($\text{m} \cdot \text{s}$)
$\sigma_{3,f}$	The conductivity between tanks 3 and the external environment in reservoir f , ($\text{m} \cdot \text{s}$)

$$\mathbf{S} = \begin{bmatrix} -\sigma_{12,f} & \sigma_{12,f} & 0 \\ \sigma_{12,f} & -\sigma_{12,f} - \sigma_{23,f} & \sigma_{23,f} \\ 0 & \sigma_{23,f} & -\sigma_{23,f} - \sigma_{3,f} \end{bmatrix}$$

$H_{0,f}$	The external drawdown in reservoir f , (m)
$H_{j,1,f}$	Drawdown in tank j in reservoir f at time $i = 1$ for all $j \in \{1, 2, 3\}$ and $f \in \{1, 2, \dots, F\}$, (m)

$H_{1,1,f}^{max}$	Maximum drawdown in tank 1 in reservoir f , (m)
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$m_{d,i}$	Historical demand in time i , (kg/s)
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$K_{1,f}$	Storage coefficient of tank 1 in reservoir f , ($\text{m} \cdot \text{s}^2$)
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$K_{2,f}$	Storage coefficient of tank 2 in reservoir f , ($\text{m} \cdot \text{s}^2$)
-----------	--

$K_{3,f}$	Storage coefficient of tank 3 in reservoir f , ($\text{m} \cdot \text{s}^2$)
-----------	--

$$\mathbf{K} = \begin{bmatrix} k_{1,f} & 0 & 0 \\ 0 & k_{2,f} & 0 \\ 0 & 0 & k_{3,f} \end{bmatrix}$$

g	gravity
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Δt	Timestep, $\Delta t = t_{t+1} - t_i$, (s)
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ρ	Density of water at 25°C, (kg/m^3)
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C_{elcet}	Price of electricity, ($\$/\text{J}$)
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C_{water}	Price of water, ($\$/\text{m}^3$)
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C_{pump}	Price of adding another pump, ($\$$)
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$P_{pump,f}$	Maximum pump power in reservoir f , (W)
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