17th and 18th century European arithmetic in an 18th century Icelandic manuscript

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ABSTRACT

Icelandic arithmetic books from the 18th century, printed and in manuscripts, adhered to the European structure of practical arithmetic textbooks, formed in the late Middle Ages: The number concept, numeration, the four operations in whole numbers and fractions, monetary and measuring units, extraction of roots, ratio, progressions and proportions. A manuscript textbook, *Arithmetica – That is reckoning art*, dated in 1721, deviates from the general model in that it does not treat monetary and measuring units but includes theoretical sections on the number concept and common notions on arithmetic. No specific model for the manuscript has been spotted, while similarities to works by authors as Ramus, Stevin, Suevus, Meichsner and Euler were found.

1 Introduction

According to Swetz (1992), basic types of texts on European arithmetics up to the fifteenth century were *theoretical tracts*, transmitting neo-Pythagorean speculations of Nicomachus of Gerasa (ca. 100); *computi*, ancestors of almanacs; *abacus arithmetic* for Roman numerals and *algorisms*, evolving from descendant translations of works of Al-Kwarizmi (ca. 825), such as *Carmen de Algorismo* (c. 1200) by Alexander Villa Dei. *Abacus arithmetic* and *algorisms* that discussed problems related to trade and commerce were called *Practica*, reflecting their practical use. Their influence prevailed in the teaching of arithmetic until the beginning of the 20th century.

Commercial arithmetics were not intended to be *theoretical tracts* devoted to philosophical speculations on the nature of number, rather; they were handbooks on readily usable mathematics. Generally, their content was numeration, monetary and measuring units, the four operations in whole numbers and fractions, i.e. addition, subtraction, multiplication, division, and often extraction of roots, progressions and proportions in the form of *Regula Trium*, the 'Rule of Three'.

There were two cathedral schools in Iceland until 1800. Their first regulations, issued in 1743, prescribed knowledge in the four arithmetic operations in whole numbers and fractions (Jónsson, 1893). The first substantial printed arithmetic textbooks in Icelandic were published in the 1780s, while textbooks in manuscripts, written in the vernacular, were dispersed from person to person in early modern times. These books generally belonged to the *Practica*.

The existence of the manuscripts shows that Icelanders made efforts to adapt European education and literature to their language as had been customary since the Middle Ages. Before the Lutheran Reformation, in 1550, translations had been made from Latin. Later, texts from the Northern-European protestant countries, written in e.g. German or Danish, seem to have been preferred for translation.

The following manuscripts of arithmetic textbooks from the seventeenth century onwards are preserved in the National and University Library of Iceland – manuscript department:

edited by Heath, 1956, 2, p. 277.		A **number** is a multitude composed of units.
Muhamed ibn-Musa al-Kwarizmi, 1992. *Kitab al-jam'val tafriq bi hisab al-Hind* (~800), Latin translation: *Dixit Algorizmi* (~1100), p. 1	Et iam patefeci in libro algebr et almucabalah, id est restaurationis et oppositionis, quod uniuersus numerus sit compositus et quod uniuersus numerus componatur super unum. Unum ergo inuenitur in uniuerso numero. Et hoc est quod in alio libro arithmetice dicitur quia **unum est radix uniuersi numeri et est extra numerum :**	And I have already explained in the book on algebra and almucabalah, that is on restoring and comparing, that every number is composite and every number is composed of the unit. The unit is therefore to be found in every number. And this is what is said in another book on arithmetic that **the unit is the origin of all numbers and is outside numbers :**
Jónsson, 1892–96. *Algorismus*, AM 544, 4to. (1306–08), p. 418.	Jafnir fingur eru fjórir; 2, 4, 6, 8 en ójafnir aðrir fjórir; 3, 5, 7, 9. **En einn er hvorki því að hann er eigi tala heldur upphaf allrar tölu.**	Even digits are four; 2, 4, 6, 8, and uneven another four; 3, 5, 7, 9. **But one is neither as it is not a number but the origin of all numbers.**
Ramus, 1569. *Arithmeticae libri Duo*, p. 1.	Numerus est, secundum quem unum quodque numeratur : ut secundum unitatem unum, secundum binarium duo, secundum ternarium tria; & sic deinceps omnes numeri: Itaque **numerus est unitatis aut multitudinis:** potesque esse minimus, ut unitas: ...	A number is that which each is counted by one, thus a unit by one, a couple by two, a triple by three, and thus in a sequence all numbers. Therefore **a number has the properties of a unit and a multitude**: it can be as small as a unity: ...
Stevin, 1585. *L'Arithmetique*, edited by Struik, 1958. pp. 494–504.	La partie est de mesine matiere qu'est son entier, Vnité est partie de multitude d'vnitez. Ergo l'vnité est de mesine matiere qu'est la multitude d'vnitez. Mais la matiere de multitude d'vnitez est nombre, Doncques la matiere d'vnité est nombre. Et qui le nie, faiĉt comme celui, qui nie qu'vne piece de pain foit du pain. Nous pourrions aussi dire ainci: Si du nombre donné l'on soubtraiĉt nul nombre, le nombre donné demeure. Soit trois le nombre donné, et de mesme soubtrahons vn, qui n'est point nombre comme tu veux. Doncques le nombre donné demeure, c'est à dire qu'il y restera encore trois, ce qui est absurd.	The part is of the same matter as its whole, unity is part of a multitude of unities, hence unity is of the same matter as the multitude of unities, But the matter of a multitude of unities is number, Hence the matter of unity is number, Who denies this behaves like one who denies that a piece of bread is bread. We can also say: If we subtract no number from a given number, then the given number remains. If three is the given number, and if from this we subtract one, which – as you claim – is no number, then the given number remains, that is three remains, which is absurd.
Cocker's ARITHMETICK (1715, first published 1677)		Unit is *Number*; for the part is of the same Matter that is his whole, the Unit is part of the Multitude of Units, therefore the Unit is of the same Matter that is the Multitude of Units; but the matter of the multitude of Units is *Number*, therefore the Matter of Unit is *Number*; for else if from a *Number* given no *Number* be subtracted, the *Number* giveth remaineth; let three be the *Number* given ; from which *Number* subtract or take away one (which as some conceive, is no number) therefore the *Number* given remaineth, that is to say, there remaineth three which is absurd.
Arithmetica –	Margir af þeim Lærdu Mathematicis	Many of the learned mathematicians

Það er reikningslist / That is Reckoning Art. ÍB 217, 4to. 1721/1750. pp. 1–2.	hafa viliad halda að 1. / unitas / væri ej Tala /numerus/ helldur væri Talann Fiølldi af 1. /:ex unitatibus / til samans lagdur vid Evcl: Elementa Lib: 7. Def. 2., hvad um 1 kyni ej seigiast, einasta álited unitatem, sem upphaf og undirrót til allrar Tølu. Adrir þar í mót meina að 1 eigi ad kallast Tal, þvj hann gieti i sier innibundid marga Parta af þessum hann sie samsettur. Þar ad auki ef 1 væri ej Tal skylldi annad Tal / til dæmis 5/ vera eins margt þó 1 væri þar af tekinn, þar þó allir skilja, ad ej þann aftur utan 4.	have maintained that 1 */unitas/* was not a number */numerus/*, rather the number was a multitude of 1 */:ex unitatibus/* added together, Evcl: Elementa Lib. 7. Def. 2., what could not be said about 1, only supposed to be *unitatem*, as the origin and basis of all number. Others, on the other hand, maintain that 1 is to be counted as a number, as it can contain many parts of which it is composed. In addition, if 1 was not a number, another number / for example 5/ should be as many, even though 1 was removed therefrom, there though everyone understands that not that one again, but 4.
Christlieb von Clausberg, C.v. 1732. *Der Demonstrativen Rechnenkunst*, pp. 14–15.	... alle diese Dinge, bey denen man eben solche Eigenschaften findet, machen gleichfalls eine **Eins** aus, und diese Einheiten zusammen genommen, geben eine **Zahl** ... Hieraus ist klar, dass die Unität oder **Eins** vor sich selbst nur der Nadir oder Wurzel der Zahlen ist, nemlich eine angenommene Grösse, wornach die Zahlen erwachsen und betrachtet werden ... indem sich keine **Zahl** benennen läst, wo nicht eine gewisse Grösse für eine **Eins** angenommen wird.	... all those things, with which one finds such characteristics, are similarly a **one**, and these units taken together, constitute a **number** ... From this it is clear that the unit or **one** for itself is only the nadir or the root of the numbers, that is an accepted magnitude from which the numbers grow and are observed ... ; as no **number** may be mentioned, without a certain quantity for a **one** being stated.

Table 1. Elaborations on the number concept in a choice of textbooks.

Clearly, the author of *Arithmetica* is aware of Stevin's reasoning that one, the unit, is indeed a number.

3.2. Common Notions

The *Common Notions* in Euclid' *Elements*, referring to magnitudes, were found in theoretical books on arithmetic, while they were seldom seen in the *Practicae*. *Arithmetica* contains thirteen *Common Notions*: 'Incontestable procedures, that is, so obvious rules that each one's understanding must acknowledge them (p. 7).' Three of these axioms match Euclid's *Common Notions* as presented in Heath's 1956 publication; while other three match Mersenne's 1644 publication of the *Elements*, Book Seven, see Table 2, below. Both lists of thirteen *Common Notions* refer to measuring, *metitur*, of *metior* : measure, in, Mersenne's in particular.

The *Common Notions* as listed in Heath's edition:

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part. (Euclid, 300 BC, vol. 1, p. 155)

Mersenne: *Universe Geometiae synopsis.* Paris, 1644, pp. 22–23.	*Arithmetica – That is Reckoning Art.* 1721/1750, p. 11.

1. *Quicumque eiusdem, vel æqualium æquemultiplices fuerint & ipsi inter se sunt æquales.* 2. *Quorum idem numerus æquemultiplex fuerit, vel quorum æquemultiplices fuerint æquales, & ipsi inter se æquales sunt.* 3. *Quicumque eiusdem numeri, vel æqualium eadem pars, vel eædem partes fuerint, & ipsi inter se sunt æquales.* 4. *Quorum idem, vel æquales numeri eadem pars, vel eædem partes fuerint, & ipsi inter se sunt æquales.* 5. *Omnis numeri pars est unitas ab eo denominata, binarij enim numeri unitas pars est ab ipso binario denominata, quæ dimidia dicitur, ternarij vero unitas est pars, quæ à ternario denominata tertia dicitur : quaternarij quarta, & ita in alijs.* 6. *Unitas omnem numerum <u>metitur</u> per unitates quæ in ipso sunt.* 7. *Omnis numerus seipsum <u>metitur</u>.* 8. *Si numerus metiatur numerum, & ille, per quem metitur, eundem <u>metietur</u> per eas, que sunt in metiente, unitates.* 9. *Quicumque numerus alium <u>metitur</u>, multiplicans eum, vel multiplicatus ab eo, per quem <u>metitur</u>, illum ipsum producit.* 10. *Si numerus numerum alium multiplicans, aliquem produxerit, multiplicans quidem productum <u>metitur</u> per unitates quæ sunt in multiplicato : multiplicatus verò <u>metitur</u> eumdem per unitates quæ sunt in multiplicante.* 11. *Quicumque numerus <u>metitur</u> duos, vel plures, <u>metietur</u> quoque eum, qui ex illis componitur.* 12. *Quicumque numerus <u>metitur</u> aliquem, <u>metietur</u> quoque eum, quem ille ipse <u>metitur</u>.* 13. *Quicumque numerus <u>metitur</u> totum & ablatum, etiam reliquum metietur.*	1. The whole is more than its part (Heath, 5). 2. The whole equals all its parts. 3. That which parts all are equal, are equals (Mersenne, 3). 4. If equals be added to equals, [they] become between themselves equal (Heath, 2). 5. If equals be subtracted from equals, the remainders are equal (Heath, 3). 6. If equals be multiplied by equals, the wholes are equal (Mersenne, 1). 7. Of equals, their halves, thirds, fourths etc. are equal. 8. The squares of equals are equal. 9. If square numbers and cubic numbers are respectively equal, then also their roots are equal. 10. Thus also their halves and doubles are equal. 11. No one can for himself <u>measure</u> all numbers. 12. All numbers have their <u>measure</u> of units (Mersenne, 7). 13. The unit neither multiplies nor divides.

Table 2: Comparison of *Common Notions*, listed in Mercenne's edition of Euclid's *Elements*, Book Seven, to those in *Arithmetica – That is Reckoning Art.*

Mersenne's edition also contains ten *Axiomata, Communes Notiones decem*, in Book One (p. 3), some of which correspond to the *Common Notions* in Heath's edition and others to no. 2 and 10 in *Arithmetica.*

3.3 Numeration

Arithmetic textbooks explain how to write numbers by place value notation, often demonstrated by large numbers. All the five examples on numeration in *Arithmetica* have corresponding examples in the German textbooks.

Arithmetica – That is Reckoning Art. (1721/1750)	Suevus (1593). *Arithmetica Historica. Die löbliche Rechenkunst.*	Meichsner G. (1625a). *Arithmetica Historica. Das ist: Rechenkunst*	Euler. L. (1738). *Einleitung zur Rechenkunst*
The number of years from the creation of the world until Christ was born: 3,970 years (p. 4).	3,970 years (p. 4)	3,970 years (p. 3)	
The cost of the building of King	13,695,380,050	13,695,380,050	13,695,380,050

Salomon's temple: 13,695,380,050 Coronatos Crowns (p. 5)	Crowns (p. 4)	Crowns (p. 2)	Crowns (p. 22)
The yearly cost of the government of the Emperor Augustinus: 1,200,000 Coronatos Crowns (p. 6)	12,000,000 Crowns (p. 5)	12,000,000 Crowns (p. 2-3)	1,200,000 Crowns (p. 22)
The fortune of Sardanapalus, the King of Assyria: 145,000,000,000 Guilders (p. 6)	154,000,000,000 Crowns (p. 6–7)	154,000,000,000 Crowns (p. 3–4)	145,000,000,000 Guilders (p. 22)
The number of grains of sand to fill the world: computed by Archimedes as 10^{63}, the unit with 63 zeros (p. 6)	$8\cdot10^{37}$ (p. 7)		10^{51} (p. 22)

Table 3. Examples on numeration in four textbooks.

Two of the above examples match Euler's text better than the two *Arithmeticas Historica*; Sardanopalus's fortune of 145 billion vs. 154 billion Guilders, and 1.2 million vs. 12 million Crowns of Emperor Augustus's yearly cost. This supports a hypothesis of a missing ancestor to both Euler's and the anonymous Icelandic text, as it is highly unlikely that one of them is copied from the other.

3.4. Addition and subtraction

The *Arithmetica* explains addition of multi-digit numbers in a well known fashion, still practiced, beginning from the right side, adding up the units, and proceeding to the left. All its examples, but one, listed in Table 4, are contained in the foreign sources by Suevus, Meichsner and Euler, which have more examples.

Arithmetica – That is Reckoning Art. (1721/1750)	Suevus (1593). *Arithmetica Historica.*	Meichsner (1625a). *Arithmetica Historica.*	Euler (1738). *Einleitung zur Rechenkunst.*
The age of Methusalem, who according to Holy Scripture was 187 years old when he begat Lamech, whereafter he lived for 782 years, to the age of 969 (p. 10).	969 years (p. 22–23)	969 years (p. 5)	969 years (p. 32)
Example revealing the 'current' year: 'I want to know how many years there are since the poet Homerus lived. Aulus Gellius writes that he lived 160 years before Rome was built, but the city of Rome was built 752 years before the birth of Christ, and the number of years since Christ was born until now is **1721** years.' (p. 10)	Number of years since the creation of Earth, 3,970 + **1,590** (current year) (p. 47)	Number of years since the creation of Earth, 3,970 + **1,625** (current year) (p. 4)	The number of years since Homerus lived, 160 plus 752 plus the current year, **1737** (p. 33)
The number of the Greeks, 880,000, and Trojans, 686,000, deceased in the Trojan War was in total 1,566,000. (p. 11)	1,566,000 men (p. 45–46)		1,566,000 men (p. 32)
Four men owe me 6,952, 8,346, 6,259 and 5,490 each, a total of 27,047 monetary units.			27,047 Rubles (p. 32)

Table 4. Comparison of addition examples in four textbooks.

The examples of the age of Methusalem and of four men owing 27,047 units with the same four amounts, in that case rix-dollars, are also found in *Arithmetica Islandica*, dated on its front page in 1716. Several examples count the present year as 1733, possibly a date of its extant copy (pp. 21, 36, 64, 66).

The example of the poet Homerus, revealing the date of all the four books, is also found in Ramus's (1569) *Arithmeticae libri Duo*, mentioned earlier. 'Ut si quaeratur quampridem vixerit Homerus, & respondeatur e Gellio, 160 annis ante conditam Romam, quæ condita sit ante natum Christum annis 752. Christum vero natum anno abhinc 1567. addantur hi tres numeri : Summa inductionis indicans Homerum annos abhinc 2479 floruisse, erit hoc modo' (p. 3). This example testifies that Ramus's book was written in 1567.

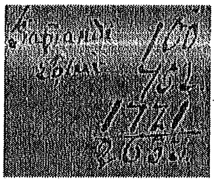

Figure 1: The example in *Arithmetica – That is Reckoning Art* on the number of years since Homerus lived.

Only three subtraction examples are found in our *Arithmetica* and none of them is historical. A demonstration follows on how subtraction and addition can be used for testing each other.

3.5. Multiplication

Arithmetica – That is Reckoning Art has several multiplication problems, similar or identical to other sources, such as on the circumference of the earth, see Table 5 below. *Arithmetica* gives an example on the average number of hours in a year, see Figure 2, as does Suevus, and Meichsner too in another book from the same year, *Arithmetica Poetica*. Euler counts the number of hours in a regular year.

Arithmetica – That is Reckoning Art (1721/1750).	Suevus (1593). *Arithmetica Historica.*	Meichsner (1625a). *Arithmetica Historica.*	Euler (1738). *Einleitung zur Rechenkunst.*
Circumference of the earth,360°·15 = 5,400 miles (p. 23).	360°·15 = 5,400 miles (p. 128).	360°·15 = 5,400 miles (p. 14–15).	360°·105 = 37,800 Werste (p. 69).
Number of hours in a year, 365·24 + 6 = 8,766 hours (p. 22).	(52·7+1)·360 +6 = 8766 hrs. (p. 127).	52·7·360 = 8,736 hrs. in 52 weeks in *Arithmetica Poetica* (1625b), (p. 17).	365·24 = 8760 hours (p. 69).
Size of a military group, 264·100 = 26,400 soldiers (p. 27).			156·97 = 15,132 soldiers (p. 69).
Fortune in King David's grave, 3000·600 = 180,000 Crowns (p. 28).	3000·600 = 180,000 Crowns (p. 171–172).		

Table 5. Comparison of multiplication examples.

The number of soldiers in the military group exceeds the number of adult men in Iceland in the early 18th century, where no army existed.

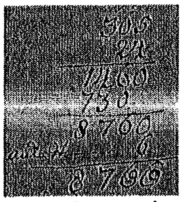

Figure 2. Computing hours in a year in *Arithmetica – That is Reckoning Art.*

3.6. Division

Only two division problems, both concerning calendar computations, are found in other books. Dates of the books are again revealed, as shown in Table 6.

Arithmetica – That is Reckoning Art (1721/1750)	Suevus (1593). *Arithmetica Historica.*	Meichsner (1625a). *Arithmetica Historica.*	Euler (1738). *Einleitung zur Rechenkunst.*
Is the coming year a leap year? Coming year, **1721**:4 = 430, remainder 1 (p. 35)	**1591**:4 = 397, remainder 3 (p. 175)	Check on which of the years 1620–1624 were leap-years (p. 25)	
Golden number of a year (**1622**+1):19 = 85, remainder 8 (p. 38)	(**1591**+1):19 = 83, remainder 15 (p. 176)		

Table 6. Comparison of division examples.

The date 1622 in *Arithmetica* could point to copying a book dated that year. Problems of this kind are not found in Euler's *Rechenkunst.*

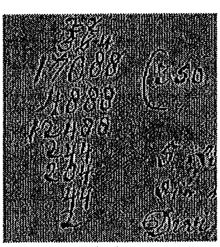

Figure 3. Dividing 17,088 by 48 in *Arithmetica – That is Reckoning Art.* The divisor is written below the dividend and disappears in a mess of different products.

3.7. Square Root and the Theorem of Pythagoras

The first example on extracting a square root in *Arithmetica – That is Arithmetic Art* concerns a general arranging in a square a group of 54,756 soldiers, exceeding the population of Iceland.

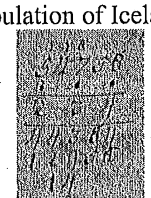

Figure 4. Extracting square root.

In continuation extracting square root is applied to the Pythagorean Theorem, a rule not found in other textbooks inspected. No tower of the kind drawn in the manuscript existed in Iceland, while stories of kings, queens, knights and princesses in castles were a favoured branch of literature.

Root extraction is not contained in the books by Suevus, Meichsner and Euler, while e.g. Gemma Frisius's (1567) *Arithmeticae practicae methodus facilis* contains extractions of square and cube roots.

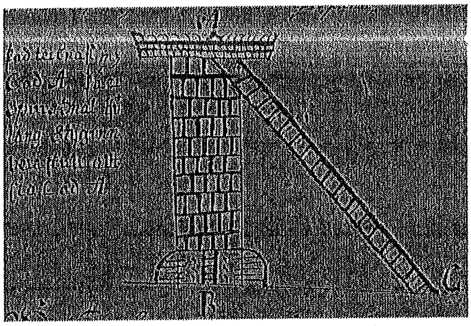

Figure 5. Demonstration of the Pythagorean Theorem:
The height of the tower AB is 30 feet and the width of the dike BC is 28 feet.
$30^2 + 28^2 = 1684$
$1684 - 41^2 = 3$
The length of the ladder AC is 41 and 3/(2·41 + 1), that is 41 3/83 feet.

3.8. The remaining text

Arithmetica – That is Reckoning Art continues with a thorough treatment of fractions and their four operations, and a large section on the *Regula Trium*, applied to proportional problems for centuries (Tropfke, 1980, pp. 359–378). The manuscript ceases abruptly late in that section, which reminds the reader that it is a copy and not an original.

The topic of fractions is not contained in the textbooks by Suevus and Meichsner, and *Regula Trium* does not exist in Euler's book. Examples using the rule, matching those in our *Arithmetica*, have not been found in Suevus's and Meichsner's books.

4. Summary and conclusions

The quotations and comparisons above witness that the European tradition of arithmetic textbooks was practiced in Iceland. The Icelandic *Arithmetica – That is Reckoning Art* has clear connections to the books *Arithmetica Historica, Die löbliche Rechenkunst* by Suevus (1593) and *Arithmetica Historica. Das ist: Rechenkunst* by Meichsner (1625a), both originating in Lutheran protestant towns.

There are even more examples common to Euler's *Einleitung zur Rechenkunst* than the other two books. Euler's book contains, however, neither root extractions nor the *Regula Trium*, and its second part is devoted to 'benannten Zahlen', monetary and measuring units, which *Arithmetica* does not touch upon. Moreover, Euler's book was published in 1738, while the Icelandic *Arithmetica* probably origins in 1721, when Euler was 14 years old.

Probably there existed more books of similar origin. More work is needed to trace the origin of *Arithmetica – That is Reckoning Art*.

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