Optimization Model for Assigning Teachers to Classes

Inga Lilja Eiríksdóttir

Thesis of 30 ECTS credits
Master of Science (M.Sc.) in Engineering Management

June 2016

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at Reykjavík University in partial fulfillment of
the requirements for the degree of
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Abstract

Before every semester the administrators of upper secondary schools in Iceland face multiple challenges, one of which is assigning teachers to classes. The assignment problem became harder for the administrators in August 2015 when a new work evaluation system was set in place. In this thesis a mathematical model is presented that is intended to help the administrators with the assignment process.

The goal of the model is to ensure equality between teachers while at the same time trying to grant their wishes of courses to teach for the semester. An unfair division of classes and the work evaluation can cause disunity and frustration between teachers so it is important that the preferences of the teachers are met in the best way possible. Job satisfaction is an important presumption for success.

The results show that an integer programming model can be used to assign teachers to classes. The model is a type of a Generalized Assignment Model. Two versions of the model are presented in this study and it can be concluded from the results that the latter model, which calculates different values for vocational teacher and academic teachers, gives a better solution. By comparing the results to real data it can be concluded that the model serves the purpose of ensuring equality between teachers reasonably well.

Keywords: Optimization, GAP, work evaluation.

Heiltölubestunarlíkan fyrir úthlutun námshópa til kennara

Inga Lilja Eiríksdóttir

júní 2016

Útdráttur

Á hverju ári standa skólastjórnendur í framhaldsskólum á Íslandi frammi fyrir mörgum vandamálum, eitt af þeim vandamálum er að raða kennurum niður á námshópa. Í ágúst 2015 var komið á nýju vinnumats-kerfi í framhaldsskólum sem gerði þessa úthlutun mun erfiðari. Í þessari ritgerð er sett fram stærðfræði-líkan sem gæti hjálpað skólastjórnendum með úthlutun á hópum.

Markmið líkansins er að tryggja jafnræði á milli kennara um leið og það reynir að koma til móts við óskir hvers og eins og tryggja gæði kennslunnar. Ósanngjörn skipting á hópum og vinnumatið sjálft getur skapað óeiningu á milli kennara svo það er mikilvægt að líkanið tryggi markmiðið á sem besta máta. Starfsánægja er mikilvæg forenda fyrir árangri í starfi.

Niðurstöðurnar sýna að heiltölubestunarlíkan getur verið góð leið til þess að raða kennurum á hópa. Tvær útgáfur af líkaninu eru settar fram í ritgerðinni og má draga þá ályktun út frá niðurstöðum að seinna líkanið, sem reiknar mismunandi gildi fyrir iðngreinakennara og bókgreinakennara, gefi betri lausn. Með því að bera saman niðurstöður líkansins og raunveruleg gögn má álykta að líkanið nái markmiði sínu að tryggja jafnræði á milli kennara nokkuð vel.

Lykilorð: Bestun, GAP, vinnumat.

I dedicate this thesis to my daughter Þorbjörg Eiríka

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Chapter 1

Introduction

Upper secondary education in Iceland is not compulsory, but everyone has the right to enter an upper secondary school if they have completed compulsory education. There are over 30 upper secondary schools in Iceland [1]. The students are predominantly from the ages of 16 to 20 years old. All students have the right to a proper education. Upper secondary schools offer around 100 courses of study that lead to certain qualification levels. The length of the courses in vocational education vary from one to ten semesters, but most prevalent are eight semester courses [2].

Every semester the administrators of the upper secondary schools face multiple challenges. Administrators have to review the students' selection of courses and evaluate the number of students selecting each course. The next step is to evaluate the number of groups depending on the number of students. A proposal of a class schedule for the next semester has to be created and groups assigned to classrooms. The quality of teaching has to be ensured and teachers assigned to each class (group within each course).

In August 1, 2015 a new work evaluation system was introduced in upper secondary schools in Iceland. The system divides the teachers' work into three aspects, A, B and C.

Aspect A includes teaching and other factors associated with teaching according to the evaluation. The educational factor is measured by: the type of teaching, the size and composition of the student group, teaching preparation, the students' credits and other variations that affect the overall work of different classes. Individual factors also influence the evaluation, such as the number of educational materials a teacher uses in his teaching and possibly more.

Aspect B includes tasks that are a permanent part of a teachers obligations at the school, other than teaching, and in aspect C are projects / additional work that teachers take on or perform in consultation with the administrators [3].

The task is to assign teachers to classes with regards to aspect A in the new evaluation system. The problem is essentially an assignment problem [4]. It is important that the assignment ensures equality between teachers and the quality of the education. How teachers are assigned to classes can cause a lot of strain or conflict among teachers, which is harmful to the work environment of the school.

How teachers are assigned to classes varies between schools. Naturally, the larger the school the more complex the division gets, however, virtually all schools use the same method; assigning teachers manually to classes. It is hard to get a clear picture of what the evaluation system will look like in the beginning of the semester since the number of students is a big part of the evaluation criteria; therefore administrators must have a good overview of the number of students before teachers are assigned to classes. Administrators of several upper secondary schools were interviewed to gain better perspective of the division process.

The question is: Can optimization be used to improve the assignment of teachers to classes in upper secondary schools?

The aim of the project is to develop a financially efficient solution for upper secondary schools that ensures equality between teachers. Since the problem is an assignment problem an integer programming model will be constructed [4].

The paper is constructed as following: Chapter 2 contains a brief overview of the background of the evaluation system and current assignment process. Chapter 3 is a literature review. In chapter 4 the model is formulated and explained. Chapter 5 describes the data used for the model. Chapter 6 contains the results and conclusions and notes for further work are listed in chapter 7.

Chapter 2

Background

In this chapter an overview of the upper secondary schools and the new evaluation system will be provided. Five interviews were taken with administrators of four upper secondary schools which all have a different school structure.

2.1 Evaluation system

The evaluation of a teacher's work-load for each class is based on a course description for each course, which is made by teachers and administrators of the schools. The following factors provide the basis of the work evaluation:

- **Dimension**: How many ECTS units the course gives.
- **Number of students**.
- **Composition of the student groups** i.e. educational status of students.
- **Teaching practices**: Timeframe, teaching format (for example distance learning), learning speed and review.
- **Preparation**: Preparation in the beginning, continuous preparation, feedback and processing. New courses, the remaking of courses and interdisciplinary teaching.
- **Assessment**: Projects, exams and feedback.
- **Teaching material**: Does teaching material need to be constructed or not.
- **Competence level**: Courses are divided into 4 competence levels.
- **Individual factors**: Number of teaching materials, number of classes (groups) in the same course (synergic effect) and more.

Aspect A is based on the work evaluation for each class. Working hours in aspect A for a full time job are 1.440 hours per year, 720 hours per semester [3].

2.1.1 Age discount

When a teacher reaches a certain age a certain "teaching discount" of aspect A is awarded. Teachers from the age of 30 - 37 years get a 12 working hour discount per semester. For 38 years old and older the discount per semester is 24 working hours. 55 - 59 year old teachers get an additional 4,17% discount of aspect A and at 60 years or older, the teachers get an additional 20,83% discount of aspect A [3].

2.1.2 Repeated courses

If a teacher teaches two or more groups in the same course the evaluation is reduced according to the following:

- The average number of students in the groups is used to calculate the evaluation.
- The evaluation is reduced by:
    - 5% for each group if two groups are the same course.
    - 6,67% for every group if three groups are the same course.
    - 7,5% for every group if four groups are the same course and so on [5].

2.2 Current assignment process

Administrators of four upper secondary schools were interviewed to gain a better perspective of the division process.

In *Fjölbrautaskóli Suðurnesja* (FS) the teachers submit their wishes for which courses they would like to teach to the head of each department before the beginning of each semester. The heads of departments then assign teachers to the classes in cooperation with the administrators. The classes taught each semester have to be ready when this is done and each class should have its spot in the schools schedule. The administrators have to consider number of students in each class to divide the workload fairly [6].

Fjölbrautaskólinn í Mosfellsbæ (FMos) is a small school so the division at FMos is not complicated. Only their main courses include more than one teacher per course but otherwise there is only one teacher per course which makes the division much easier.

Administrators calculate the evaluation system before the semester for every class and then assign teacher to the classes. They call every teacher to a meeting and show them how the evaluation will look like for next semester and the teachers get a chance to comment on the different implementations of the evaluation [7] [8].

The division of assignments in *Kvennaskólinn í Reykjavík* (Kvennó) is done in groups under the instruction of the administrator of each department; the Icelandic teachers divide the Icelandic courses, math teachers divide the math courses and so on. In some cases teachers teach more than one course, for example biology and mathematics and then these departments need to cooperate.

The teachers at Kvennó have the evaluation system in mind when they assign teachers to classes and try to make the division as fair as possible [9].

Verkmenntaskólinn á Akureyri (VMA) is a large school and the division for a school like VMA is really complicated, it is hard to see what the evaluation will look like while assigning teachers to classes. Classes available for each semester are put into an excel worksheet and teachers are assigned manually to classes like in the other schools. The problem with the division is first and foremost how late the number of students becomes clear and the division can change easily when students move between classes [10].

Chapter 3

Method

This chapter introduces a classic assignment problem, a generalized assignment problem and a few applications of the generalized assignment problem.

3.1 Integer programming

As mentioned above, the problem is in the assignment itself. An integer programming problem is a linear programming problem where at least one of the variables is restricted to integer values [11].

Integer programming is a powerful modeling framework that provides flexibility for expressing discrete optimization problems. An important use of a binary variable x is to encode a choice between two alternatives [12].

The assignment problem is a specific type of linear programming problem where the assignees are being assigned to perform tasks.

The assignment problem uses the following decision variables:

$$x_{ij} = \begin{cases} 1 & \text{if assignee } i \text{ performs task } j \\ 0 & \text{if not,} \end{cases}$$

The assignment problem model is:

$$\text{minimize } \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{3.1}$$

$$\text{subject to } \sum_{j=1}^{n} x_{ij} = 1 \qquad \text{for } i = 1, 2, ..., n, \tag{3.2}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad \text{for } j = 1, 2, ..., n, \tag{3.3}$$

$$x_{ij} \geq 0, \qquad \text{for all } i \text{ and } j. \tag{3.4}$$

The assignment of the assignees to a task is associated with a cost c_{ij}. The objective is to minimize the overall cost [4].

3.2 Generalized assignment problem

Generalized Assignment Problem (GAP) is a well-known combinatorial optimization problem. The GAP is to find the optimal assignment of n agents to m tasks where each has fixed capacity availability [13].

Ross and Soland described the GAP as follows:

> It is a generalization of the ordinary assignment problem of linear programming in which multiple assignments of tasks to agents are limited by some resource available to the agents [14].

The mathematical formulation of the GAP is:

$$\text{minimize } \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} \tag{3.5}$$

$$\text{subject to } \sum_{j=1}^{n} r_{ij} x_{ij} \leq b_i \quad \forall i \in I \tag{3.6}$$

$$\sum_{i \in I} x_{ij} = 1 \quad \forall j \in J \tag{3.7}$$

$$x_{ij} \in \{0,1\}, \forall i, j \tag{3.8}$$

$$\text{where } x_{ij} = \begin{cases} 1 & \text{if agent } i \text{ is assigned to task } j \\ 0 & \text{otherwise,} \end{cases}$$

c_{ij} is the cost of assigning agent i to task j, b_i is the capacity of agent i and r_{ij} is the weight of agent i if assigned to task j [14]. The objective function is to minimize the total assignment cost of tasks to agents. Constraint (3.6) indicates the capacity availability restriction of each agent and is referred to as the capacity constraint. Constraint (3.7) ensures that each task is assigned to exactly one agent and constraint (3.8) enforces the integrality condition on the decision variables [13].

Sahni and Gonzalez showed that the GAP is NP-hard [15].

3.3 GAP Applications

3.3.1 The weighted Assignment problem (WAP)

Öncan considers the GAP to be a special case of the WAP. The WAP is to find the optimal assignment of a set of tasks to a set of agents such that each task is performed by one and only one agent. These tasks may be completed at one of several performance levels. The WAP becomes the GAP when there is no lower limit on the resource consumption, hence, there is no boundary on the used resource and there is only a single performance level for all task-agent pairs [13].

Ross and Zoltners have discussed several problems as a special case of the WAP: The Transignment Problem, the Multiple-Choice Knapsack Problem and the GAP. They have also discussed the solution algorithms and applications of the WAP and its variations [16].

3.3.2 The elastic GAP (EGAP)

The elastic version of the GAP, Elastic GAP, is where agents are allowed to violate capacity constraints at an additional cost. A non-negative under time and overtime variables are defined, which state the unused resource of agent i and the additional resource used by agent i [17].

3.3.3 Scheduling applications

Applications of the GAP appear within many scheduling problems. Employee scheduling, machine scheduling, workforce planning, classroom scheduling, batching, etc [13].

Ferland showed how assignment-type problem and its generalized version are very suitable for formulating timetabling and scheduling problems, where time periods have to be determined for activities according to particular constraints.

Ferland shows how the GAP can be used to establish a schedule of lectures according to student registrations and lecturer and classroom availabilities. The lectures are the items i and the starting times allowed are the resources j. The problem was solved with Tabu search technique and exchange procedure. He also showed applications for Internship Scheduling, Preventive Maintenance Scheduling, Sports League Scheduling and Nurse Scheduling [18].

Chapter 4

The model

In this chapter the model is presented and explained.

4.1 Constraints

The constraints of the model cannot be violated at any cost.

The constraints of the model are:

- Every group has to have one teacher.
- Teachers must be assigned to teaching hours within their interval provided every semester. The interval takes into account each teachers employment rate:
    - 100% work: 720 teaching hours
    - 75% work: 540 teaching hours
    - 50% work: 360 teaching hours
    - 25% work: 180 teaching hours

4.2 Objective Function

The objective function is used to ensure equality between teachers while trying to grant their wishes for courses to teach. By getting the preferences of the teachers it is possible to maximize the likelihood that the teachers will be happy with the result.

The decision variables in the model will be binary, indicating whether or not a certain teacher is assigned to a certain group.

The workload for every teacher is calculated from equation (4.3).

4.3 Model description

Indices

The model uses two indices which represent the teachers and the groups.

T : set of teachers, $n = |T|$

G : set of groups, $m = |G|$

Data

The following datasets are used as inputs for the model:

V_j : Work evaluation, hours per semester, for group $j \in G$.

A_{ij} : Matrix where (i, j) gives 1 if teacher i can teach group j, 0 otherwise, for all $i \in T$ and $j \in G$.

a_i : Age discount, hours per semester, for teacher $i \in T$.

$W_{i\min}$ and $W_{i\max}$: The interval for work evaluation, hours per semester, depending on each teacher's employment rate.

P_{ij} : A preference matrix where (i, j) contains teachers preferences, ranked from 1-3, for all $i \in T$ and $j \in G$.

Decision Variables

$$x_{ij} = \begin{cases} 1 \text{ if teacher } i \text{ teaches group } j; \forall i \in T, \forall j \in G \\ 0 \text{ otherwise} \end{cases}$$

W_i is used to calculate overall workload for each teacher according to the groups he has been assigned to.

Z is the maximum teaching hours possible for all teachers. Z is higher than every W_i $\forall i \in T$.

Objective Function

$$\text{Min } \alpha Z - \sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij} x_{ij} \tag{4.1}$$

Constraints

$$\sum_{i=1}^{n} A_{ij} x_{ij} = 1, \forall j \in G \tag{4.2}$$

$$\sum_{j=1}^{m} V_j \cdot x_{ij} = W_i + a_i, \ \forall i \in T \tag{4.3}$$

$$Z - (W_i + a_i) \geq 0, \ \forall i \in T \tag{4.4}$$

$$W_{i\min} \leq W_i \leq W_{i\max}, \ \forall i \in T \tag{4.5}$$

$$x_{ij} = \{0,1\}, \ \forall i \in T, \ \forall j \in G \tag{4.6}$$

The objective function minimizes Z to ensure equality between teachers in terms of workload and maximizes the preference of the teachers. The weight factor, α, can be used to determine which should have more value; the equality between teachers or their preferences. Chapter 5 contains the description of how the matrix that contains teachers preferences, P_{ij}, is formulated. Constraint (4.2) ensures that every class has one teacher. Constraint (4.3) calculates overall workload, W, for every teacher i. Constraint (4.4) makes sure that the maximum workload for all teachers, Z, is larger than overall workload, W, for every teacher i and constraint (4.5) ensures that every teacher gets teaching hours according to their contract. Constraint (4.6) ensures that the decision variable x_{ij} is binary. Z and W_i are larger, or equal, to zero.

4.4 Limitations of the model

If a teacher teaches two or more groups of the same course a reduction to the evaluation is made according to rules shown in chapter 2.1.2. In the current formulation no constraint ensures these rules. It turned out to be quite difficult to formulate these rules so a decision was made to run the model twice to get a reasonable solution.

The solution from the first run is evaluated and the variables where teacher is assigned to two or more groups in the same course are fixed to 1, the evaluation for the courses reduced and the model run again. When this is done it is important to change the preference matrix as well if a teacher only wants to teach two groups in the same course but not more.

A possible formulation of this constraint would be to create subsets of all courses that have two or more groups. The sum over the subsets is calculated and if the sum over each subset for every teacher is larger than 1, then a corresponding binary variable is activated and a reduction would be made to the work evaluation according to the rules in chapter 2.1.2.

Chapter 5

Data

This chapter describes the arithmetic model that was used to collect data and the data that was collected from Fjölbrautaskóli Suðurnesja (FS).

5.1 Arithmetic model for the new work evaluation system

A project group was formed around the new work evaluation system by the Icelandic Teachers Union (KÍ) and the Ministry of Education. The group oversees the preparation and instillation of the evaluation system in the upper secondary schools according to wage contract. The group consists of representatives from the Association of Teachers in upper secondary schools (Félag framhaldsskólakennara), the Association of Deputy Headteachers in upper secondary schools (Félag stjórnenda í framhaldsskólum), the Ministry of Education and the Ministry of Finance.

The main work of the project group was to form five evaluation committees, which each evaluated the work for every course in their teaching area. Based on this evaluation the project group developed an excel arithmetic model (reikniverk) to use for calculations for the evaluation system [19].

5.2 Data collection

The data required for the model is: courses available, number of teachers and their ID numbers, evaluation of every group of available courses and the interval of teaching hours for every teacher.

Real data was collected from FS. The courses available for the autumn of 2015 were used to develop the model. The courses were written into the excel sheet described in chapter 5.1 and the vector V_j for evaluation for each group obtained from there. The number of groups of courses available for fall 2015 was 305.

The number of teachers in FS was used for the model, number of teachers in the autumn of 2015 were 63.

A matrix, A_{ij}, of the size 305 times 63 was created, were 305 is the number of groups and 63 is the number of teachers. Each teacher was given the number 1 for every course that he could teach and 0 if he could not teach the course. In this matrix every math teacher gets 1 for the math courses, English teacher 1 for the English courses and so on. The number of teachers that can teach each course varies from 1 to 9 teachers per course. The number of groups a teacher can teach varies from 1 to 30 groups.

The age discounts for each teacher was calculated in excel from their ID number according to the age discount rule covered in chapter 2.1.1.

$W_{i\min}$ and $W_{i\max}$ was approximated from each teachers employment contract and the work evaluation. $W_{i\min}$ and $W_{i\max}$ can vary for each teacher depending on their, or the schools, wishes. $W_{i\min}$ and $W_{i\max}$ have to be carefully calculated for every teacher taking into account the teachers' age and specialities along with employment contract and wishes.

A preference matrix, P_{ij}, of the size 305 times 63 was created. The preference matrix states if the teacher wants and is able to teach a course. A utility of 3 was given if the teacher is very qualified and wants to teach a course, 2 if the teacher is qualified to teach a course and 1 of a teacher is qualified to teach a course but doesn't necessarily want to.

The data for the model can be provided for each school by INNA, the IT system for upper secondary schools in Iceland.

Chapter 6

Results

This chapter introduces the results of the model and a suggestion of improvement of the model. Comparison of the model results and real data is then made.

6.1 Model results

The model was solved using Gurobi 6.5.1. Variables were 19279, thereof 19215 integer variables, and constraints were 557.

The current relative MIP optimality gap shows how far from the optimal solution the solution is and is used to measure the quality of the solution. If an optimal solution is not found it is preferable to be as close to the optimal solution as possible [20]. The equation for the MIP gap is:

$$\frac{ObjBound - ObjVal}{|ObjVal|} \tag{6.1}$$

Where $ObjBound$ is the MIP objective bound and the $ObjVal$ is the incumbent objective solution [20]. The result was an MIP gap of 0% which means that the obtimal solution is found. The running time for the model was just under 1 minute which is good for running the model more than one time.

The result for overall workload, W_i, can be seen in figure 6.1.

If the weight factor (α) is larger than one it gives the equality between teachers more value, if the weight factor is smaller than one the teachers' preferences get more value. The model shows minor changes in overall workload, W_i, when changing the weight factor.

Table 6.1 shows how adding a weight factor of α = 10, to add weight on Z, and a weight factor of α = 0,1, to add weight on P_{ij}, affects the teachers' preferences. The table shows the percentage of teachers getting the classes they most want to teach, the classes they are qualified to teach and least want to teach, according to the preference matrix P_{ij}. All models serve very well the purpose of assigning teachers to the classes they most want. Although all the models give a good solution the preferences of the teachers was best met with weight on P_{ij}.

Z, calculated from the formula $Z-(W_i+a_i)\geq 0,\ \forall i \in T$, got the value 1115 which corresponds to the maximum workload (number of teaching hours) a teacher was given. The Z gets this high value because of the vocational teachers. The school has few vocational teachers so the workload can't spread on as many teachers as in other courses. A solution to that problem would be to calculate two different values, to ensure equality between teachers in terms of workload, for vocational teachers and academic teachers. A third value could be calculated for teachers who are not hired for a full time job. As seen in figure 6.1 W_i was a minimum of 164 working hours for a teacher in a part time job and goes up to 1100 working hours for a teacher holding a full position.

Table 6.1: shows how adding a weight factor of α = 10, to add weight on Z, and a weight factor of α = 0,1, to add weight on P_{ij}, affects the teachers' preferences. The table shows the percentage of teachers getting the classes they most want to teach, the classes they are qualified to teach and least want to teach, according to the preference matrix P_{ij}.

Preferences	No Weight: α = 1	Weight on Z: α = 10	Weight on P: α = 0,1
3	86,2%	86,9%	87,9%
2	11,5%	10,5%	10,5%
1	2,3%	2,6%	1,6%

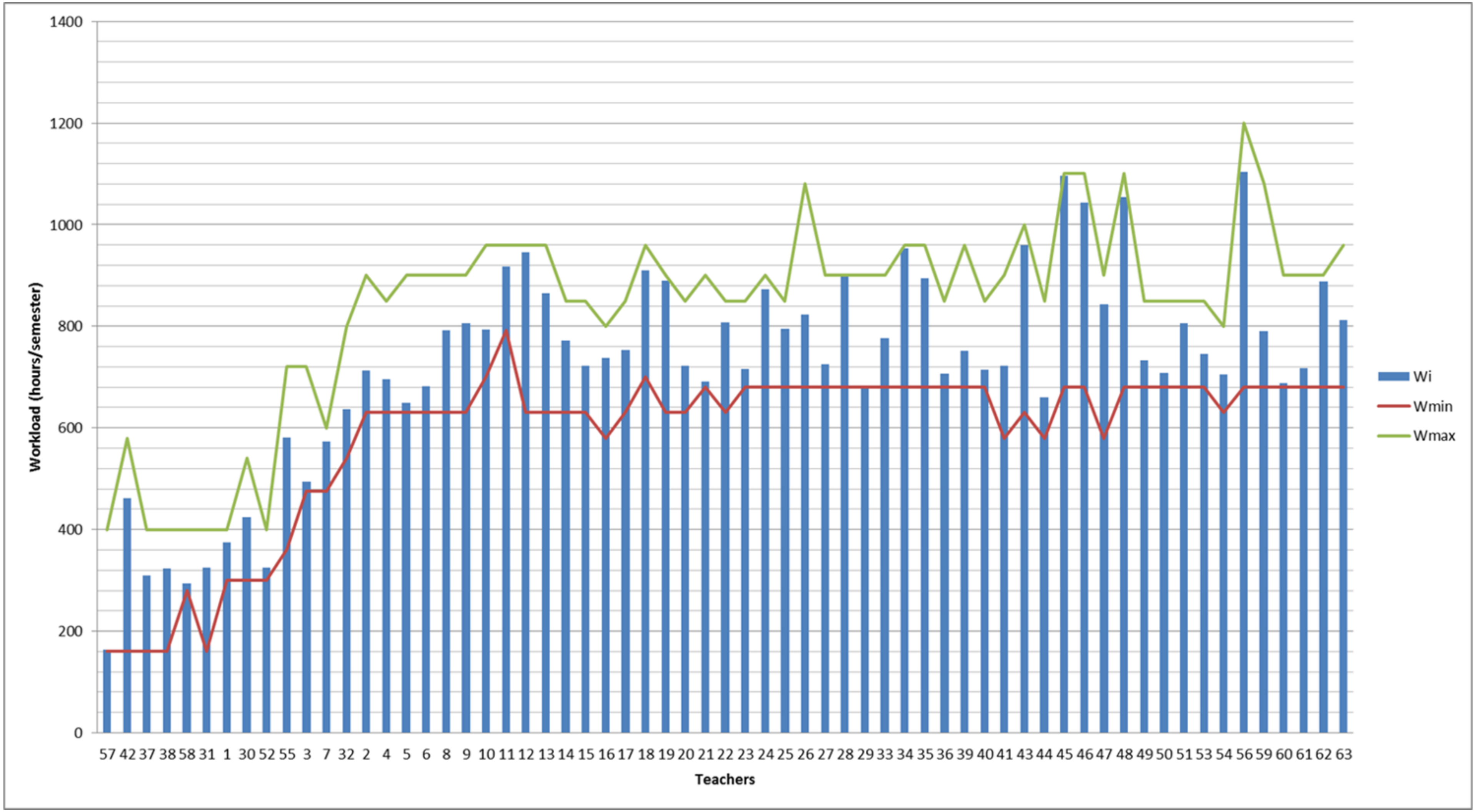

Figure 6.1: Workload, Wimin and Wimax for teachers.

6.2 Different value for vocational teachers

As described in chapter 6.1 the variable storing the maximum workload over all teachers, Z, gets a high value because of the workload of vocational teachers. A solution to this could be to calculate a different value for the vocational teachers.

The Objective function would then look like this:

$$\text{Min } \alpha(\text{Z} + \text{Y}) - \sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij} x_{ij} \tag{6.2}$$

And a constraint would be added as well:

$$Y - (W_i + a_i) \geq 0, \ \forall i \in V \tag{6.3}$$

Where V is a subset of all vocational teachers.

The MPL model for different value for vocational teachers can be seen in Appendix B.

The model was run once so the repeated courses rule from chapter 2.1.2 was ignored. The model gets a value of Z = 926,9 and Y = 1154,9. The workload for the teachers can be seen in figure 6.2.

For simplification the original model will be referred to as Model Z and the model with different value for vocational teachers as Model Z + Y.

6.3 Comparison

By comparing the results from Model Z and Model Z + Y it can be assumed that the latter one gives the better outcome. The Standard Deviation (St. Dev.) and Mean Absolute Deviation (MAD) from $W_{\min}$ and $W_{\max}$ for every teacher was calculated for both models. There is no change in the MAD but Model Z + Y gives a lower value in St. Dev. which indicates that it serves the purpose of ensuring equality between teachers slightly better. The Coefficient of Variation was calculated to get a better comparison for the models.

Table 6.2 shows a comparison of the two models presented and Mean Absolute Deviation, Standard Deviation and the Coefficient of Variation for the actual workload of teachers in FS in the autumn of 2015. By comparing the Coefficient of Variation of the three cases it can be assumed that the models presented give a better solution to the assignment problem in upper secondary shools than assigning teachers manually to classes because they give lower values for MAD, St. Dev. and the Coefficient of Variation.

Table 6.2: Comparison of MAD and St. Dev for both models and real workload.

	MAD	St. Dev.	Co. Of Var.
Model Z	133,8	92,7	69%
Model Z + Y	133,8	90,8	68%
Real workload	145,7	114,2	78%

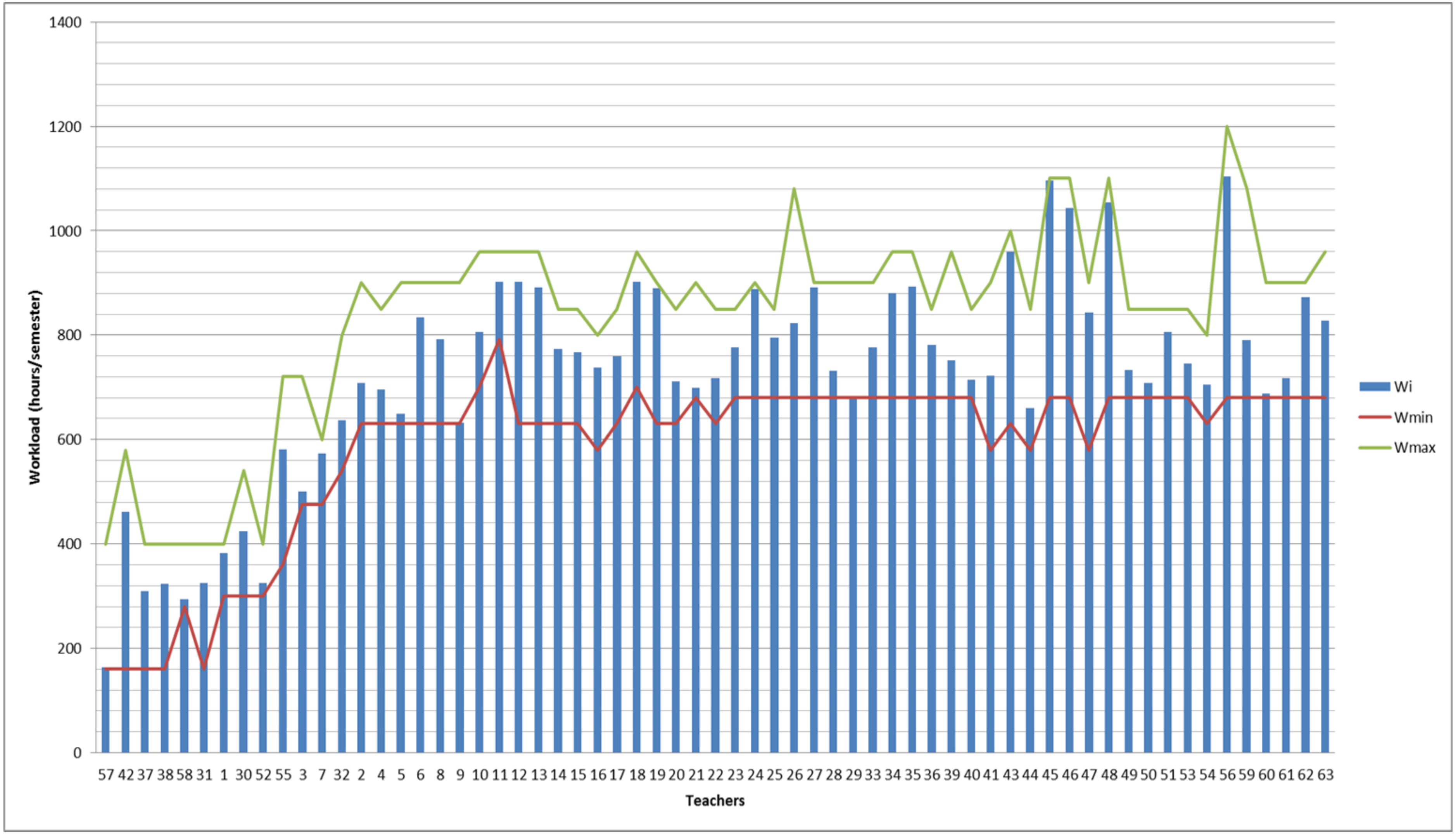

Figure 6.2: Workload, Wimin and Wimax for teachers in Model Z + Y.

Chapter 7

Conclusions

In this study a mathematical model was introduced to assign teachers to classes and distribute their workload fairly in upper secondary schools in Iceland. The results show that the model could be a good tool to help the administrators of the schools with the assignment process; the model shows better results for equality between teachers in terms of workload than assigning teacher manually to classes and is a more neutral solution. Calculating two different values to ensure equality between teachers for academic teachers and vocational teachers would be recommended since that gives a better solution. The model imitates real conditions very well and the only limitations of the model are the repeated courses rules described in chapter 2.1.2. There is no technical obstruction of formpeatulating these rules according to the ideas expressed in chapter 4.4 and it could have been done with a little more time. The short running time of the model makes it more suitable for additions.

The work evaluation for each group, V_j, can only be an approximation of what the evaluation will look like because the final evaluation is not made until three weeks into the semester. Until then students can switch courses and drop out altogether if they wish. The results from the model can therefor never be the final results but can be used as a tool for the administrators to make the assignment process neutral and effective.

The EGAP, described in chapter 3.3.2, is a powerful tool in these calculations. If the model gives an infeasible solution the EGAP can be used to get a solution. Variables for under-time and overtime are created and the model is run again. This way the administrators can see which teachers need to accept more teaching hours.

The IT system for the upper secondary schools, INNA, can provide the schools all the data needed for the model in excel. The model provides option for the administrators to work with. The administrators can use the solution provided by the optimization model and make a final personal touch on the solution based on their knowledge and measured assessment, taking into account variables that are not available using data alone. Future work with this model would be to program it into INNA and also to take into account the reduction rules for repeated courses described in chapter 2.1.2.

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Appendix A

MPL Model Z

```
TITLE      Vinnumat;

OPTIONS    ExcelWorkbook="MPL_Vinnumat.xlsx"

           ExcelSheetName="Vinnumat"

INDEX      i := EXCELRANGE("ii");

           j := EXCELRANGE("jj");

DATA       A[i,j] := EXCELRANGE("Aij");

           V[j] := EXCELRANGE("Vj");

           a[i] := EXCELRANGE("ai");

           Wmin[i] := EXCELRANGE("Wmin");

           Wmax[i] := EXCELRANGE("Wmax");

           P[i,j] := EXCELRANGE("Pij");

DECISION VARIABLES

           x[i,j] EXPORT TO EXCELRANGE("Xij");

           W[i] EXPORT TO EXCELRANGE("Wi");

           Z EXPORT TO EXCELRANGE("Z");

OBJECTIVE

           MIN Z - SUM(i,j: P * x)

SUBJECT TO

           CapG[j]: SUM(i: x) = 1;

           Workload[i]: SUM(j: V * x) - W - a = 0;

           MaxZ[i]: Z - (W + a) >= 0;

           MinW[i]: Wmin <= W ;

           MaxW[i]: W <= Wmax;
```

```
BINARY

        x;

BOUNDS

        Zerox[i,j] WHERE (A[i,j] =0): x =0;

END
```

Appendix B

MPL Model Z + Y

```
TITLE     Vinnumat;

OPTIONS   ExcelWorkbook="MPL_Vinnumat.xlsx"

          ExcelSheetName="Vinnumat"

INDEX     i := EXCELRANGE("ii");

          j := EXCELRANGE("jj");

          Voc[i] := (43,44,45,46,47,48,56,59);

          Aca[i] := i - Voc;

DATA      A[i,j] := EXCELRANGE("Aij");

          V[j] := EXCELRANGE("Vj");

          a[i] := EXCELRANGE("ai");

          Wmin[i] := EXCELRANGE("Wmin");

          Wmax[i] := EXCELRANGE("Wmax");

          P[i,j] := EXCELRANGE("Pij");

DECISION VARIABLES

          x[i,j] EXPORT TO EXCELRANGE("Xij");

          W[i] EXPORT TO EXCELRANGE("Wi");

          Z EXPORT TO EXCELRANGE("Z");

          Y EXPORT TO EXCELRANGE("Y");

OBJECTIVE

          MIN Z + Y - SUM(i,j: P * x)

SUBJECT TO

          CapG[j]: SUM(i: x) = 1;

          Workload[i]: SUM(j: V * x) - W - a = 0;

          MaxZ[Aca]: Z - (W + a) >= 0;

          MaxY[Voc]: Y - (W + a) >= 0;
```

```
        MinW[i]: Wmin <= W;

        MaxW[i]: W <= Wmax;

BINARY

        x;

BOUNDS

        Zerox[i,j] WHERE (A[i,j] =0): x =0;

END
```

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