Models for Asset Price Bubbles

Einar Halldórsson

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by

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Abstract

An asset price bubble emerges when the price of an asset exceeds its fundamental or intrinsic value. Models and theories for price bubbles have been split into two categories: rational and irrational. Rational models assume that all agents have rational expectations whereas irrational models account for psychological behaviour such as herding, moral hazard, extrapolation and informational cascades. Theoretical models for rational and irrational models are reviewed in this thesis, along with empirical tests with the main focus on the price/earnings ratio of stocks. The theoretical models provide the necessary framework to deepen our understanding of asset price bubbles and the cyclically adjusted price/earnings ratio can give the indication of an emerging bubble.

Líkön fyrir verðbólur á eignum

Einar Halldórsson

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Útdráttur

Verðbólur á eignum verða þegar markaðsverð á eign er yfir innra virði þess. Líkönum og kenningum fyrir verðbólur hefur verið skipt í 2 flokka: Rökrétt og órökrétt(e. Rational and Irrational). Rökrétt líkön gera ráð fyrir að allir fjárfestar hafi rökréttar væntingar á meðan órökrétt líkön gera ráð fyrir að fjárfestar verði fyrir áhrifum af sálfræðilegum þáttum og hegðunarmynstri. Fræðilegum líkönum fyrir rökréttar og órökréttar verðbólur verður gert skil í þessari ritgerð ásamt praktískum líkönum þar sem aðaláherslan er á hlutfall verðs og hagnaðar á verðbréfum. Fræðilegu líkönin veita nauðsynlegan grunn fyrir almennan skilning á verðbólum og hlutfall verðs og hagnaðar með hlaupandi meðaltali getur gefið til kynna ef verðbóla er að myndast.

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1 Introduction

Asset pricing is driven by the law of supply and demand, and the price is continuously changing as the market reacts to new information. The information can be new economic reports, sales figures, political events, monetary policy, wars etc. which leads to changes in expectations, consequently affecting the price.

Asset price bubbles occur when prices of assets exceed their market fundamental or intrinsic value. Kindleberger and O'Keefe (2001) described the price deviation as a temporary displacement in the asset price. Bubbles can be hard to predict and are usually only termed after they have burst. Part of the reason why asset price bubbles are not identified until after they burst is that the perception of an asset's intrinsic value tends to increase with the market price. An increase in price is thus termed as "the new normal", which can lead to irrational behaviour of investors. Price bubbles of assets are common in history and people tend to think ”This time it will be different”, repeating the same mistakes. A crises follows after a bubble has burst, with the severity depending on price escalations and size of the bubble. The crises which follow is usually called a recession or for a more severe case, depression. Forecasting the bubble beforehand can therefore be crucial to dampen the crises.

Models for price bubbles are usually categorized in to two categories: rational and irrational. Rational models assume that all agents have rational expectations whereas irrational models account for psychological behavior such as herding, moral hazard, extrapolation and informational cascades.

One of the first individuals to consider asset price bubbles was Keynes (1936), with his conclusion that bubbles could form given that investors can behave irrationally. However, most of the early literature focused on rational bubble models, where bubbles can emerge even though investors have rational expectations. With rising population growth and new technology, the economy is continuously changing. Nordhaus (2001) suggests that productivity growth and higher stock prices are related to the recent information technology revolution. Higher consumption rates and population growth can also account for rise in prices. Researchers such as Van Norden et al. (1996) and Wu (1997) have also stated that negative bubbles can emerge when assets are undervalued.

Scherbina and Schlusche (2014) derived that bubbles can not exist in prices for assets that are not infinitely-lived. The bubble would burst at the end of the asset's life, T, with no investor willing to purchase the asset at inflated prices at T-1 and by backward induction to the start of the asset's life, t_0, the bubble can not exist.

1.1 Liquidity

When interest rates are low consumers are reluctant to hold their capital in a regular savings account and often choose to invest in risky assets such as stocks and real estate, which in turn causes the prices of the assets to rise with a large enough number of investors. Liquidity and interest rates therefore have a high impact on price fluctuations. Central banks can increase the interest rates to slow down inflation and possible price bubbles, which consequently can cause widespread default on loans. Another measure which can be taken is to increase reserve capital requirements. Kindleberger and O'Keefe (2001) discuss 2 axioms which represent these measures: That inflation depends on the growth of money and asset price bubbles depend on the growth of credit.

The growth of credit, usually termed as credit expansion can be a factor in the emergence of bubbles. Bubbles with large credit expansion have been resembled to a Ponzi scheme[1] , requiring more capital and credit infinitely. Allen and Gale (2000) conclude that bubbles have 3 distinct phases.

1. **Credit Expansion**: The central bank increases lending, typically by lowering interest rates. This leads to an increase in price of assets for several years.
2. **Bubble bursts**: Asset prices collapse, often in a short period of time.
3. **Defaults on Loans**: Investors and firms default on loans after having bought assets at inflated prices.

The crises can last for years afterwards, slowing down economic growth, the severity depending on the amount of credit in play. Tirole (1985) argued that bubbles could hinder economic growth for multiple generations because of allocation of money towards more unproductive[2] assets or at least the ”useless” bubble component of the asset. The definition of a productive asset is an asset that pays dividends and/or income to the owner, either directly or indirectly.

1.2 Short Selling

Short selling is the market's solution for investors who believe that prices are too high and are willing to bet against the rising market trend. An investor who believes prices will rise will buy the asset, where as the investor who feels the prices might go down has to short sell.

Short selling involves borrowing an asset from a third party and selling it, then buying it back at a later time, anticipating that the price has fallen. This is not without problems because of premiums paid to the third party with the added risk to the investor that the third party can call back the asset at any time during the time period. Abreu and Brunnermeier (2003) outline that bubbles can persist because rational investors could be riding the bubble instead of shorting, it would bear too much risk to short the asset rather than trying to sell the asset before the bubble bursts.

Haruvy and Noussair (2006) argue that prices in experimental asset markets based on computer simulations are influenced by restrictions on short-selling capacity. Their model shows that if short sales are restricted then the price of the asset will be given by the most optimistic trader, and with short selling allowed pessimistic traders can drive down the prices.

[1]Ponzi scheme draws its name from Charles Ponzi who became infamous for the technique of paying high returns to investors by funding of new customers in the 1920's.

[2]Tirole states that while unproductive assets may pay dividends they do not increase capital accumulation, and thus future dividends in the economy.

1.3 Criticism of the Bubble term

Not everyone agrees that bubbles are a real phenomenon. Standard neoclassical economic theory assumes that all market participants are fully rational and any mispricing and potential bubbles will be eliminated by rational traders (Abreu and Brunnermeier, 2003).

The efficient-market hypothesis (EMH) states that asset prices follow a random walk[3] and on average reflect correct pricing and all available information. According to the hypothesis there are no price bubbles and it is impossible to beat the market. Fama (1970) reviews the theory and reflects on empirical work.

Santos and Woodford (1997) state that in a competitive equilibrium framework bubbles can not exist and that the conditions for bubbles are insubstantial.

1.4 Objective and Structure of Thesis

The thesis is intended to give insight into models of asset price bubbles and how conventional financial theory can explain price bubbles. Well-known rational and irrational models along with empirical tests of bubbles will be introduced with the focus on non-negative bubbles.

The structure of the thesis will begin with a short history of famous historical bubbles in chapter 2. The list is by no means exhaustive, but for an excellent survey on historical bubbles the reader is referred to Kindleberger and O'Keefe (2001). Asset pricing will be introduced in chapter 3 along with rational bubble and credit models. Irrational models and behavioural factors are reviewed in chapter 4. Chapters 3 and 4 mainly focus on theoretical models, which are often not ideal for empirical tests. The thesis moves on to some more practical and empirical tests in chapter 5 followed by conclusion and discussion in chapter 6. The main focus in the empirical tests is the Price/Earnings ratio, which shows promising signs of catching emerging price bubbles when price today is divided by cyclically adjusted earnings (CAPE ratio).

[3]Random walk considers small random deviations from the real price based on the idea that tomorrows price will depend on tomorrows news.

2 Historical Bubbles

Asset price bubbles, crashes and financial crises have occurred regularly through history. Although every bubble is different in their initiation and specific details, similarities and patterns are recurring and general assumptions can be made from them. This section provides a brief introduction of some of the more famous price bubbles.

2.1 Tulipmania

The tulip bubble usually referred to as the Tulipmania started 1634 in Holland and burst in 1637. Tulips had just been introduced into Europe from the Ottoman Empire and were thought of as a luxury item. When the flowers contracted a mosaic virus altering their appearance, the already rare tulip became more sought after and prices continued to rise, reaching a peak in 1637. Tulip buyers and speculators had been filling up their inventories and trading savings, land and real estate for bulbs causing the price to rise rapidly. At its peak the cost of one tulip could cost as much as a house in the finest street in Amsterdam (Garber, 1989), or roughly ten times the annual salary of a skilled craftsman. Investors then began to realize their profits causing a domino effect of selling and consequently prices to dive. Dealers began to dishonor their contracts and the Dutch parliament tried to intervene with offering a standard minimum rate for bulb contracts.

2.2 The South Sea Bubble

The South Sea Company was formed in 1711 and initially took over the national debt raised by the United Kingdom in War of Spanish Succession (1701-1714) in return for a promise of a monopoly of all trades to the Spanish colonies in South America. However it was well known that the Spanish blocked the trade route (Garber, 1990).

In January 1720 the price of one stock was £128. The firm had no assets but the directors of the company spread false claims of success of trading to South America and in February the price rose to £175. With the stock price rising it caused over-confidence in the company, leading to the price peak of £1050 in June. Although the South Sea bubble was the largest it was not the only bubble of 1720, which has often been named ”The Bubble year”. In July 1720 the government sanctioned the ”Bubble Act” which forbade formation of new joint-stock companies without approval of the government (Kindleberger and O’Keefe, 2001). Investors then began to sell their shares and by September the price had collapsed to £175.

2.3 The Great Crash of 1929

The 1920’s are known as ”The Roaring Twenties” for its technology and economic prosperities. Along with post-war optimism, new technology such as radio broadcasting, advances in medicine and many industrial and household inventions contributed to a rise of stock prices. The rising stock prices encouraged more people to invest, often financing their investment with borrowed credit. This over-confidence on the market and the belief that stock prices would rise even further created a speculative bubble leading to a crash on October 24, 1929, known as ”Black Thursday”. The following decade after the crash has been termed ”The Great Depression”, which affected all industrialized western countries.

2.4 Japan 1980's

In the 1980's real estate and equity prices rose rapidly in Japan. The Japanese Nikkei stock average hit an all-time high in 1989, crashing shortly afterwards. The collapse caused severe financial crises and economic stagnation for a long period. Japan had become the second largest economy in the world in the 1980's after the United States and overconfidence in the stock market along with the monetary policy of the Bank of Japan helped escalating the bubble. In 1989 the Japanese government wanted to hinder the growing asset price bubbles and tightened their monetary policy which caused investors to default on their positions triggering the collapse of the Nikkei stock price index. The fall of stock prices also caused the real estate bubble to burst. A long economic setback followed and the period between 1991-2000 has been named "The Lost Decade" in the Japanese economy (Qi, 2006).

2.5 Dot-Com Bubble

In 1991 the World Wide Web went live in the world and went more user friendly and commercial as the decade wore on. The Dot-Com bubble, where Dot-Com refers to the user domain (.com) of websites used by businesses, initiated in the mid-1990's where a huge belief in internet related stocks and assets accelerated quickly, reaching its peak in March 2000 when the bubble burst.

2.6 Housing Bubble

Up until the crash of the housing or real estate bubble in 2008, mortgages had become available for a much larger group of individuals. Sub-prime mortgages were given to consumers with poor credit history and banks would then put those mortgages into portfolios and sell other derivatives off them. When consumers defaulted it caused a domino-like effect in the system and banks heavily invested in sub-prime mortgages collapsed, along with many companies in the industry who had bought in. The bursting of the housing bubble had global consequences and feedback causing financial crashes around the world (Martin, 2011).

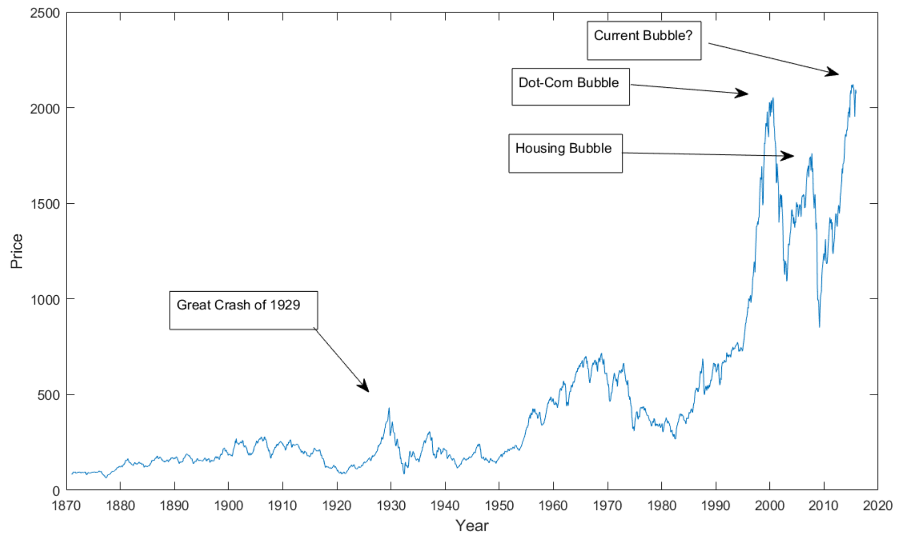


Figure 1: Price Bubbles in the S&P500 index

Figure 1 shows the S&P500 index from 1871 through 2015 presented with selected price bubbles in the US economy. Some researchers believe that a current asset price bubble is ongoing as the price today is at an all-time high as can clearly be seen in the figure. The prices are inflation adjusted which will be further introduced in chapter 5. The data contains monthly prices from Jan 1871 to Dec 2015 and is gathered from Shiller (2016).

3 Rational Bubbles

3.1 Definition

A Rational asset price bubble model assumes that all market participants have rational expectations of pricing based on a statistical expectation of the asset. All consumers have the same symmetrical information and bubbles can exist in infinite time horizon. The pricing is considered rational and there are no arbitrage opportunities. An example of an infinitely lived asset price bubble is fiat money[4], the currency which the government has declared to be legal tender without being backed by a physical commodity (Hoppe, 1994). Fiat money has zero intrinsic value and it's pricing is driven by demand and supply. Diba and Grossman (1988a) conclude that a rational bubble reflects a self-confirming belief that an asset's price depends on a variable that is intrinsically irrelevant, not part of the market fundamentals. The economist view is that the given price at any moment must reflect the market fundamentals, Blanchard and Watson (1982) explain their view of the economists:

> *It turns out that economists have overstated their case. Rationality of both behaviour and of expectations often does not imply that the price of an asset be equal to its fundamental value. In other words, there can he rational deviations of the price from this value, rational bubbles.*

Gürkaynak (2008) states that bubbles are rational if investors are willing to pay more for an asset than the present discounted value of dividends if they expect that they can sell the asset at an even higher price. However this can also hold true for irrational bubbles as will be introduced in chapter 4.

The rational bubble models introduced in this chapter show a general theoretical outline of variables and implications. In realistic models and empirical test the models would have more complex relations.

3.2 Asset Pricing

Financial theory states that the price of an asset is equal to its present expected value of dividends. If this equation holds in an infinite time horizon the fundamental pricing controls the equilibrium price. The standard pricing model for assets is

$$S_t = \frac{1}{1+r}\mathbb{E}_t(S_{t+1} + D_{t+1}). \tag{1}$$

Where S_t is the price, $\frac{1}{1+r}$ is the discount factor with the interest rate r and $\mathbb{E}_t(S_{t+1} + D_{t+1})$ the present value of expected price and dividends at time $t+1$ with $\mathbb{E}()$ as the conditional expectations operator. To illustrate, consider a stock which is expected to pay a dividend of 6 with a expected price of 60, one year from now. The interest rate or the required rate of return is 5%, the price at t=0 is:

$$S_0 = \frac{6}{1+0.05} + \frac{60}{1+0.05} = 62.86.$$

Solving equation 1 for the equilibrium price S_t with no bubbles and the transversality condition:

$$\lim_{i\to\infty}\Big(\frac{1}{1+r}\Big)^i\mathbb{E}_t(S_{t+i}) = 0, \tag{2}$$

[4]The United States dollar was backed up by gold until 1972, consumers could exchange their money for a gold before the gold standard was dropped.

the equation can be solved to

$$S_t = \sum_{i=1}^{\infty} \Big(\frac{1}{1+r}\Big)^i \mathbb{E}_t(D_{t+i}). \tag{3}$$

The transversality condition rules out bubbles and the price is always equal to the fundamental without any bubble component. Now, if an investor could sell the asset at a higher price than the discounted value of dividends, there exists a bubble and $\mathbb{E}_t(S_{t+1})$ is not zero. This would cause the price to go back to the fundamental value to ensure equilibrium.

A bubble model contains two parts, along with the standard definition of pricing there is a bubble component which is equal to the asset's expected price of the next period. The bubble term B_t is now introduced into the model:

$$S_t = \sum_{i=1}^{\infty} \Big(\frac{1}{1+r}\Big)^i \mathbb{E}_t(D_{t+i}) + B_t. \tag{4}$$

Where the bubble term satisfies:

$$B_t = \frac{1}{1+r} \mathbb{E}_t(B_{t+1}). \tag{5}$$

The new price is now the equilibrium price. The rational bubble is a basic component of the asset value in (4) rather than a irrational deviation of the price. B_t is independent from expected dividends. Equation (5) eliminates all arbitrage opportunities.

Example: There exists an asset which is expected to pay constant dividends of 5 for 3 years. The interest rate is also assumed to be a constant 5% for the period. The price of the asset today is 15, putting this into equation (4) gives

$$15 = \Big(\frac{1}{1+0.05}\Big)5 + \Big(\frac{1}{1+0.05}\Big)^2 5 + \Big(\frac{1}{1+0.05}\Big)^3 5 + B_0,$$
$$B_t = 1.38.$$

So there exists a bubble of 1.38 on the asset, rational investors may hold the asset if they expect the interest rate or dividends to change. Equation (5) shows that the bubble component must grow at the rate of interest, so the expected bubble at year 1 becomes

$$1.38 = \frac{1}{1+0.05} \mathbb{E}_0(B_1),$$
$$\mathbb{E}_0(B_1) = 1.45.$$

Which rules out arbitrage opportunities and ensures equilibrium pricing. In previous example the dividends remained constant for the asset, however for stocks of large companies it can be assumed that they must grow at a constant rate and therefore their dividends as well. The Gordon growth model named after Myron J. Gordon can be used for large companies or companies that are growing at a steady rate, the formula for the growth model is

$$S_t = \frac{\mathbb{E}(D_{t+1})}{r-g} \tag{6}$$

where g is the growth rate of dividends, the model can be extended to infinite horizon

$$S_0 = \sum_{t=1}^{\infty} D_0 \frac{(1+g)^t}{(1+r)^t}. \tag{7}$$

Here the r is the required rate of return, which must be estimated and can be calculated with the Capital Asset Pricing Model(CAPM):

$$r = r_f + \beta(r_m - r_f). \tag{8}$$

CAPM expresses that investors must be compensated for both time value of money (r_f) and risk, $\beta(r_m - r_f)$. The risk free rate r_f denotes the interest rate which the investor is relatively certain to get with no risk, usually a 10-year government bond. The risk measure β measures the volatility of a stock relative to the market over a period of time, stocks with β=1 follow the markets volatility, stocks with β<1 have less volatility and stocks with β>1 have higher volatility than the market and are more risky. The market return r_m is the market return over a period of time. The difference between r_m and r_f is the market risk premium.

Using CAPM, the required return for a stock with $\beta = 1.5$ (Risky) in a market with r_m=8% and the risk free rate, r_f=5% is

$$r = 0.05 + 1.5(0.08 - 0.05) = 0.095$$

The stock price today is 150 and last dividend paid was 2, the growth rate for the company has been a steady 8%. Using equation 6 the value of the stock can be calculated

$$S_0 = \frac{2(1 + 0.08)}{0.095 - 0.08} = 144,$$

which suggests that the price today is overvalued. Using equation (7) the model is extended for 5 years in Table 1 where the investor sells the stock at the end of year 5.

Table 1: Present Value with the Growth Model

Year	**1**	**2**	**3**	**4**	**5**	**Total PV**
Dividends	2.16	2.33	2.52	2.72	2.94	
PV of Dividends	1,97	1,95	1,92	1,89	1,87	9,60
Stock Price at year 5					211.58	
PV of Stock					134.40	134.40
Stock Price Today						144

Of course, some companies decide to hold their dividends for various reasons such as expansion and takeovers. For a company that pays consistent dividends but suddenly decides to retain it, equation (6) can be expanded by adding a earnings variable, E

$$S = E_1 \frac{\frac{D}{E}}{r - g}. \tag{9}$$

The dividends/earnings ratio, D/E is the expected long term ratio and E_1 the earnings of next year. The disadvantage of the growth model is the uncertainty of the constant growth and assuming that dividends are being consistently paid.

For comparison of stocks it can be useful to look at the Price/Earnings ratio or P/E ratio which shows the price relative to earnings. A company with higher a P/E ratio usually has a higher growth rate although it can give the wrong impression if the company has a very high debt. The P/E ratio will be further discussed in chapter 5.2.

3.3 Optimization Problem

The maximization problem faced by consumers can be used to derive the basic asset pricing formula. In this model consumers wish to maximize expected utility in infinite time horizon, followed by the framework of Diba and Grossman (1988b) the optimization problem is described by

$$\begin{aligned} &\text{Max } \mathbb{E}_t\Big(\sum_{t=1}^{\infty}\beta^t U(C_t)\Big) \\ &\text{subject to } C_t + P_t(x_{t+1} - x_t) \leq E_t + D_t x_t \end{aligned} \tag{10}$$

Where U is the utility function and C is a stochastic process representing the consumption of a single perishable good. The constant discount factor β, satisfies $0<\beta<1$. The consumer receives endowment or earnings E_t and dividend D_t at the end of each period. The consumer can buy x_t units of the asset priced at P_t

The payoffs E_t and D_t is considered exogenous to the model and assumed stationary. The first-order condition for optimality is

$$P_t U'(C_t) = \beta\mathbb{E}_t[(P_{t+1} + D_{t+1})U'(C_{t+1})] \tag{11}$$

Diba and Grossman (1988b) describe the left side of the equation as the marginal utility from selling the share at time t and the right side as the period's expectation of the marginal utility from selling a share at time $t+1$. When the shares are normalized to unit, the market clearing condition becomes

$$C_t = E_t + D_t \text{ for } t \leq T. \tag{12}$$

Following the model from Lucas Jr (1978), (12) is substituted into (11) and leads to the price equation

$$\begin{aligned} P_t U'(E_t + D_t) - \beta\mathbb{E}_t[(P_{t+1}U'(E_{t+1} + D_{t+1})] = \\ \beta\mathbb{E}_t[U'(E_{t+1} + D_{t+1})D_{t+1}]. \end{aligned} \tag{13}$$

The general solution to (13) is

$$Q_t = B_t + F_t, \tag{14}$$

where $Q_t = U'(E_t + D_t)P_t$ and B_t is the rational bubble component with the market fundamental, F_t

$$F_t = \sum_{i=1}^{\infty}\beta^t\mathbb{E}_t[U'(E_{t+i} + D_{t+i})D_{t+i}] \tag{15}$$

The bubble term B_t is the solution to the homogeneous equation,

$$\mathbb{E}_t B_{t+1} - \beta^{-1}B_t = 0. \tag{16}$$

The solutions to the equation satisfy the stochastic difference equation

$$B_{t+1} - \beta^{-1}B_t = z_{t+1} \text{ with } \mathbb{E}_t(z_{t+1}) = 0. \tag{17}$$

The stochastic z_t term has zero mean and auto-correlation.

The general solution of (17) is

$$B_t = \beta^{-t}B_0 + \sum_{s=1}^{t}\beta^{s-t}z_s. \tag{18}$$

Rational bubbles can not be negative and must grow at the rate of interest. Therefore they can not start and if they exist, they must have been there since the beginning and are unable to restart after they have burst.

To show how the bubble must grow at the rate of interest consider the the cash flow (CF) paid at each time t and the expected bubble component B_t

$$P_t = \mathbb{E}_t\Big(\sum_{t=0}^{\infty} \frac{CF_t}{(1+r)^t}\Big) + \lim_{T\to\infty} \mathbb{E}_t\Big(\frac{B_T}{(1+r)^{T-t}}\Big) \tag{19}$$

If the bubble component grows at an interest rate r_B then the bubble at time T is

$$B_T = B_t(1+r_B)^{T-t},$$

then the present value of the bubble component becomes:

$$\lim_{T\to\infty} \mathbb{E}_t\Big(\frac{B_t(1+r_B)^{T-t}}{(1+r)^{T-t}}\Big) \tag{20}$$

It is straightforward to show that if the interest rate r_B is higher than r then the present value would be infinite. If r_B is less then r than the limit approaches zero and the bubble can not exist, therefore the interest on the bubble must be equal to the rate of interest.

3.4 Stochastic Differential Equation for Asset Prices

Asset price model can be derived by a standard stochastic differential equation(SDE), driven by a standard Brownian motion often called a Wiener process (W), which catches the behaviour of a risky asset at time t

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \tag{21}$$

where S_t is the asset value at time t and hence dS_t the change in price, dt is the timestep used, μ is the growth rate, and σ the volatility. The Wiener process is nowhere differentiable and requires its own calculus. The process is a Markov process, meaning that past values are irrelevant for future values and expected future value at time *t+1* depends only on the current value at time *t*.

To solve equation 21 define f as a continuous differentiable function of S

$$df(S_t) = \Big(\frac{\delta f}{\delta t} + \mu S\frac{\delta f}{\delta s} + \frac{\sigma^2 s^2}{2}\frac{\delta^2 f}{\delta s^2}\Big)dt + \sigma S\frac{\delta f}{\delta s}dW. \tag{22}$$

By using Ito's Lemma and defining f(t)=logS_t it derives

$$dlogS_t = (\mu - \frac{\sigma^2}{2})dt + \sigma dW_t. \tag{23}$$

Now integrating both sides from t to T provides:

$$logS_T = logS_t + (\mu - \frac{\sigma^2}{2})(T-t) + \sigma(W(T) - W(t)). \tag{24}$$

The function has normal distribution and after applying the exponential function to both sides

$$S_T = S_t e^{(\mu - \frac{\sigma^2}{2})(T-t) + \sigma(W(T)-W(t))}, \tag{25}$$

the result is a log-normal distributed function which follows the Geometric Brownian Motion(GBM). The expected value and variance is

$$E(S_t) = S_0 e^{\mu t} \tag{26}$$

$$Var(S_t) = S_0^2 e^{2\mu t}(e^{\sigma^2 t} - 1). \tag{27}$$

To show the behavior of the Geometric Brownian Motion a few simulations will be made based on the data in figure 1, which shows the inflation-adjusted monthly price of the S&P500 from 1871-2015. To simulate, μ and σ need to be estimated.

Now, logarithmic returns are defined by

$$r_t = log\Big(\frac{S_t}{S_{t-1}}\Big) \tag{28}$$

Using the moment matching estimation method as shown in Chin et al. (2013), r_i=1,2,...,N in equation 28 are independent and identically distributed. Given the expected value and variance of returns

$$\mathbb{E}\Big[log\Big(\frac{S_t}{S_{t-1}}\Big)\Big] = \Big(\mu - \frac{1}{2}\sigma^2\Big)dt \tag{29}$$

$$Var\Big[log\Big(\frac{S_t}{S_{t-1}}\Big)\Big] = \sigma^2 dt, \tag{30}$$

the estimation of μ and σ are given by

$$\Big(\mu - \frac{1}{2}\sigma^2\Big)dt = \overline{r}$$

and

$$\sigma^2 dt = \frac{1}{(N-1)dt}\sum_{t=1}^{N}(r_t - \overline{r})^2,$$

where

$$\overline{r} = \frac{1}{N}\sum_{t=1}^{N} r_t.$$

Therefore, we have

$$\mu = \frac{1}{dt}\Big(\overline{r} + \frac{1}{2(N-1)}\sum_{t=1}^{N}(r_t - \overline{r})^2\Big) \tag{31}$$

$$\sigma = \sqrt{\Big(\frac{1}{(N-1)dt}\sum_{t=1}^{N}(r_t - \overline{r})^2\Big)} \tag{32}$$

Now the parameters for the simulations of the Geometric Brownian Motion er calculated from the data in figure 1, and presented in table 2

Table 2: Parameters for the Simulations of the Geometric Brownian Motion

Parameter	S_0	μ	σ	d_t	T
Value	85	3.21%	14.2%	1/12	10

The timestep, d_t=1/12 is used here as the data from figure 1 is presented in monthly prices. However if the simulation were to account for daily price changes or trading days in a calendar year which average about 252 days a year, d_t=1/252 might be more appropriate. Figure 2 shows 1,000 alternate price paths from the starting price, S_O=85, simulated over 10 years.

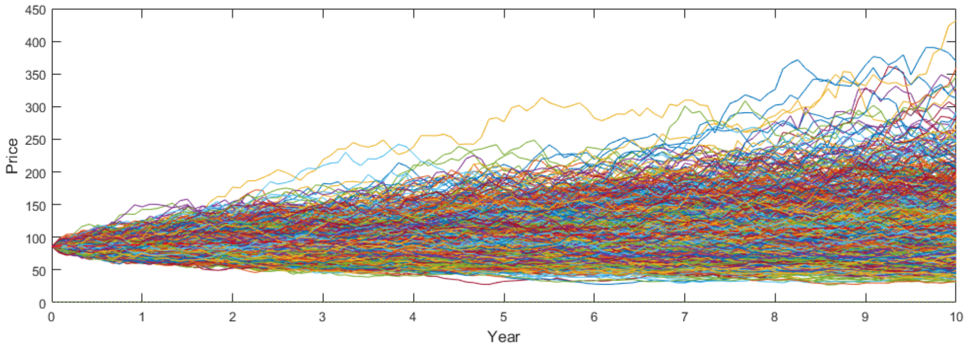


Figure 2: 1,000 simulations of the Geometric Brownian Motion with μ=3.21% and σ=14.2%

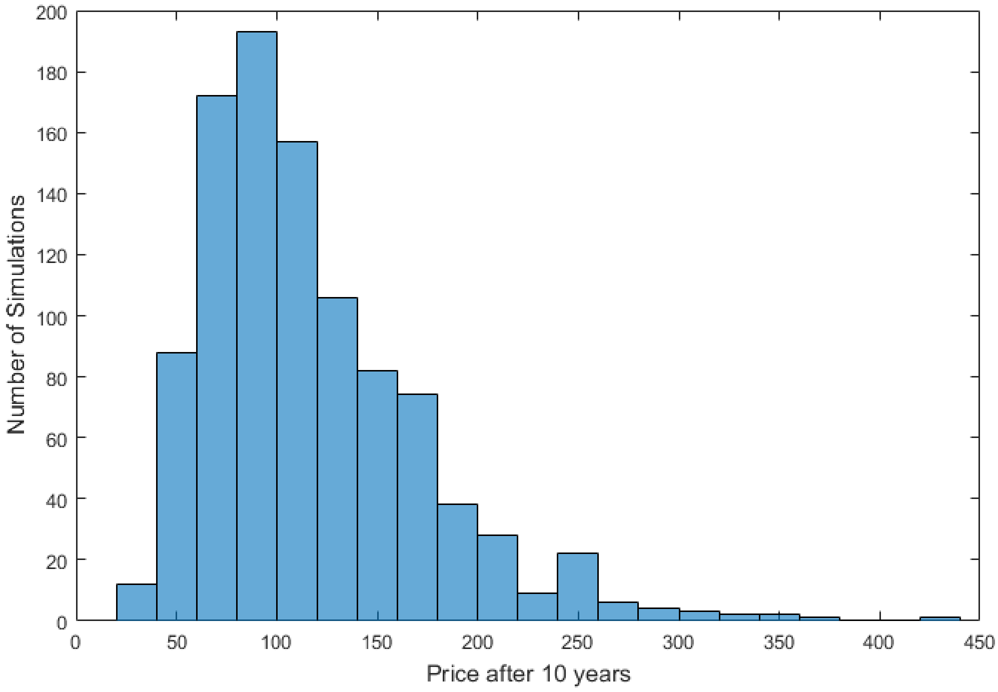


Figure 3: Distribution of 1,000 simulations of the Geometric Brownian Motion with μ=3.21% and σ=14.2%

The price distribution at the end of the tenth year, S_{10} is shown in figure 3. The graph shows how the prices are log-normally distributed at the end of the simulation. This model does not consider any bubbles but is a rather useful tool to simulate asset prices.

3.4.1 Options

An option is a contract which gives the buyer the right to sell or buy an underlying asset at a specific strike price, K either before or at a certain date, depending on the type of option. This section will introduce basic option pricing as the framework for future work as some researchers believe that the price of options can be an indicator for emerging bubbles. Cox and Hobson (2005) consider option pricing in markets with bubbles and conclude that the Put-call parity fails. Protter et al. (2007) stated that European put options never contain bubbles but European call options might.

The price of call and put options can be calculated with the Black-Scholes-Merton formula. The price of a put option with strike price K and asset price S is:

$$P(S,t) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1). \tag{33}$$

The price of a call option with strike price K and asset price S is:

$$C(S,t) = SN(d_1) - Ke^{-rt}N(d_2), \tag{34}$$

where

$$d_1 = \frac{ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{35}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{36}$$

The function *N()* is the cumulative standard normal distribution function. The Black-Scholes-Merton model calculates the price of option in a risk-neutral world, meaning that the r in the model is the risk-free interest rate at time t. The risk-free interest is often thought of as the interest rate of long-term government bonds.

An investor who purchases an option has the right to exercise it but is not obligated to do so. The investor buys the right at the price of the option which is called the option premium. Many types of options are available, the most common are the European and American type options. American options gives the buyer the right to exercise over the whole lifetime of the options but the European options can only be exercised at maturity T. The investor takes a long position if he buys options and a short position if he sells options.

The payoff for a long position in a put option is:

$$Put_{Payoff} = max(K - S_t, 0). \tag{37}$$

The profit/loss of the position is found if the price of the put premium is subtracted from the payoff:

$$\text{Profit/Loss} = max(K - S_t, 0) - P(S,t) \tag{38}$$

The payoff for a long position in a call option is:

$$Call_{Payoff} = max(S_t - K, 0), \tag{39}$$

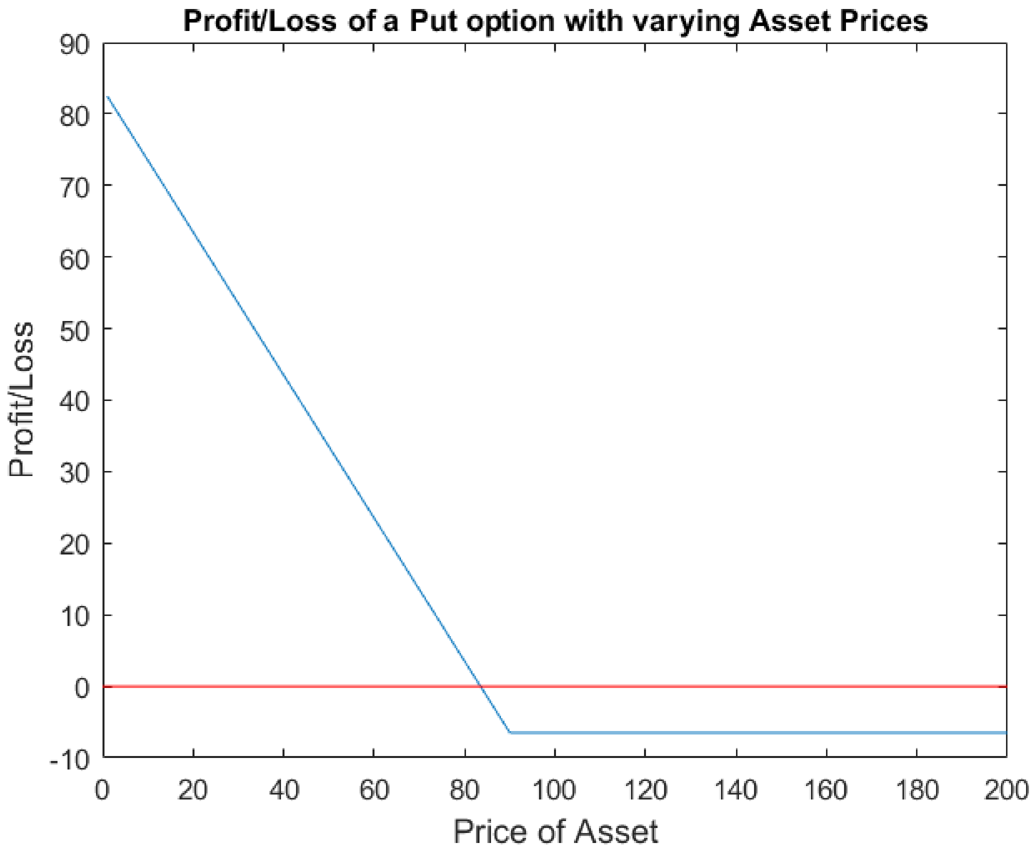
with the profit/loss of

$$\text{Profit/Loss} = max(S_t - K, 0) - C(S,t). \tag{40}$$

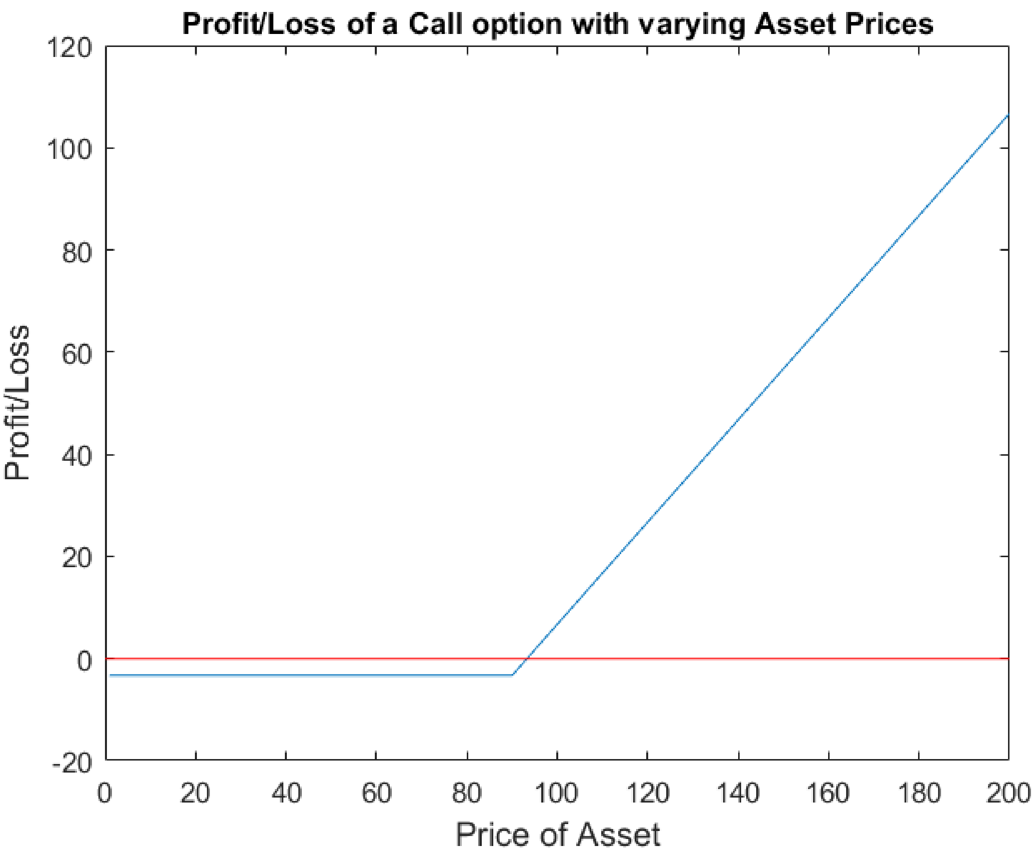
Using the same parameters as in table 2 with the risk free rate r_f=2% the price of a call and put option with strike K=90 are calculated:

Table 3: Parameters of the Call and Put Options

Parameter	S_0	K	r_f	σ	T	P(S,t)	P(S,t)
Value	85	90	2%	14.2%	1	6.50	3.28

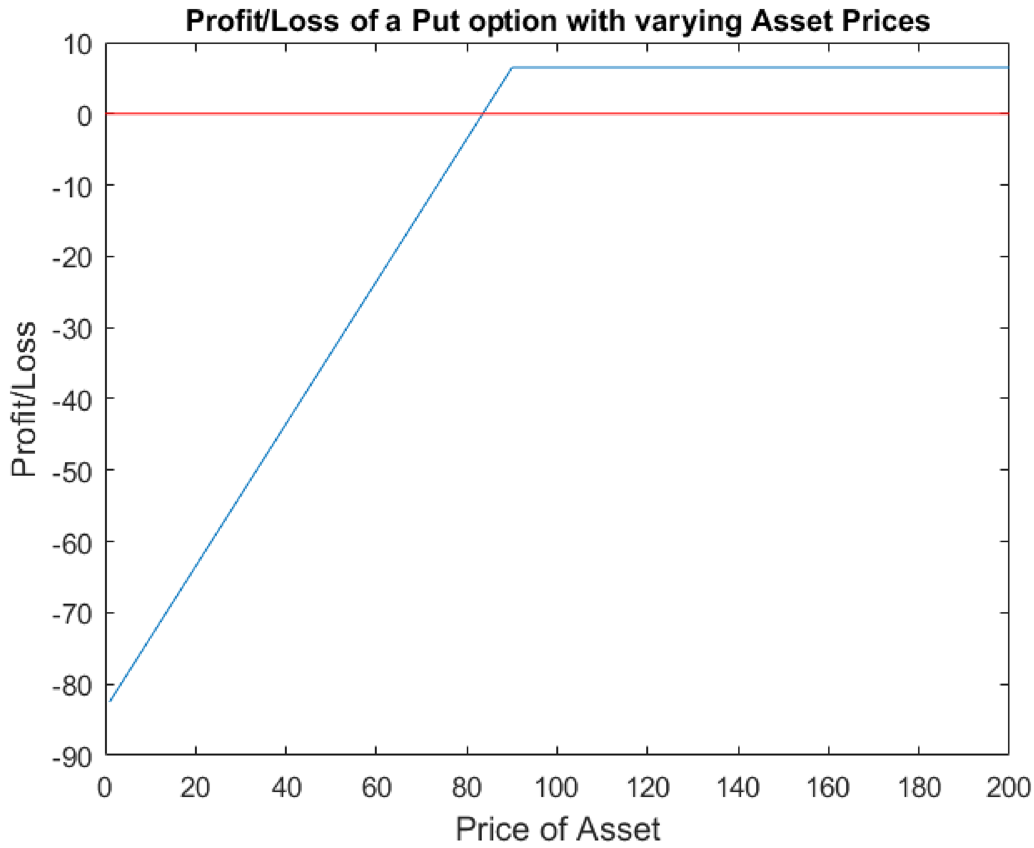


(a) Put Option with K=90

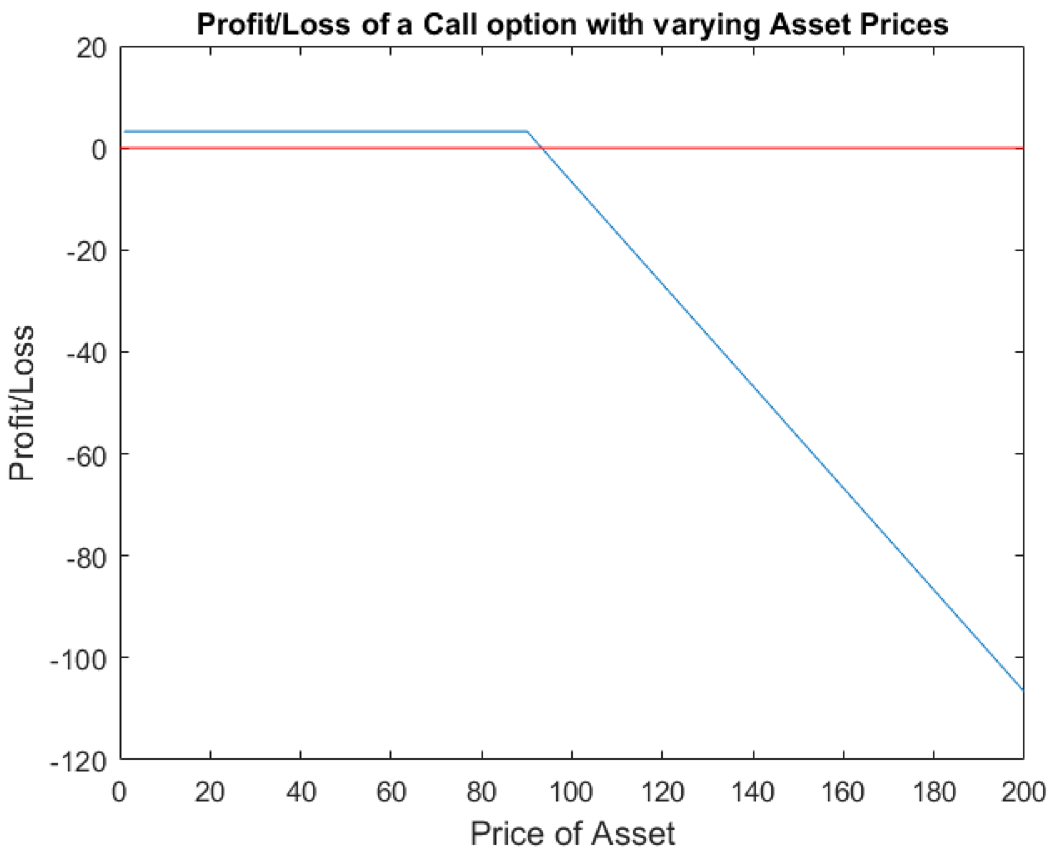


(b) Call Option with K=90

Figure 4: Profit/Losses of a long position in options



(a) Put Option with K=90



(b) Call Option with K=90

Figure 5: Profit/Losses of a short position in options

Figure 4 shows how the profit and losses of a long position in both a call and put option with varying asset price S_t. The loss can never be more than what the investor paid for the option premium.

In contrast the graphs in figure 5 shows how the profit and losses of a short position in both a call and put option with varying asset price S_t. The investor now sells options on the asset and the profit can never be more than the option premium.

3.5 Rational Bubble Model with Credit

Credit risk is the risk taken by a lender that the borrower defaults on his loan. Credit expansion means that more consumers are taking out more loans. An increase in the number of loans means there is more capital floating around in the system. Credit expansion can be a driving factor in escalating asset prices. Firms and investors will borrow capital if they estimate that they have leverage: the gains from borrowing money are higher than the interest paid on the loan. Allen and Gale (2000) derived a model with 2 main theoretical innovations:

- Risk Shifting: Prices can be driven up when investors can borrow capital to invest in assets, causing risky assets to be priced above their fundamental value. When investors default on their loan the risk is transferred to the lender, the higher rewards on risky assets are appealing to investors with little downside if they default.

- Credit Expansion: Greater credit expansion can increase the likelihood of crises following a bubble. Encouraging investors to invest in risky assets interacts with risk shifting and drives up prices.

An agency problem [5] arises in the credit driven economy when investors who supply capital have little control of how it is invested, for example in a hedge fund. If the fund manager does well and has high returns he is highly compensated bringing in more investors. If his assets do badly the worst thing that can happen to him is that he is fired. The fund managers have therefore limited liability and are highly motivated to buy more risky assets without having to deal with the downside the outcome.

The outline of Allen and Gale (2000) model is discussed in chapters 3.5.2 and 3.5.3.

3.5.1 A Simple Numerical Example

Before discussing the equation-heavy credit model a simpler numerical example of the model is discussed here as presented by Allen (2005). There are 2 dates, t=1,2. Investors can choose between a safe asset in a variable supply, x and a risky asset in fixed supply. The return on each safe asset is 50%. For every 1 unit invested at t=1, it returns 1.5 at t=2. There is only 1 risky asset available, and for each unit invested at t=1, it returns 6 with probability of 0.25 and 1 with probability of 0.75. The expected payoff for the risky asset is therefore

$$6 * 0.25 + 1 * 0.75 = 2.25.$$

[5] The agency problem in corporate finance is the conflict of interest between a firm and the firm's stakeholders. The manager of the firm represents the agent for the stakeholders and his duty is to maximize stakeholders wealth.

Table 4: Expected Payoffs for the Assets

		t=1	t=2
Asset	Supply	Investment	Expected Payoff
Safe	x	1	1.5
Risky	1	P	2.25

If each investor has 1 wealth at t=1 and invests the marginal returns of the two assets presented in table 4 must be the same since the investors are risk neutral

$$\frac{2.25}{P_F} = \frac{1.5}{1},$$

where

$$P_F = \frac{2.25}{1.5} = 1.5.$$

The value of the asset is 1.5 and represents the discounted present value of the payoff, here denoted as the fundamental value, P_F. The discount rate is the opportunity cost of the investor. Any price, P above 1.5 can be regarded as a bubble.

Now if the investors have no capital on their own, they require funding from banks to finance investments. The banks are risk neutral along with the investors, and have B>0 units of goods to lend. The bank makes no distinction between assets and has to lend to investors as much as they require at the going rate of interest. To ensure equilibrium the rate of interest is equal to the return on the safe asset. There is a problem with risk shifting since the bank can not distinguish between investors safe or risky decisions and bears the risk of the borrower defaulting on the loan. Investors are therefore tempted to buy the risky asset and the asset price rises above its intrinsic value.

In this case investors have no wealth at t=0, and require funding. They can choose to borrow capital to buy assets with an interest rate of 33.33%. They can only borrow 1 unit and pay 1.33 unless they default. Using the same values as in table 4 the marginal return to the safe asset is

$$1.5 - 1.33 = 0.17.$$

If a investor borrows 1 and chooses the risky asset which costs 1.5, he purchases 1/1.5 units. The marginal return to the risky asset is

$$0.25\Big(\frac{1}{1.5} * 6 - 1.33\Big) + 0.75 * 0 = 0.67$$

When the payoff is 1, the investor defaults and the entire payoff goes to the lender. The marginal return of the risky asset is higher than that of the safe asset, 0.67>0.17. The lender obtains

$$0.25 * 1.33 + 0.75 * 1 * (1/1.5) = 1.5 - 0.67 = 0.83$$

on the risky asset.

Table 5: Example of Risk Shifting

	Expected Payoff	
Asset	Borrower	Lender
Safe	0.17	1.33
Risky	0.67	0.83
Risk shift	+0.5	-0.5

The equilibrium value is when the risky asset P, has been bid up because of limited supply, until the expected profit is the same for both the risky and safe asset

$$0.25(\frac{1}{P} * 6 - 1.33) + 0.75 * 0 = 1.5 - 1.33$$
$$P = 3$$

This shows a bubble as the fundamental price, P_F of the risky asset is 1.5.

Now to include the amount of credit and level of interest rate, assume that a central bank sets the level of credit B, available to banks. In this example there is one bank and number of investors set to one. Now, x units of the consumption good at t=1 produces *f(x)* units at t=2. Assume that *f(x)* is

$$f(x) = 3x^{0.5} = 3(B - P)^{0.5}. \tag{41}$$

The interest rate r is equal to the marginal product of investment due to competition on the loan market, investors will bid up the interest rate to

$$r = f'(B - P) = 1.5(B - P)^{-0.5}. \tag{42}$$

The price P, that investors are willing to pay for the risky asset considering the expected payoffs in table 4 is

$$0.25\Big(\frac{1}{P} * 6 - r\Big) + 0.75 = 0.$$

Using equation 42 yields

$$P = 4(B - P)^{0.5}$$

Now, solving for P

$$P = 8(-1 + \sqrt{1 + 0.25B})$$

Although monetary policy and central banks try to determine the amount of credit in the system they have limited control of it, meaning that B is uncertain and random. Allen (2005) mentions that this uncertainty is particularly great in countries undergoing financial liberalisation, but changes in policy administration and external factors can also increase uncertainty. In this example there is a prior date, t=0 and the uncertainty is between dates 0 and 1. At date 1 there is 0.5 probability that B=5 and 0.5 probability that B=7. The pricing equation at date 0 is

$$0.5\Big(\frac{1}{P_0} * 5.27 - r_0\Big) + 0.5 * 0 = 0,$$

where r_0 is the interest rate at date 0 is calculated with equation 42, and B and P replaced by B_0 and P_0.

$$P_0 = \frac{5.27}{1.5(B_0 - P_0)^{-0.5}}$$

If it is assumed that the starting credit is B_0=6, it results in r_0=1.19 and P_0=4.42. Although the expected payoff at date 2 is 2.25 the price of the risky asset is much higher, or 4.42.

Table 6: Example with Financial Risk

Probability	B	P	r
0.5	5	4	1.5
0.5	7	5.27	1.14
Exp.	6	4.42	1.19

In the following 2 sections the model will be presented with more complex relations.

3.5.2 Asset Pricing with Uncertainty Generated by the Real Sector

The model is based on assumptions to make risk shifting, bubbles and default on loans as clear as possible for the reader. The first model leaves out credit expansion and focuses on random asset returns. The model has the following attributes

- There are 2 dates, t=1,2 and a single consumption good at each date
- Investors choose between 2 assets either safe asset X_S in variable supply or in a risky asset X_R in fixed supply.
- All investors are treated symmetrically with interest rate r
- The variable B is the total amount available to borrow.

The safe asset pays a fixed return r times the x units held of X_S at date 2. There is only one risky asset, X_R=1. The risky asset returns Rx units, where R is a random variable with continuous positive density function h(R) on the support [0,R_{MAX}] and a mean $\overline{R}$. The return is determined by the marginal product of capital in the economy. The investors are risk-neutral and x units of the consumption good at date 1 are transformed into f(x) units at date 2.

Since the investor's portfolio consists of the units held of the safe asset and risky asset respectively the payoff at date 2 is:

$$rX_s + RX_R - r(X_S + PX_R) = RX_R - rPX_R,$$

or the interest paid off the loan subtracted from the return on the risky investment. The return and interest paid off the safe asset is the same and it drops out the equation. The interest rate R*=rP is the critical value of return when the investor defaults. There is a cost, c(x) for investing in the risky asset. The investment cost is to set boundaries on individual portfolios and initiates at date 1. The maximization problem faced by the investor is

$$\max_{X_R \geq 0} \int_{R*}^{R_{MAX}} (RX_R - rPX_R)h(R)dR - c(X_R) \tag{43}$$

The market clearing condition for the one risky asset:

$$X_R = 1 \tag{44}$$

Market clearing in the loan market, where (P=X_RP)

$$X_S + P = B \tag{45}$$

Market clearing for capital goods, with return on the safe asset as the marginal product of capital:

$$r = f'(X_S) \tag{46}$$

Now, inserting (44) in (43), the maximization problem becomes

$$\int_{R*}^{R_{MAX}} (R - rP)h(R)dR = c'(1). \tag{47}$$

When substituting from the budget constraint $X_S = B - PX_R = B - P$, equation 46 becomes

$$r = f'(B - P) \tag{48}$$

If there was no risk shifting the risky asset's intrinsic or fundamental value would be equal to the value a risk neutral investor would pay for it. The investor has borrowed an amount B which and has the following maximization problem:

$$\begin{aligned} &\max_{(X_S, X_R)\geq 0} \quad \int_0^{R_{MAX}} (RX_R - rPX_R)h(R)dR - c(X_R) \\ &\text{subject to} \quad X_S + PX_R \leq B \end{aligned} \tag{49}$$

There is no possibility of default in (49).

$$\int_0^{R_{MAX}} Rh(R)dR - rP = c'(X_R) \tag{50}$$

The fundamental price $\overline{P}$ is found by setting $X_R = 1$ in the first order condition of (50), the amount which an investor would be willing to pay for a risky asset with his own capital. The fundamental price is the discounted value of net returns.

$$\overline{P} = \frac{1}{r}[R - c'(1)] \tag{51}$$

A classic definition of a bubble is when the equilibrium price is greater than the fundamental. The fundamental price is the discounted value of expected future dividends. By arranging the equilibrium condition the price equation comes to:

$$P = \frac{1}{r}\Big[\frac{\int_{R^*}^{R_{MAX}} Rh(R)dR - c'(1)}{Pr(R \geq R^*)}\Big] \tag{52}$$

The function Pr(R≥ R^*) denotes the probability that returns are higher or equal to the critical value of interest and no default. The probability of default is Pr(R<R^*).

Proposition 1: There exists a bubble in the equilibrium and the equilibrium price, P is at least as high as the fundamental price, $\overline{P}$. P is strictly higher than $\overline{P}$ if probability of default is positive, Pr(R<R^*)>0.

The proof of proposition 1 is when equation (52) is rewritten as:

$$rP = \frac{r\overline{P} - \int_0^{R^*} Rh(R)dR}{Pr(R \geq R^*)} \tag{53}$$

Now

$$\int_0^{R^*} Rh(R)dR \geq R^* Pr(R < R^*) \tag{54}$$

Combining these 2 equations gives:

$$rP \geq \frac{r\overline{P} - rPPr(R < R^*)}{Pr(R \geq R^*)} \tag{55}$$

To simplify Pr(R≥ R^*)=1-Pr(R<R^*) is used to obtain:

$$P \geq \overline{P}$$

If P>$\overline{P}$(default possibility) the inequality is strict and

$$P > \overline{P}$$

Since investors are treated symmetrically they all default when R<R^* leading to a financial crises. In realistic models there would be more diverse investors and only a proportion of them would default.

Proposition 2: (r,P) and (r',P') are the equilibrium interest rate and price of risky assets before and after a mean-preserving spread in the distribution of the return R. Then either:

1. (r,P)=(r',P') the equilibrium stays the same or
2. r'>r, P'>P, the fundamental value falls $\overline{P'}$<$\overline{P}$, size of the bubble increases and probability of default increases as a consequence.

3.5.3 Asset Pricing with Uncertainty Generated by the Financial Sector

The former model is now extended to show how uncertainty of credit expansion can increase the size of the bubble further. The model now adds a prior date t=0 and extends to three dates, with t=0,1,2 and a single consumption good at each date. The equilibrium price at time t=1 is

$$P_1 = \frac{1}{r_1}[\overline{R} - c'(1)]. \tag{56}$$

The maximization problem faced by the investor is now

$$\max_{X_{0,R}\geq 0} \int_{B_1^*}^{B_{1,MAX}} [P_1(B_1)X_{0,R} - r_0 P_0 X_{0,R}]k(B_1)dB_1 - c(X_{0,R}). \tag{57}$$

Where k(B) is a positive continuous density function on [0,$B_{1,MAX}$]. The price of the risky asset at date 1,$P_1(B_1)$ is also a random variable. The critical value of B_1 when the investor defaults at date 1 is B_1^*

$$P_1(B_1^*) = r_0 P_0 \tag{58}$$

With the market clearing conditions:

$$X_{0,R} = 1, \tag{59}$$

$$X_{0,S} + P_0 X_{0,R} = B_0, \tag{60}$$

$$r_0 = f'(X_{0,S}). \tag{61}$$

The unique equilibrium still exists if E[$P_B(B_1)$]>$c'(1)$ and the equilibrium conditions is reduced to three

$$\int_{B_1^*}^{B_{1,MAX}} [P_1(B_1) - P_1(B_1^*) - c'(1)]k(B_1)dB_1 = 0 \tag{62}$$

$$r_0 = f'(B - P_0)$$

$$P_1(B_1^*) = r_0 P_0$$

An owner-investor with B units of wealth is now introduced and considered at which price $\overline{P_0}$ he would be willing to hold one unit of the risky asset. The maximization becomes

$$\begin{aligned} \max_{X_{0,S}X_{0,R}\geq 0} \quad & \int_0^{B_{1,MAX}} [r_0 X_{0,S} + P_1(B_1)X_{0,R}]k(B_1)dB_R - c(X_{0,R}) \\ \text{subject to} \quad & X_{0,S} + PX_{0,R} \leq B \end{aligned} \tag{63}$$

The fundamental value of the risky asset is derived from the first order conditions

$$\overline{P_0} = \frac{1}{r_0} E[P_1(B_1)] - c'(1) \tag{64}$$

or

$$P = \frac{1}{r_0}\Big[\frac{\int_{B^*}^{B_{1,MAX}} P_1(B_1)k(B_1)dB_1 - c'(1)}{Pr(B_1 \geq B_1^*)}\Big]. \tag{65}$$

Proposition 3: The equilibrium values for the intermediated economy are $(r_o, P_0, B_1^*, X_{0,S}, X_{0,R})$ and $\overline{P_0}$ is the fundamental price of the risky asset. The inequality $P_0 \geq \overline{P_0}$ is strict if the possibility of default is positive.

Example: Assume that B_1 has a uniform distribution on [0,2]. The loan amount at t=0 is B_0=1, the function of the safe asset given by $f(X_S)$=4$X_S^{0.5}$ with the interest mean $\overline{R}$-c'(1)=4. Focusing on positive prices the price $P_1(B_1)$ becomes

$$P_1(B_1) = 2[(1 + B_1)^{0.5} - 1]$$

Now substituting $P_1(B_1)$ in equation 62, $k(B_1)$=0.5 and drawing c'(1) to the right side, the equation becomes:

$$\int_{B_1^*}^{2} \Big(2[(1 + B_1)^{0.5} - 1] - 2[(1 + B_1^*)^{0.5} - 1]\Big) * 0.5dB_1 = c'(1) \tag{66}$$

Now the amount of credit B_1^*, demanded to avoid financial crises can be calculated by setting the cost, c'(1), associated with purchasing the risky asset to some value. As c'(1) approaches zero the default rate B_1^* approaches $B_{1,Max}$ as can be seen in table 7.

Table 7: A Numerical Example of the Allen and Gale Model

c'(1)	B_1^*	Probability of a crises	Intermediated P_0	Fundamental $\overline{P_0}$	Bubble P_0-$\overline{P}_0$
0.2	0.90	0.45	0.31	0.25	0.06
0.1	1.21	0.61	0.38	0.27	0.11
0.01	1.74	0.87	0.47	0.29	0.18

The level of B_0 is 1 at t=0, when c'(1)=0.2 the credit needed is 0.9 so the credit remains the same. In the other instances where c'(1) is equal to 0.1 and 0.01 the credit needs to expand to 1.21 and 1.74 respectively to avoid financial crises.

4 Irrational Bubbles

4.1 Definition

A rational bubble model where all consumers are considered to act rationally can be missing key elements of psychological behaviour that contribute towards the creation of a bubble. When the present value model failed to explain asset prices, the deviations from the prices were introduced as bubbles, with the early literature primarily assuming that investors in the model were rational. Later literature relaxed the rationality assumption and and the idea of prices that are driven by human emotion and behaviour were introduced. Models which consider effects of speculation and behavioural effects have been developed to account for irrational pricing for assets. Vissing-Jorgensen (2004) state that there are 4 fields of behavioural finance:

1. Finding price patterns which can not be explained by the efficient market theory and rational consumers.

2. Finding irrational behaviour of investors that is inconsistent with traditional finance theory.

3. Develop new theories for the price patterns and irrational behaviour based on psychology and experiments.

4. Limits to arbitrage literature: Rational investors are not able to arbitrage prices back to fundamental value due to the behaviour of irrational investors.

Alan Greenspan used the term "Irrational Exuberance" to denote what later would be known as the Dot-com bubble in the late 1990's and was later used for a title of a book by Robert Shiller in 2000. Irrational exuberance refers to bubbles that can be explained by overconfidence, social contagion and often controlled by the greater fool theory i.e. buying an asset purely to sell at later time to" a greater fool" at a higher price. Irrational Bubbles are often named fads and and are usually seen as high rapid price movements, created by social forces or mass belief in an asset's potential earnings which drives the price up from its intrinsic value (Meltzer, 2002).

4.2 Psychological factors

4.2.1 Herding

Investors buy or sell in the direction of the market trend. Investors only buy stocks on the rise, and hedge fund or mutual fund managers might feel pressured by their investors and the fear that they might move on to another fund if they do not follow some trend.

4.2.2 Moral Hazard

Investor who can default on their loan are subject to moral hazard. They might not act as if they were fully exposed to the risk causing them to buy a more risky asset. Allen and Gale (2000) describe the risk shifting to the bank from the investor, the bank has the risk of bankruptcy of the investor. Large corporations and banks could also be subject to moral hazard if they feel that the government will bail them out in an attempt to save the economy.

4.2.3 Extrapolation

Estimating future asset prices based on past growth rates i.e. extrapolating prices based on historical data. Barberis et al. (2015) present an extrapolative model of bubbles. Investors in the model form their demand for a risky asset by weighing two signals: an average of the asset's past price changes and the asset's degree of overvaluation. Lansing et al. (2006) shows how an investor might be "locked-in" to a extrapolative forecast if a fundamental based model has a lower accuracy i.e. more forecasting errors, and other investors are using the same approach.

4.2.4 Information Cascade

Prices are driven up because of information in the social media, news or "word of mouth" effects (Shiller, 2015). Barberis et al. (2015) conclude that nearly all bubbles occur on the back of good news and confidence in the economy for example the Tulipmania, South Sea Bubble, the 1929 U.S. stock market and the Dot-Com bubble. Variables of over-estimation of prices due to positive feedback have a tendency to grow faster than the exponential function (Hüsler et al., 2013).

4.3 Simple Behavioral Model

DeLong (2009) derived a model which he denoted "The Simplest Possible Behavioral Model" to outline price bubbles in the stock market with manias, panics and crashes. In his model there are no rational investors who arbitrage the bubble aside, but rather compare returns of stock and bonds in the market and choose the asset with greater return. The investors randomly encounter each other and compare returns at a rate denoted by λ. The expected price P, at t+1 is given by the equation:

$$P_{t+1} = P_t + \lambda P_t(1 - P_t)[(\frac{P_t - P_{t-1} + D_t}{P_{t-1}}) - r] \tag{67}$$

The model is here regarded as irrational due to the herding behaviour of investors when comparing returns. The stocks pay a stochastic dividend D_t with a probability π, and no dividend with probability $(1-\pi)$, meaning that if stock investor encounters a bond investor at time t he would switch to bonds, if they do not encounter each other the investors continue with their former strategy. The bonds pay a fixed rate of return r. The price P_t also accounts for the number and share of agents buying stocks, the total wealth invested in stocks. Unconditional expectations give:

$$\mathbb{E}(\Delta P_{t+1}) = \lambda P_t(1 - P_t)\Big(\frac{\mathbb{E}(\Delta P_t) + \pi\delta}{P_{t-1}} - r\Big). \tag{68}$$

When $\mathbb{E}(\Delta P_t) = 0$, then $\mathbb{E}(\Delta P_{t+1}) = 0$. Then P_{t-1} is

$$P_{t-1} = \frac{\pi\delta}{r}, \tag{69}$$

which is the fundamental or intrinsic value in the model. The fundamental value will be denoted P^*. By formulating the model in Matlab a simulation with the parameters in Table 8, the Matlab code is given in Appendix A.

Figure 6 shows the first simulation of the model. Here the fundamental value is

$$P^* = \frac{\pi\delta}{r} = \frac{0.5 * 0.05}{0.05} = 0.5.$$

Table 8: Parameters for the Simulation of the DeLong Model

Parameter	π	1-π	δ	r	λ	P^*	P_0
Value	0.5	0.5	0.05	0.05	1.5	0.5	0.25

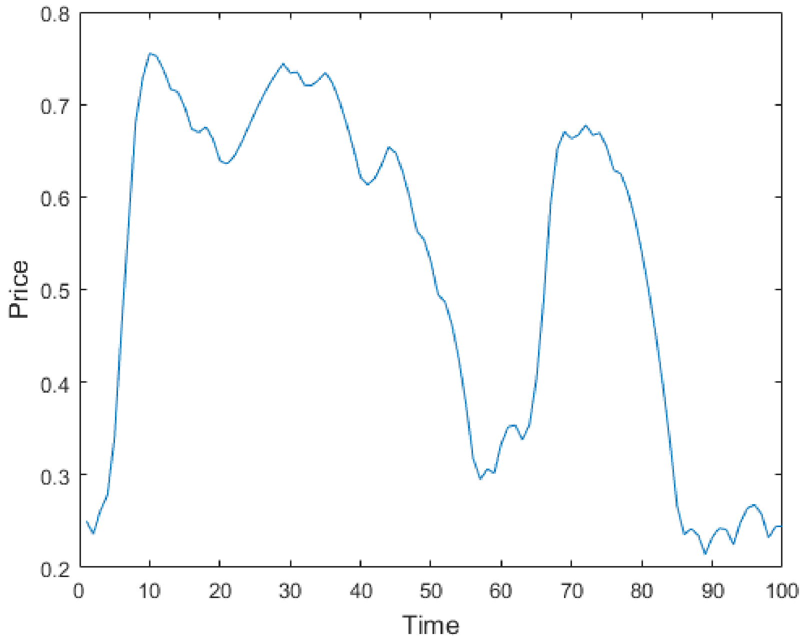


Figure 6: Simulation of the DeLong Model

The simulation presents bubbles where the price exceeds 0.5 and recession when the price drops below. The initial market price is set to half of the the fundamental value, P_0=0.25 to assume a initial displacement of the stock price. The price path is simulated over a time period of 100 which can be regarded as years. As seen in the figure the graph presents both price bubbles and recessions which are usually briefer. When prices are high the dividend yield is low and more investors choose bonds with more return, if dividends are not paid at that time for more than 1 time period it will cause a crash in the model.

Figure 7 shows 2 further simulations of the same parameters as in figure 6. These simulations show similar behaviour as figure 6, although they do not present as distinctive panics and crashes. The price decline happens in a rather slow manner as a stock investor has to meet a bond investor to compare returns and switch his purchasing method. When prices are high there are too few bond investors left to switch to stocks, which puts a certain cap on the price preventing it to go higher. The λ parameter controls the bubble manias and crashes, as illustrated in figure 8.

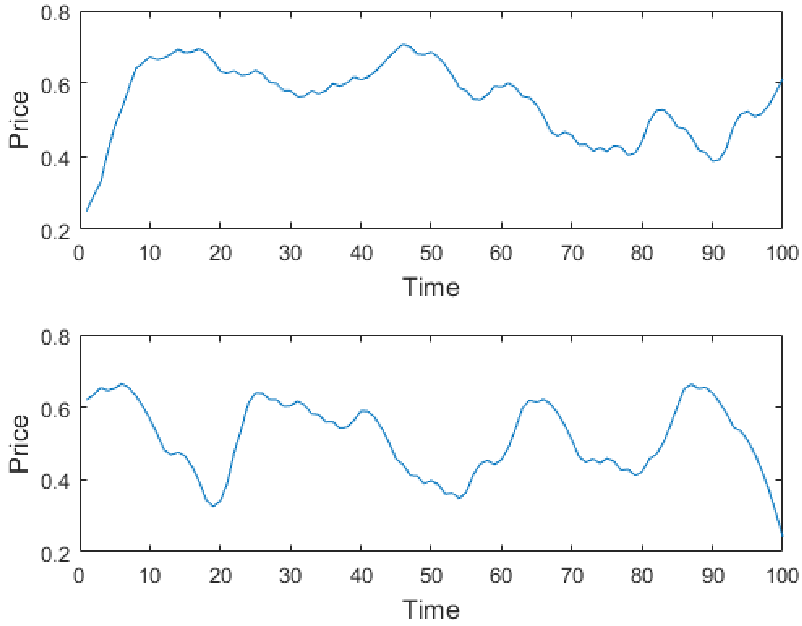
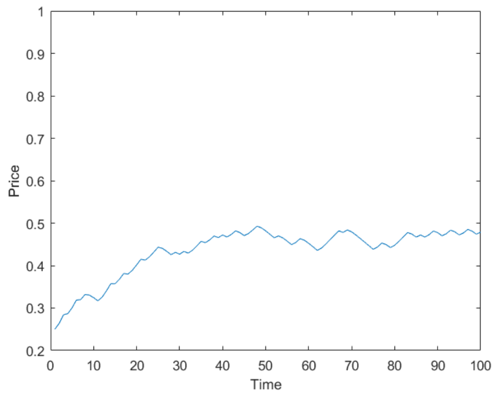
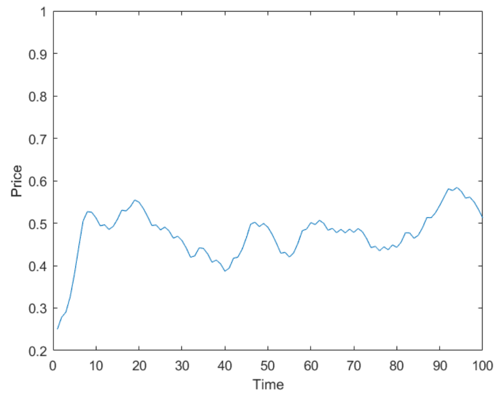


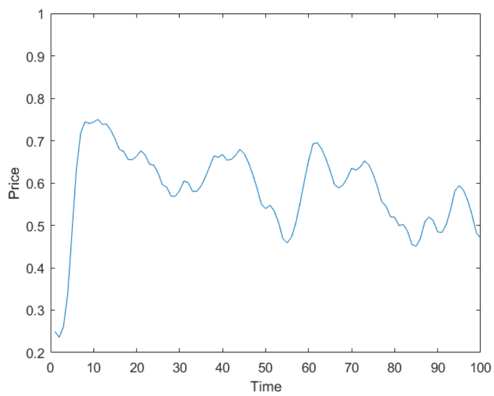
Figure 7: Additional Simulations of the DeLong Model



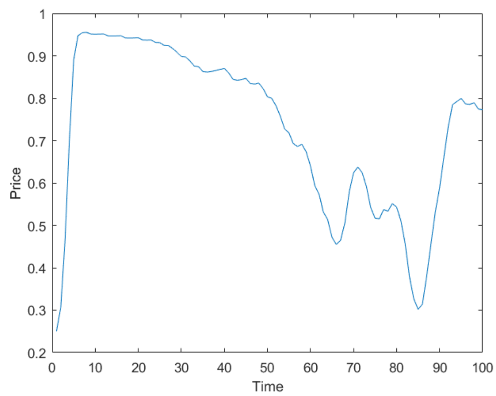
(a) λ=0.5



(b) λ=1



(c) λ=1.5



(d) λ=2

Figure 8: Simulations with different values of the imitation parameter λ.

Figure 8 shows how the imitation parameter λ controls how suddenly the price movements happen, higher values of λ have more steep price movements where as the lower values eliminate positive feedback and take more time to find a new equilibrium price.

The model is quite simple as the name suggests and has its limitation. Adding rational investors could alter the model as well as adding more options for investors than bonds or stocks i.e. real estate or commodities. The model assumes that investors can only compare the last periods return with their own, if they could compare historical returns it would make the model considerably more realistic. The dividend stochastic process is likewise rather simple where the dividend is either paid or not at all. To get more distinctive swift downward price movements with panic and crashes capital requirements, margin calls, portfolio insurance could be added as well as many other financial instruments where investors can be bankrupt.

4.4 Heterogeneous-Beliefs Model

Another type of model are the heterogeneous beliefs of investors regarding asset prices that can create a bubble. Heterogeneous beliefs are common for just about everything in life, opinions of people differ on politics, sports, religion and for these models: future asset prices. Investors can be optimistic or pessimistic of an asset's future value and in a dynamic model the the price can exceed the evaluation of the most optimistic investor. If an optimistic investor buys an asset at time t, he can resell it at t+1, t+2 etc. when he becomes less optimistic. Investors who were more pessimistic are now more optimistic and are willing to buy the asset (Harrison and Kreps, 1978). Heterogeneous beliefs models are more applicable in a market with short-sale constraints, where pessimistic investors are prevented from shorting assets i.e. arts, real estate or where shorting is restricted (Brunnermeier and Oehmke, 2012). Ofek and Richardson (2003) document substantial short sale restrictions for Internet stocks as one of the reasons for the emergence of the Dot-Com bubble. Scheinkman and Xiong (2003) model a heterogeneous-belief model which is accompanied by high trading volume and price volatility. They name the Dot-Com bubble as an example of a bubble with high trading volume and price volatility.

The role of credit has been incorporated in to heterogeneous beliefs models. Geanakoplos (2010) presents a leverage cycle, in economic booms leverage becomes too high when certain group of investors with limited wealth use leveraged borrowing which drives up asset prices. If the investors lose wealth or the ability to borrow they will buy less and assets will be held by more pessimistic investors and valued lower. This in turn causes leverage to be too high in boom times and too low in recessions. From the equilibrium leverage Geanakoplos (2010) outlines the anatomy of a crash after investors have received bad news:

1. Asset prices go down following bad news.
2. The optimistic investors who were leveraged lose a large portion of their wealth and are forced to sell their assets.
3. Asset prices go further down when the supply is greater than the demand causing more losses in wealth for the optimistic investors.
4. As assets prices are about to reach some equilibrium price the margin requirements are tightened trying to control the uncertainty.
5. Forced sales drive asset prices further down
6. The optimistic investors lose all their wealth and default on their loans.

7. Spillovers exist when optimistic investors in one asset are forced to sell other assets which they are also optimistic for.

8. Investors who survive have a great chance to profit from low asset prices.

The key in this context is the heterogeneous beliefs amongst investors. The optimistic investors want to buy more of the asset than the general public. There could be many reasons why the investors would want to do so, a few examples include that they could be less risk averse, have more access to hedging techniques, get utility from holding the asset or they are simply more optimistic of asset prices in general.

A simple binomial tree can be used to price assets. Consider a stock with an initial price S_0 that undergoes a random walk represented in figure 9.

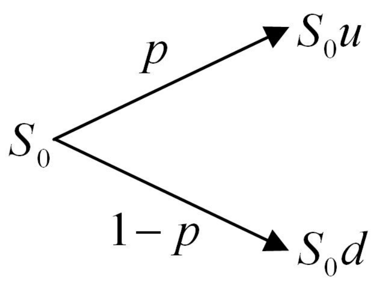


Figure 9: One-Step Binomial Tree

The price of the stock can go up with a probability p which results in a final value of S_0u or the price can go down with a probability 1-p with a final value S_0d.

Example: A stock is priced at S_0=70 and expected to rise in value to 100 with a probability of p=0.6. The stock price is expected to go down to 50 with a probability of 1-p=0.4. The expected payoff to the investor is:

$$E(S_0) = 0.6 * 100 + 0.4 * 50 = 80.$$

Now adapting a model illustrated by Brunnermeier and Oehmke (2012), the probabilities p and 1-p are represented by heterogeneous beliefs of 2 investors, A and B. The tree is expanded to 2 time steps and the investors beliefs of the stocks represented by π^A and π^B at each time.

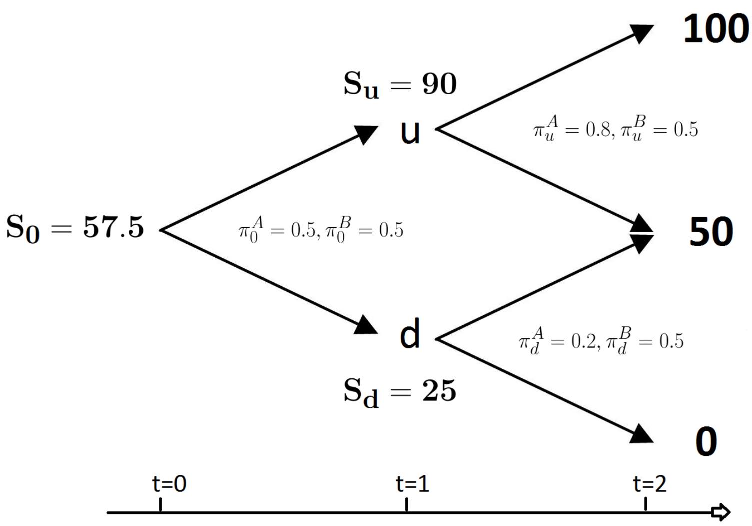


Figure 10: Two-Step Binomial Tree for Investors A and B.

Both investors start with the same belief that the price of an asset will rise or fall with equal probability π_0^A=π_0^B=0.5. At time t=1 investor A becomes optimistic of future prices in the event of an upswing and pessimistic in case of a fall in price. Investor B holds the same belief as at t=0. The tree is represented in figure 10.

In the event of rise in prices at node u the expected payoff for the investors is:

$$E_u^A = 0.8 * 100 + 0.2 * 50 = 90,$$

$$E_u^B = 0.5 * 100 + 0.5 * 50 = 75.$$

In the event of fall in prices at node d the expected payoff for the investors is:

$$E_d^A = 0.2 * 50 + 0.2 * 0 = 10,$$

$$E_d^B = 0.5 * 50 + 0.5 * 0 = 25.$$

Finally their expected payoff at the first node at t=0:

$$E_0^A = 0.5 * 90 + 0.5 * 10 = 50,$$

$$E_0^B = 0.5 * 75 + 0.5 * 25 = 50.$$

Their expected payoff is the same if they hold the asset until t=2. However, if they have the option of selling the asset at t=1 the expected payoff changes. In the event of node u at t=1, investor B can sell the asset to investor A who is now optimistic.

In the event of node d investor A is now pessimistic and can sell the asset to investor B who is now optimistic compared to A. For the both nodes u and d, their expected payoff at t=0 is now:

$$E_0 = 0.5 * 90 + 0.5 * 25 = 57.5.$$

When the investors have the option of selling the asset at t=1, both A and B are willing to pay 57.5 despite that their individual payoff is lower. This simple example shows how heterogeneous beliefs can drive up asset prices and create bubbles.

5 Empirical Tests

5.1 Introduction

Testing for bubbles can be difficult, a proposed model could be missing some variables that is the underlying reason behind the bubble. An overly simple model of fundamentals with the hypothesis of no bubbles could be rejected with an alternative structure (Gürkaynak, 2008).

Empirical tests for rational bubbles with the present value of dividends model have been used many times with different techniques. Campbell and Shiller (1987) test for co-integration between price and dividends concluding that the deviations can be explained as rational bubbles. Craine (1993) used augmented Dickey-Fuller test to find a unit root in the price-dividend ratio which shows a rational bubble.

Cuñado et al. (2005) use fractional integration to show a rational bubble when using monthly data on price-dividend ratios, however they reject them when looking at using daily or weekly data possibly due to size of their sample. The relationship between price and dividends will be briefly examined in chapter 5.3 but more thought will be given to the Price/Earnings ratio in chapter 5.2.

5.2 Price/Earnings Ratio

Because the price of assets depends on expected future dividends it is useful to compare the two variables. As described in chapter 3.2 not every company pays dividends so price relative to earnings might prove to be a better method. The Price/Earnings or P/E ratio measure the stock price relative to earnings per share:

$$\text{P/E Ratio} = \frac{\text{Market Value per share}}{\text{Earnings per share(EPS)}}. \tag{70}$$

Considering a company currently trading at 50 per share and with earnings over the last 12 months of 5 would have the P/E ratio of:

$$\frac{50}{5} = 10.$$

The investor is therefore paying 10 units for every 1 unit earned. Using the most recent 12 months earnings is called a 'trailing' P/E ratio and is most commonly used. Forward P/E ratio uses expected earnings over the next 12 months. High P/E ratio is not necessarily bad as the company could be in a growth period where earnings are lower as the company expands and investors are expecting higher growth of the company in the future.

5.2.1 P/E Ratio of the S&P500

Figure 11 shows the real price and real earnings of the S&P500[6] gathered from Robert J.Shiller's website, with "real" denoting that values have been adjusted for inflation. Inflation is the rate at which general level of prices of goods and services is rising over a period of time, consequently lowering the purchasing power of consumers. A measure to examine the inflation is the consumer price index, CPI. The CPI is calculated by averaging price changes in a predetermined basket of goods and services, where they are weighted according to their importance.

[6]The S&P500 is an American stock market index of 500 large stocks chosen for market size, liquidity and other factors.

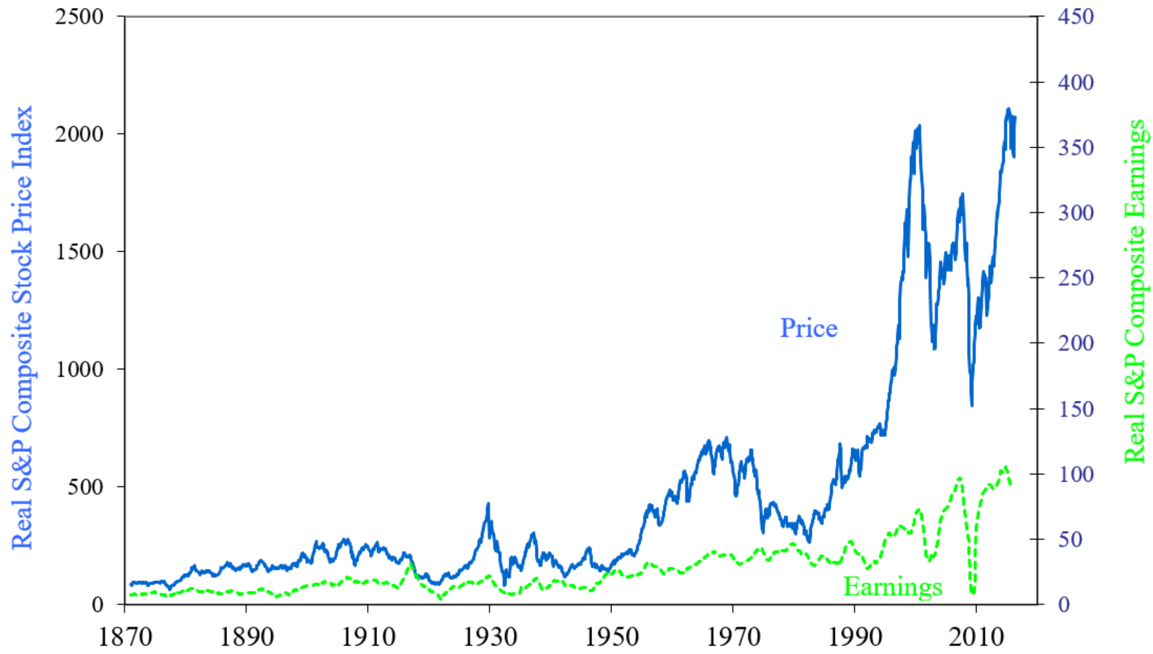


Figure 11: Real Price and Real Earnings of the S&P500. Adapted from Shiller (2016)

To calculate the inflation adjusted price of time t, the price, P_t is multiplied by the CPI of time T and then divided by the CPI of time t. The formulas for the real price and real earnings of time t in respect to the inflation of time T is

$$P_{t,Real} = P_t * \frac{CPI_T}{CPI_t}, \tag{71}$$

$$E_{t,Real} = E_t * \frac{CPI_T}{CPI_t}. \tag{72}$$

For example the price of the S&P500 in January 1871 was 4.44 and the CPI 12.46. To calculate the equivalent price in January 2016, where the CPI was 236.92, the inflation adjusted price is

$$P_{1871,Real} = P_{1871} * \frac{CPI_{2016}}{CPI_{1871}} = 4.44 * \frac{236.92}{12.46} = 84.40$$

Figure 13 shows the P/E ratio of the S&P500 index through the period from 1871 up until 2016. There are 2 notable peaks, first one right after the Dot-Com bubble in 2000 and a large peak when the housing bubble burst in 2008. Earnings dropped more than stock prices which causes the shifts in the ratio. Although the graph shows the bubbles it does so too late, and it is hard to make any kind of predictions of the bubbles beforehand. To adjust the figure a simple moving average can be introduced to filter out noise from random price fluctuations.

$$P/E_{t,MA} = \frac{Price_t}{(\sum_{i=1}^{120} Earnings_{t-i})/120} \tag{73}$$

Equation 73 shows the 10 year moving average of the P/E ratio where the price at month t is divided by the average earning of the past 120 months. This particular moving average of

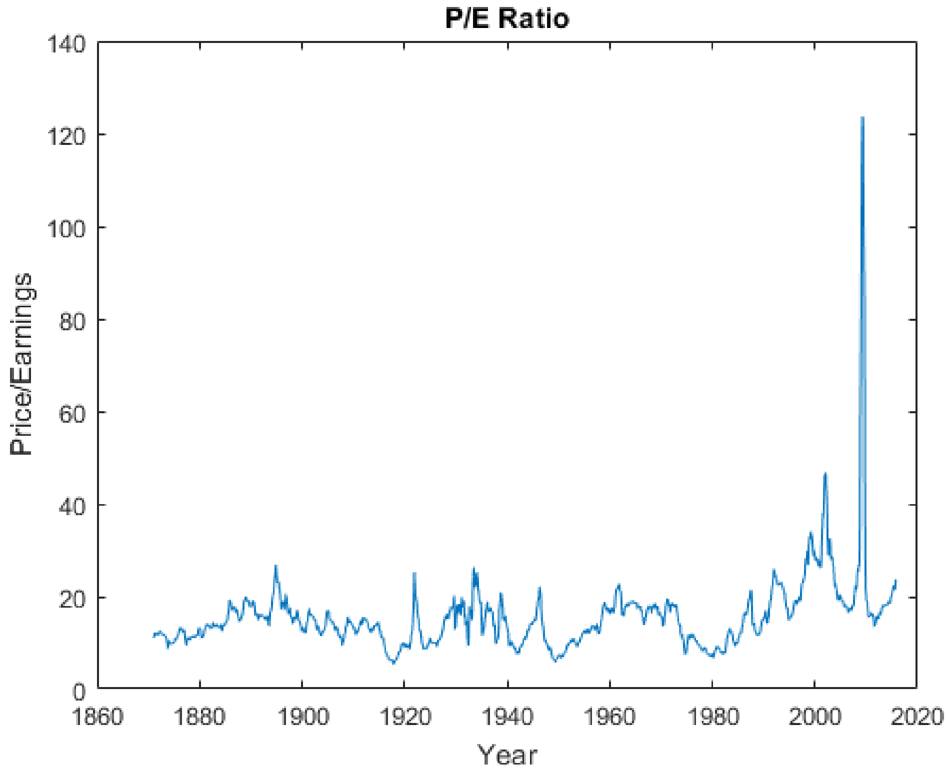


Figure 12: Price/Earnings Ratio of the S&P500 1871-2015

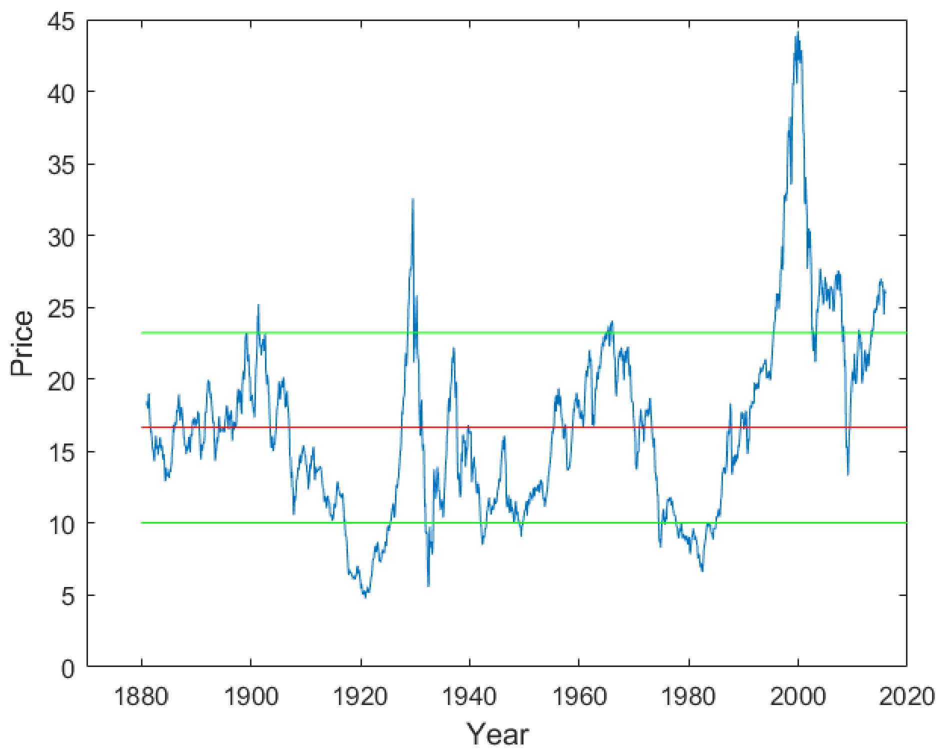


Figure 13: P/E ratio of the S&P500 with 10 year moving average

the S&P500 with inflation adjusted values has been denoted as CAPE (Shiller, 2015) or the cyclically adjusted price-to-earnings ratio.

The graph in figure 13 now shows peaks before the great crash of 1929, bursting Dot-Com bubble and the housing bubble . The average P/E ratio is 16.65 presented with a red line with a standard deviation 6.61 presented with green upper and lower bound. Values over the upper bound can be thought of as a price bubble, catching bubbles in 1900, 1927,1969 and almost all prices after 1997 apart from the bursting of the Dot-Com and housing bubble respectively.

5.2.2 Price/Earnings ratio of the Icelandic Market

The same method is now applied on the Icelandic market. Figure 14 shows the inflation adjusted price and earnings of the selected share index which consists of ICEX-15, OMXI6 and OMXI8 from March 1995 throughout April 2016. The indices consist of companies with the highest market capitalization for a given period of time. The data is gathered from Brynjar Örn Ólafsson's website (Ólafsson, 2016a).

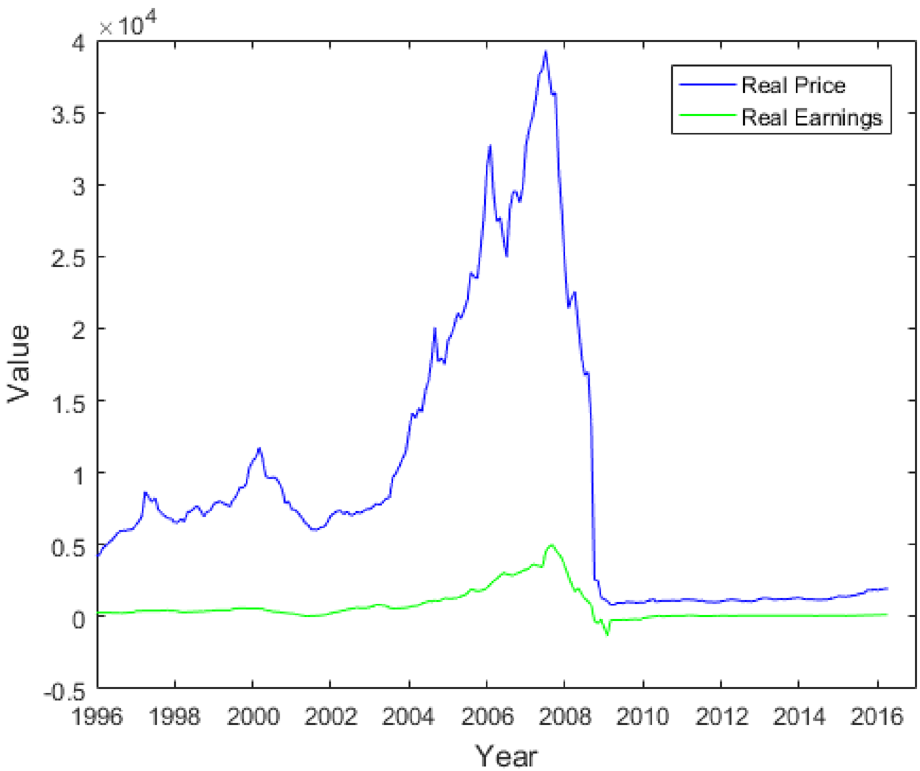


Figure 14: Real Price and Real Earnings of the selected share index consisting of ICEX-15, OMXI6 and OMXI8.

Bursting of the housing bubble in the United states had a domino-like effect on other markets, including Iceland. Prices began to drop in 2007 and crashed to their lowest point in over a decade in 2008. Both prices and earnings have been in a steady increase since.

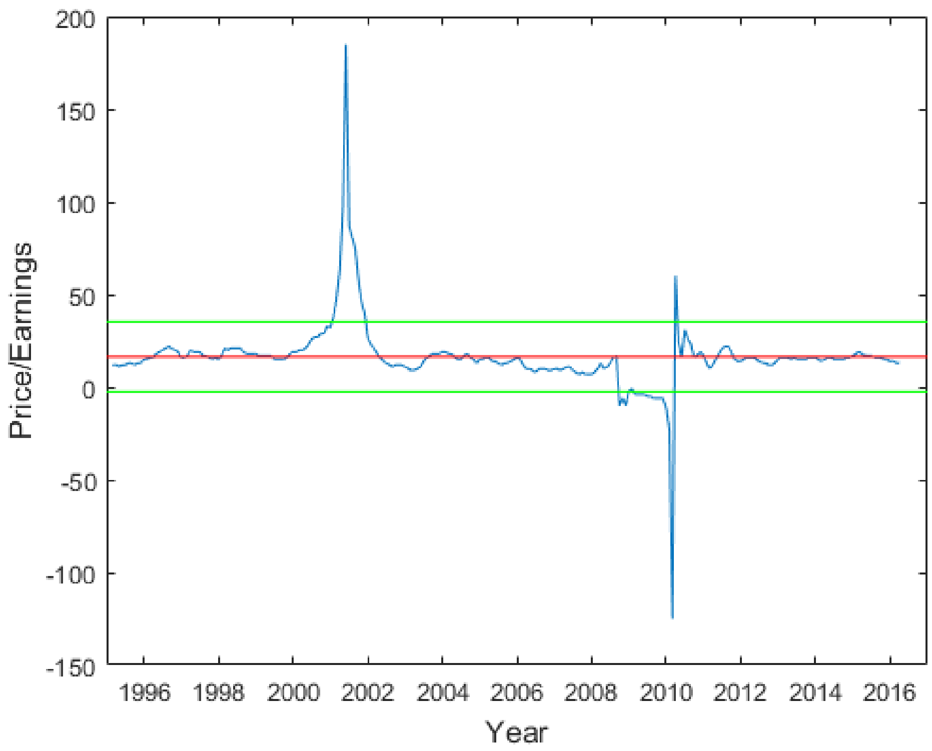


Figure 15: Price/Earnings ratio of the selected share index consisting of ICEX-15, OMXI6 and OMXI8.

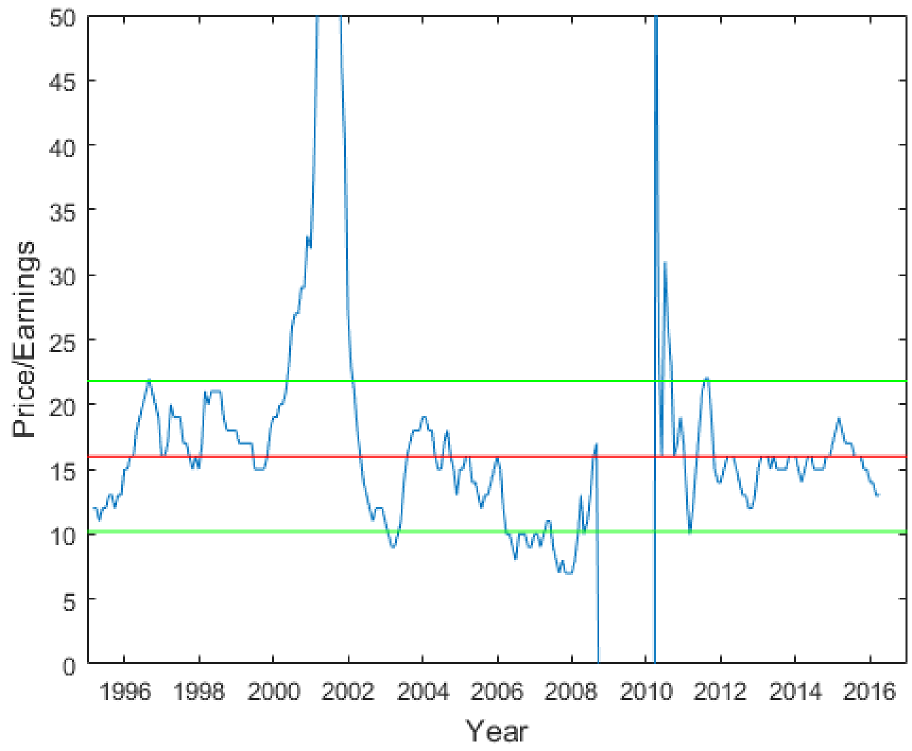


Figure 16: Price/Earnings ratio of the selected share index with a fixed upper and lower limit

The Price/Earnings ratio of the index shown in figure 15 has an average ratio of 16.4 with quite high standard deviation of 18.9. The index undergoes constituent changes every 6 months and as reviewed by Ólafsson (2016b) the proportion of financially based firms and companies was 60% of the index in July 2003 and roughly 90% in July 2007, whereas only 1 financially based company was part of the index in March 2016. Considering the heavy changes the index has undertaken, negative and P/E values over 50 have been treated as outliers and left out of calculations in figure 16. The average ratio is now 16 with a more decent deviation of 5.8.

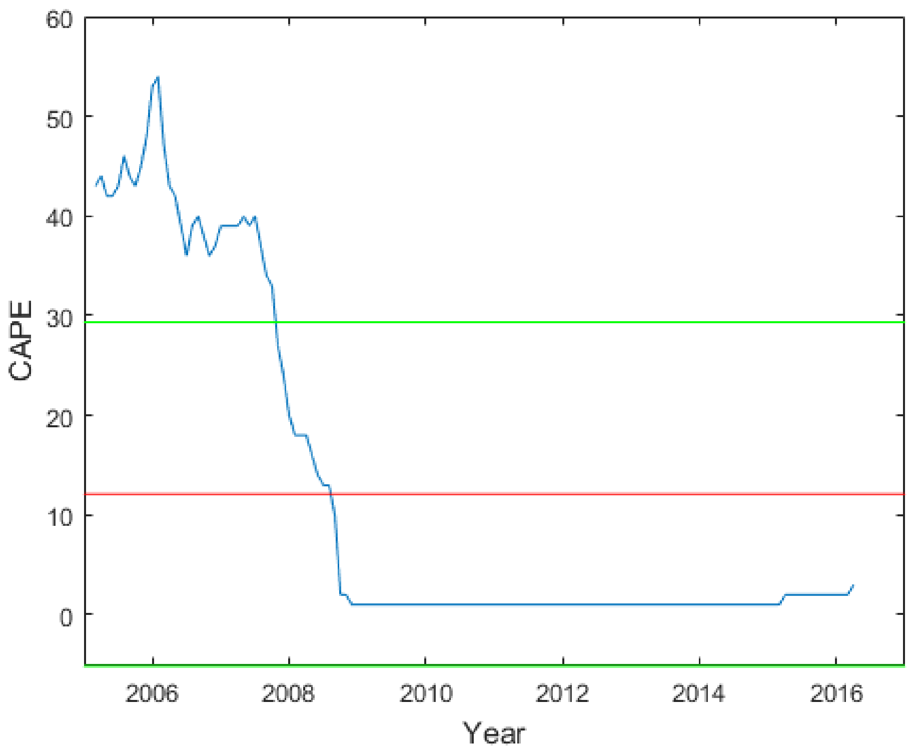


Figure 17: CAPE ratio of the selected share index consisting of ICEX-15, OMXI6 and OMXI8.

Figure 17 shows the CAPE ratio of the index from March 2015 throughout April 2016. The bubble is clearly noticeable with values well over 30 before the collapse. Because the CAPE ratio is a 10 year moving average of the earnings it is still recovering from the losses endured in 2008. Considering that factor and the fact that the price/earnings history does not go very far back, a 5 year moving average is presented here as a more appropriate tool to capture the market trend and possible price bubbles.

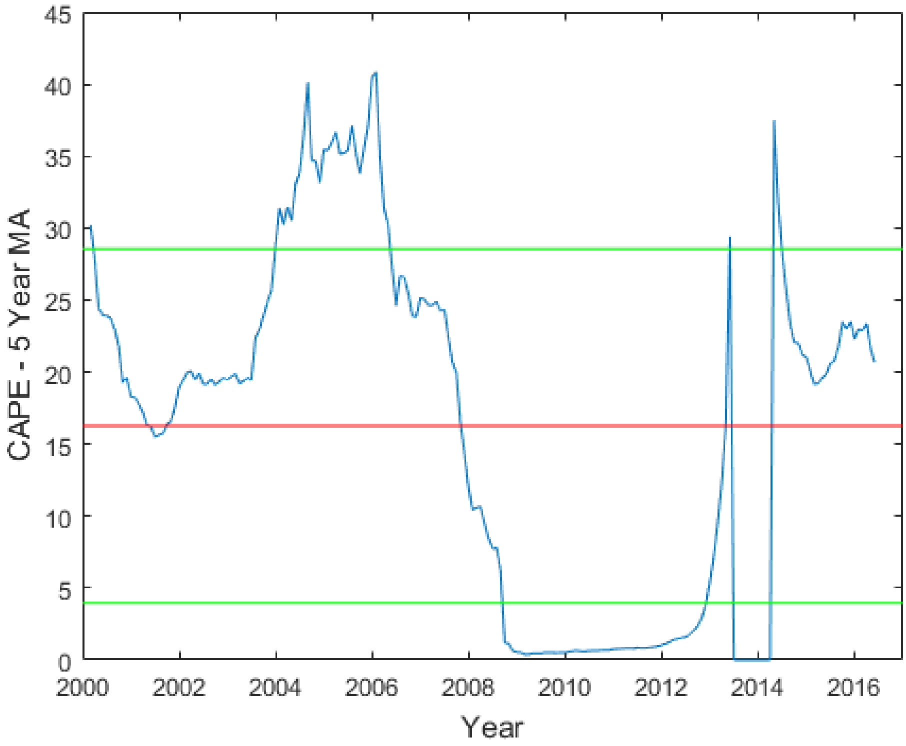


Figure 18: 5 year CAPE ratio of the selected share index consisting of ICEX-15, OMXI6 and OMXI8.

Disregarding negative outliers between 2013 and 2014 the average 5 year CAPE ratio is 16.3 with a standard deviation of 12.3. The ratio captures values in March 2000 and from 2004-2007 over the standard deviation which happen to be the internet and housing bubble respectively. The ratio shoots over 30 early in 2014 suggesting that there may have been an overvaluation on the market, but quickly adjusts with rising earnings, suggesting that there is no bubble present in 2016.

5.3 Variance Bound Tests

Variance bound tests of the present value model were developed by (Shiller, 1981), LeRoy and Porter (1981), they were initially derived to criticise the efficient market hypothesis and the present value model but it can be used for empirical bubble tests as stated by Blanchard and Watson (1982). Variance bound tests are based on the present value model of expected future dividends.

Shiller (1981) modeled a variance test to see if stock prices move too much to be justified by subsequent changes in dividends. The model presents a way of estimating a ex-post rational price series p^* and comparing with the actual price series p_t. Starting with the market fundamental pricing with expected dividends:

$$P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i \mathbb{E}_t(D_{t+i}) \tag{74}$$

And now denoting P_t^*, as the price with present value of actual dividends or ex-post rational price,

$$P_t^* = \sum_{i=1}^{\infty} \Big(\frac{1}{1+r}\Big)^i D_{t+i}. \tag{75}$$

The difference between P_t and P_t^* cannot be seen under rational expectations, this difference is denoted by ϵ_t,

$$P_t^* = \sum_{i=1}^{\infty} \Big(\frac{1}{1+r}\Big)^i [\mathbb{E}_t(d_{t+i}) + \epsilon_i] = P_t + \sum_{i=1}^{\infty} \Big(\frac{1}{1+r}\Big)^i \epsilon_i \tag{76}$$

The variance bound tests assumes that ϵ_t is not correlated with P_t and all information available at time t. The variance of P_t^* is

$$V(P_t^*) = V(P_t) + \varphi V(\epsilon_t) \geq V(P_t) \text{ with} \tag{77}$$

$$\varphi = \frac{(\frac{1}{1+r})^2}{1 - (\frac{1}{1+r})^2}$$

The ex post price in (74) is difficult to determine as future dividends are unrealised. Historical data can be used to determine the present value in (75).

Using the same data as in figure 13 the first thing is to determine the constant interest rate used to discount the dividends. The interest rate r_t at each time is here regarded as the price increase from t to t+1 of the stock along with the dividend D_t paid at time t:

$$r_t = \frac{P_{t+1} - P_t + D_t}{P_t}. \tag{78}$$

Then by using logarithmic returns the average return r is calculated

$$r = exp\Big(\frac{\sum_{t=0}^{N} log(1 + r_t)}{N}\Big) - 1. \tag{79}$$

The present value P^* in equation (75) is altered so the price at time t is equal to the current dividends D_t and discounted future price P_{t+1}^*.

$$P_t^* = D_t + \frac{P_{t+1}^*}{1+r}. \tag{80}$$

The last price in the series at time T, P_{t+1}=P_T^* needs to be estimated as it can not be calculated as it uses backwards induction from the last price in the series.

Figure 19 shows the real and estimated prices on monthly data of the S&P500 from 1871 to 2015, the last value of P_T^* is estimated to be the value of the index in January 2016. The parameters can be seen in Table 9. Defining a bubble as the deviation in prices between the price P_t and P_t^* produces the size of the bubble at time t.

Table 9: Parameters for the price P_t and estimated price P_t^*.

Parameter	P_0	P_T	P_0^*	P_T^*	r
Value	85.1	2054.08	85.03	1918.6	0.046

Disregarding negative bubbles and focusing on positive price bubbles the size of the bubble is shown in figure 20. The major downside of this model is that the price P_t^* relies heavily on the last price P_T^*, which has to be estimated and can alter the price path significantly.

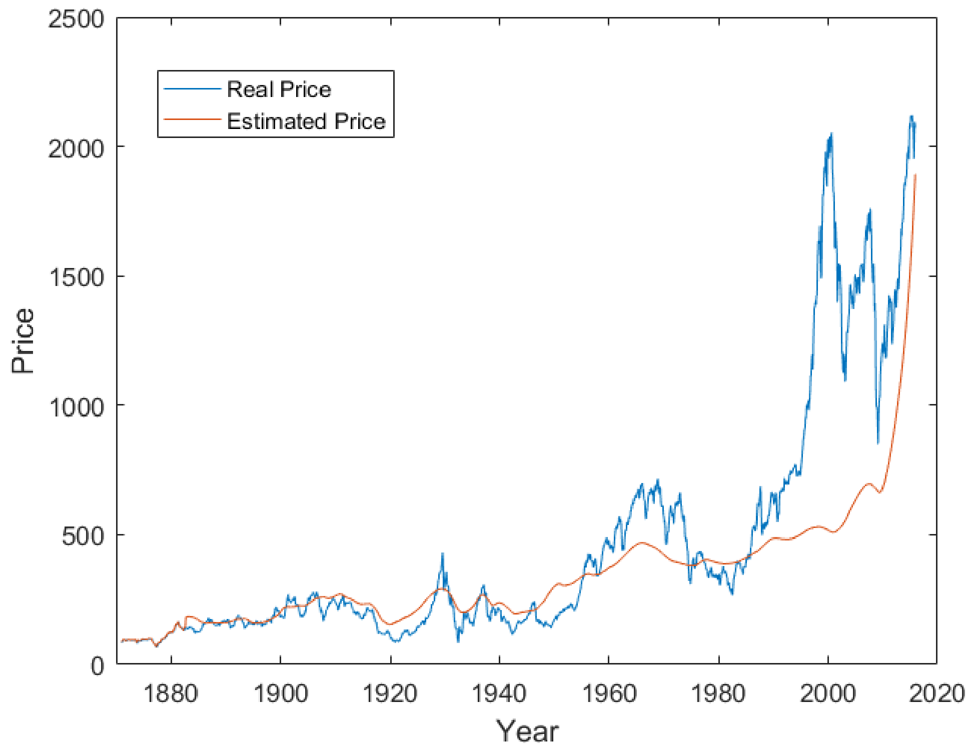


Figure 19: Real Price, P_t and estimated price, P_t^* of the S&P500 from 1871-2015.

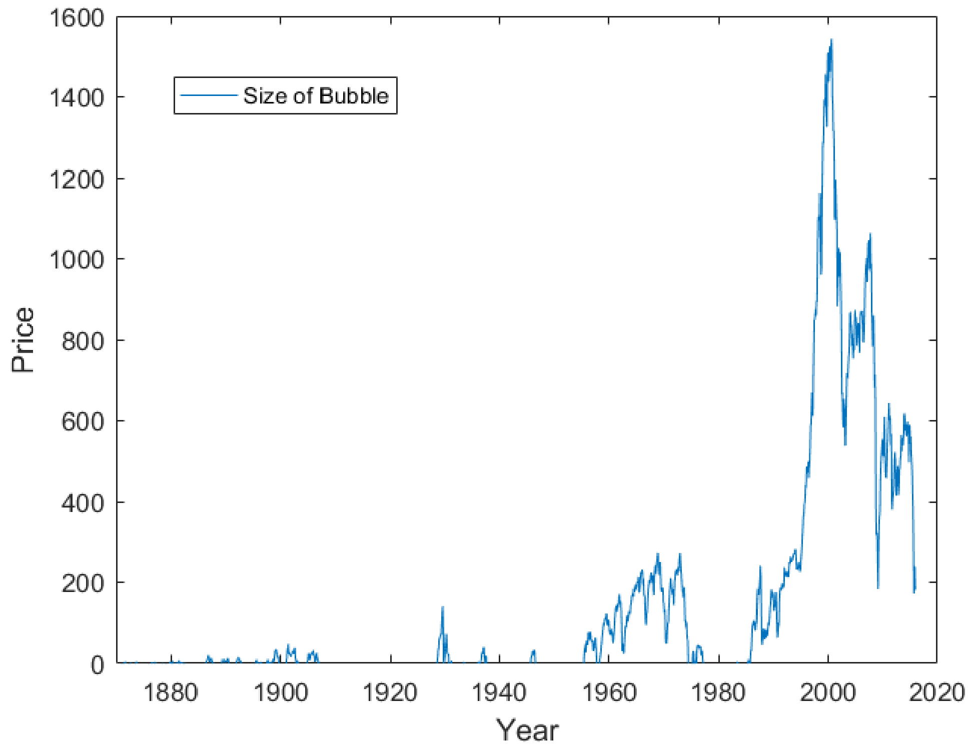


Figure 20: Size of the bubble, P_t- P_t^*.

6 Discussion and Conclusion

For now a perfect model to capture asset price bubbles does not exist. The world and the economy is constantly changing and asset price bubble models must adapt to new circumstances. However, history tends to repeat itself and asset price bubbles seem to have a few common indicators. The theoretical models discussed in this thesis can provide the necessary framework to deepen understanding of asset price bubbles.

To summarise a list of a few indicators which might contribute to the emergence of price bubbles are presented:

1. **Monetary Policy and amount of Credit:** Low long-term interest rates help to escalate asset price bubbles as investors have access to more credit and there will be more capital in play. Tightening monetary policy and increasing the interest rate often sets up a sequence of defaults leading to financial crises.

2. **Risk Shifting:** the risk shifting problem associated with credit expansion shows how an investor can shift the risk to the lender while retaining any upside returns. With enough investor with those incentives in the system they can create a bubble when driving up prices of risky assets.

3. **High CAPE ratio:** Because it is cyclically adjusted it captures the market trend better than the Price/Earnings ratio. CAPE ratio well over 20 should be reviewed and investigated as a possible bubble.

4. **High volume trading and high price volatility:** The psychological factors such as information cascades and herding can cause high volume trading and price volatility and can be an indicator of an emerging bubble.

These factors can give the indication of a bubble, however each instance should be investigated further and other possible outside factors considered which could explain high prices. Future work could include adding more parameters to the simple behavioural model, including different types of assets and allowing investors in the model to compare historical returns.

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Appendix A

Matlab code for the behavioral model of De Long:

```
function simplestbehavior(T, PriceToday, lambda, r,dividend)

if (nargin < 5)
    T = 100;
    PriceToday = 0.25;
    lambda = 1.5;
    r = 0.05;
    PriceLastT = 0.25;
    dividend=0.05;
end



for t = 1:T

    %Bernoulli distribution, probability that dividend is paid
    p = 0.5;
    U = rand;
    div = (U < p);

     %dividend paid, binary
    if (div > p)
        d = 1;
    else
        d = 0;
    end
    % calculating the new stock price


    PriceNextT = PriceToday + lambda*(PriceToday*(1-PriceToday))*((PriceToday-PriceLastT+d*dividend)/PriceLastT-r);


    % new price becomes old price


    PriceLastT=PriceToday;
    q(t)=[PriceToday]; %Store in a Matrix

    PriceToday = PriceNextT;


end


xlabel ('Time','fontsize',12);
ylabel ('Price','fontsize',12);


plot(q)
ylim([0.1 max(q)+0.1])
```

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