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in Economics

Exchange rate intervention in small open economies

Bayesian estimation of a DSGE model for Iceland

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Abstract

In this thesis the welfare effects of exchange rate intervention in small open economies will be examined. A dynamic stochastic general equilibrium model is built that incorporates the basic features of these economies. A monetary policy that responds to the inflation rate and the output gap is compared to monetary policies that additionally respond to the real exchange rate. The reaction of the economy to various shocks is examined and the welfare loss is estimated in order to compare monetary policies. Historical observations of various parameters for the Icelandic economy are used to estimate the parameters of the model using Bayesian estimation.
This thesis shows that in order to reduce the welfare loss introduced by various exogenous shocks exchange rate intervention is necessary. Exchange rate intervention reduces the observed volatility in the output gap, the domestic inflation and in the interest rate when used in response to certain exogenous shocks.

Keywords: DSGE model, Bayesian estimation, Iceland, Icelandic economy, exchange rate intervention, monetary policy, small open economy, dynamic stochastic, general equilibrium, exchange rate, interest rate.

Preface

This dissertation is in fulfillment of the requirements for the M.Sc. degree in Economics at the University of Iceland. The dissertation is to the value of 30 ECTS credits. The M.Sc. degree requires 90 ECTS credits.

My supervisor for this dissertation was Helgi Tómasson at the Faculty of Economics of the University of Iceland and I would like to express my gratitude to him for his support. All errors are my own.

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Steinar Björnsson

Contents

Nomenclature

This chapter defines the terminology used in the dissertation, in alphabetical order.

Variable	**Description**
a_t	Percent deviation from steady state of labor productivity.
A_t	Specific labor productivity.
$\overline{A}$	The steady state of labor productivity.
$b^{*\prime}_{r,t}$	Percent deviation from the steady state of real foreign assets. Denominated in the domestic currency.
$b^{F}_{r,t}$	Percent deviation from the steady state of real foreign debt. Denominated in the domestic currency.
B_t	Nominal net domestic debt of households, domestic currency.
B^*_t	Domestic nominal holding of foreign assets, denominated in the foreign currency.
$B^{*\prime}_t$	Domestic nominal holding of foreign assets, denominated in the domestic currency.
B^F_t	Foreign nominal debt of the domestic economy, denominated in the domestic currency.
$B^*_{r,t}$	Domestic real holding of foreign assets, denominated in the foreign currency.
$B^{*\prime}_{r,t}$	Domestic real holding of foreign assets, denominated in domestic currency.
$B^F_{r,t}$	Foreign real debt of the domestic economy, denominated in domestic currency.
$\overline{B}^*_{r,t}$	The steady state of real foreign assets. Domestic currency.
$\overline{B}^F_{r,t}$	The steady state of real foreign debt. Domestic currency.
c_t	Percent deviation of the private consumption from its steady state.
$c_{F,t}$	Percent deviation of the imports from the steady state.
$c^*_{H,t}$	Percent deviation of the exports from the steady state.
C_t	The composite consumption index of foreign and domestically produced goods. Equation 3.2.
$C_{F,t}$	The aggregate consumption index of foreign produced goods. Equation 3.20.

$C_{H,t}$	The aggregate consumption index of domestically produced goods. Equation 3.43.
C_t^*	Aggregate foreign private consumption. Equation 3.28.
$\overline{C}^*$	Steady state foreign private consumption.
$\overline{C}_H^*$	Steady state of exports from the domestic economy.
$\overline{C}_F$	Steady state of imports from the foreign economy.
CA_t	Nominal current account, equation 3.44.
e_t	Percent deviation from steady state of the nominal exchange rate, ξ_t.
$\mathbb{E}_t\{x_{t+1}\}$	The expected value of x one period ahead, taken at time t.
f_t	Percent deviation from steady state of the real net foreign debt.
F_t	Net real foreign debt of the domestic economy. Equation 3.46.
$\overline{F}$	Steady state net real foreign debt, denominated in domestic currency.
h	External habit formation of the optimizing household.
mc_t	Percent deviation from steady state of the domestic production firms' real marginal cost.
MC_t	Total domestic real marginal cost. Equation 3.33.
n_t	Percent deviation from steady state of hours of labor.
N_t	Hours of labor.
$\overline{N}$	The steady state of hours of labor.
NMC_t	Nominal marginal cost of domestic producers.
p_t	Percent deviation from steady state of the domestic price level.
$P_{H,t}$	The price level of domestically produced goods. Equation 3.40.
$P_{F,t}$	The price level of imported goods. Equation 3.41.
P_t	The domestic consumer price index. Equation 3.10.
P_t^*	The foreign consumer price index.
$P'_{H,t}$	The price level that optimizing producing firms set each period.
$P'_{F,t}$	The price level that optimizing importing firms set each period.
$\overline{P}$	Steady state domestic price level.
$\overline{P}_F$	The steady state of the price level of imported goods.
$\overline{P}_H$	The steady state of the price level of domestically produced goods.
Q_t	The real exchange rate. Equation 3.24.
$\overline{Q}$	The steady state of the real exchange rate.
R'_t	The nominal domestic interest rate, in percentages.
R_t	Scaled domestic interest rate. Equal to $1 + R'_t$.
$R_t^{*\prime}$	The foreign nominal interest rate.
R_t^*	Scaled foreign interest rate. Equal to $1 + R_t^{*\prime}$. Equation 3.29.
$\overline{R}'_{real}$	Steady state real interest rate, in percentages.
$\overline{R}_{real}$	Steady state scaled real interest rate of the domestic economy. Equal to $1 + \overline{R}'_{real}$.
$\overline{R}$	Steady state domestic nominal scaled interest rate level.
s_t	Percent deviation from steady state of the terms of trade.
S_t	The terms of trade. Equation 3.23.

TC_t	Total domestic real production cost. Equation 3.32.
w_t^{real}	The percent deviation from the steady state of real wages.
w_t	The percent deviation from the steady state of nominal wages.
W_t	The nominal wages. Equation 3.21.
$\overline{W}$	The steady state of nominal wages.
X_t^M	Real imports, denominated in the foreign currency.
y_t	Percent deviation from the steady state of real GDP.
Y_t	Real gross domestic product of the domestic economy. Equation 3.48.
$\overline{Y}$	Steady state gross domestic product.

α	The degree of openness of the domestic economy.
α^*	Degree of openness of the foreign economy.
α_1	Rate of interest rate smoothing, $\alpha_1 \in [0, 1]$.
α_2	Weight on inflation, in the monetary policy.
α_3	Weight on the output gap, in the monetary policy.
α_4	Weight on the real exchange rate level, in the monetary policy.
α_5	Weight on the rate of change of the real exchange rate, in the monetary policy.
β	The rate of time preference.
γ	Long term risk premium for the domestic economy.
ϵ	The elasticity of substitution between varieties of different goods, assumed to be the same for foreign and domestically produced goods.
ϵ_t^a	Gaussian shock to the labor productivity.
ϵ_t^f	Gaussian shock to the net foreign debt.
ϵ_t^q	Gaussian shock to the real exchange rate.
ϵ_t^{prem}	Gaussian shock to the risk premium.
$\epsilon_t^{\pi_F}$	Gaussian shock to the inflation in imported goods.
ϵ_t^{ψ}	Gaussian shock to the LOP gap.
$\epsilon_t^{c^*}$	Gaussian shock to the foreign consumption.
$\epsilon_t^{r^*}$	Gaussian shock to the foreign interest rate.
$\epsilon_t^{\pi^*}$	Gaussian shock to the foreign inflation.
η	The elasticity of substitution between home and foreign goods.
η^*	Elasticity of substitution between home and foreign goods, seen from the foreign economy.
θ_F	Fraction of importing firms unable to reset their prices optimally.
θ_H	Fraction of domestic producers unable to reset their prices optimally.
μ	How an individual values between the lagged term and the *scaled-terms of trade factor*, in equation 3.43.
ν_t^a	A latent shock variable for the labor productivity.
ν_t^f	A latent shock variable for the net foreign debt.
ν_t^q	A latent shock variable for the real exchange rate.
ν_t^{prem}	A latent shock variable for the risk premium.
$\nu_t^{\pi_F}$	A latent shock variable for the inflation in imported goods.
ν_t^{ψ}	A latent shock variable for the LOP gap.
$\nu_t^{c^*}$	A latent shock variable for the foreign private consumption.

$\nu_t^{r^*}$	A latent shock variable for the foreign interest rate.
$\nu_t^{\pi^*}$	A latent shock variable for the foreign inflation.
ξ_t	The nominal exchange rate. Foreign currency to domestic currency.
π_t	Percent deviation from the steady state of domestic inflation.
π_t^*	Percent deviation from the steady state of foreign inflation.
Π_t^*	Foreign inflation. Defined as $\frac{P_t^*}{P_{t-1}^*}$.
Π_t	Domestic inflation. Equal to $\frac{P_t}{P_{t-1}}$.
$\overline{\Pi}^T$	CBI's inflation target.
$\overline{\Pi}$	Steady state domestic inflation.
$\overline{\Pi}^*$	Steady state foreign inflation.
ρ_a	Autocorrelation coefficient for the labor productivity.
ρ_f	Autocorrelation coefficient for the net foreign debt.
ρ_{prem}	Autocorrelation coefficient for the risk premium.
ρ_{π_F}	Autocorrelation coefficient for the inflation in imported goods.
ρ_ψ	Autocorrelation coefficient for the LOP gap.
ρ_q	Autocorrelation coefficient for the real exchange rate.
ρ_{π^*}	Autocorrelation coefficient for the foreign inflation.
ρ_{r^*}	Autocorrelation coefficient of the nominal foreign interest rate.
ρ_{c^*}	Autocorrelation coefficient for the foreign consumption.
σ	The inverse elasticity of intertemporal substitution.
ϕ	The inverse elasticity of labor supply.
Φ	The risk premium of the domestic economy. Equation 3.27.
ψ_t	Percent deviation from steady state of the law of one price gap, Ψ_t.
Ψ	The law of one price gap. Equation 3.25.

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1 Introduction

'Central banks in small open economies should openly recognize that exchange rate stability is part of their objective function'[1]
February 12, 2010,
Olivier Blanchard,
Chief economist at the International Monetary Fund

The same laws do not apply in the design of monetary policies for small open economies as for large developed ones. Small open economies have to deal with stronger volatility in international financial markets and international trade. High variability of country risk premiums and commodity prices play an important role and central banks must take notice to these factors when implementing the monetary policy.

One of the key variables is the real exchange rate through which the fluctuations of international markets are transmitted to the small open economy. External shocks can alter the real exchange rate which may lead to increased cost of external debt service, the income of commodity exports, the cost of imports and other factors. A key factor is that change in the real exchange rate may alter the expected path of domestic inflation and the central bank must make appropriate adjustments.

The goal of this thesis is to estimate empirically how the monetary policy in a small open economy interacts with shocks to the economy, mainly external shocks. A welfare analysis will be made to compare different monetary policies. A model of a typical small open economy is built and estimated using historical data of the Icelandic economy. The model has to be sufficiently general to incorporate the basic structures observed in small open economies. A dynamic stochastic general equilibrium (DSGE) model is built and estimated with Bayesian techniques to estimate all the equations and shocks simultaneously. We will also look at impulse response functions to see how different structural variables respond to different shocks, which gives a little insight into the dynamics of the model.

[1] See Blanchard, Dell'Ariccia, and Mauro [2010].

The model considers imperfect capital markets using a risk premium that depends on net foreign debt. Nominal and real rigidities, exports and imports of goods, imperfect pass-through of the exchange rate and wage indexation will also be considered. The foreign economy is taken to be exogenous to the domestic economy.

This thesis approaches four general questions. 1) What are the welfare effects of using the real exchange rate as one of the factors when deciding the central banks interest rate, in small open economies? 2) Should central banks in small open economies with floating currencies use more exchange rate intervention? 3) To what kind of shocks should the central banks react to by using exchange rate intervention? 4) How do different monetary policies compare in terms of welfare loss: monetary policies that respond to the exchange rate level, monetary policies that respond to the rate of change of the real exchange rate or monetary policies that allow the currency to float freely?

2 Literature Survey

In academics, central banks and various international policy institutions, in recent years, there has been a growing interest in open economy macroeconomic models called Dynamic Stochastic General Equilibrium (DSGE) models based on the new Keynesian framework. A few noted institutions that have developed DSGE models are for example (see Tovar [2008]): Bank of Canada, Bank of England, Central bank of Brazil, Central bank of Chile, Central Bank of of Peru, European Central Bank, Norges Bank, Sveriges Riksbank, US federal reserve and the IMF.

Much of the literature on monetary policy in open economies has focused on whether the central banks responded to the real exchange rate or not. Empirical studies indicate that many countries include the real exchange rate in their policy reaction functions. The evidence is not conclusive though, New Zealand and Australia for example did not incorporate the exchange rate in their policy reaction function (Lubik and Schorfheide [2007]). Welfare analysis has produced contradictory results depending on the model used (Bergin, Shin, and Tchakarov [2007]). It has been found that having the monetary policy respond to the real exchange rate marginally improves macroeconomic performance of central banks, see for example Ball [1999].
But other studies such as Wollmershauser [2006], Morón and Winkelried [2005] and Cavoli [2009] show that defending the exchange rate may be useful in a context of financial instability or as a response to fear of floating.

In a recent paper on the subject, Gonzalez and Garcia [2010], it was found that risk premium shocks explain most of the variance of the exchange rate. The changes in the real exchange rate causes important reallocation of resources across sectors in the short run. In the paper it was found that when a shock to the risk premium occurs the central bank can avoid excess volatility by raising the interest rate. But an important result from Gonzalez and Garcia [2010] is that in order to reduce the observed volatility of inflation and in the output gap, more exchange rate intervention is necessary, in small open economies. The volatility can be greatly reduced by changing the interest rate when the exchange rate is fluctuating due to a risk premium shock.

Recent contributors to DSGE estimations of small open economies are for example Adolfson, Laseén, Lindé, and Villani [2007a], Dib, Gammoudi, and Moran [2008], Justiniano and Preston [2004a], Liu [2006] and Lubik and Schorfheide [2005].

Kydland and Prescott [1982] originally used the term DSGE in their seminal contribution on the Real Business Cycle (RBC) model. Later research in DSGE models included Keynesian short run macroeconomic features called nominal rigidities, such as Calvo [1983] type staggering pricing behavior and Taylor [1980] type wage contracts. This new DSGE modeling framework is called new-neoclassical synthesis or new-Keynesian modeling paradigm, see for example Clarida, Galí, and Gertler [1999], Galí and Gertler [2007], Goodfriend [2007], Goodfriend and King [1997], Mankiw [2006] and Lubik and Schorfheide [2005].

This DSGE modeling framework uses micro-foundations of both households and firms optimization problems with both nominal and real (price/wage) rigidities that provide short-run dynamic macroeconomic fluctuations and combine it with a description of the monetary policy transmission mechanism, for instance see Christiano, Eichembaum, and Evans [2005] and Smets and Wouters [2004].
The key advantage of modern DSGE models, over traditional reduced form macroeconomic models, is that the structural interpretation of their parameters allows to overcome the famous *Lucas critique*. In Lucas [1976] and Lucas and Sargent [1979] it is argued that if private agents behave according to a dynamic optimization approach and use available information rationally, they should respond to economic policy announcements by adjusting their supposed behavior. Hence reduced form parameters are subject to the Lucas critique. But, DSGE models are based on optimizing agents. Deep parameters of these models are therefore less susceptible to this critique.

The model derived in the next chapter is a DSGE model consistent with Kolasa [2008], Liu [2006], Galí and Monacelli [2005] and Lubik and Schorfheide [2005], to name a few. It is also worth noting that the Central Bank of Iceland has very recently published a working paper on a DSGE model for Iceland, and the CBI will most likely switch to the DSGE model from their Quarterly Macroeconomic Model in the near future, see Seneca [2010].

3 The Model

Now we will define the small open economy model that is used in the estimation. Derivation of key structural equations is laid out. The work of earlier literature on the subject is used as a foundation and a small model is constructed to capture as much dynamic as possible for the small open economy[1]. A small open economy is an economy that participates in international trade, but is small enough, compared to other economies, that its policies do not alter the world prices, interest rates or incomes. Countries with small open economies are therefore price takers.

The economy consists of utility optimizing households and profit maximizing firms but the government is excluded and the representative social planner form is used. There is an import/export sector and we consider nominal and real rigidities in prices and wages. The foreign economy is exogenous to the small open economy. The uncovered interest rate parity with a risk premium is used to model the real exchange rate and a dynamic asset equation is used to model the net foreign assets. Capital is assumed to be fixed, for simplification. The model is defined in the following sections.

We begin by taking a look at an economic property that will be used when we define the model. This property is called *Constant elasticity of substitution*, (CES), and it is a desirable property of some economic functions. It refers to a particular type of aggregator function which combines two or more inputs into an aggregated quantity. The aggregator function has the general form:

$$y = \left[\sum_{i=1}^{n} a_i^{\rho_1} x_i^{\rho_2} \right]^{\frac{1}{\rho_2}}$$

Where y is the output, $0 \leq a_i \leq 1$ are the share parameters and $x_i \geq 0$ are input factors. The parameters $\rho_1 \in \mathbb{R}$ and $-\infty < \rho_2 < 1$ define the shape of the

[1] Previous work includes for example Smets and Wouters [2004], Monacelli [2005] and Galí and Monacelli [2005].

function. The parameter ρ_2 is also a measure of substitutability between inputs. For example as ρ_2 approaches zero the aggregate function approaches the *Cobb-Douglas* functional form. If we choose $\rho_1 = \frac{1}{\sigma}$ and $\rho_2 = \frac{\sigma-1}{\sigma}$ the aggregate function becomes the general form of the CES production function. Where $\sigma > 0$ is the elasticity of substitution between inputs.
The aggregator function exhibits constant elasticity of substitution, it is therefore often called the *CES function.* The elasticity of substitution measures the percentage change in the input ratio divided by the percentage change in the technical rate of substitution[2] (TRS), with output being held fixed.

3.1 Households

We imagine a representative household who seeks to maximize[3]:

$$\mathbb{E}_{t=0}\left\{\sum_{t=0}^{\infty}\beta^t\{U(C_t,H_t)-V(N_t)\}\right\} \tag{3.1}$$

Where:

$$U(C_t,H_t) = \frac{(C_t-H_t)^{1-\sigma}}{1-\sigma} \text{ and } V(N_t) = \frac{N_t^{1+\phi}}{1+\phi}$$

Where $\mathbb{E}_t$ is expectation at time t, β is the rate of time preference, σ is the inverse elasticity of intertemporal substitution and ϕ is the inverse elasticity of labor supply. N_t denotes hours of labor, C_t is private consumption at time t and we define $H_t \equiv hC_{t-1}$ which represents external habit formation of the optimizing household[4]. We have $h \in (0,1)$.

Private consumption, C_t, is the composite consumption index of foreign and domestically produced goods[5]:

$$C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{3.2}$$

This functional form arises as a utility function in consumer theory[6]. Where $\alpha \in [0,1]$ corresponds to the share of domestic consumption allocated to imported

[2] Also referred to as *Marginal rate of technical substitution.*
[3] The work of Galí and Monacelli [2005] is followed.
[4] For more information see Justiniano and Preston [2004a] and Bouakez and Ruge-Murcia [2005].
[5] This definition of the consumption index is quite common, and used by, for example, Haider and Khan [2008], Liu [2006] and Monacelli [2005].
[6] A CES utility function is one of the cases considered by Avinash Dixit and Joseph Stiglitz in their study of optimal product diversity in a context of monopolistic competition.

goods. It is also in this sense that α represents a natural index of openness[7].
We have $\eta > 0$ which is the elasticity of substitution between home and foreign goods, from the viewpoint of the domestic consumer. Note that when $\sigma \to 0$ the consumption goods $C_{F,t}$ and $C_{H,t}$ are perfect substitutes. The consumption index has constant elasticity of substitution.

Each country produces a continuum of differentiated goods, represented by the unit interval. The variable $C_{H,t}$ is an index of consumption of domestic goods given by the CES function:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon}{\epsilon-1}} dj \right)^{\frac{\epsilon-1}{\epsilon}} \tag{3.3}$$

Where $j \in [0,1]$ denotes the good variety and where $\epsilon > 1$ denotes the elasticity of substitution between varieties of goods produced within any given country.
The variable $C_{F,t}$ is an index of imported goods, given by:

$$C_{F,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\kappa}{\kappa-1}} di \right)^{\frac{\kappa-1}{\kappa}} \tag{3.4}$$

Where κ measures the substitutability between goods produced in different foreign countries. And $C_{i,t}$ an index of the quantity of goods imported from country i and consumed by households of the domestic economy. It is given by an analogous CES function:

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\epsilon}{\epsilon-1}} dj \right)^{\frac{\epsilon-1}{\epsilon}} \tag{3.5}$$

The household's budget constraint is constant at time t and is given by[8]:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + R_t B_t \leq \mathbb{E}_t\{B_{t+1}\} + W_t N_t \tag{3.6}$$

for $t = 1, 2, ..., \infty$. $P_{i,t}(j)$ is the price of variety j imported from country i, denominated in the domestic currency. $P_{H,t}$ is the domestic price index. W_t are the nominal wages and N_t are the total hours of labor.
B_t is the nominal net debt of households, denominated in domestic currency and B_t can become negative or positive, depending on if the household is a net borrower or a net owner of assets, if $B_t > 0$ the household is in net debt.
R is defined as $R \equiv 1 + R'$, where R' is the domestic nominal interest rate, in percentages.

The domestic price index is given by the following CES function:

$$P_{H,t} = \left(\int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \tag{3.7}$$

[7] So $\alpha = 0$ means a closed economy.

[8] A similar budget constraint is used by Monacelli [2005]. The net debt expression in the budget constraint is from Wickens [2008].

Where ϵ denotes the elasticity of substitution between varieties produced within any given country like before. The price index for goods imported from country i, in the domestic currency, is given by:

$$P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \tag{3.8}$$

For all $i \in [0,1]$. Finally the price index for imported goods, expressed in the domestic currency is given by:

$$P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\kappa} di \right)^{\frac{1}{1-\kappa}} \tag{3.9}$$

The overall domestic *Consumer price index*, CPI, is defined as[9]:

$$P_t \equiv \left\{ (1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right\}^{\frac{1}{1-\eta}} \tag{3.10}$$

Where η is the elasticity of substitution between home and foreign goods like before.
Total consumption expenditures by domestic households is given by:

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \tag{3.11}$$

Using the preceding expressions, and following Monacelli [2005], we rewrite the budget constraint assuming symmetry across all j goods as:

$$P_t C_t + R_t B_t \leq \mathbb{E}_t\{B_{t+1}\} + W_t N_t \tag{3.12}$$

We will also make the assumption that ϵ, the elasticity of substitution between varieties of goods, is assumed to be the same in the foreign and home economies so we have $\epsilon = \kappa$. The assumption is irrelevant because domestic consumption of foreign goods has negligible effect on the foreign economy.

Now we will derive the optimal expression for consumption of domestically produced products, $C_{H,t}$, and for consumption of imported products, $C_{F,t}$. We begin by setting up the household Lagrangian, where we maximize the utility function and use the households budget constraint as a constraint of the maximization, equation 3.13.
The household's optimizing problem then becomes:

$$\begin{aligned} \mathcal{L} = \mathbb{E}_{t=0} & \left\{ \sum_{t=0}^{\infty} e^{-\beta t} \{ U(C_t, H_t) - V(N_t) \} \right\} \\ & + \sum_{t=0}^{\infty} \lambda_t \left[B_{t+1} + W_t N_t - R_t B_t - P_t C_t \right] \end{aligned} \tag{3.13}$$

[9] See for example Monacelli [2005], where the same definition of the CPI is used.

We use equation 3.2 to rewrite the budget constraint in equation 3.13, eliminating C_t, we get the following Lagrangian expression:

$$\mathcal{L} = \mathbb{E}_{t=0}\left\{\sum_{t=0}^{\infty} e^{-\beta t}\{U(C_t, H_t) - V(N_t)\}\right\} \tag{3.14}$$
$$+\sum_{t=0}^{\infty}\lambda_t\left[B_{t+1} + W_{t+j}N_t - R_tB_t - P_t\left[\left((1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right]\right]$$

We also use equation 3.11 two rewrite the budget constraint in equation 3.13, eliminating C_t, we get the yet another Lagrangian expression:

$$\mathcal{L} = \mathbb{E}_{t=0}\left\{\sum_{t=0}^{\infty} e^{-\beta t}\{U(C_t, H_t) - V(N_t)\}\right\} \tag{3.15}$$
$$+\sum_{t=0}^{\infty}\lambda_t\left[B_{t+1} + W_tN_t - R_tB_t - (P_{H,t}C_{H,t} + P_{F,t}C_{F,t})\right]$$

Taking the partial derivative of equation 3.14 with respect to $C_{H,t}$ gives us:

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta C_{H,t}} &= -\lambda_t P_t\frac{\eta}{\eta-1}\left[\left((1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1}\right](1-\alpha)^{\frac{1}{\eta}}\frac{\eta-1}{\eta}C_{H,t}^{\frac{\eta-1}{\eta}-1} \\ &+ e^{-\beta t}\frac{\delta}{\delta C_{H,t}}U(C_t, H_t)\end{aligned} \tag{3.16}$$

We can rewrite equation 3.16, using equation 3.2, as:

$$\frac{\delta\mathcal{L}}{\delta C_{H,t}} = -\lambda_t P_t(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}-1}C_tC_t^{-\frac{\eta-1}{\eta}} + e^{-\beta t}\frac{\delta}{\delta C_{H,t}}U(C_t, H_t) \tag{3.17}$$

Taking the partial derivative of equation 3.15 with respect to $C_{H,t}$ gives us:

$$\frac{\delta\mathcal{L}}{\delta C_{H,t}} = -\lambda_t P_{H,t} + e^{-\beta t}\frac{\delta}{\delta C_{H,t}}U(C_t, H_t) \tag{3.18}$$

Combining equations 3.17 and 3.18 and solving for $C_{H,t}$ gives us:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t \tag{3.19}$$

Taking the partial derivatives of equations 3.14 and 3.15 with respect to $C_{F,t}$ and combining them, like we did for $C_{H,t}$, yields:

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t \tag{3.20}$$

Which concludes the derivation of the optimal expressions for imports and domestically produced goods.

Solving the household's optimizing problem, maximizing equation 3.13 with respect to C_t, N_t and B_{t+1}, yields the following two first order conditions (FOC's)[10]:

$$\frac{W_t}{P_t} = N_t^{\phi}(C_t - H_t)^{\sigma} \tag{3.21}$$

$$(C_t - H_t)^{-\sigma} e^{\beta} = \mathbb{E}_t \left\{ (C_{t+1} - H_{t+1})^{-\sigma} \frac{R_{t+1}}{\Pi_{t+1}} \right\} \tag{3.22}$$

These two equations will be used to model the wages and the consumption of households, in the domestic economy.

3.2 The Foreign Economy

The home economy is very small compared to the foreign economy so the foreign economy is taken to be exogenous. Imports and exports from the home economy have negligible effect on the foreign economy. In this section we start by examining the connection to the foreign economy in terms of the exchange rate and related variables, then we examine foreign production, inflation and the foreign interest rate.

3.2.1 The Exchange Rate and Terms of Trade

We begin by defining the *terms of trade*, which is defined as[11]:

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{3.23}$$

The terms of trade is the price of imports divided by the price of exports, it is a measure of competitiveness of the home economy. An increase in S_t corresponds to an increase in competitiveness.

We define ξ_t as the nominal exchange rate in units of foreign currency to domestic currency. So an increase in ξ_t corresponds to an appreciation of the domestic currency. The real exchange rate then becomes[12]:

$$Q_t = \xi_t \frac{P_t}{P_t^*} \tag{3.24}$$

Where P_t^* is the foreign price level, in units of foreign currency. Taking the partial derivative with respect to ξ_t on both sides gives us:

$$\frac{\delta Q_t}{\delta \xi_t} = \frac{\delta}{\delta \xi_t} \xi_t \frac{P_t}{P_t^*} = \frac{P_t}{P_t^*} > 0$$

[10] See Appendix A for a derivation of equations 3.21 and 3.22.

[11] This definition of the terms of trade is quite common and used by for example Haider and Khan [2008].

[12] The definition of the real exchange rate comes from Wickens [2008], page 147.

So an increase in the real exchange rate, Q_t, can be considered as an appreciation of the domestic currency, because of the positive relationship between Q_t and ξ_t.

We define the *law of one price gap* as[13]:

$$\Psi = \frac{P_t^*}{\xi_t P_{F,t}} \tag{3.25}$$

The *law of one price* holds if $\Psi = 1$, then we have $P_{F,t} = \frac{P_t^*}{\xi_t}$. The *LOP* gap is a wedge between the foreign price of goods and the domestic price of these imported foreign goods.

3.2.2 The Uncovered Interest Rate Parity

A simple *uncovered interest rate parity* (UIP) has been shown to be rejected empirically[14]. The UIP comes from the idea that border free financial markets make the yield between interest bearing accounts highly competitive since it's possible to choose between domestic and foreign bank accounts and investments. If a risk premium is added to the UIP relationship it becomes more empirically stable, so we have UIP with a risk premium[15]:

$$\Phi_t R_t \frac{\xi_{t+1}}{\xi_t} = R_t^* \tag{3.26}$$

Where $R_t^* = 1 + R_t^{*\prime}$, where $R_t^{*\prime}$ is the foreign nominal interest rate, in percentages. Φ is the risk premium needed for the UIP to become more empirically stable. The risk premium is thought to be correlated to the net foreign debt of the economy[16]. This means that a domestic surplus indicates a lower risk, so foreign investors accept lower yield. The risk premium captures the default risk as perceived by investors with the domestic interest rate being higher than the world interest rate, if the economy is a net borrower. The risk premium has the following expression[17], where the risk premium is proportional to net foreign debt, F_t, domestically denominated:

$$\Phi_t = e^{-\gamma \frac{F_t}{Y_t}} \tag{3.27}$$

Where $\gamma \geq 0$ is the neutral risk premium factor, depending on the country's history of risk. γ is assumed to be a constant. F_t is the real net foreign debt[18], denominated in domestic currency and Y_t is the real gross domestic product (GDP).

[13] This form of the LOP gap is used in Liu [2006], for example.
[14] See for example Adolfson, Vredin., Lindé, and Villani [2007b].
[15] Following Post [2007].
[16] According to Lane and Milesi-Ferretti [2001].
[17] See Post [2007] page 45.
[18] Defined by equation 3.46.

3.2.3 Foreign Consumption, Inflation and Interest Rate

It is customary when modeling DSGE models for small open economies that the foreign sector is taken to be exogenous. The home country has negligible effect on the outside world. The foreign private consumption, the foreign inflation and the foreign interest rate are taken as exogenous. The variables are subject to shocks but revert to their steady state at a certain pace, determined by a autocorrelation coefficient ρ[19]. We assume that the variables have a defined steady state, we will discuss the steady states further in chapter 4.
The foreign private consumption is defined as[20]:

$$\frac{C_t^*}{\overline{C}^*} = \left(\frac{C_{t-1}^*}{\overline{C}^*}\right)^{\rho_{C^*}} e^{\epsilon_t^{C^*}} \tag{3.28}$$

Where C_t^* is real foreign private consumption and $\overline{C}^*$ is the steady state of foreign private consumption[21]. We have $\rho_{C^*} \in (0,1)$ and $\epsilon_t^{C^*}$ is a Gaussian shock with non-zero mean and variance $\sigma^2_{\epsilon^{C^*}}$[22]. Note that the shocks can also be interpreted as measurements error.

Foreign inflation and interest rate are defined in the same way:

$$\frac{\Pi_t^*}{\overline{\Pi}^*} = \left(\frac{\Pi_{t-1}^*}{\overline{\Pi}^*}\right)^{\rho_{\Pi^*}} e^{\epsilon_t^{\Pi^*}} \text{ and } \frac{R_t^*}{\overline{R}^*} = \left(\frac{R_{t-1}^*}{\overline{R}^*}\right)^{\rho_{R^*}} e^{\epsilon_t^{R^*}} \tag{3.29}$$

Where $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$ is foreign inflation and $R^* \geq 1$ is the scaled nominal foreign interest rate. $R^* = 1 + R^{*\prime}$ where $R^{*\prime}$ is the foreign nominal interest rate, in percentages. $\overline{R}^*$ is the steady state of the scaled foreign nominal interest rate and $\overline{\Pi}^*$ is the steady state of foreign inflation. $\epsilon_t^{R^*}$ and $\epsilon_t^{\Pi^*}$ are Gaussian shocks. The parameters $\rho_{R^*} \in (0,1)$ and $\rho_{\Pi^*} \in (0,1)$ are autocorrelation coefficient's.

[19]Note that when equations 3.28 and 3.29 are log-linearized they become an AR(1) process, see chapter 4.

[20]For the foreign consumption, interest rate and inflation I follow preceding work on DSGE models for small open economies, see for example Liu [2006], Haider and Khan [2008] and Justiniano and Preston [2004b]. When equations 3.28 and 3.29 are log-linearized they follow an AR(1) process which is customary for the exogenous processes, see chapter 4 where the equations are log-linearized.

[21]The real foreign private consumption is a growth variable, but it is thought to have a steady state for a short period of time, since we are only interested in the dynamics of the model, not the growth. This problem will be addressed further in chapter 4 where the model is log-linearized.

[22]See chapter 5, page 35 for further explanation.

3.3 Firms

Domestic producers inhabit the domestic economy along with households. They are identical monopolistically competitive firms, producing differentiated goods. There is a continuum of firms, indexed by $j \in (0,1)$ where each firm maximizes its profits, subject to an isolated demand curve and the firms only use a homogeneous type of labor for production, the capital is assumed to be fixed and is therefore left out.

3.3.1 Production Technology and Cost

We have domestic firms with the same CRS-technology, so we have a linear production function with only labor as input. Firm number j produces a differentiated good, $Y(j)$[23]:

$$Y_t(j) = A_t N_t(j) \tag{3.30}$$

Where A_t is the specific labor productivity. Aggregate output can be written as[24]:

$$Y_t = \left[\int_0^1 Y_t(j)^{-(1-\epsilon)} dj \right]^{\frac{1}{-(1-\epsilon)}} \tag{3.31}$$

Since capital is omitted the only cost of firms is the wage cost so the real total cost becomes:

$$TC_t \equiv \frac{W_t}{P_{H,t}} N_t = \frac{W_t}{P_{H,t}} \frac{Y_t}{A_t} \tag{3.32}$$

Where P_H is the price of domestically produced products. The real marginal cost becomes:

$$\frac{\delta TC_t}{\delta Y_t} \equiv MC_t = \frac{W_t}{P_{H,t} A_t} \tag{3.33}$$

3.3.2 Calvo-Type Price Setting Behavior

This section explains the equations and relationships that define the price level and inflation in domestically produced goods and imported goods. For the model, firms set prices according to a Calvo type staggered-price setting[25].

[23] See Monacelli [2005] page 9 where the same production function is used.

[24] This is a CES-functional form, as is done in Monacelli [2005].

[25] See Calvo [1983] and Monacelli [2005] for more information on the equations and derivations for price level of domestically produced goods and imported goods.

3.3.2.1 Domestic Price Level

Domestic differentiating goods are subject to a Calvo-price setting. In any period a $(1-\theta_H)$ fraction of firms are able to reset their prices optimally, $\theta_H \in [0,1]$. While the other fraction, θ_H, can not[26].
The latter fraction is assumed to adjust their prices, $P^I_{H,t}(j)$, by indexing it to the inflation in the last period[27]:

$$P^I_{H,t}(j) = P_{H,t-1}(j)\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H} \tag{3.34}$$

It is assumed that the degree of past inflation is the same as the probability of resetting prices[28]. We only consider the symmetric equilibrium where the prices for the firms are the same, $P_{H,t}(j) = P_{H,t}(k) \forall j,k$. So we let $P'_{H,t}$ denote the price level that optimizing firms set each period. The aggregate domestic price level becomes:

$$P_{H,t} = \left\{(1-\theta_H)(P'_{H,t})^{1-\epsilon} + \theta_H\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\epsilon}\right\}^{\frac{1}{1-\epsilon}} \tag{3.35}$$

Where $\epsilon > 1$ denotes the elasticity of substitution between varieties of goods produced within any given country, like before and equation 3.35 is in a CES functional form. Firms re-optimize their prices and maximize their profits, in aggregate, after setting the new price $P'_{H,t}(j)$ at time t as[29]:

$$max \sum_{k=0}^{\infty} \mathbb{E}_t\left[(\theta_H)^k\left\{D_{t,t+k}\left(Y_{H,t+k}\left[P'_{H,t} - NMC_{H,t+k}\right]\right)\right\}\right] \tag{3.36}$$

Where $D_{t,t+k}$ is a discount factor, considered as the price of a discount bond that pays one unit of the domestic currency at time $t+k$. We maximize equation 3.36 with respect to $P'_{H,t}(j)$, subject to the following demand function:

$$Y_{H,t+k} \leq \left(C_{H,t+k} + C^*_{H,t+k}\right)\left[\frac{P'_{H,t}}{P_{H,t+k}}\right]^{-\epsilon}$$

Where $NMC_{H,t+k}$ is the nominal marginal cost. Demand comes from both consumption of domestic products, $C_{H,t}$, and from imported products, $C_{F,t}$. The first order condition from the maximization problem, equation 3.36, becomes:

$$\sum_{k=0}^{\infty} \mathbb{E}_t\left[(\theta_H)^k\left\{D_{t,t+k}\left(Y_{H,t+k}\left[P'_{H,t} - \frac{\epsilon}{\epsilon-1}NMC_{H,t+k}\right]\right)\right\}\right] = 0 \tag{3.37}$$

[26] The average duration of a price is given by $\frac{1}{1-\theta_H}$.
[27] See Appendix 2 in Galí and Monacelli [2005] for more details.
[28] This assumption ensures that the Phillips curve is vertical in the long run.
[29] According to Calvo [1983].

Where $\frac{\epsilon}{\epsilon-1}$ is the real marginal cost if prices were fully flexible, a frictionless markup[30]. Now we divide through equation 3.37 by $P_{H,t-1}$, and write $\Pi_{H,t+k} = \frac{P_{H,t+k}}{P_{H,t-1}}$ and $MC_{H,t+k} = \frac{NMC_{H,t+k}}{P_{H,t+k}}$. Equation 3.37 can therefore be written as:

$$\sum_{k=0}^{\infty} \mathbb{E}_t \left[(\theta_H)^k \left\{ D_{t,t+k} \left(Y_{H,t+k} \left[\frac{P'_{H,t}}{P_{H,t-1}} - \frac{\epsilon}{\epsilon-1} \Pi_{H,t+k} MC_{H,t+k} \right] \right) \right\} \right] = 0 \quad (3.38)$$

Now we use the fact that[31]:

$$D_{t,t+k} = \beta^k \mathbb{E}_t \left\{ \left(\frac{P_t}{P_{t+k}} \right) \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \right\} \quad (3.39)$$

And we rewrite equation 3.38 as:

$$\sum_{k=0}^{\infty} (\beta\theta_H)^k \left[\mathbb{E}_t \left\{ \left(\frac{C_{t+k}^{-\sigma}}{P_{t+k}} Y_{H,t+k} \left[\frac{P'_{H,t}}{P_{H,t-1}} - \frac{\epsilon}{\epsilon-1} \Pi_{H,t+k} MC_{H,t+k} \right] \right) \right\} \right] = 0 \quad (3.40)$$

We will come back to this equation in chapter 4, where we will log-linearize the equation and solve for inflation in domestically produced products, Π_H.

3.3.2.2 Prices of Imported Goods

At the wholesale level for imports, the assumption is made that the *law of one price* (LOP) holds, but endogenous fluctuations from *purchasing power parity* (PPP) in the short run arise because of monopolistically competitive importers. Domestic prices of imports are therefore over and above the marginal cost. The LOP fails to hold at the retail level for imports because of this. Importers purchase foreign goods at world market prices and then sell to domestic consumers and a markup is charged over their cost, which creates a wedge between domestic and import prices of foreign goods, measured in the domestic currency. We therefore have a LOP gap, equation 3.25.

Following the domestic producers with sticky prices, the optimal price setting behavior for the domestic monopolistically competitive importer is defined as, similar to equation 3.40[32]:

$$\sum_{k=0}^{\infty} (\beta\theta_F)^k \left[\mathbb{E}_t \left\{ \left(\frac{C_{t+k}^{-\sigma}}{P_{t+k}} Y_{F,t+k} \left[\frac{P'_{F,t}}{P_{F,t-1}} - \frac{\epsilon}{\epsilon-1} \Pi_{F,t+k} MC_{F,t+k} \right] \right) \right\} \right] = 0 \quad (3.41)$$

[30] See also Galí [2008] for further detail.

[31] This equation of the discount factor is obtained from the households optimizing problem on page 5 in Monacelli [2005] where a conventional stochastic Euler equation is derived and solved for the discount factor.

[32] This form of the price level of imported goods is also used in Liu [2006] and Haider and Khan [2008].

Where $\theta_F \in [0,1]$ is the stickiness parameter of importing retailers that do not re optimize their prices every period. Equation 3.41 can be linearized and solved for inflation in the prices of imported goods. This is done in chapter 4.

3.3.3 The Import / Export Sector

Competition in the world market is assumed to bring import prices equal to marginal cost at the wholesale level, but rigidities arising from inefficient distribution networks and monopolistic retailers allow domestic import prices to deviate from the world price[33].

The import relationship for the economy has been derived here above, equation 3.20:

$$C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

The magnitude of imports depends on the elasticity of substitution between foreign and domestic goods, η, the degree of openness, α and the share of the price level for imported goods to the aggregated price level, P_F/P. Imports also depend on the total level of private consumption, C_t.

Now we need an expression for exports. We begin by writing the import function for the foreign economy, which is also the export function for the domestic economy, analogous to equation 3.20[34]:

$$C^*_{H,t} = \alpha^* \left(\frac{P_{H,t}}{P^*_t}\xi_t\right)^{-\eta^*} C^*_t \tag{3.42}$$

Where $C^*_{H,t}$ is the export of the domestic economy and the import of the foreign economy, α^* is the degree of openness for the foreign economy, η^* is the elasticity of substitution between home and foreign goods (seen from the foreign economy) and C^*_t is the aggregate private consumption of the foreign economy.

If the foreign consumers had perfect information, the fraction of all buyers who would purchase from the representative Icelandic firm would be $\alpha^* \left(\frac{P_{H,t}}{P^*_t}\xi_t\right)^{-\eta^*}$. The assumption is made that an individual Icelandic firm is small relative to the domestic market and that it competes with other Icelandic firms in the same way as it competes with foreign firms with the same market share. The stock of buyers, $\frac{C^*_{H,t}}{C^*_t}$, is assumed to adjusts slowly towards its long term equilibrium, so we add

[33] Similar argument is used by Burstein, Neves, and Rebelo [2003], which they support using United States data.

[34] This same method for obtaining an expression for the import function of the foreign economy is used by Liu [2006], page 13.

a sticky component to equation 3.42[35]. We also assume that the LOP holds for exports so that

$$\Psi_t = 1 => P_{F,t} = \frac{P_t^*}{\xi_t} => \frac{P_{H,t}}{P_t^*}\xi_t = \frac{1}{S_t}$$

The export function then becomes:

$$\frac{C_{H,t}^*}{C_t^*} = \left[\alpha^* \left(S_t\right)^{\eta^*}\right]^{\mu} \left(\frac{C_{H,t-1}^*}{C_{t-1}^*}\right)^{1-\mu} \tag{3.43}$$

Where $\mu \in [0,1]$ is the factor that notes how an individual values between the lagged term and the scaled terms of trade factor, $\alpha^* \left(S_t\right)^{\eta^*}$.

3.4 The Dynamic Asset Equation

Now an expression for the asset accumulation of the economy is derived. We start by looking at the *Current account* (CA) for the small open economy, it is defined as[36]:

$$CA_t = P_t C_{H,t}^* - \frac{P_t^*}{\xi_t} X_t^m + R_t^* \frac{B_t^*}{\xi_t} - R_t B_t^F = \frac{\Delta B_{t+1}^*}{\xi_t} - \Delta B_{t+1}^F \tag{3.44}$$

Where CA_t is the nominal current account, X_t^M is real imports denoted in foreign currency, B_t^* is the domestic nominal holding of foreign assets expressed in foreign currency and B_t^F is the foreign holding of domestic assets expressed in domestic currency but we will use it as the foreign debt of the domestic economy to the foreign economy, denominated in the domestic currency. $C_{H,t}^*$ is the real exports of domestic goods expressed in the domestic currency, like before.
To obtain the CA in real terms we divide by P_t through the equation above and obtain:

$$C_{H,t}^* - \frac{P_t^*}{\xi_t P_t} X_t^m + (1 + R_t^{*\prime}) \frac{B_t^*}{\xi_t P_t} - (1 + R_t^{\prime}) \frac{B_t^F}{P_t} = \frac{B_{t+1}^*}{\xi_t P_t} - \frac{B_{t+1}^F}{P_t}$$

We remember the definition of the real exchange rate from equation 3.24 so we get:

$$C_{H,t}^* - \frac{X_t^m}{Q_t} + (1 + R_t^{*\prime}) \frac{B_t^*}{\xi_t P_t} - (1 + R_t^{\prime}) \frac{B_t^F}{P_t} = \frac{B_{t+1}^*}{\xi_t P_t} - \frac{B_{t+1}^F}{P_t}$$

Imports in foreign currency divided by the real exchange rate becomes imports in domestic currency, so we get: $\frac{X_t^m}{Q_t} = C_{F,t}$. We define domestic holdings of foreign

[35] Here the work of Gottfries [2002] is followed. Export sluggishness and the sticky component is emphasized in literature like Phelps and Winter [1970] and Gottfries [1991]. They derive customer flow equations similar to equation 3.43 assuming that customers have imperfect information about prices charged by different suppliers.

[36] We use the definition of the CA from Wickens [2008], Chapter 7: The Open Economy.

assets in domestic currency as: $B_t^{*\prime} = \frac{B_t^*}{\xi_t}$, we also remember that $R_t = 1 + R_t'$ and $R_t^* = 1 + R_t^{*\prime}$. Now we rewrite the *Current account equation* as:

$$C_{H,t}^* - C_{F,t} + R_t^* \frac{B_t^{*\prime}}{P_t} - R_t \frac{B_t^F}{P_t} = \frac{B_{t+1}^{*\prime}}{P_t} - \frac{B_{t+1}^F}{P_t}$$

We multiply the B_{t+1} variables by $\frac{P_{t+1}}{P_{t+1}}$ and get:

$$C_{H,t}^* - C_{F,t} + R_t^* \frac{B_t^{*\prime}}{P_t} - R_t \frac{B_t^F}{P_t} = \frac{B_{t+1}^{*\prime}}{P_t} \frac{P_{t+1}}{P_{t+1}} - \frac{B_{t+1}^F}{P_t} \frac{P_{t+1}}{P_{t+1}}$$

We remember that $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ so the CA equation in real terms becomes:

$$C_{H,t}^* - C_{F,t} + R_t^* \frac{B_t^{*\prime}}{P_t} - R_t \frac{B_t^F}{P_t} = \frac{B_{t+1}^{*\prime}}{P_{t+1}} \Pi_{t+1} - \frac{B_{t+1}^F}{P_{t+1}} \Pi_{t+1} \tag{3.45}$$

Net foreign debt, F_t, was used in equation 3.27, we now define it as (in real terms):

$$F_t = \frac{B_t^F}{P_t} - \frac{B_t^{*\prime}}{P_t} \tag{3.46}$$

Equations 3.45 and 3.46 form an expression for the asset/debt accumulation of the economy.

3.5 Market Equilibrium

The goods market clearing for the domestic economy requires that domestic output is equal to domestic private consumption plus exports of domestic goods but minus imports of foreign goods.
We write the national identity as:

$$Y_t = C_t + C_{H,t}^* - C_{F,t} \tag{3.47}$$

If we put the expression for imports and exports into equation 3.47 we get the following relationship:

$$Y_t = C_t + C_t^* \left[\alpha^* (S_t)^{\eta^*}\right]^\mu \left(\frac{C_{H,t-1}^*}{C_{t-1}^*}\right)^{1-\mu} - \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

If we collect C_t we get:

$$Y_t = C_t^* \left[\alpha^* (S_t)^{\eta^*}\right]^\mu \left(\frac{C_{H,t-1}^*}{C_{t-1}^*}\right)^{1-\mu} + \left(1 - \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\right) C_t \tag{3.48}$$

Which is the expression for the gross domestic product of the economy.

3.6 Monetary Policy and Welfare

The *monetary policy* is a very important component of the model. We want to use a monetary policy, that is similar to the monetary policy of the *Central Bank of Iceland* (CBI), and compare it to a monetary policy that additionally responds to the real exchange rate. We will define a loss function for the economy and use it to estimate the difference in welfare loss between different monetary policies.

3.6.1 A basic Monetary Policy

Let's begin by looking at the monetary policy of the *Central Bank of Iceland*. The official Central Bank of Iceland's main objective is price stability. It is defined as a 12-month rise in the *Consumer Price Index* of 2.5%[37].
The Central Bank's main instrument for attaining its inflation target, at least before late 2008, was the interest rate on its loans to the financial undertakings against collateral. The Bank can also buy or sell foreign currency in the interbank market with the aim of influencing the exchange rate of the króna and the domestic inflation.
The Icelandic króna has been floating for two decades, up until 2001 the Central Bank of Iceland tried to control the float, by exchange rate intervention. But since March 2001 it has not been an official objective to control the exchange rate, until the financial crisis of 2008. The exchange rate did have some effect on CBI's actions from 2001, but the króna was allowed to float freely and the official Monetary Policy did not incorporate exchange rate directly when modeling the interest rate.

The CBI uses a Quarterly Macroeconomic Model (QMM) to model the Icelandic economy[38]. The interest rate expression used in the model is as follows[39]:

$$RS_t = 0.6RS_{t-1} + 0.4\left[(RRN_t + IT) + 1.5\left(INF_{t+4} - IT\right) + 0.5GAPAV_t\right] \quad (3.49)$$

Where RS is the *Short-term interest rate*, RRN is the *Real neutral interest rate* (exogenous), IT is the *Inflation Target*, 2.5%, (exogenous), INF_{t+4} is the *Four-quarter CPI inflation rate* (rational expectations) and $GAPAV$ is the *Annual average of output gap*. The factors 0.6 and $1 - 0.6 = 0.4$ in the CBI's model are interest rate smoothing factors.

We want to make a similar model of the CBI's interest rate that can be used in a DSGE model. The default interest rate is the long term real interest rate plus the

[37] Inflation targeting was used up until the financial crisis of 2008 and still is but capital controls have been in place since the crisis and a temporary exchange rate target is being used.

[38] The CBI is currently working on a Bayesian DSGE model for the Icelandic economy and it will likely replace the current QMM model, for inflation targeting at least.

[39] This is the short term interest rate model as it appears in Daníelsson, F. Gudmundsson, Haraldsdóttir, Ólafsson, Pétursdóttir, Pétursson, and Sveinsdóttir [2009].

inflation target. The model reacts to changes in inflation relative to the inflation target, and it reacts to percent changes in GDP relative to long term GDP growth. The DSGE reaction function is defined as[40]:

$$R_t = R_{t-1}^{\alpha_1} \left\{ \overline{R}_{real} \overline{\Pi}^T \left(\frac{\Pi_{t+1}}{\overline{\Pi}^T} \right)^{\alpha_2} \left(\frac{Y_t/Y_{t-1}}{\overline{GDP}} \right)^{\alpha_3} \right\}^{1-\alpha_1} \tag{3.50}$$

We will refer to equation 3.50 as *monetary policy 1.* R_t is the scaled domestic nominal interest rate in period t. We have $\alpha_1 \in [0,1]$ which is the interest rate smoothing parameter, $\alpha_2 \geq 0$ is the weight on inflation and $\alpha_3 \geq 0$ is the weight on output gap. Π_{t+1} is the rational expectation of inflation, for the next period. Remember that $\Pi_t = 1 + \Pi_t'$ where Π_t' is the percent change in the domestic price level. $\overline{\Pi}^T$ is the CBI's inflation target. $\overline{GDP}$ is the steady state economic growth of the economy, but since we do not allow growth in our model we have $\overline{GDP} = 1$, we will talk more about the growth variables in later chapters. The variable $\overline{R}_{real}$ is the scaled equilibrium real interest rate defined as one plus the percentage rate. We have $\overline{R}_{real} = 1 + \overline{R}_{real}'$ where $\overline{R}_{real}'$ is the steady state real interest rate, in percentages.

3.6.2 Monetary Policy and the Real Exchange Rate

Now we have defined the basic monetary policy. We want to define a new one that also responds to the *Real Exchange Rate*. Recent literature on the subject is followed[41] and the real exchange rate is added to equation 3.50 as follows:

$$R_t = R_{t-1}^{\alpha_1} \left\{ \overline{R}_{real} \overline{\Pi}^T \left(\frac{\Pi_{t+1}}{\overline{\Pi}^T} \right)^{\alpha_2} \left(\frac{Y_t/Y_{t-1}}{\overline{GDP}} \right)^{\alpha_3} \left(\frac{Q_t}{\overline{Q}} \right)^{-\alpha_4} \right\}^{1-\alpha_1} \tag{3.51}$$

We will refer to equation 3.51 as *monetary policy 2.* Q_t is the real exchange rate from equation 3.24 and $\overline{Q}$ is the steady state real exchange rate. This monetary policy therefore responds to the level of the real exchange rate, and tries to maintain exchange rate equilibrium. $\alpha_4 \geq 0$ is the weight on the real exchange rate level. It has a minus sign because a rise in the real exchange rate denotes appreciation of the domestic currency and the interest rate should be lowered[42].

We will also examine a similar monetary policy, that in addition to reacting to the real exchange rate level, it also reacts to the rate of change in the real exchange rate between periods[43]:

$$R_t = R_{t-1}^{\alpha_1} \left\{ \overline{R}_{real} \overline{\Pi}^T \left(\frac{\Pi_{t+1}}{\overline{\Pi}^T} \right)^{\alpha_2} \left(\frac{Y_t/Y_{t-1}}{\overline{GDP}} \right)^{\alpha_3} \left(\frac{Q_t}{\overline{Q}} \right)^{-\alpha_4} \left(\frac{Q_t}{Q_{t-1}} \right)^{\alpha_5} \right\}^{1-\alpha_1} \tag{3.52}$$

[40] When this monetary policy is log-linearized it becomes like the monetary policies used in Haider and Khan [2008] and Liu [2006], for example. See chapter 4 for the log-linearization.

[41] Here Gonzalez and Garcia [2010] is followed, where a monetary policy with the same form is used.

[42] We calculate the optimal value for the monetary policy parameters in chapter 5.

[43] See Gonzalez and Garcia [2010], page 9.

We will refer to equation 3.52 as *monetary policy 3*. α_5 is the weight on the rate of change of the real exchange rate.

3.6.3 The Loss Function

The three monetary policies stated above will be compared. We will define a loss function that measures the *Welfare Loss* of the economy. The loss function is a function of fluctuations of the GDP from its steady state, fluctuations of the domestic inflation from its steady state and fluctuations of the domestic interest rate from its steady state. The loss function is defined as[44]:

$$LF = \sigma_\pi^2 + \frac{1}{2}\sigma_y^2 + \frac{1}{5}\sigma_r^2 \tag{3.53}$$

Where σ_π^2 is the variance of the deviations of inflation from its steady state, σ_y^2 is the variance of the deviations of the GDP from its steady state and σ_r^2 is the variance of the deviations of the interest rate from its steady state. The dynamics for the deviations of the GDP, inflation and interest rate from their steady states are derived in chapter 4. The lower the Loss Function's value, the greater the *Welfare*[45].

[44] See Gonzalez and Garcia [2010] and Hunt [2006] for similar loss functions.

[45] The international financial crisis of 2007-2010 shows us that when an economy deviates from its steady state by many percentages it can have severe consequences, and the dampening of these economic fluctuations are necessary, see for example Stiglitz [2010].

4 Linearization

In general, nonlinear systems like the expressions derived in chapter 3 cannot be solved analytically. However, their solution can be very well approximated by a corresponding set of linear equations.
The equations for the model are *log-linearized* around the steady state of the variables. The variables become deviations from the steady state. The deviations may not become large because a Taylor approximation is used. This chapter goes through the basics in the linearization, demonstrates basic concepts and derives the model in log-linear form. The model is simulated in the linear form[1].

4.1 The basics of Log-Linearization

The idea is to use Taylor series approximations. In general, any nonlinear function $F(x_t, y_t)$ can be approximated around any point (x_t^*, y_t^*) using the formula:

$$
\begin{aligned}
F(x_t, y_t) = F(x_t^*, y_t^*) &+ F_x(x_t^*, y_t^*)(x_t - x_t^*) + F_y(x_t^*, y_t^*)(y_t - y_t^*) + \\
&F_{xx}(x_t^*, y_t^*)(x_t - x_t^*)^2 + F_{xy}(x_t^*, y_t^*)(x_t - x_t^*)(y_t - y_t^*) + \\
&F_{yy}(x_t^*, y_t^*)(y_t - y_t^*)^2 + ...
\end{aligned}
$$

If the gap between (x_t, y_t) and (x_t^*, y_t^*) is small, then high order terms and cross-terms will all be very small and can be ignored. But if the linearization is around a point that is 'far away' from (x_t, y_t) then this approximation will not be accurate. Since we are linearizing around the steady state, we are linearizing around zero, $(0, 0, ...)$, because our variables are deviations from the steady state and zero deviation is the equilibrium.

When linearizing for DSGE models we take logs and then linearize the logs of variables around a steady state path in which all real variables are growing at the same rate. The steady state path is relevant because the stochastic economy

[1] See chapter 6 for details on the simulation

will, on average, tend to fluctuate around the values given by this path, making the approximation an accurate one. This gives us a set of linear equations in the deviations of the logs of these variables from their steady state values. An important approximation is the following:

$$log(X) - log(Y) \approx \frac{X - Y}{Y} \tag{4.1}$$

This approach has the advantage that variables are expressed in terms of their percentage deviations from the steady state paths. So we have a system that can be thought of as the *business cycle component* of the model. Coefficients can be thought of as elasticities and *impulse response functions* (IRF's) are easy to interpret. This method doesn't require taking a lot of derivatives.

Lower case letters will generally denote deviations of variables from their steady state:

$$x_t \equiv \frac{X_t - \overline{X}}{\overline{X}} \approx log(X_t) - log(\overline{X}) \tag{4.2}$$

An important identity is that every variable can be written as:

$$X_t = \overline{X}\frac{X_t}{\overline{X}} = \overline{X}e^{x_t} \tag{4.3}$$

Taking the first order Taylor approximation we get:

$$X_t = \overline{X}e^{x_t} \approx \overline{X}(1 + x_t) \tag{4.4}$$

Another important approximation is the following:

$$X_t Y_t \approx \overline{XY}(1 + x_t)(1 + y_t) \approx \overline{XY}(1 + x_t + y_t) \tag{4.5}$$

Setting the cross terms, $x_t y_t$, equal to zero is a good approximation because we are looking at small deviations from the steady state.

Still another important approximation that will be used is:

$$log(1 + x_t) \approx x_t \text{ and } log(1 + x_t + y_t) \approx x_t + y_t$$

We assume that log-linear technology follows an AR(1)[2] process:

$$a_t = \rho_{a_t} a_{t-1} + \epsilon_t^{a_t}$$

Where $a_t = log(A_t) - log(\overline{A})$ and $\overline{A}$ is the steady state technology. The parameter $\epsilon_t^{a_t}$ is a Gaussian shock with non-zero mean and variance $\sigma^2_{a_t^*}$. Technology is the source of all long run growth in the economy, so there is no trend growth in our model because we consider $\overline{A}$ as a constant. This means that the steady state variables are all constants. It is possible to have a trend growth in the model but we skip it for simplification, because we are mainly interested in the dynamics of the model, not the growth[3].

[2]Autoregressive process with one lag.

[3]More information on the trend growth and the linearization can be found in Uhlig [1995]

4.2 Log-linearizing the Model

Now the equations derived in chapter 3 are log-linearized. This is a set of few key equations and we will linearize them one by one. Basic steps are shown in most cases, but methods in the above section are mainly used.

4.2.1 Wages

We begin by linearizing equation 3.21. We get:

$$\frac{\overline{W}}{\overline{P}}(1 + w_t - p_t) = \left(\overline{N}(1 + n_t)\right)^{\phi} \left(\overline{C}(1 + c_t) - \overline{H}(1 + h_t)\right)^{\sigma}$$

Taking logs on both sides and using the fact that $H_t = hC_{t-1}$ we get:

$$\begin{aligned} &log\left(\frac{\overline{W}}{\overline{P}}\right) + log(1 + w_t - p_t) = \\ &\quad \phi\left(log(\overline{N}) + log(1 + n_t)\right) + \sigma\left(log(\overline{C}(1 + c_t) - h\overline{C}(1 + c_{t-1}))\right) \end{aligned}$$

Now we use the Taylor approximation:

$$log(\overline{C}(1 + c_t) - h\overline{C}(1 + c_{t-1})) \approx log(\overline{C} - h\overline{C}) + c_t\frac{1}{1-h} - c_{t-1}\frac{h}{1-h}$$

Inserting the Taylor approximation into the equation we get:

$$\begin{aligned} log\left(\frac{\overline{W}}{\overline{P}}\right) + log(1 + w_t - p_t) &= \phi\left(log(\overline{N}) + log(1 + n_t)\right) \\ &+ \sigma\left(log(\overline{C} - h\overline{C}) + c_t\frac{1}{1-h} - c_{t-1}\frac{h}{1-h}\right) \end{aligned} \tag{4.6}$$

The steady state of equation 3.21 is:

$$\frac{\overline{W}_t}{\overline{P}_t} = \overline{N}_t^{\phi}(\overline{C}_t - h\overline{C})^{\sigma}$$

Taking logs on both sides:

$$log(\overline{W}) - log(\overline{P}) = \phi log(\overline{N}) + \sigma log(\overline{C} - h\overline{C}) \tag{4.7}$$

Subtracting equation 4.7 from equation 4.6 we get:

$$w_t^{real} = w_t - p_t = \phi n_t + c_t\frac{\sigma}{1-h} - c_{t-1}\frac{\sigma h}{1-h} \tag{4.8}$$

Where we have used the approximation that $log(1 + x) \approx x$. Note that $w_t = log(W_t) - log(\overline{W})$, $p_t = log(P_t) - log(\overline{P})$, $n_t = log(N_t) - log(\overline{N})$ and $c_t = log(C_t) - log(\overline{C})$.

4.2.2 Consumption

Linearizing equation 3.22, the consumption equation, we get:

$$e^{\beta}\overline{\Pi}(1+\pi_{t+1})\left(\overline{C}(1+c_t)-h\overline{C}(1+c_{t-1})\right)^{-\sigma} = \\ \overline{R}(1+r_{t+1})\left(\overline{C}(1+c_{t+1})-h\overline{C}(1+c_t)\right)^{-\sigma}$$

Taking logs on both sides we get:

$$\beta + log(\overline{\Pi}) + \pi_{t+1} - \sigma log(\overline{C}(1+c_t) - h\overline{C}(1+c_{t-1})) = \\ -\sigma log(\overline{C}(1+c_{t+1}) - h\overline{C}(1+c_t))$$

Using the Taylor approximation like we did before, and subtracting the log expression of the steady state formula we get:

$$c_t = c_{t-1}\frac{h}{1+h} + c_{t+1}\frac{1}{1+h} - (r_{t+1} - \pi_{t+1})\frac{1-h}{1+h}\frac{1}{\sigma} \tag{4.9}$$

Which is our log-linear consumption equation.

4.2.3 The Terms of Trade

The log-linearized form of the terms of trade formula, equation 3.23 is:

$$s_t = p_{F,t} - p_{H,t} \tag{4.10}$$

Subtracting the equation in period $(t-1)$ from the equation in period t yields:

$$s_t - s_{t-1} = p_{F,t} - p_{F,t-1} - (p_{H,t} - p_{H,t-1}) = \pi_{F,t} - \pi_{H,t} \tag{4.11}$$

Where we have used the fact that $\pi_t = p_t - p_{t-1}$.

4.2.4 The Consumer Price Index

Now we log-linearize the CPI, equation 3.10. We rewrite the equation as:

$$P_t^{1-\eta} - \alpha P_{F,t}^{1-\eta} = (1-\alpha)P_{H,t}^{1-\eta}$$

Rewriting like before, we get:

$$\overline{P}^{1-\eta}(1+p_t)^{1-\eta} - \alpha\overline{P_F}^{1-\eta}(1+p_{F,t})^{1-\eta} = (1-\alpha)\overline{P_H}^{1-\eta}(1+p_{H,t})^{1-\eta}$$

Now we say that the steady states of the price levels are equal, so we get $\overline{P} = \overline{P_F} = \overline{P_H}$. Canceling these terms out we get:

$$(1+p_t)^{1-\eta} - \alpha(1+p_{F,t})^{1-\eta} = (1-\alpha)(1+p_{H,t})^{1-\eta} \tag{4.12}$$

Taking logs on both sides gives:

$$log\left[(1+p_t)^{1-\eta} - \alpha(1+p_{F,t})^{1-\eta}\right] = log(1-\alpha) + (1-\eta)log(1+p_{H,t}) \tag{4.13}$$

Now we use the following Taylor approximation:

$$log\left[(1+p_t)^{1-\eta} - \alpha(1+p_{F,t})^{1-\eta}\right] \approx log(1-\alpha) + \frac{1-\eta}{1-\alpha}p_t - \alpha\frac{1-\eta}{1-\alpha}p_{F,t}$$

Putting the Taylor approximation into equation 4.13 we get:

$$log(1-\alpha) + \frac{1-\eta}{1-\alpha}p_t - \alpha\frac{1-\eta}{1-\alpha}p_{F,t} = log(1-\alpha) + (1-\eta)log(1+p_{H,t})$$

Canceling out $(1-\eta)$, noting that $log(1+p_{H,t}) \approx p_{H,t}$ and rewriting the terms gives:

$$p_t = (1-\alpha)p_{H,t} + \alpha p_{F,t} \tag{4.14}$$

Subtracting period $(t-1)$ from period t in equation 4.14 gives:

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} \tag{4.15}$$

Which is our log-linearized expression of the aggregate domestic inflation. Putting together equations 4.10 and 4.14 we get:

$$p_t = p_{H,t} + \alpha s_t \tag{4.16}$$

4.2.5 The Real Exchange Rate

Log-linearizing the real exchange rate, equation 3.24 yields:

$$q_t = e_t + p_t - p_t^* \tag{4.17}$$

Where $e_t \equiv log(\xi_t) - log(\overline{\xi})$ and $q_t \equiv log(Q_t) - log(\overline{Q})$. We will use equation 4.17 when we log-linearize the uncovered interest rate parity.

4.2.6 The Law of One Price Gap

Log-linearizing the LOP gap, equation 3.25, yields:

$$\psi_t = p_t^* - e_t - p_{F,t} \tag{4.18}$$

Where $\psi_t = log(\Psi_t) - log(\overline{\Psi})$. Using equation 4.17 to eliminate p_t^* from equation 4.18 yields:

$$\psi_t = -q_t + p_t - p_{F,t} \tag{4.19}$$

Using equation 4.16 to eliminate p_t gives:

$$\psi_t = -q_t + p_{H,t} + \alpha s_t - p_{F,t}$$

And finally using equation 4.10 to eliminate $p_{F,t} - p_{H,t}$ and rewriting gives us the log-linearized LOP gap:

$$\psi_t = -q_t - (1-\alpha)s_t \tag{4.20}$$

Now we want to be able to apply a shock to the LOP gap and model the response of the economy to this shock. The shock can also be thought of as a measurement error. We define a shock variable ν_t^{ψ} as follows:

$$\nu_t^{\psi} = \rho_{\psi}\nu_{t-1}^{\psi} + \epsilon_t^{\psi}$$

The shock variable follows an AR(1) process where $0 < \rho_{\psi} < 1$ is the autocorrelation coefficient. The parameter ϵ_t^{ψ} is a Gaussian shock with non-zero mean and variance σ_{ψ}^2. The shock and the autocorrelation coefficient will be defined further in chapter 5. The LOP gap equation with a shock variable becomes:

$$\psi_t = -q_t - (1-\alpha)s_t + \nu_t^{\psi} \tag{4.21}$$

4.2.7 The Uncovered Interest Rate Parity

Log-linearizing the *UIP*, equation 3.26, gives us:

$$-\gamma\frac{\overline{F}}{\overline{\overline{Y}}}(1 + f_t - y_t) + log(\overline{R}) + r_t + e_{t+1} - e_t = log(\overline{R}^*) + r_t^*$$

Subtracting the steady-state like before, gives us:

$$-\gamma\frac{\overline{F}}{\overline{\overline{Y}}}(f_t - y_t) + r_t + e_{t+1} - e_t = r_t^*$$

Using equation 4.17 to eliminate e_{t+1} and e_t gives us:

$$q_{t+1} - q_t = r_t^* - \pi_{t+1}^* - (r - \pi_{t+1}) + \gamma\frac{\overline{F}}{\overline{\overline{Y}}}(f_t - y_t) \tag{4.22}$$

We want to be able to shock the real exchange rate but also the risk premium. We divide equation 4.22 into two separate equations as follows:

$$q_{t+1} - q_t = r_t^* - \pi_{t+1}^* - (r - \pi_{t+1}) + prem_t + \nu_t^q \tag{4.23}$$

$$prem_t = \gamma\frac{\overline{F}}{\overline{\overline{Y}}}(f_t - y_t) + \nu_t^{prem} \tag{4.24}$$

Equations 4.23 and 4.24 form our log-linear expression for the uncovered interest rate parity with a risk premium. Where $\overline{F}$ is the steady state net foreign debt of

the economy and $\overline{Y}$ is the steady state GDP. And using the general terminology we have $\pi_{t+1} = log(\Pi_{t+1}) - log(\overline{\Pi})$ and $f_t = log(F_t) - log(\overline{F})$.
We have also added a shock, ν_t^q, to the real exchange rate and a shock ν_t^{prem} to the risk premium, as we did with the LOP gap equation. The shocks follow an AR(1) process and are defined as follows:

$$\nu_t^q = \rho_q \nu_{t-1}^q + \epsilon_t^q \text{ and } \nu_t^{prem} = \rho_{prem} \nu_{t-1}^{prem} + \epsilon_t^{prem}$$

Where $0 < \rho_q, \rho_{prem} < 1$ are the autocorrelation coefficients. The parameters ϵ_t^q and ϵ_t^{prem} are Gaussian shocks with non-zero mean and variance σ_q^2 and σ_{prem}^2, respectively.

4.2.8 Foreign Consumption, Inflation and Interest Rate

The log-linear form of foreign consumption, equation 3.28, becomes an AR(1) process:

$$c_t^* = \rho_{c^*} c_{t-1}^* + \epsilon_t^{c^*} \tag{4.25}$$

Foreign inflation and the foreign interest rate also become an AR(1) process, equation 3.29:

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \epsilon_t^{\pi^*} \tag{4.26}$$

$$r_t^* = \rho_{r^*} r_{t-1}^* + \epsilon_t^{r^*} \tag{4.27}$$

Where $\rho_{c^*}, \rho_{\pi^*}, \rho_{r^*} \in (0,1)$ are the autocorrelation coefficients. The parameters $\epsilon_t^{c^*}$, $\epsilon_t^{\pi^*}$ and $\epsilon_t^{r^*}$ are Gaussian shocks with non-zero mean and variance $\sigma_{c^*}^2$, $\sigma_{\pi^*}^2$ and $\sigma_{r^*}^2$, respectively. These shocks will be defined further in chapter 5.

4.2.9 The Production Function, Marginal Cost and Technology

Assuming a symmetric equilibrium across all j firms, the first order log-linear approximation of the aggregate production function, equation 3.31, becomes:

$$y_t = a_t + n_t \tag{4.28}$$

Where $y_t = log(Y_t) - log(\overline{Y})$ and $a_t = log(A_t) - log(\overline{A})$.

In the beginning of this chapter we assumed that log-linear technology follows an AR(1) process:

$$a_t = \rho_{a_t} a_{t-1} + \epsilon_t^{a_t} \tag{4.29}$$

Where $\epsilon_t^{a_t}$ is a Gaussian shock with non-zero mean and variance $\sigma_{a_t^*}^2$.

When log-linearizing real firms' marginal cost, equation 3.33, we get:

$$\begin{aligned} mc_t &= w_t - p_{H,t} - a_t \\ &= (w_t - p_t) + (p_t - p_{H,t}) - a_t \end{aligned}$$

Using equation 4.8 to eliminate $w_t - p_t$ and equation 4.16 to eliminate $p_t - p_{H,t}$ we get:

$$mc = \phi n_t + c_t \frac{\sigma}{1-h} - c_{t-1} \frac{\sigma h}{1-h} + \alpha s_t - a_t$$

Now using equation 4.28 to eliminate n_t we get the marginal cost expression as:

$$\begin{aligned} mc &= \phi(y_t - a_t) + c_t \frac{\sigma}{1-h} - c_{t-1} \frac{\sigma h}{1-h} + \alpha s_t - a_t \\ &= \phi y_t + c_t \frac{\sigma}{1-h} - c_{t-1} \frac{\sigma h}{1-h} + \alpha s_t - (1+\phi) a_t \end{aligned} \tag{4.30}$$

4.2.10 Domestic Inflation

Now we log-linearize equation 3.40, inflation in domestically produced goods. The log-linearization is done around the steady state to obtain the decision rule for $P'_{H,t}$ and we get[4]:

$$p'_{H,t} = p_{H,t-1} + \sum_{k=0}^{\infty} \left\{ (\beta\theta_H)^k \left[E_t(\pi_{H,t+k}) + (1-\beta\theta_H) E_t(mc_{t+k}) \right] \right\} \tag{4.31}$$

So firms set their prices according to the future discounted sum of inflation and deviations of real marginal cost from its steady state[5]. We rewrite the equation as:

$$\begin{aligned} p'_{H,t} &= p_{H,t-1} + \pi_{H,t} + (1-\beta\theta_H) mc_t \\ &\quad + (\beta\theta_H) \sum_{k=0}^{\infty} \left\{ (\beta\theta_H)^k \left[E_t(\pi_{H,t+k+1}) + (1-\beta\theta_H) E_t(mc_{t+k+1}) \right] \right\} \\ &= p_{H,t-1} + \pi_{H,t} + (1-\beta\theta_H) mc_t + \beta\theta_H (p'_{H,t+1} - p_{H,t}) \end{aligned}$$

In the first line we split up the summation into two terms, at time t and at from time $t+1$ to ∞. The second line rewrites the last term using equation 4.31. Now we rearrange to obtain the following expression:

$$p'_{H,t} - p_{H,t-1} = \beta\theta_H E_t(\pi_{H,t+1}) + \pi_{H,t} + (1-\beta\theta_H) mc_t \tag{4.32}$$

Subtracting period $t-1$ form period t in equation 4.32 and rearranging we obtain an expression for the inflation in domestically produced goods:

$$\pi_{H,t} = \beta(1-\theta_H) E_t(\pi_{H,t+1}) + \theta_H \pi_{H,t-1} + \lambda_H mc_t \tag{4.33}$$

[4] Following Galí and Monacelli [2005].

[5] Since we are holding capital fixed and the banking sector and money multiplier are not considered.

Where $\lambda_H = \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H}$. Equation 4.33 is the familiar *New Keynesian Phillips Curve* (NKPC) that we derived using the Calvo pricing structure. So domestic inflation has both a forward looking component and a backward looking component. If all firms were able to adjust their prices at each period ($\theta_H = 0$) the inflation process would be purely forward looking and disinflationary policy would be completely costless. The real marginal cost for firms is an important determinant of domestic inflation.

Now we want to obtain a similar expression for the inflation in imported goods, Π_F. We log-linearize equation 3.41 in the same way as we did for inflation in domestically produced goods and the price setting behavior for the domestic imports becomes[6]:

$$p'_{F,t} = p_{F,t-1} + \sum_{k=0}^{\infty} \left\{ (\beta\theta_F)^k \left[E_t(\pi_{F,t+k}) + (1-\beta\theta_F) E_t(\psi_{t+k}) \right] \right\} \tag{4.34}$$

We follow the same steps as for the inflation in domestically produced products. Analogous to equation 4.33, the log-linear inflation in prices of imported goods arising from the Calvo-pricing structure becomes:

$$\pi_{F,t} = \beta(1-\theta_F)E_t(\pi_{F,t+1}) + \theta_F \pi_{F,t-1} + \lambda_F \psi_t + \nu_t^{\pi_F} \tag{4.35}$$

Where $\lambda_F = \frac{(1-\beta\theta_F)(1-\theta_F)}{\theta_F}$. We have also added a shock variable, $\nu_t^{\pi_F}$, which is defined as follows:

$$\nu_t^{\pi_F} = \rho_{\pi_F} \nu_{t-1}^{\pi_F} + \epsilon_t^{\pi_F}$$

The shock variable follows an AR(1) process where $0 < \rho_{\pi_F} < 1$ is the autocorrelation coefficient. The parameter $\epsilon_t^{\pi_F}$ is a Gaussian shock with non-zero mean and variance $\sigma_{\pi_F}^2$.

Equations 4.15, 4.33 and 4.35 complete the inflation dynamics for the small open economy. In sticky-price models, inflation dynamics are mainly driven by firms' preference for smoothing their pricing decisions. This gives rise to nominal rigidities that would not be present if prices were fully flexible.
The cost of inflation in this case is the cost to the economy because prices are not able to adjust, hence the classification of such models as 'New Keynesian'[7]. From the social planner's perspective, optimal policy is one that minimizes deviations of marginal cost and the LOP gap from its steady state.

4.2.11 Imports and Exports

Log-linearizing equation 3.20, the import equation, gives us:

$$c_{F,t} = \eta(p_t - p_{F,t}) + c_t \tag{4.36}$$

[6] Following Galí and Monacelli [2005] as before.

[7] The cost of adjustment argument for a firm's pricing decision yields a similar NKPC expression, see Yun [1996].

Inserting equation 4.19 into equation 4.36 yields:

$$c_{F,t} = \eta(\psi_t + q_t) + c_t \tag{4.37}$$

Which gives us a log-linear expression for imports.

Log-linearizing the export function, equation 3.43, gives us the log-linearized export function:

$$c^*_{H,t} = c^*_t + \mu\eta^* s_t + (1-\mu)(c^*_{H,t-1} - c^*_{t-1}) \tag{4.38}$$

Where $c^*_{H,t} = log(C^*_{H,t}) - log(\overline{C}^*_H)$ is the log-linear real export of the domestic economy.

4.2.12 The Dynamic Asset Equation

Now the asset equation is log-linearized, equation 3.45, step by step. We begin with the following:

$$\begin{aligned} &\overline{C}^*_H(1 + c^*_{H,t}) - \overline{C}_F(1 + c_{F,t}) + \overline{R}^*\overline{B}^{*\prime}_r(1 + r^*_t + b^{*\prime}_{r,t}) \\ &\quad -\overline{RB}^F_r(1 + r_t + b^F_{r,t}) = \overline{B}^{*\prime}_r\overline{\Pi}(1 + b^{*\prime}_{r,t+1} + \pi_{t+1}) \\ &\quad - \overline{B}^F_{r,t+1}\overline{\Pi}(1 + b^F_{r,t+1} + \pi_{t+1}) \end{aligned} \tag{4.39}$$

Where $B^F_{r,t} \equiv \frac{B^F_t}{P_t}$ and $B^{*\prime}_{r,t} \equiv \frac{B^{*\prime}_t}{P_t}$ is foreign debt and assets in real terms. We also have $b^F_{r,t} = log(B^F_{r,t}) - log(\overline{B}^F_r)$ and $b^{*\prime}_{r,t} = log(B^{*\prime}_{r,t}) - log(\overline{B}^{*\prime}_r)$ as usual. Now we write the asset equation in equilibrium terms:

$$\overline{C}^*_H - \overline{C}_F + \overline{R}^*\overline{B}^{*\prime}_r - \overline{RB}^F_r = \overline{B}^{*\prime}_r\overline{\Pi} - \overline{B}^F_r\overline{\Pi} \tag{4.40}$$

Now we subtract equation 4.40 from equation 4.39 and get:

$$\begin{aligned} &\overline{C}^*_H(c^*_{H,t}) - \overline{C}_F(c_{F,t}) + \overline{R}^*\overline{B}^{*\prime}_r(r^*_t + b^{*\prime}_{r,t}) - \overline{RB}^F_r(r_t + b^F_{r,t}) = \\ &\quad \overline{B}^{*\prime}_r\overline{\Pi}(b^{*\prime}_{r,t+1} + \pi_{t+1}) - \overline{B}^F_r\overline{\Pi}(b^F_{r,t+1} + \pi_{t+1}) \end{aligned} \tag{4.41}$$

Equilibrium export is equal to equilibrium import, so we have $\overline{C}^*_H = \overline{C}_F$. We rewrite equation 4.41 as:

$$\begin{aligned} &\overline{C}^*_H(c^*_{H,t} - c_{F,t}) + \overline{R}^*\overline{B}^{*\prime}_r r^*_t - \overline{RB}^F_r r_t = \\ &\quad \overline{\Pi}(\overline{B}^{*\prime}_r b^{*\prime}_{r,t+1} - \overline{B}^F_r b^F_{r,t+1}) + \overline{RB}^F_r b^F_{r,t} - \overline{R}^*\overline{B}^{*\prime}_r b^{*\prime}_{r,t} \\ &\quad +(\overline{B}^{*\prime}_r\overline{\Pi} - \overline{B}^F_r\overline{\Pi})\pi_{t+1} \end{aligned} \tag{4.42}$$

Log-linearizing equation 3.46, net foreign debt, yields:

$$\overline{F}f_t = \overline{B}^F_r b^F_{r,t} - \overline{B}^{*\prime}_r b^{*\prime}_{r,t} \tag{4.43}$$

Now we make the following approximation:

$$\overline{RB}_r^F b_{r,t}^F - \overline{R}^* \overline{B}_r^{*\prime} b_{r,t}^{*\prime} \approx \frac{\overline{R} + \overline{R}^*}{2} \overline{F} f_t \tag{4.44}$$

Inserting equations 4.43 and 4.44 into equation 4.42 yields:

$$\begin{aligned} \overline{C}_H^* (c_{H,t}^* - c_{F,t}) + \overline{R}^* \overline{B}_r^{*\prime} r_t^* - \overline{RB}_r^F r_t = \\ -\overline{\Pi F} f_{t+1} + \frac{\overline{R} + \overline{R}^*}{2} \overline{F} f_t + \overline{\Pi} (\overline{B}_r^{*\prime} - \overline{B}_r^F) \pi_{t+1} \end{aligned}$$

Which we rewrite as:

$$f_{t+1} = \frac{\overline{R} + \overline{R}^*}{2\overline{\Pi}} f_t - \frac{\overline{C}_H^*}{\overline{\Pi F}} (c_{H,t}^* - c_{F,t}) - \frac{\overline{R}^* \overline{B}_r^{*\prime}}{\overline{\Pi F}} r_t^* + \frac{\overline{RB}_r^F}{\overline{\Pi F}} r_t + \frac{\overline{B}_r^{*\prime} - \overline{B}_r^F}{\overline{F}} \pi_{t+1} + \nu_t^f \tag{4.45}$$

So equation 4.45 is the dynamic net foreign debt equation[8] of the economy, in real terms. We have added a shock variable like before, ν_t^f, which is defined as follows:

$$\nu_t^f = \rho_f \nu_{t-1}^f + \epsilon_t^f$$

The shock variable follows an AR(1) process where $0 < \rho_f < 1$ is the autocorrelation coefficient. The parameter ϵ_t^f is a Gaussian shock with non-zero mean and variance σ_f^2.

4.2.13 The Gross Domestic Product

Now we log-linearize the market equilibrium expression, equation 3.48. We begin by noting that equilibrium imports are equal to equilibrium exports:

$$\overline{C}_F = \delta \left(\frac{\overline{P}}{\overline{P}_F} \right)^\eta \overline{C} = \overline{C}_H^* = \overline{C}^* (\delta^* \overline{S}^{\eta^*})^\mu \left(\frac{\overline{C}_H^*}{\overline{C}^*} \right)^{1-\mu} \tag{4.46}$$

Log-linearizing equation 3.48 and using the expression in equation 4.46 we get:

$$\begin{aligned} \overline{Y}(1 + y_t) = (1 + c_t) \left(\overline{C} - \overline{C}_H^* (1 + p_t - p_{F,t})^\eta \right) \\ + \overline{C}_H^* (1 + c_t^*)(1 + s_t)^{\mu\eta^*} \left(1 + c_{H,t-1}^* - c_{t-1}^* \right)^{1-\mu} \end{aligned} \tag{4.47}$$

Since exports are equal to imports in the long run we get:

$$\overline{Y} = \overline{C} + \overline{C}_H^* - \overline{C}_F = \overline{C} \tag{4.48}$$

Taking logs on both sides of equation 4.47 and using a first order Taylor approximation around zero[9] for the right hand side we get:

$$\begin{aligned} log(\overline{C}) + y_t \approx f(\vec{0}) + c_t f_{c_t}(\vec{0}) + p_t f_{p_t}(\vec{0}) + p_{F,t} f_{p_{F,t}}(\vec{0}) \\ + s_t f_{s_t}(\vec{0}) + c_{H,t-1}^* f_{c_{H,t-1}^*}(\vec{0}) + c_{t-1}^* f_{c_{t-1}^*}(\vec{0}) + c_t^* f_{c_t^*}(\vec{0}) \end{aligned} \tag{4.49}$$

[8] But we sometimes refer to it as the *dynamic asset equation*.

[9] Also known as *Maclaurin series*.

Where:

$$f(c_t, p_t, p_{F,t}, s_t, c^*_{H,t-1}, c^*_{t-1}, c^*_t) \equiv log((1+c_t)\left(\overline{C} - \overline{C}^*_H(1+p_t-p_{F,t})^\eta\right) + \overline{C}^*_H(1+c^*_t)(1+s_t)^{\mu\eta^*}\left(1+c^*_{H,t-1}-c^*_{t-1}\right)^{1-\mu}) \quad (4.50)$$

And $f_x \equiv \frac{\delta f}{\delta x}$. We get:

$$\begin{aligned} log(\overline{C}) + y_t \approx & \overbrace{log(\overline{C} + \overline{C}^*_H - \overline{C}_F)}^{=log(\overline{C})} + \\ & (c_t(\overline{C} - \overline{C}_F) + p_t(-\eta\overline{C}_F) + p_{F,t}(\eta\overline{C}_F) + s_t(\overline{C}^*_H\mu\eta^*) \\ & + c^*_{H,t-1}(\overline{C}^*_H(1-\mu)) + c^*_{t-1}(-\overline{C}^*_H(1-\mu)) + c^*_t(\overline{C}^*_H))\frac{1}{\overline{C}} \end{aligned} \quad (4.51)$$

We rewrite equation 4.51 as:

$$y_t\frac{\overline{C}}{\overline{C^*_H}} = c_t(\frac{\overline{C}}{\overline{C}^*_H} - 1) - \eta(p_t - p_{F,t}) + s_t\mu\eta^* + c^*_{H,t-1}(1-\mu) - c^*_{t-1}(1-\mu) + c^*_t \quad (4.52)$$

Now we use the import equation, equation 4.36, to replace $p_t - p_{F,t}$ and we get:

$$y_t\frac{\overline{C}}{\overline{C^*_H}} = c_t(\frac{\overline{C}}{\overline{C}^*_H} - 1) + (c_t - c_{F,t}) + s_t\mu\eta^* + c^*_{H,t-1}(1-\mu) - c^*_{t-1}(1-\mu) + c^*_t \quad (4.53)$$

We rewrite the equation once again and get:

$$(y_t - c_t)\frac{\overline{C}}{\overline{C^*_H}} = c^*_t - c_{F,t} + s_t\mu\eta^* + (c^*_{H,t-1} - c^*_{t-1})(1-\mu) \quad (4.54)$$

Which concludes the market equilibrium expression for the economy.

4.2.14 The Monetary Policy

The log-linearizing monetary policy 1, equation 3.50, gives us[10]:

$$r_t = \alpha_1(r_{t-1}) + (1-\alpha_1)\{\alpha_2(\pi_{t+1}) + \alpha_3(y_t - y_{t-1})\} \quad (4.55)$$

Where $r_t = log(R_t) - log(\overline{R})$.

The log-linearized form of monetary policy 2, equation 3.51, becomes:

$$r_t = \alpha_1(r_{t-1}) + (1-\alpha_1)\{\alpha_2(\pi_{t+1}) + \alpha_3(y_t - y_{t-1}) - \alpha_4(q_t)\} \quad (4.56)$$

The log-linearized form of monetary policy 3, equation 3.52, becomes:

$$r_t = \alpha_1(r_{t-1}) + (1-\alpha_1)\{\alpha_2(\pi_{t+1}) + \alpha_3(y_t - y_{t-1}) - \alpha_4(q_t) + \alpha_5(q_t - q_{t-1})\} \quad (4.57)$$

Which concludes the log-linearization of the model. In the next chapter we calibrate the model.

[10] Which is a very similar form of the monetary policy that was used in Hunt [2006] where a New Keynesian model of the Icelandic economy was used.

5 Calibration

In the previous chapter we derived the dynamic model for the small open economy in log-linearized form. The equations defined there describe the dynamics of the economy in terms of deviations from the steady state. This chapter calibrates the constants used and we also define the *prior distributions* for the parameters in the model. To estimate the model parameters historical observables are used. Since the model has implications for the log-deviations from the steady state of the variables we have to preprocess the data before the estimation stage.
The observables are defined and we use historical time series for the observables as input to the model. When the model has been calibrated it can be estimated and that is done in chapter 6.

5.1 Observables and Steady states

The variables in the model are defined in terms of deviations from their steady states. We have defined many steady state variables in the preceding chapter. Variables with a bar over them are steady states, $\overline{X}$, and they will be calibrated using historical time series of the observables. Quarterly observations of the Icelandic economy are used but we also have two observables of the foreign economy.

5.1.1 Growth Variables

Observations from the period 1997 to 2009 are used. We make the approximation that there is a constant steady state for the whole period, so we have to filter the growth out of the time series in order to obtain an approximation for the steady states. We have five steady states of growth variables: $\overline{C}_H^*$, $\overline{B}_r^{*\prime}$, $\overline{B}_r^F$, $\overline{C}$ and $\overline{Y}$. We have to detrend these time series so there is no growth in the period. The growth variables in real terms are shown in figure 5.1. The prices are fixed at the beginning of the year 1996. The drop in foreign assets and foreign debt in 2008, as seen in the figure, is because deposit institutions in winding-up proceedings have been taken out. So the assets and debt of these institutions are not included[1].

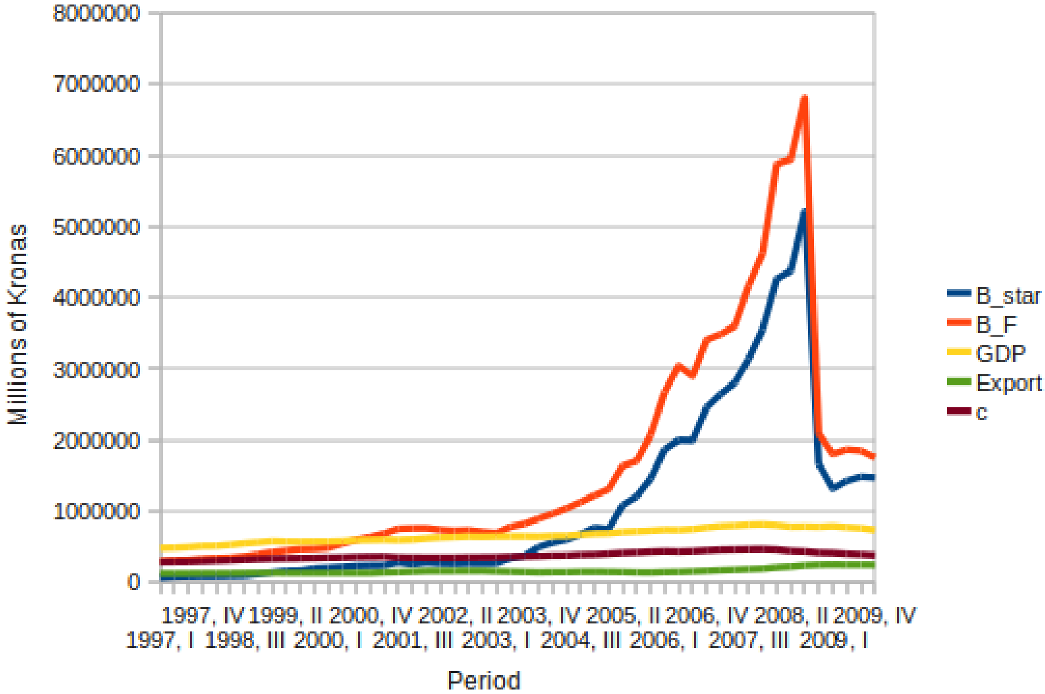

Figure 5.1: Foreign assets (B star), foreign debt (B F), exports, GDP and private consumption in real terms.

We calculate the average growth in GDP per quarter. The average growth is 1.376% per quarter. We filter the growth of the economy out of the series using the following method. If there are n observations in a time series, then observation number k in the detrended time series becomes:

$$x'_k = \frac{x_k}{(1.01376)^{(k-1)}}$$

Where x'_k is observation number k in the detrended series, and x_k is observation number k in the original series.

This is done to every observation of all the time series that experience growth. We detrend the GDP (Y_t), private consumption (C_t), foreign assets of the economy ($B^{*\prime}_{r,t}$), foreign debt ($B^F_{r,t}$) and exports ($C^*_{H,t}$). The result is shown in figure 5.2.

We see that GDP, private consumption and exports have become stable but foreign assets and debt are not. We can now obtain a steady state for the period by taking

[1]The time series for the foreign assets and foreign debt were obtained from the Central Bank of Iceland, *www.sedlabanki.is*. The time series for the private consumption, exports and GDP were obtained from Statistics Iceland, *www.statice.is*.

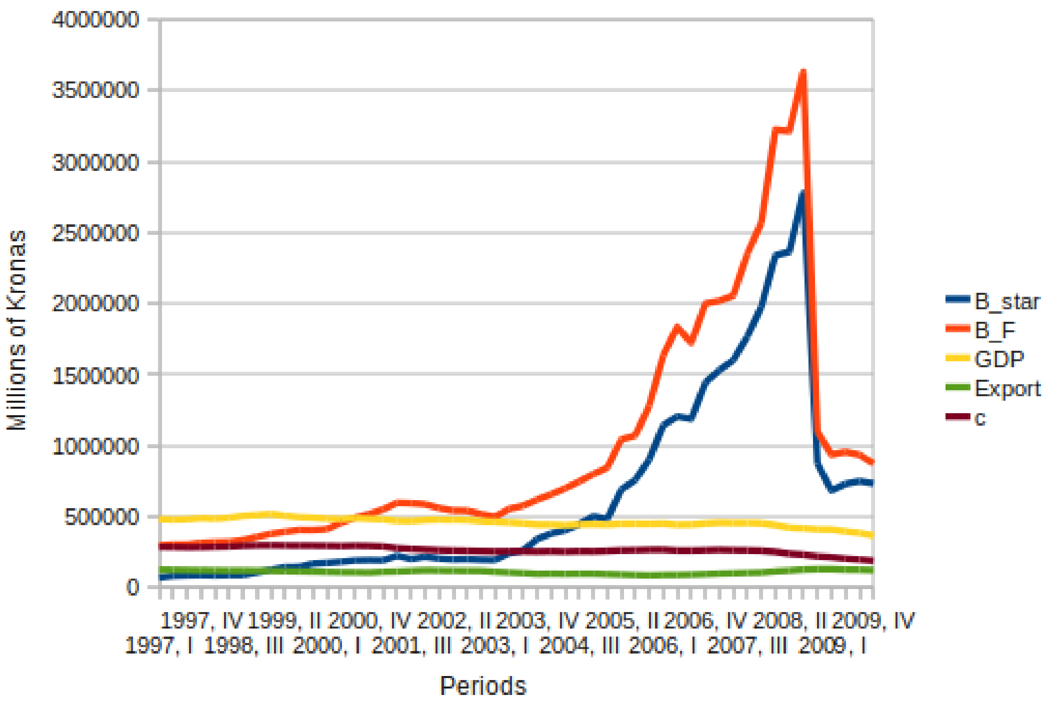

Figure 5.2: Detrended foreign assets, foreign debt, exports, GDP and private consumption.

the average of these detrended time series. We will take the average of the whole period for GDP, exports and private consumption to obtain an approximation for the steady state. But we use only the period from 1997 to 2005 for foreign assets and foreign debt. Foreign assets and foreign debt is relatively stable in that period, and then a bubble begins. Calculating the steady states this way, we get the results shown in table 5.1. These are the average values of the real detrended time series.

According to equation 4.48 the steady state GDP is equal to steady state private consumption, since we don't have investments and government expenditure. We can see that the estimated steady state for $\overline{Y}$ in table 5.1 is higher than the one for $\overline{C}$ so we have to lower the value for $\overline{Y}$ so that it becomes equal to $\overline{C}$. The dynamics of the Y_t time series are still used, but the steady state is scaled because of the simplifications in our model.

		M.kr.
$\overline{B}_r^{*\prime}$	$=$	256,744
$\overline{B}_r^{F}$	$=$	552,579
$\overline{Y}$	$=$	459,789
$\overline{C}_H^{*}$	$=$	110,050
$\overline{C}$	$=$	264,403

Table 5.1: Steady states of the growth variables.

5.1.2 Non Growth Variables

Now we need to calculate the other steady states used in the model. We have quarterly observations for the same period, 1997 to 2009 for the variables: F_t, R_t, R_t^* and Π_t.

For the net foreign debt steady state, $\overline{F}$, we use the foreign debt and foreign asset steady state:

$$\overline{F} = \overline{B}_r^F - \overline{B}_r^{*\prime} = 552,579 - 256,744 = 295,835 M.kr.$$

We cannot use the net foreign debt as an observable because of its large deviation from equilibrium. We only use the steady state of the net foreign debt, and the steady state is calculated as the average of the net foreign debt from 1997 until 2005. After 2005 it deviates largely from the steady state. The net debt time series is shown in figure 5.3.

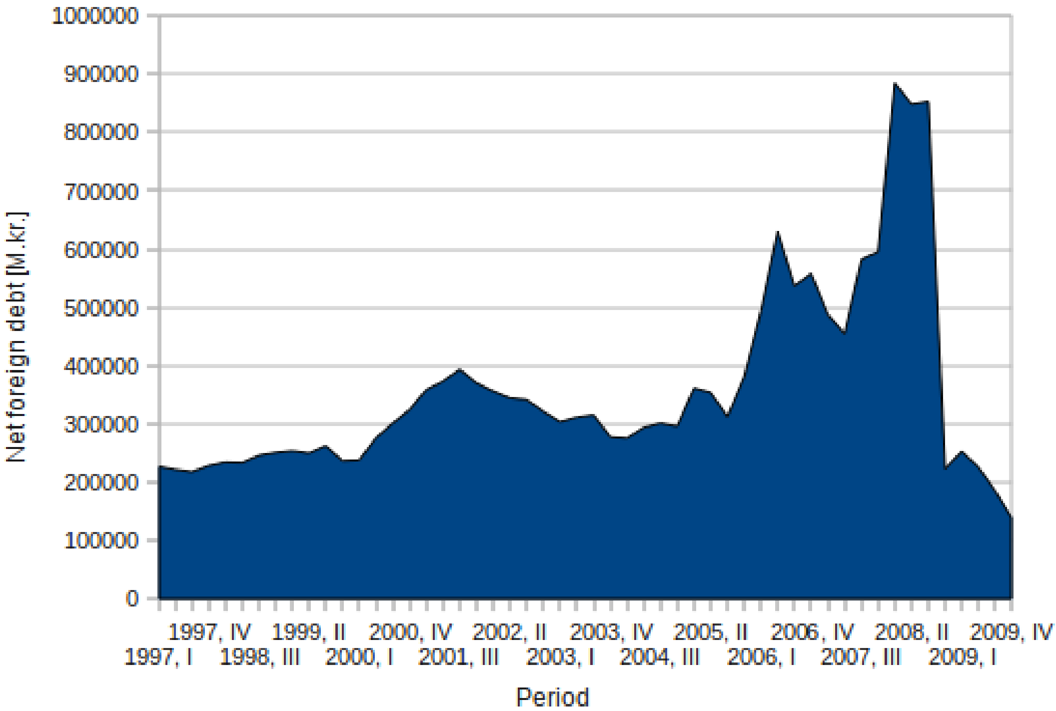

Figure 5.3: Real net foreign debt of the domestic economy, F_t.

The domestic central bank interest rate is an observable for the model. We use a time series of the nominal unindexed interest rate and it can be seen in figure 5.4[2]. We calculate the steady state of the interest rate as the average of the time series over the whole period. $\overline{R} = 1.099$.
We also use domestic inflation as an observable. We use a time series of the

[2]The time series for the interest rate are obtained at the Central Bank of Iceland, *www.sedlabanki.is*.

consumer price index, a plot of the series is shown in figure 5.4[3]. We calculate the average of the series to obtain the steady state, $\overline{\Pi} = 1.054$.
The foreign interest rate is also used as an observable. Around 50% to 60% of the exports from Iceland are sold in euros, and around 30% to 40% is in US dollars. We therefore only use the interest rate of the US dollar and the euro as the foreign interest rate, for simplicity. The foreign interest rate is defined as:

$$R_t^* = 0.6R_t^{Eur} + 0.4R_t^{US}$$

Where R_t^{Eur} is the interest rate on the Euro and R_t^{US} is the interest rate on the US dollar. We use the nominal central bank interest rate[4]. The foreign central bank interest rate is shown in figure 5.4. We use the average of the time series as the steady state, $\overline{R}^* = 1.039$.

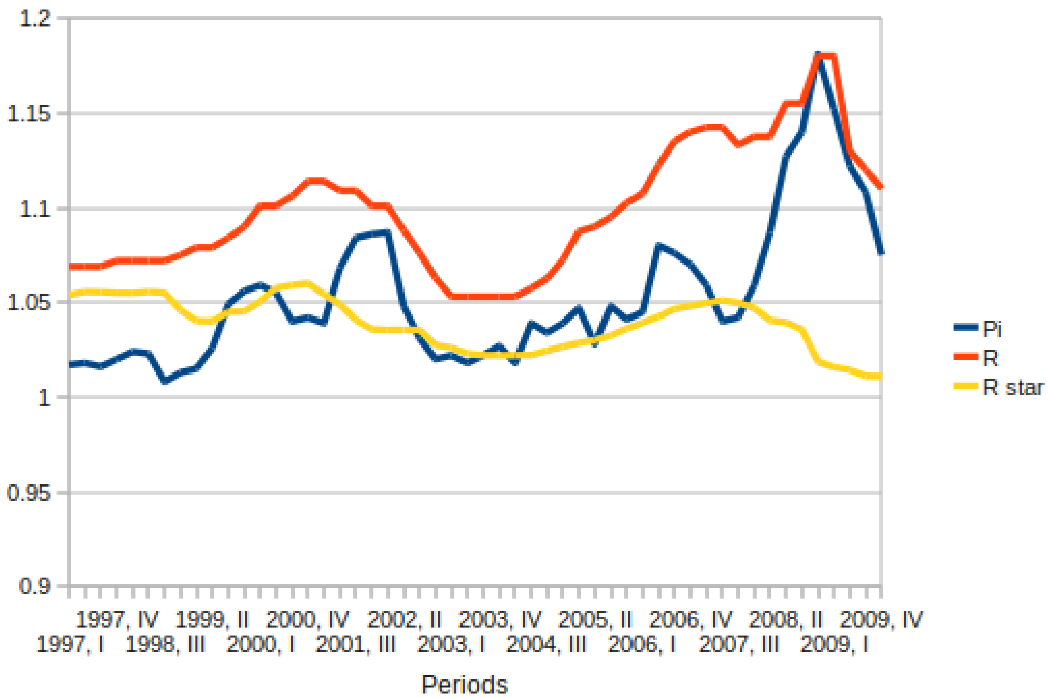

Figure 5.4: Domestic interest rate (R), foreign interest rate (R star) and domestic inflation (Pi).

We take the real exchange rate of the domestic economy as an observable[5]. The real exchange rate can be seen in figure 5.5. We also use foreign inflation as an observable for the model. For simplicity we only use the consumer price index for the Euro Area and for the United States, like we did with the foreign interest rate. We define the foreign inflation as:

$$\Pi_t^* = 0.6\Pi_t^{Eur} + 0.4\Pi_t^{US}$$

[3]The time series for the CPI is obtained at Statistics Iceland, *www.statice.is*.

[4]The Euro interest rate is obtained at the European Central Bank, *www.ecb.int*, and the US interest rate is obtained at the Federal Reserve, *www.federalreserve.gov*.

[5]The real exchange rate time series is obtained at the Central Bank of Iceland, *www.sedlabanki.is*.

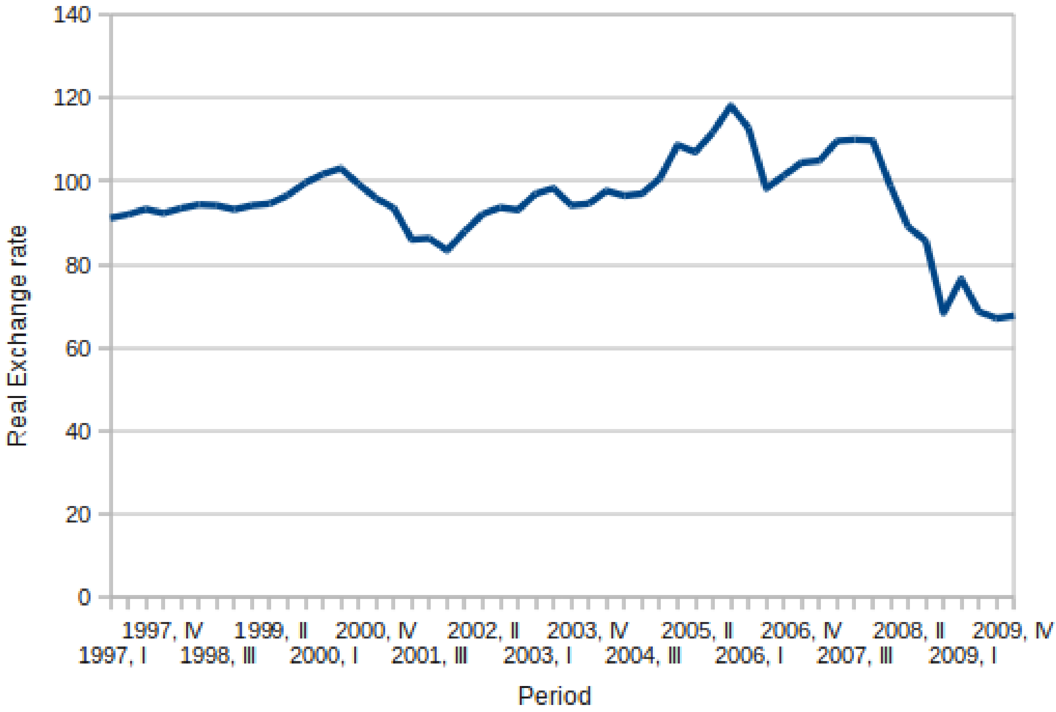

Figure 5.5: The real exchange rate, Q_t.

Where Π_t^{Eur} is the consumer price inflation in the Euro Area and Π_t^{US} is the consumer price inflation in the United States[6]. Foreign consumer price inflation, Π_t^* is shown in figure 5.6. All the steady states used in the model are shown in table 5.2. Note that foreign inflation is an observable, but we don't use its steady state in the model. All the observables are summarized in table 5.3.

The steady states			
$\overline{R}$	=	1.099	
$\overline{R}^*$	=	1.039	
$\overline{\Pi}$	=	1.054	
$\overline{F}$	=	295,835	M.kr.
$\overline{B}_r^{*\prime}$	=	256,744	M.kr.
$\overline{B}_r^F$	=	552,579	M.kr.
$\overline{Y}$	=	459,789	(264,403) M.kr.
$\overline{C}_H^*$	=	110,050	M.kr.
$\overline{C}$	=	264,403	M.kr.

Table 5.2: The Steady states of the model.

[6]The US consumer price index was obtained at the *U.S. Bureau of Labor Statistics* and the Euro Area consumer price index was obtained at *Eurostat*.

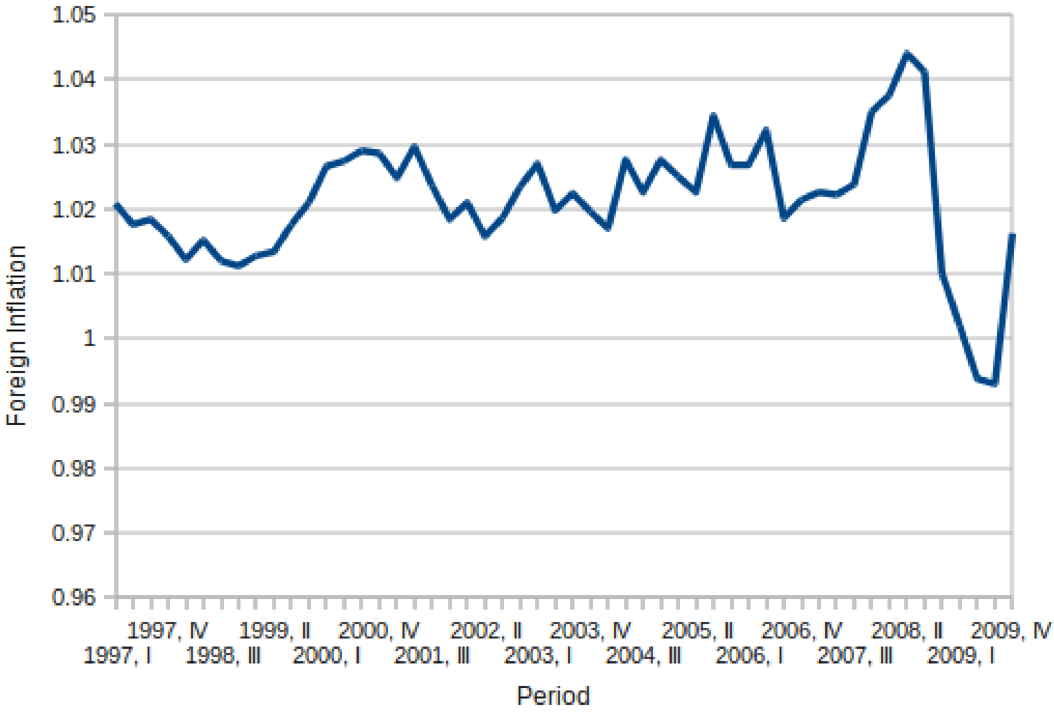

Figure 5.6: Foreign inflation, Pi star.

	The Observables	Figure
Π_t	Domestic inflation	5.4
R_t	Domestic interest rate	5.4
Π_t^*	Foreign inflation	5.6
R_t^*	Foreign interest rate	5.4
Q_t	Domestic real exchange rate	5.5
$C_{H,t}^*$	Domestic exports	5.2
C_t	Domestic private consumption	5.2
Y_t	Domestic GDP	5.2

Table 5.3: The observables of the model.

5.2 Parameters: Priors and Constants

In this section we calibrate the parameters and constants used in the model. Priors can reflect strongly held beliefs about the validity of economic theories. The priors incorporate the researchers beliefs about possible ranges regarding the nature and behavior of the variables[7]. In practice, priors are chosen based on observation, facts or from existing empirical literature. We will use existing literature and optimal policy calculations for the choice of priors, more on that here below. The Bayesian estimation will be discussed in chapter 6 page 49.

[7]See for example Smets and Wouters [2004].

5.2.1 Households

For the households we have a few parameters. We have the rate of time preference β, the inverse elasticity of inter temporal substitution σ, the inverse elasticity of labor supply ϕ and the external habit formation h. See chapter 3 for more detail. We choose the priors in line with earlier literature[8], they are defined in table 5.4. We also have η, the elasticity of substitution between home and foreign goods. We fix the discount factor as $\beta = 0.95$.

Parameter	Distribution	Mean	S.E.	Domain
σ	Gamma	2.00	0.10	$\Re^+$
ϕ	Gamma	1.00	0.30	$\Re^+$
h	Beta	0.50	0.20	$[0, 1]$
η	Gamma	1.00	0.30	$\Re^+$

Table 5.4: Priors for households' parameters.

5.2.2 Firms

We have the price setting fractions for firms, θ_H and θ_F. The priors for these two parameters are shown in table 5.5, and they are chosen in line with earlier literature[9] like the priors for households.

Parameter	Distribution	Mean	S.E.	Domain
θ_H	Beta	0.50	0.25	$[0, 1]$
θ_F	Beta	0.50	0.25	$[0, 1]$

Table 5.5: Priors of the parameters for firms.

5.2.3 The Foreign Economy

For the foreign economy we have the factor that notes how an individual values between terms in the export function, equation 3.43, μ. We also have the elasticity of substitution between the home and foreign goods seen from the foreign economy, η^*. Then we have the degree of openness, α. The priors for these parameters are shown in table 5.6 [10]. Since η^* is the elasticity of substitution of the rest of the world, it should have a higher value than η. The reason is that it's likely for the

[8] See Liu [2006], Haider and Khan [2008] and Gonzalez and Garcia [2010].

[9] See for example Haider and Khan [2008], where the same priors are used.

[10] The priors for the foreign economy are in line with Gottfries [2002].

rest of the world to have more substitutes to choose from than the small open economy. We also have the risk premium, we fix the long term risk premium as $\gamma = 0.01$. But the risk premium varies with net foreign debt to GDP as shown by equation 3.27.

Parameter	Distribution	Mean	S.E.	Domain
μ	Beta	0.10	0.20	$[0,1]$
η^*	Gamma	3.00	0.30	$\Re^+$
α	Beta	0.40	0.25	$[0,1]$

Table 5.6: Priors for the parameters in the foreign economy.

5.2.4 Shocks

The shocks are a very important factor in the model. In chapter 4 when the model was log-linearized we added shocks to some equations. These shocks will be defined here. We have added shocks to various equations and we want to estimate how the model responds to these shocks. We want to estimate how various monetary policies respond to various shocks. We have a stationary stochastic model so that shocks to it are only allowed to be temporary. A permanent shock cannot be accommodated because the model needs to revert to the steady state, and the steady state is considered constant. Furthermore, shocks can only hit the system today, as the expectation of future shocks must be zero. The shock follows an AR(1) process and propagates throughout the economy until it reaches the steady state. To do that we use a 'latent shock variable' and we add it to the relationship that we want to shock, like we did when we log-linearized the model. The shock variable is an AR(1) process. For example if we wanted to add a shock to the LOP gap (equation 4.21) we add the shock variable:

$$\psi_t = -q_t - (1-\alpha)s_t + \nu_t \tag{5.1}$$

The ν_t variable is endogenous but we add en exogenous shock to it:

$$\nu_t = \rho\nu_{t-1} + \epsilon_t$$

Where ϵ_t is a Gaussian shock with variance $Var(\epsilon_t) = \sigma_\epsilon^2$ and non-zero mean. The parameter $\rho \in (0,1)$ defines how fast the shock variable goes to zero, called *Autocorrelation Coefficient*. This way we can shock the LOP gap for a certain period until it reverts to the steady state.
Note that in order to avoid stochastic singularity, there must be at least as many shocks or measurement errors in the model as there are observed variables[11]. We

[11] The estimation will be discussed in detail in chapter 6, but for more information on DSGE technicalities, see for example Hamilton [1994] or Canova [2007].

have eight observables and we define a total of nine shocks to the model. We add a shock variable to the log-linear expression of the variable that we want to shock, like we do in equation 5.1. The shocks to endogenous variables are defined in table 5.7 and shock to the exogenous processes are defined in table 5.8. When we add a shock to the exogenous processes we don't use a latent shock variable but we can add the shock, ϵ_t, directly, since the exogenous processes are AR(1) processes.

Variable	Shock to variable
q_t	$\nu_t^q = \rho_q \nu_{t-1}^q + \epsilon_t^q$
f_t	$\nu_t^f = \rho_f \nu_{t-1}^f + \epsilon_t^f$
$\pi_{F,t}$	$\nu_t^{\pi_F} = \rho_{\pi_F} \nu_{t-1}^{\pi_F} + \epsilon_t^{\pi_F}$
ψ_t	$\nu_t^{\psi} = \rho_{\psi} \nu_{t-1}^{\psi} + \epsilon_t^{\psi}$
$prem_t$	$\nu_t^{prem} = \rho_{prem} \nu_{t-1}^{prem} + \epsilon_t^{prem}$

Table 5.7: Shocks to the endogenous variables.

Variable	Shock to variable
a_t	$a_t = \rho_a a_{t-1} + \epsilon_t^a$
c_t^*	$c_t^* = \rho_{c^*} c_{t-1}^* + \epsilon_t^{c^*}$
r_t^*	$r_t^* = \rho_{r^*} r_{t-1}^* + \epsilon_t^{r^*}$
π_t^*	$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \epsilon_t^{\pi^*}$

Table 5.8: Shocks to the exogenous processes.

The autocorrelation coefficients, ρ_x, will also be estimated with Bayesian techniques. In previous literature the coefficients are generally defined the same way[12]. The priors for the autocorrelation coefficients are defined in table 5.9. The priors are all defined the same way.

Parameter	Distribution	Mean	S.E.	Domain
ρ_x	Beta	0.50	0.10	$[0, 1]$

Table 5.9: The priors of the autocorrelation coefficients.

We define the shocks to the model, ϵ, as Gaussian with non-zero mean. We define the prior distribution for the shocks, the mean and standard deviation. When we simulate the model a random shock is taken from the prior distribution and the shock is applied at $t = 0$, and then we estimate the response functions. The response functions are often called *Impulse response functions*, because the shock is only applied at $t = 0$ as an impulse, but not at later times. This is done in detail in chapter 6. The shocks are defined in table 5.10[13].

Note that the shock to the net foreign debt has a positive mean, so the net foreign debt increases on average when shocked. The shock to the real exchange rate has a negative mean so the currency is depreciating and the shock to foreign private

[12]See for example Gonzalez and Garcia [2010].
[13]Where S.D. is the standard deviation of the prior.

Shock	Distribution	Mean	S.D.
ϵ_t^f	Normal	0.05	0.02
ϵ_t^ψ	Normal	0.01	0.01
ϵ_t^q	Normal	-0.05	0.02
$\epsilon_t^{\pi_F}$	Normal	0.01	0.01
ϵ_t^a	Normal	0.01	0.01
ϵ_t^{prem}	Normal	0.05	0.02
$\epsilon_t^{\pi^*}$	Normal	0.05	0.02
$\epsilon_t^{r^*}$	Normal	0.05	0.02
$\epsilon_t^{c^*}$	Normal	-0.05	0.02

Table 5.10: The priors of the shocks.

consumption does also have a negative mean so the consumption is decreasing on average when the shock hits. We have defined a smaller mean of the prior for the shocks to ψ_t, $\pi_{t,F}$ and a_t. This is because we are mostly interested in the shocks to the foreign variables: $prem_t$, π_t^*, r_t^* and c_t^*. We want to see how the three monetary policies compare when these shocks are applied to the economy.

5.3 Optimal Policy

Finally we calibrate the monetary policies used in the model, equation 3.50 to 3.52. We want to calibrate the parameters, α_1 ... α_5. We use an algorithm that searches numerically for the best value for the coefficients of the policy[14]. The algorithm minimizes an objective function. The objective function is a weighted sum of variances and we calibrate the objective function so it becomes like the loss function, equation 3.53. We iterate through the model by changing the policy parameters until a minimum of the objective function has been reached. We will use the optimal policy calculations as guidelines for choosing the priors, along with earlier literature.

We apply all the shocks defined in table 5.10 and iterate until the objective function has been minimized. We use the weights used in the loss function where inflation variance has weight equal to 1, output variance has weight 0.5 and the interest rate variance has weight 0.2. We use these weights for our optimal policy objective function.

We calculate the values of the parameters using the optimal policy algorithm, minimizing the welfare loss. The results are displayed in table 5.11.

Note that we put a minus sign on α_4 when defining the monetary policies, equations

[14]Using Dynare, we use a function called Optimal simple rule (OSR) that does this numerical calculation.

Parameter	M.P. 1	M.P. 2	M.P. 3
	Optimal values		
α_1	0.40	0.56	0.50
α_2	3.00	3.03	2.75
α_3	0.50	0.36	0.30
α_4	-	0.12	0.40
α_5	-	-	0.50

Table 5.11: Optimal policy parameters for monetary policies 1,2 and 3, equations 3.50, 3.51 and 3.52.

3.51 and 3.52, because we wanted a negative relationship between the interest rate and the level of the real exchange rate because when the exchange rate appreciates the interest rate should be lowered. The optimal policy calculations imply that this was correct, since the optimal value for α_4 is positive, and hence the relationship is negative.

Looking at recent literature, Gonzalez and Garcia [2010] use a very similar monetary policy to monetary policy 3, where it also responds to the real exchange rate. They use a DSGE model to examine the role of risk premium shocks to small open economies. Let's take a look at the priors used there for the monetary policy, they are shown in table 5.12.

Parameter	Distribution	Mean	S.D.
α_1	Beta	0.50	0.05
α_2	Gamma	2.00	0.10
α_3	Gamma	0.50	0.10
α_4	Gamma	0.25	0.10
α_5	Gamma	0.25	0.10

Table 5.12: Priors for the monetary policy parameters, from recent literature.

Taking note of table 5.12 and the optimal policy parameters derived above, we choose the priors for the monetary policies' parameters. We choose different priors for the monetary policies so that each monetary policy is calibrated as optimal as possible. The choice of priors for the monetary policies parameters can be found in tables 5.13, 5.14 and 5.15.

Parameter	Distribution	Mean	S.D.
α_1	Gamma	0.40	0.10
α_2	Gamma	3.00	0.10
α_3	Gamma	0.50	0.10

Table 5.13: The priors for the parameters in monetary policy 1, equation 3.50.

Parameter	Distribution	Mean	S.D.
α_1	Gamma	0.56	0.10
α_2	Gamma	3.03	0.10
α_3	Gamma	0.36	0.10
α_4	Gamma	0.12	0.10

Table 5.14: The priors for the parameters in monetary policy 2, equation 3.51.

Parameter	Distribution	Mean	S.D.
α_1	Gamma	0.60	0.10
α_2	Gamma	2.00	0.10
α_3	Gamma	0.40	0.10
α_4	Gamma	0.75	0.10
α_5	Gamma	0.40	0.10

Table 5.15: The priors for the parameters in monetary policy 3, equation 3.52.

All the priors have the same standard deviation, or 0.10. Monetary policy 3 has the highest rate of interest smoothing but the lowest weight on inflation variance. Also note that monetary policy 3 puts relatively high weight on the real exchange rate level while monetary policy 2 puts low weight on it. When the model has been estimated we can compare the posteriors.

This concludes the calibration of the model and now it can be estimated. In chapter 6 the model will be simulated and the posteriors estimated. Then in chapter 7 come the results from the simulation and we compare the welfare loss between different monetary policies.

6 Estimation

Now the model that we derived and calibrated in the preceding chapters will be estimated. We will simulate the log-linearized model for a number of periods and examine the models response to various shocks. The posteriors of the parameters will be estimated and we will examine the impulse response functions. Different monetary policies will be used and we will examine the difference in response of the model, based on what monetary policy is used. We will use the loss function defined in chapter 3 to measure the difference in welfare loss between different monetary policies. The results of the estimation will be discussed in chapter 7.

The working procedure of the estimation, in short, is that we use the log-linear model from chapter 4 and we apply shocks to it at period $t = 0$. The shocks are drawn from the prior distributions defined in table 5.10. The parameters are also drawn from their prior distributions. We use the historical time series from section 5.1 and use them to find the likelihood function that describes the density of the data, given the model and its parameters. We use the likelihood function together with the priors to obtain the posterior distributions, using Bayesian estimation.
When we have the posterior distributions we simulate the model and iterate the simulation a number of times, drawing the shocks and the parameters from the posterior distributions and obtain a medium response of the system. We use this response to calculate the welfare using the loss function, equation 3.53 and to compare monetary policies.

6.1 Bayesian Estimation

Bayesian estimation has become increasingly popular in the field of macroeconomics. Recent literature on DSGE models commonly uses Bayesian estimation to estimate the models. Central banks around the world are using Bayesian estimated DSGE models extensively, for inflation targeting and other purposes. The Bayesian estimation is used for technical reasons, and because of greater computer power it is becoming more frequently used.

There are a number of advantages of Bayesian estimation. For example, the Bayesian estimation is based on the likelihood, which is generated by the DSGE system, but not on the indirect conflict between the implied DSGE and VAR impulse response functions. But our model must not be mis-specified, which can be a disadvantage.

The Bayesian estimation uses priors which work as weights in the estimation process so that the posterior distribution avoids peaking at strange points where the likelihood peaks. Because the DSGE models can be misspecified, the likelihood often peaks in regions of the parameter space that do not fit to common observations, leading to the *dilemma of absurd parameter estimates.*
It helps us to identify parameters to include priors. But when estimating the model, identification can be a problem. It can be summarized by different values of structural parameters leading to the same joint distribution for observables. This can happen when the posterior distribution is flat over a subspace of parameter values. But weighting the prior densities with the likelihood can lead to adding just enough curvature in the posterior distribution to assist numerical maximization.
By including shocks the Bayesian estimation explicitly addresses model misspecification, which can be interpreted as observation errors, in the structural equations.

6.1.1 The Basic Mechanics of Bayesian estimation

Bayesian estimation can be thought of as a bridge between calibration and maximum likelihood. The calibration of models is done by the specification of priors. The maximum likelihood approach comes from the estimation process based on using the model together with data. Priors can be thought of as weights on the likelihood function in order to give more importance to certain areas of the parameter subspace. The priors and likelihood functions are linked by Bayes' rule, as we will describe here below.

We describe the priors by a density function of the form:

$$p(\theta_{\mathcal{A}} \mid \mathcal{A})$$

Where $\mathcal{A}$ is a specific model, $\theta_{\mathcal{A}}$ represents the parameters of model $\mathcal{A}$ and $p(\cdot)$ stands for a probability density function (pdf). The likelihood function describes the density of the observed data, given the model and the parameters:

$$\mathcal{L}(\theta_{\mathcal{A}} \mid \mathbf{Y}_T, \mathcal{A}) \equiv p(\mathbf{Y}_T \mid \theta_{\mathcal{A}}, \mathcal{A})$$

Where $\mathbf{Y}_T$ are the observations until period T, and the likelihood in our case is recursive and we write it as:

$$p(\mathbf{Y}_T \mid \theta_{\mathcal{A}}, \mathcal{A}) = p(y_0 \mid \theta_{\mathcal{A}}, \mathcal{A}) \prod_{t=1}^{T} p(y_t \mid \mathbf{Y}_{t-1}, \theta_{\mathcal{A}}, \mathcal{A})$$

Using *Bayes theorem* we obtain the density of the parameters, knowing the data. Generally we have:

$$p(\theta \mid \mathbf{Y}_T) = \frac{p(\theta; \mathbf{Y}_T)}{p(\mathbf{Y}_T)}$$

We know that:

$$p(\mathbf{Y}_T \mid \theta) = \frac{p(\theta; \mathbf{Y}_T)}{p(\theta)} \Longleftrightarrow p(\theta \mid \mathbf{Y}_T) = p(\mathbf{Y}_T \mid \theta) p(\theta)$$

Now we can combine the prior density and the likelihood function to get the posterior density:

$$p(\theta_{\mathcal{A}} \mid \mathbf{Y}_T, \mathcal{A}) = \frac{p(\mathbf{Y}_T \mid \theta_{\mathcal{A}}, \mathcal{A}) p(\theta_{\mathcal{A}}, \mathcal{A})}{p(\mathbf{Y}_T \mid \mathcal{A})}$$

Where $p(\mathbf{Y}_T \mid \mathcal{A})$ is the *marginal density* of the data conditional on the model:

$$p(\mathbf{Y}_T \mid \mathcal{A}) = \int_{\theta_{\mathcal{A}}} p(\theta_{\mathcal{A}}; \mathbf{Y}_T \mid \mathcal{A}) d\theta_{\mathcal{A}}$$

The posterior density that is not normalized (*posterior kernel*) corresponds to the numerator of the posterior density:

$$p(\theta_{\mathcal{A}} \mid \mathbf{Y}_T, \mathcal{A}) \propto p(\mathbf{Y}_T \mid \theta_{\mathcal{A}}, \mathcal{A}) p(\theta_{\mathcal{A}} \mid \mathcal{A}) \equiv \mathcal{K}(\theta_{\mathcal{A}} \mid \mathbf{Y}_T, \mathcal{A})$$

This is the main equation that we use to rebuild all posterior moments of interest. Matlab®[1] is used for all calculations together with a toolbox called Dynare[2], which is a software that estimates and solves DSGE models. A program of the model is written, which is solved and estimated using Dynare and Matlab. The general working of the Dynare estimation is discussed here below. The likelihood function will be estimated using the *Kalman filter* and then we simulate the *posterior kernel* using a Monte Carlo method called the *Metropolis-Hastings*. These topics will also be explained further in the following section.

6.1.2 Bayesian Estimation of DSGE models

A DSGE model can be thought of as a collection of first order conditions and equilibrium conditions, we can write it in the form:

$$\mathbb{E}_t\{f(y_{t+1}, y_t, y_{t-1}, u_t)\} = 0, \ \mathbb{E}(u_t) = 0 \text{ and } \mathbb{E}(u_t u_t') = \Sigma_u$$

[1]Matlab is a numerical computing environment, see *www.mathworks.com* for more information.

[2]Dynare is a pre-processor and a collection of Matlab and GNU Octave routines which solve non-linear models with forward looking variables. See www.dynare.org for more information.

Where y is vector of endogenous variables of any dimension and u is a vector exogenous stochastic shocks of any dimension. The *policy function* is a solution to this system. It is a set of equations relating variables in the current period to the past state of the system and to current shocks, that satisfy the original system. We write the *policy function* as $y_t = g(y_{t-1}, u_t)$.

We write the solution to a DSGE model as follows:

$$\begin{aligned}
y_t^* &= M\bar{y}(\theta) + M\hat{y}_t + N(\theta)x_t + \eta_t \\
\hat{y}_t &= g_y(\theta)\hat{y}_{t-1} + g_u(\theta)u_t \\
\mathbb{E}(\eta_t\eta_t') &= V(\theta) \\
\mathbb{E}(u_t u_t') &= Q(\theta)
\end{aligned}$$

Where $\hat{y}_t$ are variables in deviations from steady state, $\bar{y}$ is the vector of steady state values and θ is the vector of deep (structural) parameters to be estimated. The variable η_t is the process error. M is a $m \times m_1$ vector of constants, where m is the number of endogenous variables and m_1 is the number of η_t shocks. $N(\theta)$ is a $m \times m_2$ matrix where m_2 is the number of u_t shocks and x_t is a variable to allow for a linear trend, to allow for the most general case[3]. y^* is observable and it is related to the true variables win an error η_t. The first and second equation here above make up a system of state equations.

Now we estimate the likelihood of the DSGE solution system. The equations above are non-linear in deep parameters, θ, but they are linear in endogenous and exogenous variables so the likelihood can be evaluated with a linear prediction error algorithm, and we will use the *Kalman filter*. The Kalman filter is a mathematical method and its purpose is to use measurements, observed over time, that contain noise and other inaccuracies. The Kalman filter produces estimates of the true values of measurements and their associated calculated values by predicting a value, estimating the uncertainty of the predicted value and computing a weighted average of the predicted value and the measured value. Kalman filters are based on linear dynamic systems who are discrete in the time domain. They are modeled on a Markov Chain built on linear operators who are shocked by Gaussian noise.

Following is how the Kalman filter recursion goes[4]:
For $t = 1, ..., T$ and with initial values y_1 and P_1 we have:

$$\begin{aligned}
v_t^* &= y_t^* - \bar{y}^* - M\hat{y}_t - Nx_t \\
F_t &= MP_tM' + V \\
K_t &= g_yP_tg_y'F_t^{-1} \\
\hat{y}_{t+1} &= g_y\hat{y}_t + K_tv_t \\
P_{t+1} &= g_yP_t(g_y - K_tM)' + g_uQg_u'
\end{aligned}$$

[3]See Hamilton [1994] and Canova [2007] for more information on solving DSGE models.
[4]For more information on the Kalman filter see for example Canova [2007] and Hamilton [1994].

From the Kalman filter recursion, it is possible to derive the *log-likelihood* given by:

$$ln\left\{\mathcal{L}(\vec{\theta} \mid \mathbf{Y}_T^*)\right\} = -\frac{Tk}{2}ln(2\pi) - \frac{1}{2}\sum_{t+1}^{T}|F_t| - \frac{1}{2}v_t' F_t^{-1} v_t$$

Where the vector $\vec{\theta}$ contains the parameters we have to estimate: θ, $V(\theta)$ and $Q(\theta)$. The variable $\mathbf{Y}_T^*$ expresses the set of observable endogenous variables y_t^* found in the measurement equation.

The log-likelihood above is one step in finding the posterior distribution of our parameters. The *log-posterior kernel* can be expressed as:

$$ln\{\mathcal{K}(\vec{\theta} \mid \mathbf{Y}_T^*)\} = ln\{\mathcal{L}(\vec{\theta} \mid \mathbf{Y}_T^*)\} + ln\{p(\vec{\theta})\} \tag{6.1}$$

Where the first term on the right hand side is known after carrying out the Kalman filter recursion. Now to finish calculating the posteriors we need to use the priors.

To find the mode of the posterior distribution we maximize the *log posterior kernel*, equation 6.1, with respect to θ. This is done using numerical methods. The likelihood function is not Gaussian with respect to θ but to functions of θ as they appear in the state equation. The posterior distribution will be given by the kernel equation above but its a nonlinear function of the θ parameters. Therefore we cannot find an explicit form of $\hat{\theta}_{ML}$. Instead we use a sampling-like method called the *Metropolis-Hastings*, which is well known in the Bayesian literature. The Metropolis-Hastings algorithm simulates the posterior distribution using a 'rejection sampling algorithm' that generates a sequence of samples from a distribution that is unknown at the outset[5].
We have the posterior mode but we are often interested in the mean and the variance of estimators of θ. To calculate the mean and variance the algorithm builds on the fact that under general conditions the distribution of the deep parameters will be asymptotically normal. The algorithm constructs a Gaussian approximation around the posterior mode and uses a scaled version of the asymptotic covariance matrix as the covariance matrix for the proposal distribution. This allows for an efficient exploration of the posterior distribution, at least in the neighborhood of the mode[6]. The *Metropolis-Hastings algorithm* implements the following steps:

1. A *starting point* is chosen, $\vec{\theta}^0$, which is typically chosen as the posterior mode. The loop 2-3-4 is then run.

2. A *proposal*, $\vec{\theta}^*$, is drawn from a jumping distribution

   $$J(\vec{\theta}^* \mid \vec{\theta}^{t-1}) = \mathcal{N}(\vec{\theta}^{t-1}, c\Sigma_m)$$

   Where Σ_m is the inverse of the Hessian computed at the posterior mode and c is the *scale factor*.

[5] Known as the Markov Chain process.
[6] See An and Schorfheide [2006], page 18, for more information on these calculations.

3. The acceptance ratio is computed:

$$r = \frac{p(\vec{\theta}^* \mid \mathbf{Y}_T)}{p(\vec{\theta}^{t-1} \mid \mathbf{Y}_T)} = \frac{\mathcal{K}(\vec{\theta}^* \mid \mathbf{Y}_T)}{\mathcal{K}(\vec{\theta}^{t-1} \mid \mathbf{Y}_T)}$$

4. Finally we accept or discard the proposal $\vec{\theta}^*$ according to the following rule:

$$\vec{\theta}^t = \begin{cases} \vec{\theta}^* & \text{with probability } min(r,1) \\ \vec{\theta}^{t-1} & \text{otherwise} \end{cases}$$

   Then we update, if necessary, from the jumping distribution.

We have such a complicated acceptance rule to be able to visit the entire domain of the posterior distribution. It is not good to throw out the candidate giving a lower value of the posterior kernel too quickly. Using that candidate for the mean of the drawing distribution allows us to leave a local maximum and travel towards the global maximum. The idea is therefore to allow the search to turn away from taking a small step up, and instead take a few small steps down in the hope of being able to take a big step up in the near future. An important parameter in the searching procedure is the variance of the jumping distribution and in particular the *scale factor*. If the scale factor is too small, the *acceptance rate* will be too high and the Markov Chain of candidate parameters will 'mix slowly', meaning that the distribution will take a long time to converge to the posterior distribution since the chain is likely to get 'stuck' around a local maximum. But if the scale factor is too large, the acceptance rate will be very low and the chain will spend too much time in the tails of the posterior distribution.

Using the methods in this section we are able to calculate the posterior distributions of the parameters with the help of Matlab and the Dynare toolbox[7]. In the next section we calculate the posteriors for the parameters in our *small open economy model.*

6.2 The Parameters

We have a number of parameters in the model that we want to estimate using Bayesian techniques. We have declared the priors in chapter 5 and there we also defined observables and we introduced historical time series of the observables. Now we use these priors and observables to estimate the posterior distributions of the parameters, given the measurements, the priors and the model. We will use the methods discussed in the preceding section.

[7]For more information on how Dynare solves DSGE models see Griffoli [2007] and Adjemian, Juillard, Mihoubi, Perendia, and Villemot [2009].

We begin with the log-linearized model from chapter 4. We use the constants and priors from chapter 5. We then apply all the 9 shocks to the model, the shocks are defined in table 5.10. We use the historical data from chapter 5 for the following eight observables: Y_t, Π_t, R_t, R_t^*, Q_t, $C_{H,t}^*$, Π_t^* and C_t.

We use the Metropolis-Hastings algorithm and we use two independent chains, each of length 100.000 units. We have to do three runs, one for each monetary policy we want to use. The monetary policies used are defined by equations 3.50, 3.51 and 3.52. We obtain the posteriors for all the structural parameters and for all the shocks. The numerical results from the Bayesian estimation are displayed in Appendix B page 65.

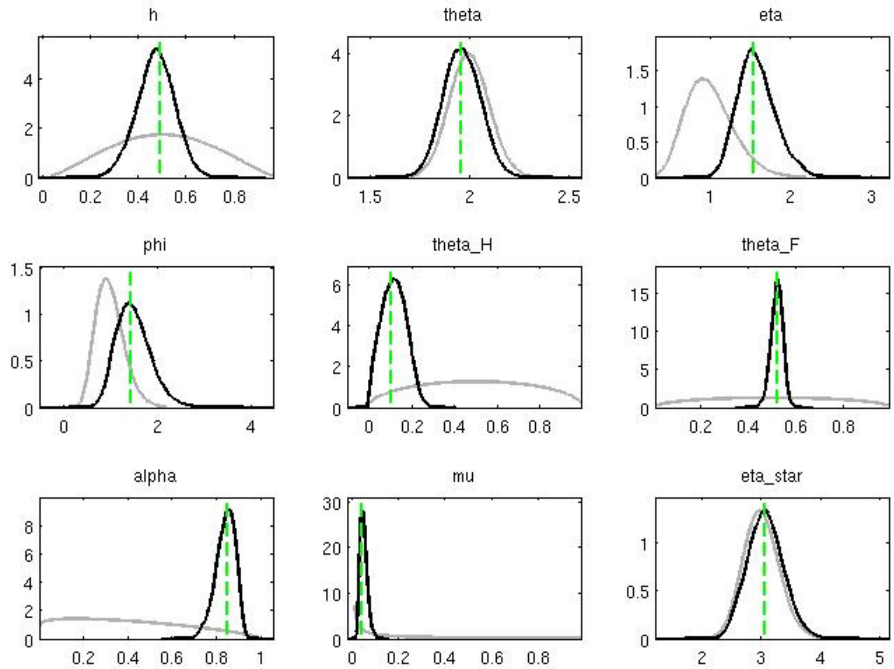

Figure 6.1: A few priors and posteriors, when monetary policy 1 was used, equation 3.50. See Appendix C for all the posteriors.

Graphical results of all the priors and all the posteriors are in Appendix C page 69. In figure 6.1 we can see the priors and posteriors for a few structural parameters, when monetary policy 1 was used.
Now we have estimated the posteriors and in the next section we take look at the models response to the shocks applied.

6.3 Simulation and Impulse Response Functions

Now we take a little look at the dynamics of the model. We have defined nine shocks that we apply to the model at time $t = 0$, the shocks are defined in table 5.10. We estimate the response using the posteriors obtained in the preceding section. Parameters and shocks are taken at random from the posterior distributions and the model is simulated for one hundred periods, $T = 100$.

For each shock we do 500 iterations, so we draw 500 times from the same posterior distribution, for the shocks and the parameters, and simulate the response. Then a medium response is calculated from the 500 simulations, with a confidence interval. We look at the response of the observables. Since there are two foreign observables we model the response for six domestic observables: Y_t, Π_t, R_t, Q_t, $C^*_{H,t}$ and C_t and possibly the foreign observable being shocked.

This is done for each monetary policy. We have nine shocks and three monetary policies, a total of 27 response graphs, each graph with six to seven response functions. The response functions of the observables, when a shock is applied to the risk premium, can be seen in figures 6.2, 6.3 and 6.4.

All the figures of the impulse response functions are shown in Appendix D. There we get a graphical display of the dynamics of the economy. We can see the response of the observables to the shocks, for every monetary policy. We can also see a confidence interval for the path of the variable, shown in gray.

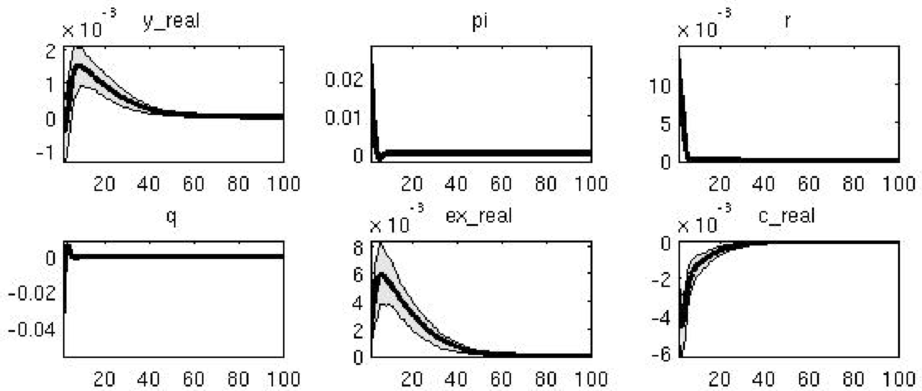

Figure 6.2: Response functions to a risk premium shock, when monetary policy 1 was used, equation 3.50.

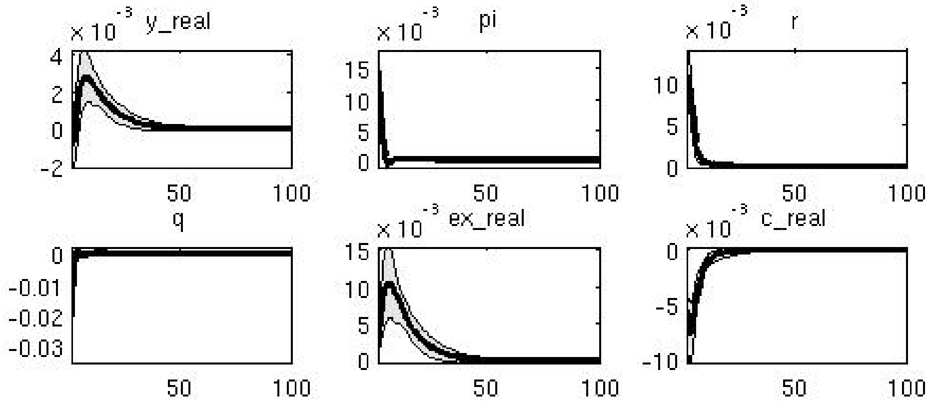

Figure 6.3: Response functions to a risk premium shock, when monetary policy 2 was used, equation 3.51.

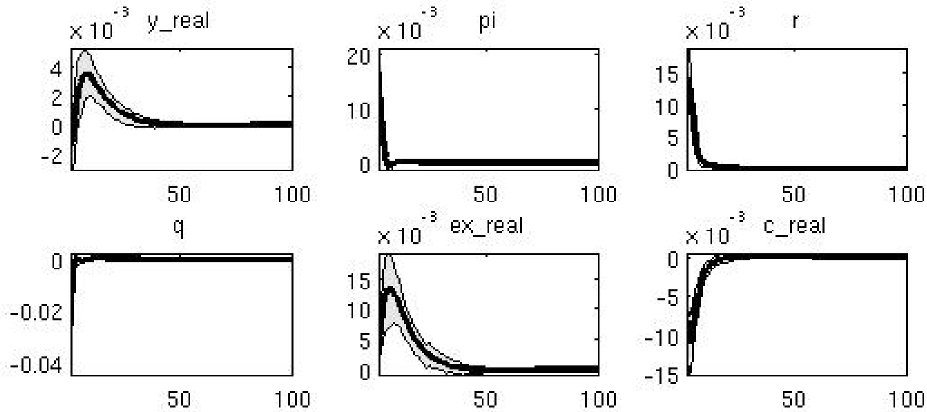

Figure 6.4: Response functions to a risk premium shock, when monetary policy 3 was used, equation 3.52.

Now we have estimated the model using various shocks. We have obtained an average response of the observables to all the shocks, for 100 periods. It is the variance of the average response that we are interested in. We have estimated the model using three different monetary policies and we want to know which monetary policy dampens the deviations of the variables from their steady states the most. We will look at the deviations and variances in the next chapter, where the results of the estimation are put forth and discussed.

7 Results

In the preceding chapter we estimated the model using Bayesian estimation. Shocks were applied to the model and the posterior distributions were estimated. From these posterior distributions parameters and shocks were drawn to simulate the model. The model was simulated and an average response was calculated. The response functions are displayed graphically in Appendix D and the results from the posterior estimation are in Appendix B and Appendix C.

We are interested in the deviations from the steady state. In chapter 3 we defined a loss function that we will use to compare the different monetary policies. The monetary policies are defined by equations 3.50, 3.51 and 3.52. The loss function, equation 3.53, was defined as follows:

$$LF = \sigma_\pi^2 + \frac{1}{2}\sigma_y^2 + \frac{1}{5}\sigma_r^2$$

The average response estimated in the preceding chapter gives us the time series for the observables of the model. This time-series gives us the deviations from steady state, over 100 periods. We can see from the figures in Appendix D that the variables have reached the steady state in most cases before the 100 periods are over, in most cases even well before. If we look at figures 6.2, 6.3 and 6.4 for example, we see the response when the risk premium experiences a shock. In Appendix B we can see the posteriors for the shocks. We see that the average risk premium shock is around 5% upward deviation from the steady state. In the figures we see that this leads to a depreciation of the currency, a quite big depreciation in the first periods. This then leads to an increase in exports. Private consumption decreases but it decreases less that than the increase in exports so we have a net increase in the GDP. The inflation rate goes up in the first periods and the interest rate as well, as we would have expected. The dynamics are similar for all three monetary policies but the difference is how much the variables deviate from the steady state. This is what we are interested in. The rest of the figures of the response functions are in Appendix D.

We calculate the variance of the time-series for the average response. We do this for the response in all the observables. For example when the risk premium shock is applied we have a response in six observables, like we saw in the three figures. We calculate the variance of the average response, which is the middle line in the response graphs. The results of these calculations are in Appendix E. We have eight observables and nine shocks, which gives a total of 72 response time-series, for each monetary policy. The variance of all these time series are displayed in the appendix. This gives us the variance of the deviations from the steady state. We can see that sometimes the variance is zero, this is because shocks to domestic variables do not affect the foreign economy.

Now we use the results for the variances in Appendix E and we calculate the *Welfare Loss* using equation 3.53. We use the variance in GDP, inflation and interest rate for each shock. The results are in tables 7.1 and 7.2.

	M.P. 1	M.P. 2	M.P. 3
Shock		Welfare loss	
ϵ_q	3.5706×10^{-6}	8.1687×10^{-6}	7.9939×10^{-6}
ϵ_a	14.837×10^{-6}	15.459×10^{-6}	15.782×10^{-6}
ϵ_ψ	3.1393×10^{-6}	3.0762×10^{-6}	3.8138×10^{-6}
ϵ_{π_F}	3.0721×10^{-6}	3.4076×10^{-6}	5.4307×10^{-6}
ϵ_f	12.320×10^{-6}	11.816×10^{-6}	8.0799×10^{-6}
Average:	7.3878×10^{-6}	8.3855×10^{-6}	8.2201×10^{-6}

Table 7.1: The welfare loss for the three different monetary policies, when shocks are applied to the domestic variables .

	M.P. 1	M.P. 2	M.P. 3
Shock		Welfare loss	
ϵ_{c^*}	0.3656×10^{-6}	0.3799×10^{-6}	0.3693×10^{-6}
ϵ_{r^*}	0.6733×10^{-6}	0.6658×10^{-6}	0.5286×10^{-6}
ϵ_{π^*}	0.2142×10^{-6}	0.2226×10^{-6}	0.1457×10^{-6}
ϵ_{prem}	7.4090×10^{-6}	3.6898×10^{-6}	4.7804×10^{-6}
Average:	2.1655×10^{-6}	1.2395×10^{-6}	1.4560×10^{-6}

Table 7.2: The welfare loss for the three different monetary policies, when shocks are applied to the foreign variables.

For the welfare loss when a shock is applied to the domestic variables we see that monetary policy 1 is almost always better than the other two. About 10% better on average. This is what we would have expected since exchange rate intervention is not expected to be used against shocks in domestic variables.

But in table 7.2, when we have shocks to the foreign variables we see that monetary polices 2 and 3 are always better in terms of welfare loss, except when there is a shock to the foreign consumption then all the monetary policies yield similar results.

8 Conclusion

When the results are examined we see that the monetary policies that respond to the real exchange rate fare better in terms of welfare loss. The monetary policies yield similar results when there is a shock to foreign private consumption. Monetary policy 3, that responds to both the level of the real exchange rate and the rate of change of the real exchange rate, yields the least welfare loss when shocks are applied to the foreign interest rate and to foreign inflation. The welfare loss of monetary policy three is 78% and 68% of the welfare loss of monetary policy 1 when responding to foreign inflation and interest rate shocks. We conclude that exchange rate intervention reduces welfare loss greatly in response to shocks in foreign interest rate and foreign inflation.

We see that a shock to the risk premium has the greatest effect, of the foreign shocks, on the welfare loss. This is in line with Gonzalez and Garcia [2010] where it was found that risk premium shocks explained most of the variability of the real exchange rate which has important reallocation effects in the short run and they concluded that more exchange rate intervention is necessary in order to reduce volatility produced by risk premium shocks.
Monetary policy 2 fares best against the risk premium shocks. The welfare loss observed using monetary policy 2 is only 50% of the welfare loss when monetary policy 1 is used, when faced with a positive risk premium shock. We conclude that exchange rate intervention reduces observed volatility greatly when the domestic economy faces risk premium shocks.

Net foreign debt is all the foreign debt of the economy minus the foreign assets, denominated in the domestic currency. When a positive shock is applied to this variable, so that net foreign debt increases, we can see in table 7.1 that the monetary policies that use exchange rate intervention fare a lot better in terms of welfare loss. The welfare loss of monetary policy 3 is only around 65% of the welfare loss of using monetary policy 1 against the same shock. We conclude that in order to reduce the welfare loss introduced to the economy through foreign assets and foreign debt volatility, exchange rate intervention is necessary.

Note that when the economy is hit by a real exchange rate shock, depreciation, the welfare loss is around 50% less using monetary policy 1 that the other ones. When we examine the posterior distributions of the shocks[1] we see that the mean of the real exchange rate shock is -0.0198 when monetary policy 1 is used but the mean is -0.0495 and -0.0547 for monetary policies 2 and 3, respectively. The welfare loss is greater for monetary policies 2 and 3 because the domestic inflation rises much more than when monetary policy 1 is used. We conclude that exchange rate intervention only increases the welfare loss when the economy experiences a depreciation shock because of the effect the intervention has on domestic inflation, in the short run.

The general conclusion is therefore that small open economies should use exchange rate intervention instead of letting its currency float freely since it is likely to reduce the welfare loss introduced by external shocks significantly.

Further Extensions

Further extensions could include adding a banking sector to the model. It would also be possible to add capital and investment. The governments budget constraint could also be added to account for government expenditures and government investment. Doing this we might get a look at how the money multiplier works in the economy, and the financial acceleration. This might help us understand the monetary policy better and the working of the economy in terms of interest rates and exchange rate interventions.

A housing sector could also be added along with a model of the financial market and the banking sector. Then it could be examined if the central bank should monitor asset price inflation along with consumer price inflation, and the effects of exchange rate intervention on asset prices and inflation.

[1] The numerical results for the posterior estimation are in Appendix B.

Appendix A

We set up the Lagrangian for the households optimizing problem, where we maximize utility and the households budget constraint is used as a maximizing constraint.

$$\mathcal{L} = \mathbb{E}_{t=0}\left\{\sum_{t=0}^{\infty} e^{-\beta t}\{U(C_t, H_t) - V(N_t)\}\right\} \tag{1}$$
$$+\sum_{t=0}^{\infty} \lambda_t \left[B_{t+1} + W_t N_t - R_t B_t - P_t C_t\right]$$

We begin by differentiating with respect to C_t:

$$\frac{\delta \mathcal{L}}{\delta C_t} = e^{-\beta t}\left((C_t - H_t)^{-\sigma}\right) - \lambda_t P_t \equiv 0 \tag{2}$$

Rewriting gives:

$$\left((C_t - H_t)^{-\sigma}\right) = \lambda_t P_t e^{\beta t} \tag{3}$$

Differentiating the Lagrangian with respect to N_t gives us:

$$\frac{\delta \mathcal{L}}{\delta N_t} = e^{-\beta t}\left(-N_t^{\phi}\right) + \lambda_t W_t \equiv 0 \tag{4}$$

Rewriting gives:

$$N_t^{\phi} = \lambda_t W_t e^{\beta t} \tag{5}$$

Solving equations 3 and 5 together gives us:

$$\frac{W_t}{P_t} = N_t^{\phi}(C_t - H_t)^{\sigma} \tag{6}$$

Which is the same as equation 3.21.

Now we differentiate the Lagrangian, equation 1, with respect to B_{t+1}:

$$\frac{\delta \mathcal{L}}{\delta B_{t+1}} = \lambda_t - R_{t+1}\lambda_{t+1} \equiv 0 \tag{7}$$

Rewriting gives us:

$$\lambda_{t+1} R_{t+1} = \lambda_t \tag{8}$$

Now we write equation 3 for the period $t + 1$:

$$\left((C_{t+1} - H_{t+1})^{-\sigma}\right) = \lambda_{t+1} P_{t+1} e^{\beta(t+1)} \tag{9}$$

Now we insert equations 3 and 9 into equation 8, eliminating λ_t and λ_{t+1}, we get:

$$\frac{((C_{t+1} - H_{t+1})^{-\sigma})}{P_{t+1} e^{\beta(t+1)}} R_{t+1} = \frac{((C_t - H_t)^{-\sigma})}{P_t e^{\beta t}} \tag{10}$$

We rewrite equation 10 using the fact that $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ and we get:

$$(C_t - H_t)^{-\sigma} e^{\beta} = \mathbb{E}_t \left\{ (C_{t+1} - H_{t+1})^{-\sigma} \frac{R_{t+1}}{\Pi_{t+1}} \right\} \tag{11}$$

Which is the same as equation 3.22.

Appendix B

This appendix holds the numerical results of the posterior estimation for the model. The numerical results for all the structural parameters and all the shocks are displayed, one set for each monetary policy. For a graphical display of the priors and posteriors see Appendix C.

Parameters	Prior mean	Post. mean	95% conf. interval		Prior	Prior S.D.
h	0.500	0.4716	0.3420	0.6011	beta	0.2000
θ	2.000	1.9618	1.8082	2.1121	gamm	0.1000
η	1.000	1.5822	1.1978	1.9563	gamm	0.3000
ϕ	1.000	1.4956	0.9235	2.0843	gamma	0.3000
θ_H	0.500	0.1177	0.0197	0.2021	beta	0.2500
θ_F	0.500	0.5203	0.4804	0.5608	beta	0.2500
α	0.400	0.8400	0.7705	0.9098	beta	0.2500
μ	0.100	0.0497	0.0261	0.0734	beta	0.2000
η^*	3.000	3.0657	2.5736	3.5586	gamma	0.3000
α_1	0.400	0.6360	0.5640	0.7097	gamma	0.1000
α_2	3.000	2.9526	2.7984	3.1139	gamma	0.1000
α_3	0.500	0.5526	0.4042	0.6967	gamma	0.1000
ρ_{π^*}	0.500	0.6076	0.5014	0.7211	beta	0.1000
ρ_q	0.500	0.6211	0.5185	0.7252	beta	0.1000
ρ_a	0.500	0.8756	0.8153	0.9392	beta	0.1000
ρ_{c^*}	0.500	0.5949	0.4499	0.7495	beta	0.1000
ρ_{r^*}	0.500	0.7665	0.7012	0.8330	beta	0.1000
ρ_{prem}	0.500	0.2298	0.1397	0.3140	beta	0.1000
ρ_ψ	0.500	0.8233	0.7586	0.8897	beta	0.1000
ρ_f	0.500	0.8181	0.7467	0.8939	beta	0.1000
ρ_{π_F}	0.500	0.8182	0.7648	0.8706	beta	0.1000

Table 1: Posterior results of the structural parameters using monetary policy 1, equation 3.50.

Shocks	Prior mean	Post. mean	95% conf. interval		Prior	Prior S.D.
ϵ_{π^*}	0.050	0.0078	0.0064	0.0091	norm	0.0200
ϵ_{prem}	0.050	0.0478	0.0340	0.0625	norm	0.0200
ϵ_q	-0.050	-0.0198	-0.0289	-0.0099	norm	0.0200
ϵ_{r^*}	0.050	0.0066	0.0054	0.0078	norm	0.0200
ϵ_{c^*}	-0.050	-0.0198	-0.0245	-0.0152	norm	0.0200
ϵ_ψ	0.010	0.0270	0.0206	0.0335	norm	0.0100
ϵ_a	0.010	0.0393	0.0307	0.0480	norm	0.0100
ϵ_{pi_F}	0.010	0.0299	0.0223	0.0378	norm	0.0100
ϵ_f	0.050	0.0937	0.0708	0.1168	norm	0.0200

Table 2: Posterior results of the shocks using monetary policy 1, equation 3.50.

Parameters	Prior mean	Post. mean	95% conf. interval		Prior	Prior S.D.
h	0.500	0.4607	0.3318	0.5884	beta	0.2000
θ	2.000	1.9604	1.7940	2.1186	gamma	0.1000
η	1.000	1.5958	1.2264	1.9802	gamma	0.3000
ϕ	1.000	1.4418	0.8552	2.0198	gamma	0.3000
θ_H	0.500	0.1241	0.0203	0.2148	beta	0.2500
θ_F	0.500	0.5167	0.4777	0.5576	beta	0.2500
α	0.400	0.8226	0.7398	0.9068	beta	0.2500
μ	0.200	0.0522	0.0250	0.0780	beta	0.2000
η^*	3.000	3.0564	2.5828	3.5392	gamma	0.3000
α_1	0.560	0.6481	0.5807	0.7192	gamma	0.1000
α_2	3.030	2.9834	2.8160	3.1458	gamma	0.1000
α_3	0.360	0.4562	0.2810	0.6170	gamma	0.1000
α_4	0.120	0.0241	0.0000	0.0493	gamma	0.1000
ρ_{π^*}	0.500	0.6122	0.5073	0.7191	beta	0.1000
ρ_q	0.500	0.2270	0.1396	0.3145	beta	0.1000
ρ_a	0.500	0.8811	0.8249	0.9467	beta	0.1000
ρ_{c^*}	0.500	0.6051	0.4622	0.7505	beta	0.1000
ρ_{r^*}	0.500	0.7632	0.6991	0.8312	beta	0.1000
ρ_{prem}	0.500	0.6139	0.5107	0.7145	beta	0.1000
ρ_ψ	0.500	0.8314	0.7668	0.8971	beta	0.1000
ρ_f	0.500	0.8173	0.7460	0.8992	beta	0.1000
ρ_{π_F}	0.500	0.8168	0.7661	0.8701	beta	0.1000

Table 3: Posterior results of the structural parameters using monetary policy 2, equation 3.51.

Shocks	Prior mean	Post. mean	95% conf. interval		Prior	Prior S.D.
ϵ_{π^*}	0.050	0.0078	0.0065	0.0092	norm	0.0200
ϵ_{prem}	0.050	0.0199	0.0107	0.0288	norm	0.0200
ϵ_q	-0.050	-0.0495	-0.0634	-0.0346	norm	0.0200
ϵ_{r^*}	0.050	0.0066	0.0054	0.0077	norm	0.0200
ϵ_{c^*}	-0.050	-0.0201	-0.0249	-0.0153	norm	0.0200
ϵ_ψ	0.010	0.0268	0.0203	0.0333	norm	0.0100
ϵ_a	0.010	0.0394	0.0307	0.0484	norm	0.0100
ϵ_{π_F}	0.010	0.0302	0.0231	0.0371	norm	0.0100
ϵ_f	0.050	0.0922	0.0690	0.1150	norm	0.0200

Table 4: Posterior results of the shocks using monetary policy 2, equation 3.51.

Parameters	Prior mean	Post. mean	95% conf. interval		Prior	Prior S.D.
h	0.500	0.4716	0.3380	0.6056	beta	0.2000
θ	2.000	1.9312	1.7684	2.0932	gamma	0.1000
η	1.000	1.5781	1.2072	1.9563	gamma	0.3000
ϕ	1.000	1.5067	0.9295	2.0800	gamma	0.3000
θ_H	0.500	0.0997	0.0123	0.1779	beta	0.2500
θ_F	0.500	0.5320	0.4968	0.5673	beta	0.2500
α	0.400	0.8185	0.7490	0.8897	beta	0.2500
μ	0.100	0.0450	0.0234	0.0656	beta	0.2000
η^*	3.000	3.0920	2.5980	3.5795	gamma	0.3000
α_1	0.500	0.6421	0.5741	0.7098	gamma	0.1000
α_2	2.750	2.7629	2.5986	2.9243	gamma	0.1000
α_3	0.300	0.3424	0.1844	0.5004	gamma	0.1000
α_4	0.400	0.1953	0.1256	0.2624	gamma	0.1000
α_5	0.500	0.3347	0.2551	0.4165	gamma	0.1000
ρ_{π^*}	0.500	0.5887	0.4825	0.6926	beta	0.1000
ρ_q	0.500	0.2659	0.1776	0.3574	beta	0.1000
ρ_a	0.500	0.8781	0.8170	0.9431	beta	0.1000
ρ_{c^*}	0.500	0.6142	0.4729	0.7642	beta	0.1000
ρ_{r^*}	0.500	0.7669	0.7028	0.8340	beta	0.1000
ρ_{prem}	0.500	0.6463	0.5536	0.7465	beta	0.1000
ρ_ψ	0.500	0.8302	0.7684	0.8917	beta	0.1000
ρ_f	0.500	0.7816	0.6933	0.8800	beta	0.1000
ρ_{π_F}	0.500	0.8368	0.7859	0.8882	beta	0.1000

Table 5: Posterior results of the structural parameters using monetary policy 3, equation 3.52.

Shocks	Prior mean	Post. mean	95% conf. interval		Prior	Prior S.D.
ϵ_{π^*}	0.050	0.0078	0.0065	0.0091	norm	0.0200
ϵ_{prem}	0.050	0.0244	0.0133	0.0345	norm	0.0200
ϵ_q	-0.050	-0.0547	-0.0702	-0.0391	norm	0.0200
ϵ_{r^*}	0.050	-0.0064	-0.0075	-0.0054	norm	0.0200
ϵ_{c^*}	-0.050	-0.0192	-0.0233	-0.0150	norm	0.0200
ϵ_ψ	0.010	0.0278	0.0214	0.0341	norm	0.0100
ϵ_a	0.010	0.0396	0.0308	0.0480	norm	0.0100
ϵ_{π_F}	0.010	0.0253	0.0196	0.0308	norm	0.0100
ϵ_f	0.050	0.0914	0.0671	0.1159	norm	0.0200

Table 6: Posterior results of the shocks using monetary policy 3, equation 3.52.

Appendix C

This appendix shows graphical results for all the priors and posteriors for all the shocks and structural parameters, when estimated with the three different monetary policies. The gray line is the prior, the darker line is the posterior and the dotted line is the posterior mean.

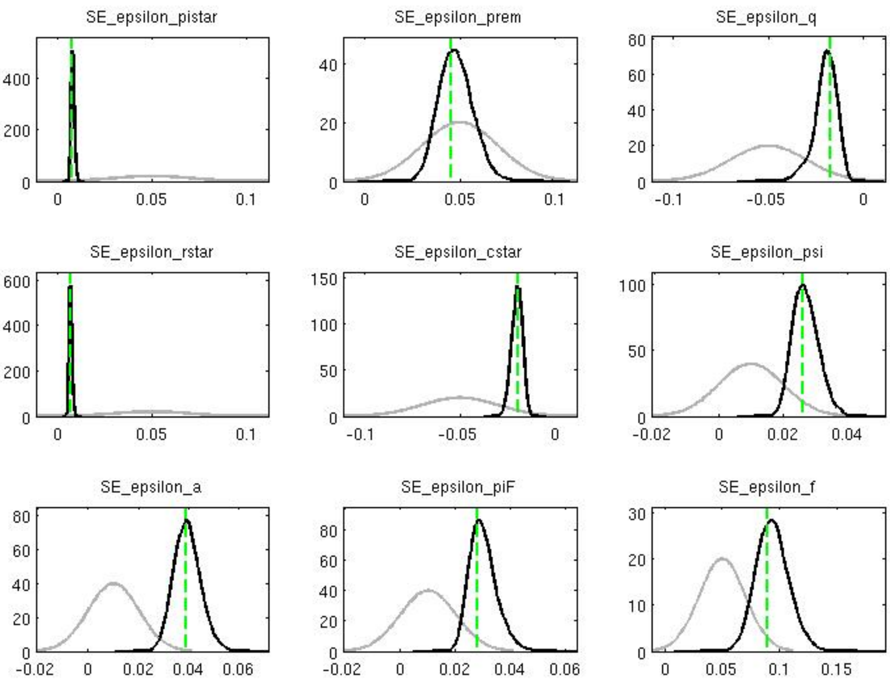

Figure 1: Priors and posteriors for all the shocks, when monetary policy 1 was used, equation 3.50.

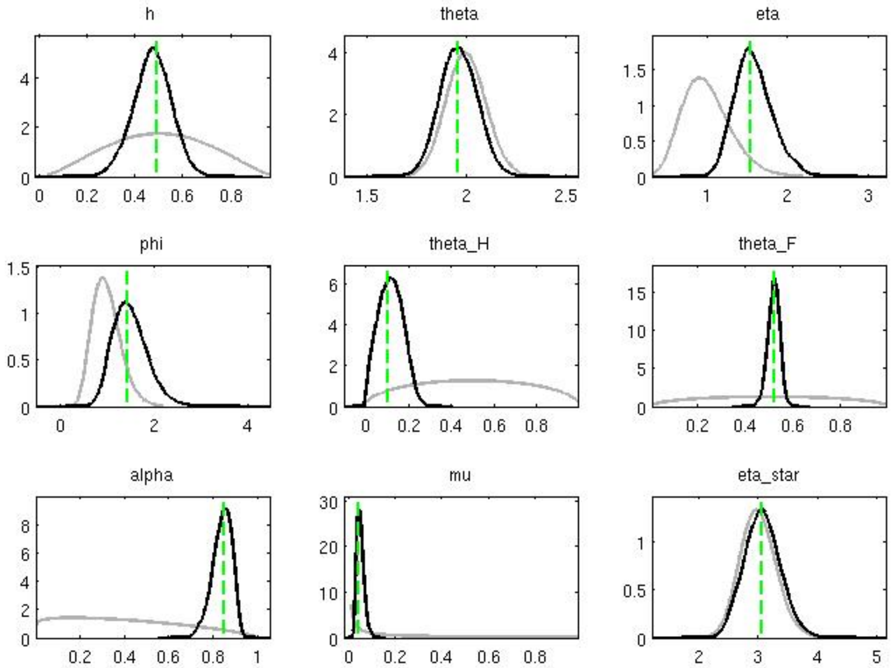

Figure 2: Priors and posteriors for the structural parameters, when monetary policy 1 was used, equation 3.50. Figure 1/3.

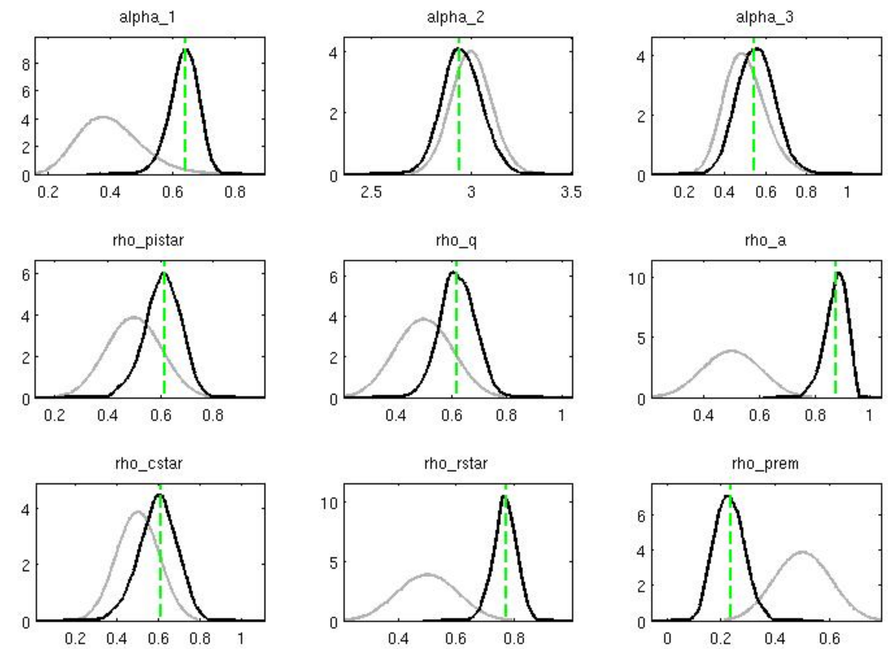

Figure 3: Priors and posteriors for the structural parameters, when monetary policy 1 was used, equation 3.50. Figure 2/3.

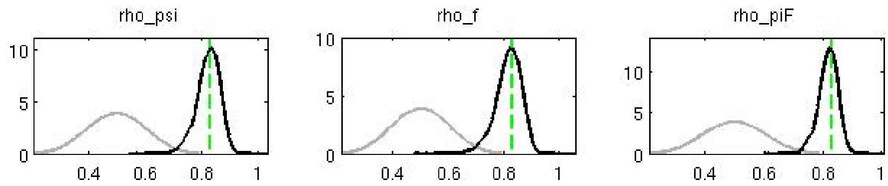

Figure 4: Priors and posteriors for the structural parameters, when monetary policy 1 was used, equation 3.50. Figure 3/3.

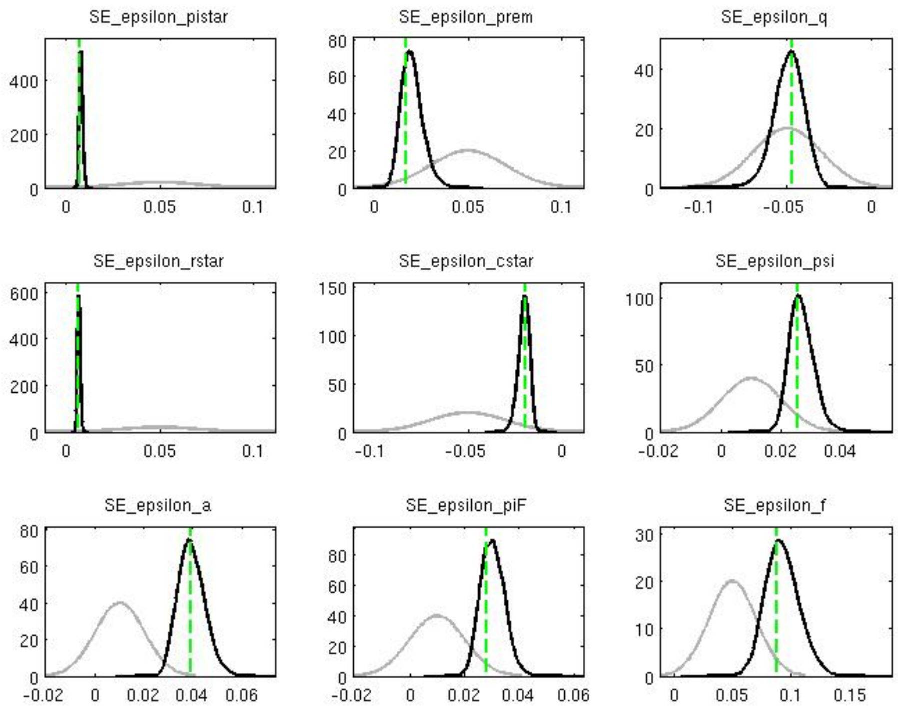

Figure 5: Priors and posteriors for all the shocks, when monetary policy 2 was used, equation 3.51.

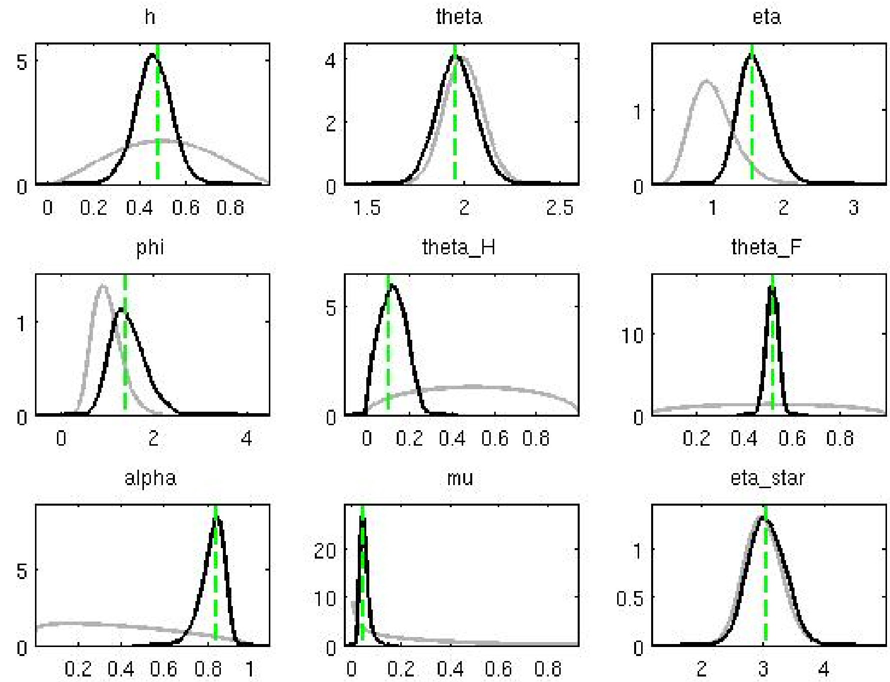

Figure 6: Priors and posteriors for the structural parameters, when monetary policy 2 was used, equation 3.51. Figure 1/3.

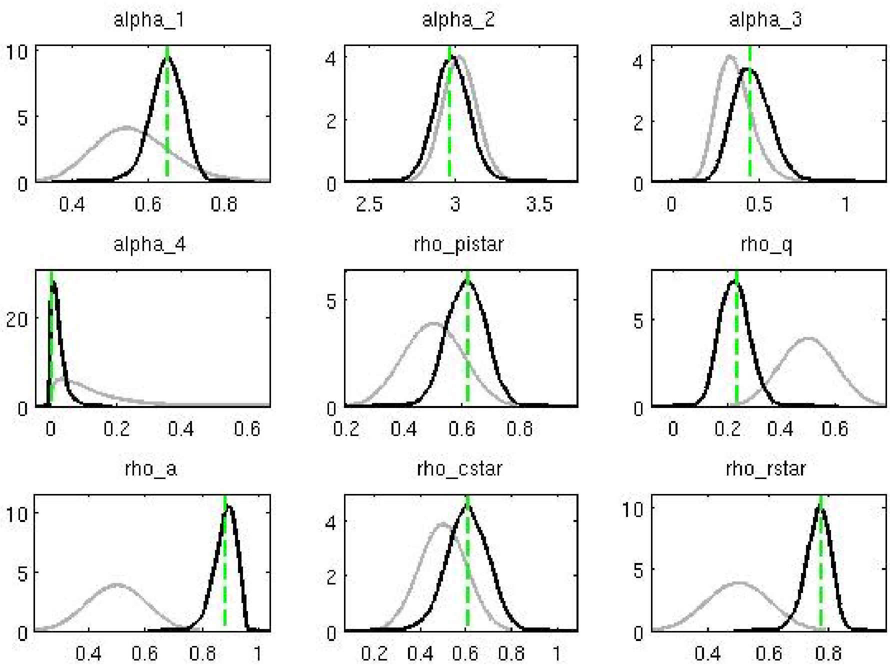

Figure 7: Priors and posteriors for the structural parameters, when monetary policy 2 was used, equation 3.51. Figure 2/3.

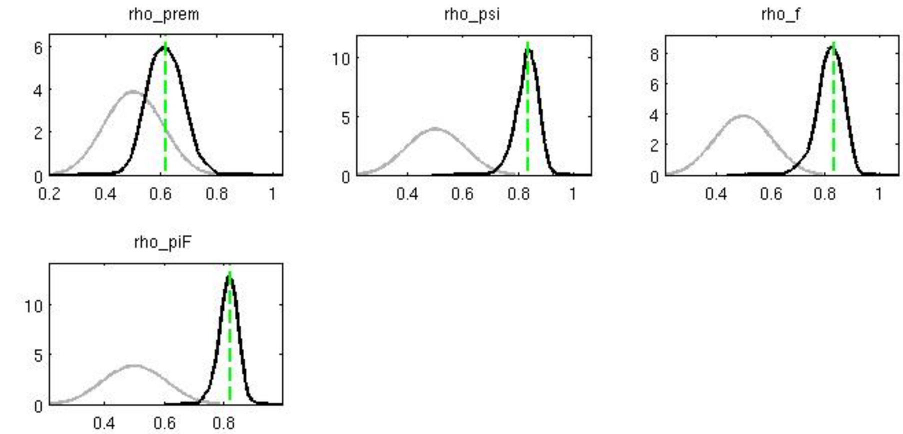

Figure 8: Priors and posteriors for the structural parameters, when monetary policy 2 was used, equation 3.51. Figure 3/3.

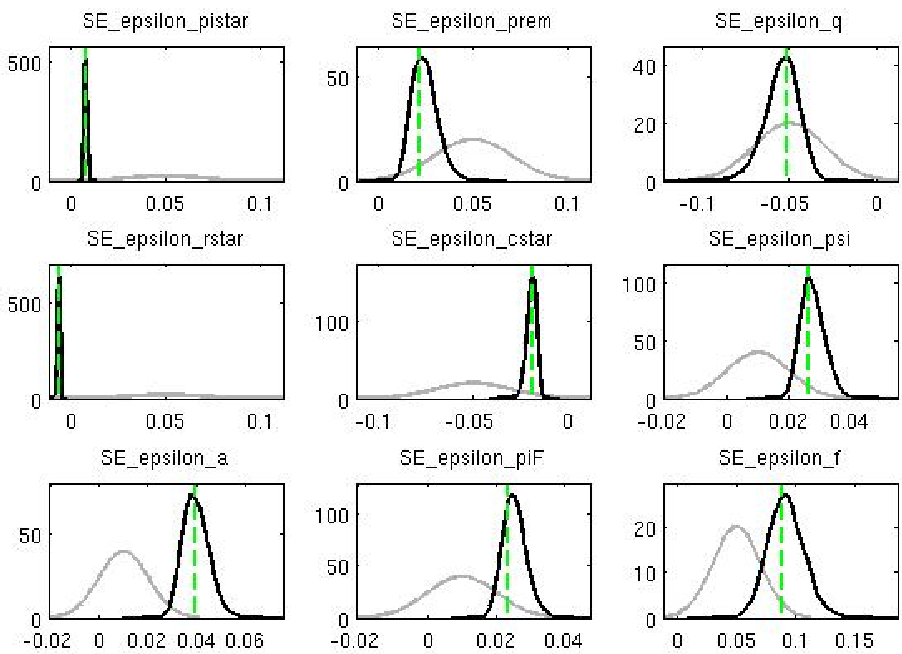

Figure 9: Priors and posteriors for all the shocks, when monetary policy 3 was used, equation 3.52.

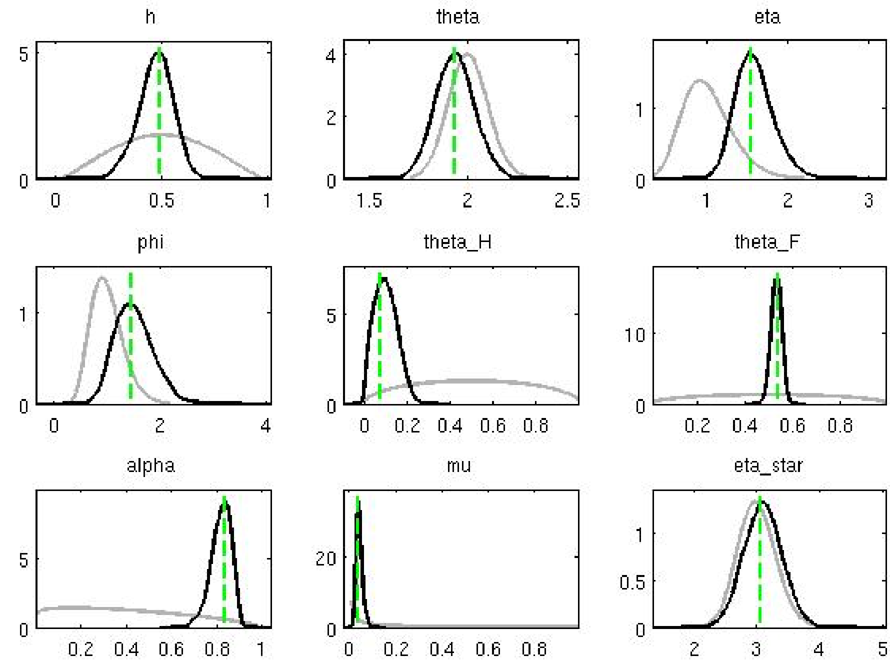

Figure 10: Priors and posteriors for the structural parameters, when monetary policy 3 was used, equation 3.52. Figure 1/3.

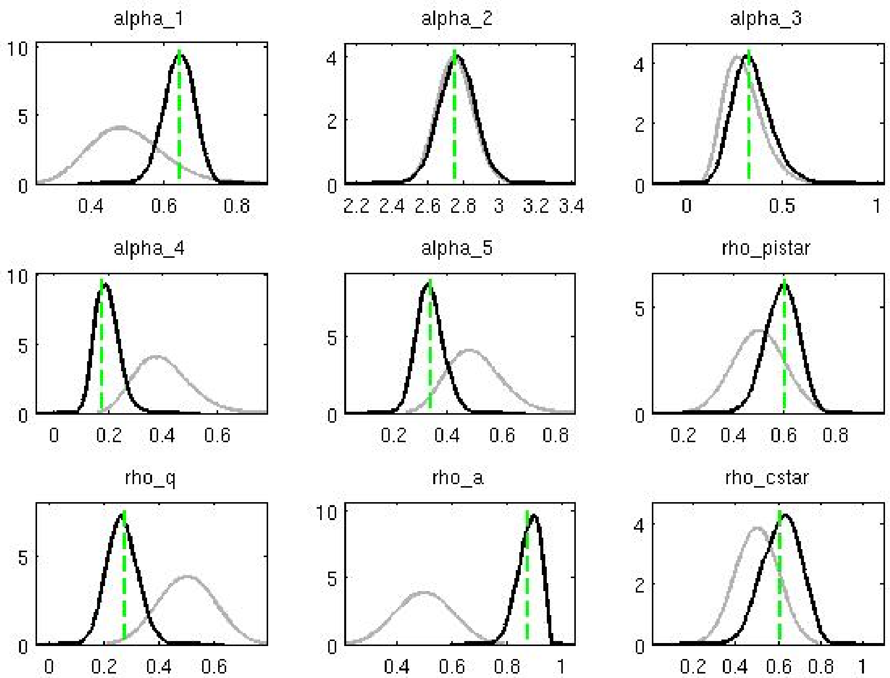

Figure 11: Priors and posteriors for the structural parameters, when monetary policy 3 was used, equation 3.52. Figure 2/3.

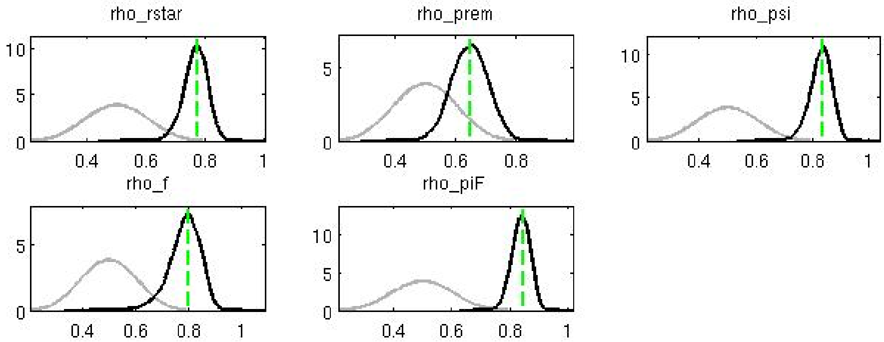

Figure 12: Priors and posteriors for the structural parameters, when monetary policy 3 was used, equation 3.52. Figure 3/3.

Appendix D

This appendix shows all the response function graphs. There are 9 shocks and they are all applied to the three different monetary policies, which gives a total of 27 graphs. Each graph displaying six to seven response functions. The shocks are drawn from their posterior distributions, see Appendix B.

Monetary policy 1

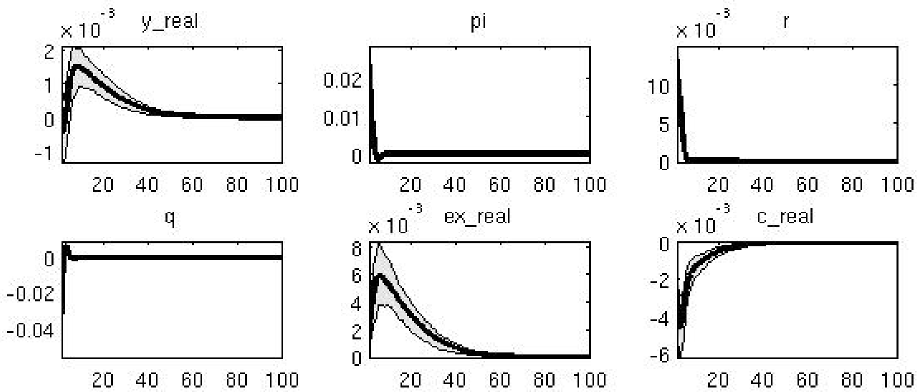

Figure 13: Response functions when a shock, ϵ_{prem}, is applied to the risk premium, $prem_t$. Monetary policy 1 is used here, equation 3.50. Figure 1/9.

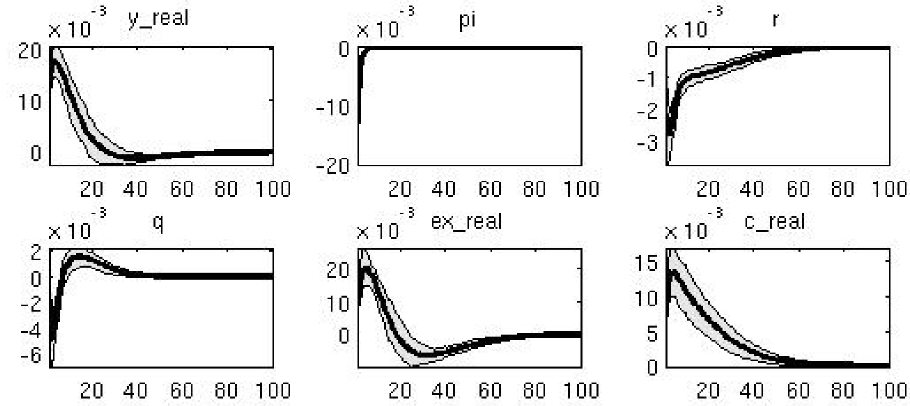

Figure 14: Response functions when a shock, ϵ_a, is applied to the labor productivity, a_t. Monetary policy 1 is used here, equation 3.50. Figure 2/9.

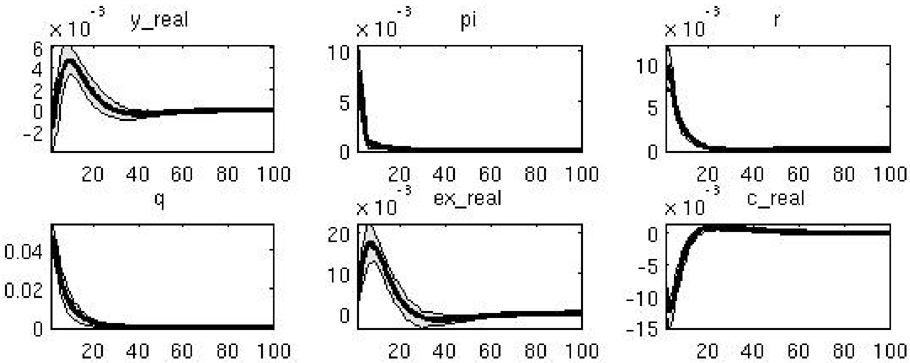

Figure 15: Response functions when a shock, ϵ_{π_F}, is applied to the inflation in imported goods, $\pi_{F,t}$. Monetary policy 1 is used here, equation 3.50. Figure 3/9.

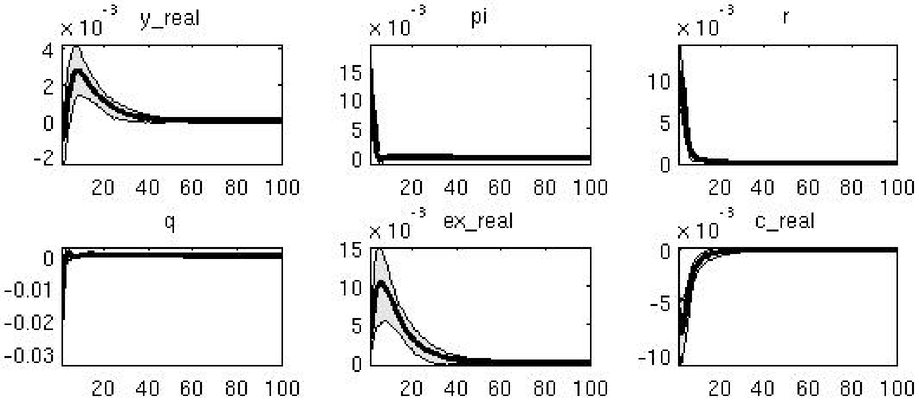

Figure 16: Response functions when a shock, ϵ_q, is applied to the real exchange rate, q_t. Monetary policy 1 is used here, equation 3.50. Figure 4/9.

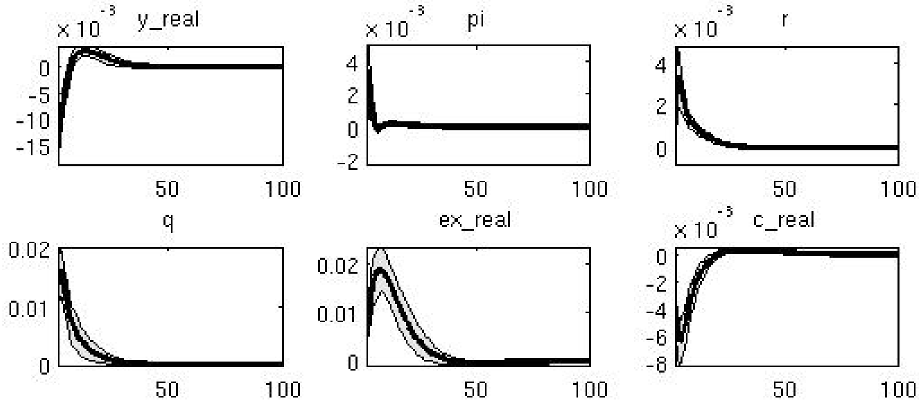

Figure 17: Response functions when a shock, ϵ_ψ, is applied to the *law of one price gap*, ψ_t. Monetary policy 1 is used here, equation 3.50. Figure 5/9.

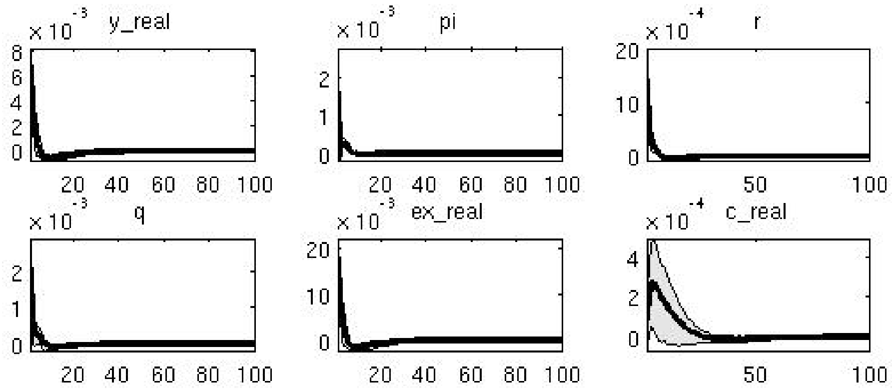

Figure 18: Response functions when a shock, ϵ_{c^*}, is applied to the foreign private consumption, c_t^*. Monetary policy 1 is used here, equation 3.50. Figure 6/9.

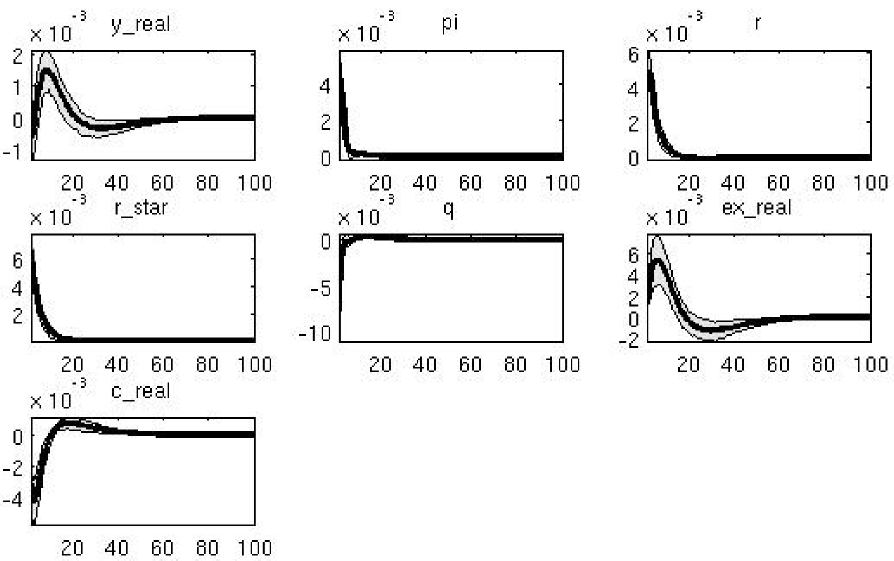

Figure 19: Response functions when a shock, ϵ_{r^*}, is applied to the foreign interest rate, r_t^*. Monetary policy 1 is used here, equation 3.50. Figure 7/9.

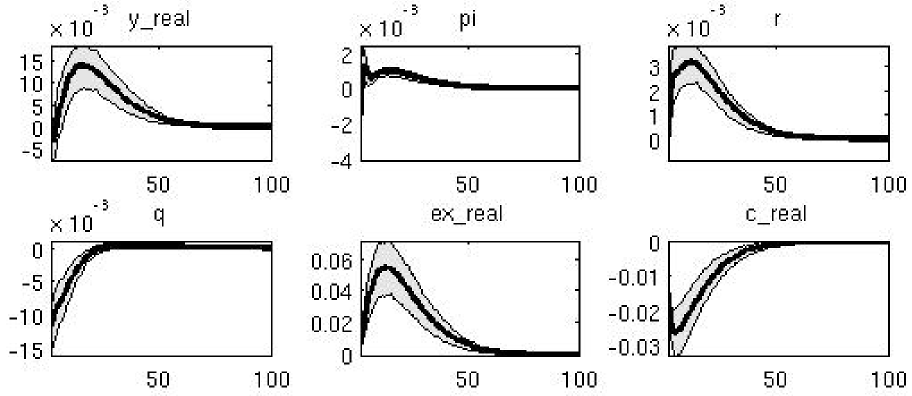

Figure 20: Response functions when a shock, ϵ_f, is applied to the net foreign debt, f_t. Monetary policy 1 is used here, equation 3.50. Figure 8/9.

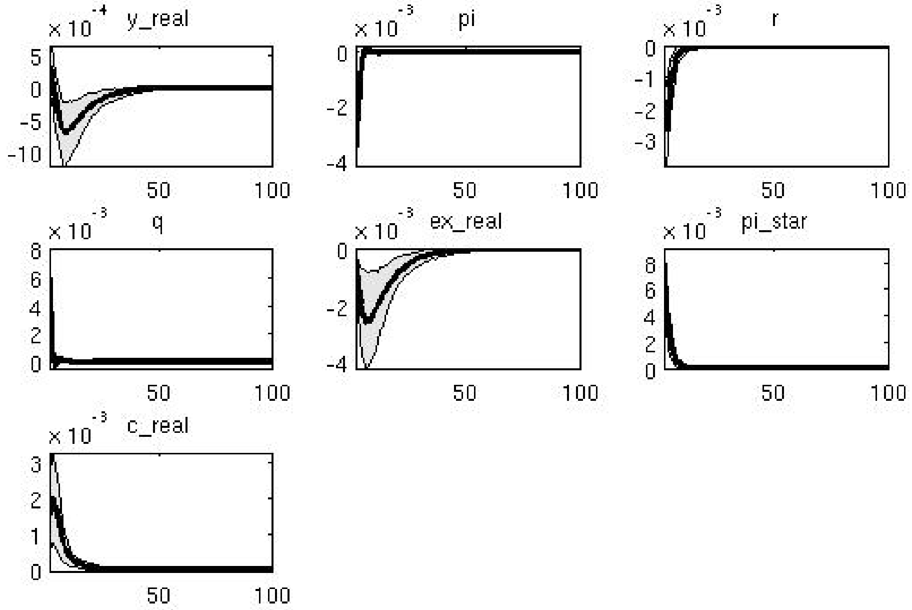

Figure 21: Response functions when a shock, ϵ_{π^*}, is applied to the foreign inflation, π^*_t. Monetary policy 1 is used here, equation 3.50. Figure 9/9.

Monetary policy 2

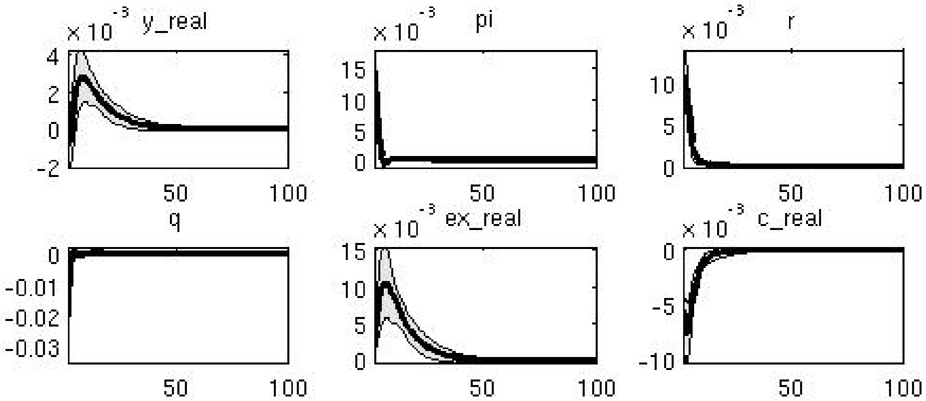

Figure 22: Response functions when a shock, ϵ_{prem}, is applied to the risk premium, $prem_t$. Monetary policy 2 is used here, equation 3.51. Figure 1/9.

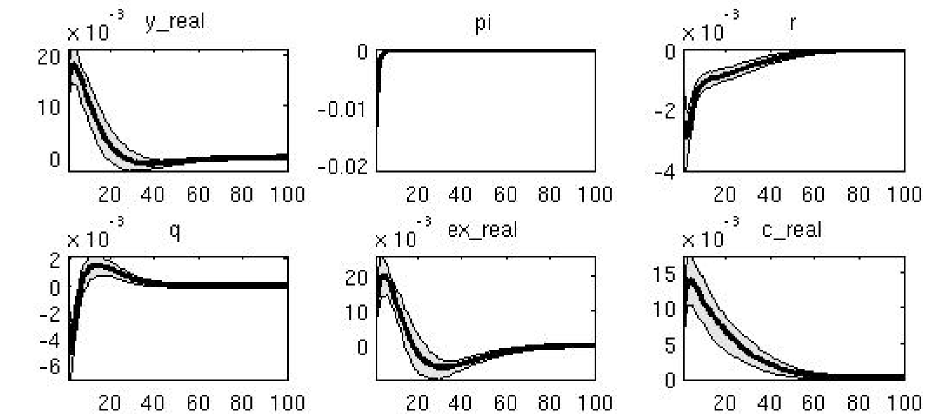

Figure 23: Response functions when a shock, ϵ_a, is applied to the labor productivity, a_t. Monetary policy 2 is used here, equation 3.51. Figure 2/9.

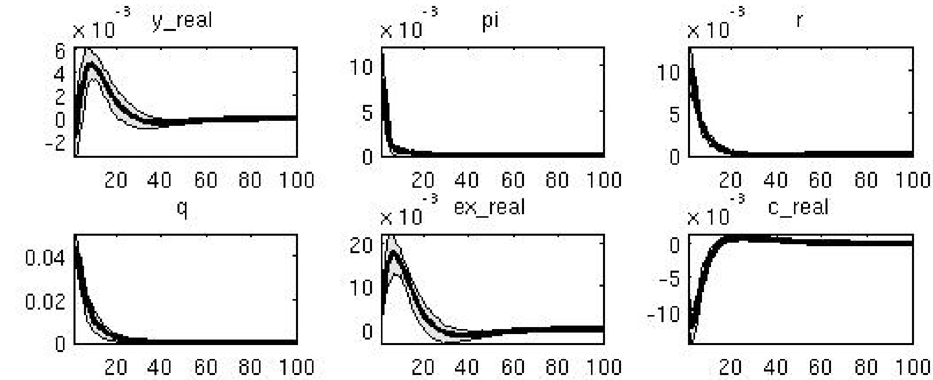

Figure 24: Response functions when a shock, ϵ_{π_F}, is applied to the inflation in imported goods, $\pi_{F,t}$. Monetary policy 2 is used here, equation 3.51. Figure 3/9.

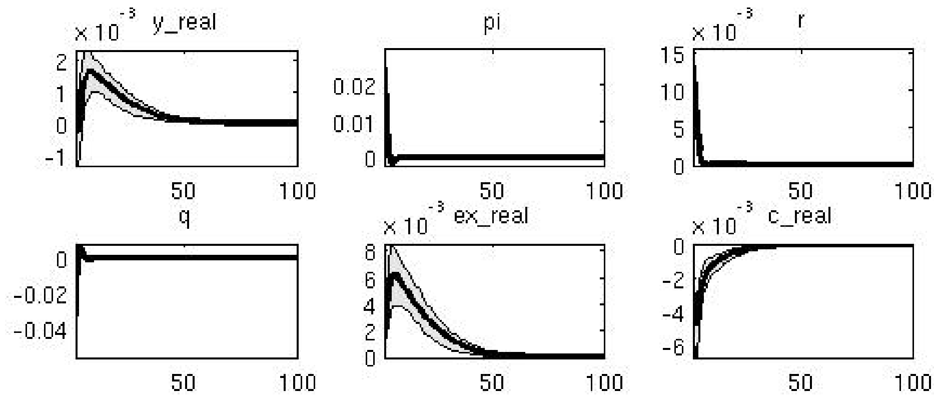

Figure 25: Response functions when a shock, ϵ_q, is applied to the real exchange rate, q_t. Monetary policy 2 is used here, equation 3.51. Figure 4/9.

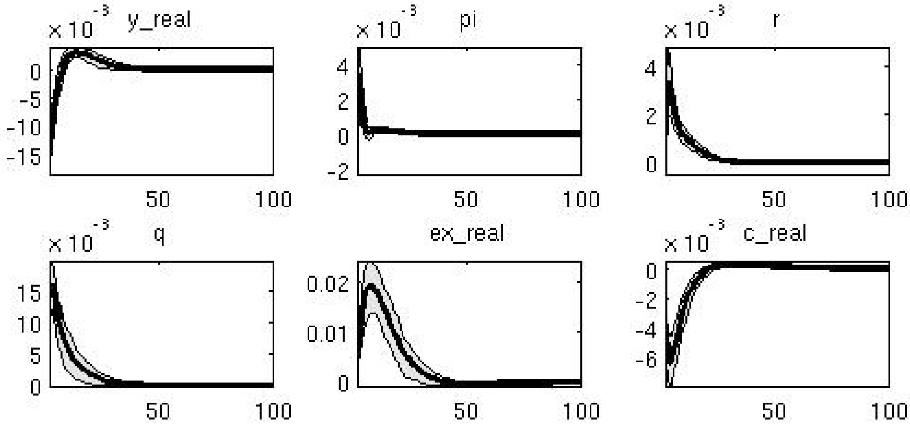

Figure 26: Response functions when a shock, ϵ_{ψ}, is applied to the *law of one price gap*, ψ_t. Monetary policy 2 is used here, equation 3.51. Figure 5/9.

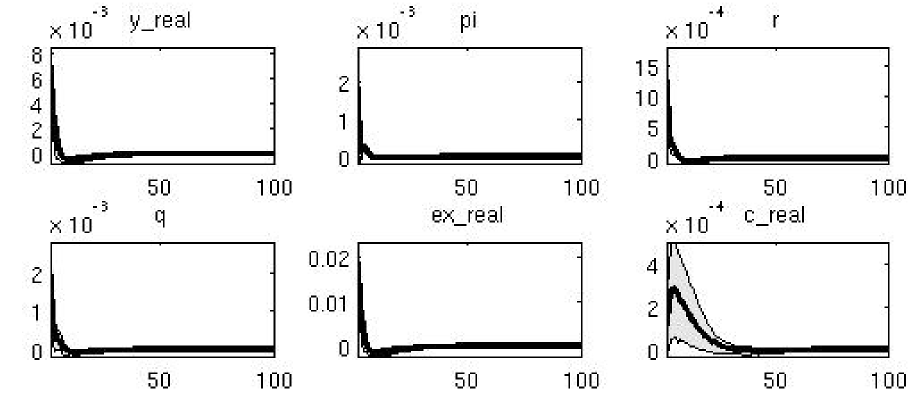

Figure 27: Response functions when a shock, ϵ_{c^*}, is applied to the foreign private consumption, c^*_t. Monetary policy 2 is used here, equation 3.51. Figure 6/9.

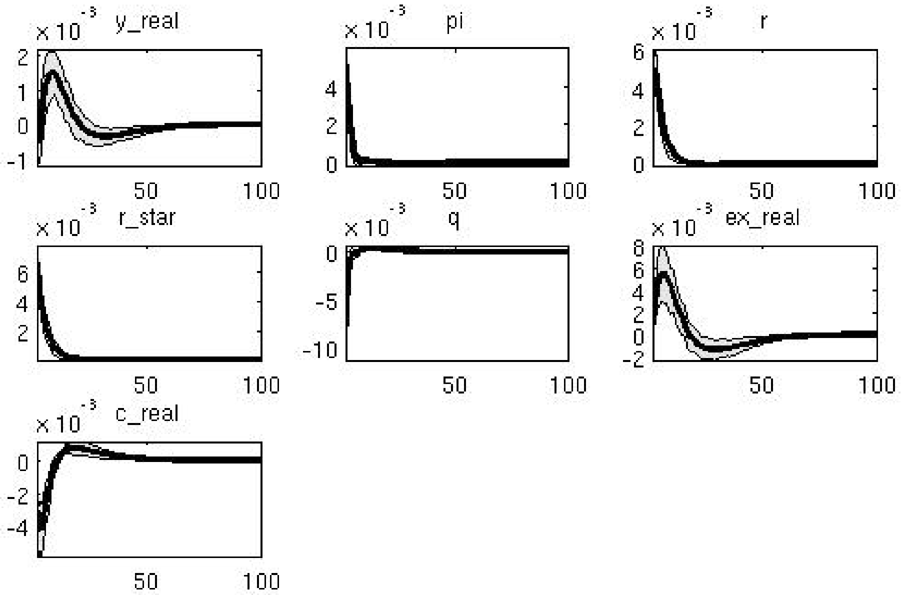

Figure 28: Response functions when a shock, ϵ_{r^*}, is applied to the foreign interest rate, r_t^*. Monetary policy 2 is used here, equation 3.51. Figure 7/9.

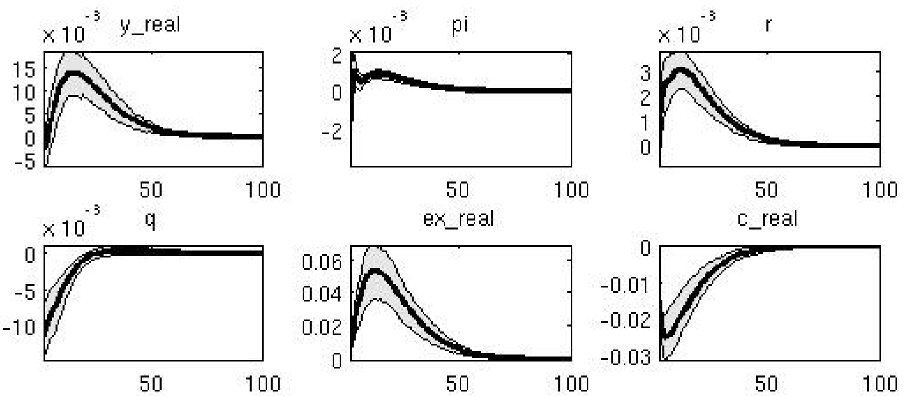

Figure 29: Response functions when a shock, ϵ_f, is applied to the net foreign debt, f_t. Monetary policy 2 is used here, equation 3.51. Figure 8/9.

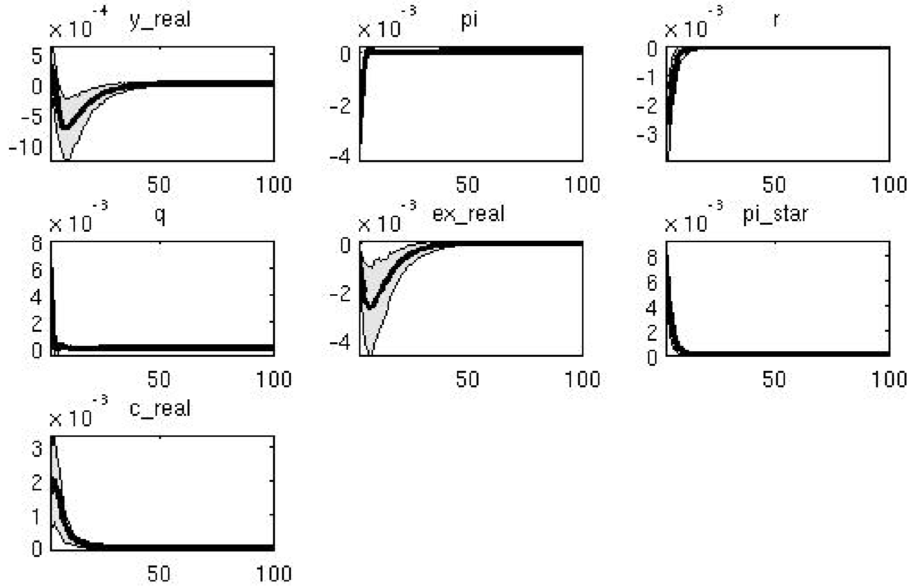

Figure 30: Response functions when a shock, ϵ_{π^*}, is applied to the foreign inflation, π_t^*. Monetary policy 2 is used here, equation 3.51. Figure 9/9.

Monetary policy 3

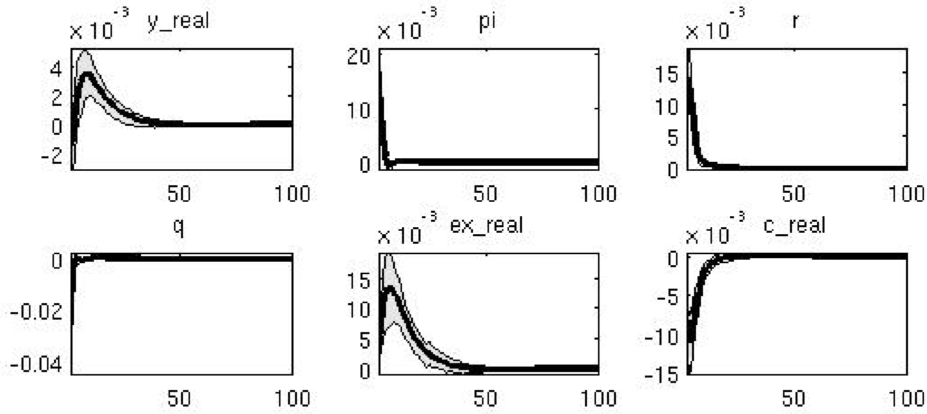

Figure 31: Response functions when a shock, ϵ_{prem}, is applied to the risk premium, $prem_t$. Monetary policy 3 is used here, equation 3.52. Figure 1/9.

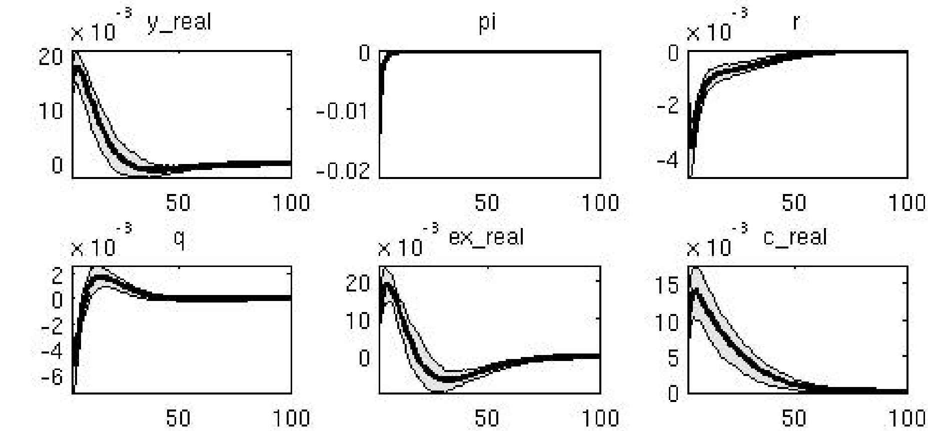

Figure 32: Response functions when a shock, ϵ_a, is applied to the labor productivity, a_t. Monetary policy 3 is used here, equation 3.52. Figure 2/9.

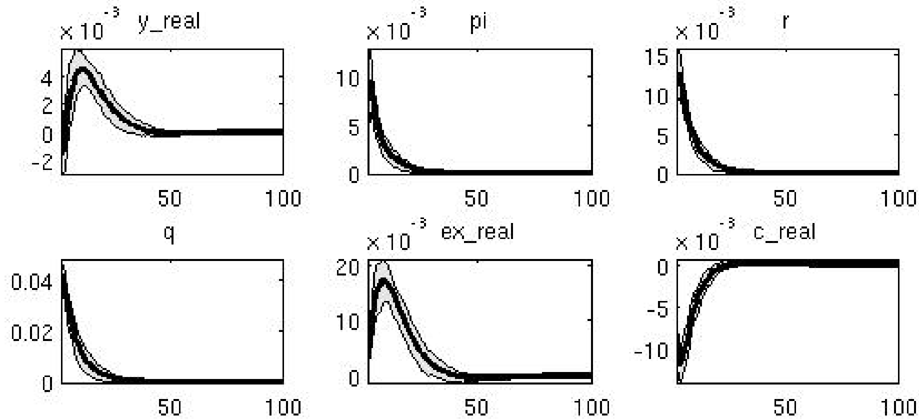

Figure 33: Response functions when a shock, ϵ_{π_F}, is applied to the inflation in imported goods, $\pi_{F,t}$. Monetary policy 3 is used here, equation 3.52. Figure 3/9.

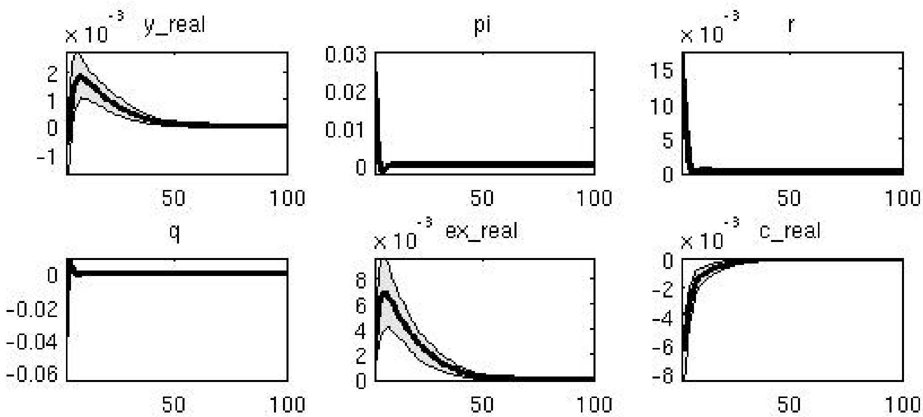

Figure 34: Response functions when a shock, ϵ_q, is applied to the real exchange rate, q_t. Monetary policy 3 is used here, equation 3.52. Figure 4/9.

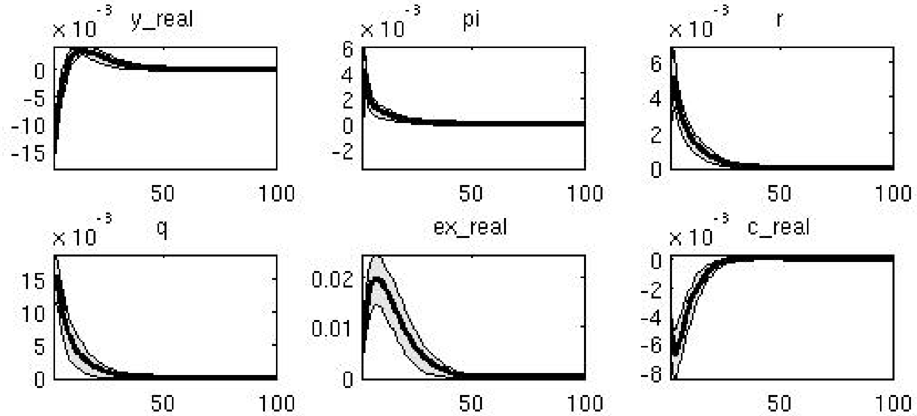

Figure 35: Response functions when a shock, ϵ_{ψ}, is applied to the *law of one price gap*, ψ_t. Monetary policy 3 is used here, equation 3.52. Figure 5/9.

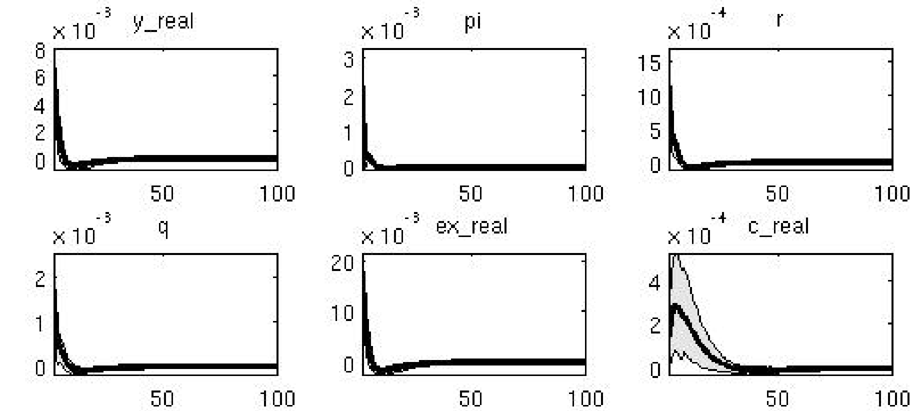

Figure 36: Response functions when a shock, ϵ_{c^*}, is applied to the foreign private consumption, c_t^*. Monetary policy 3 is used here, equation 3.52. Figure 6/9.

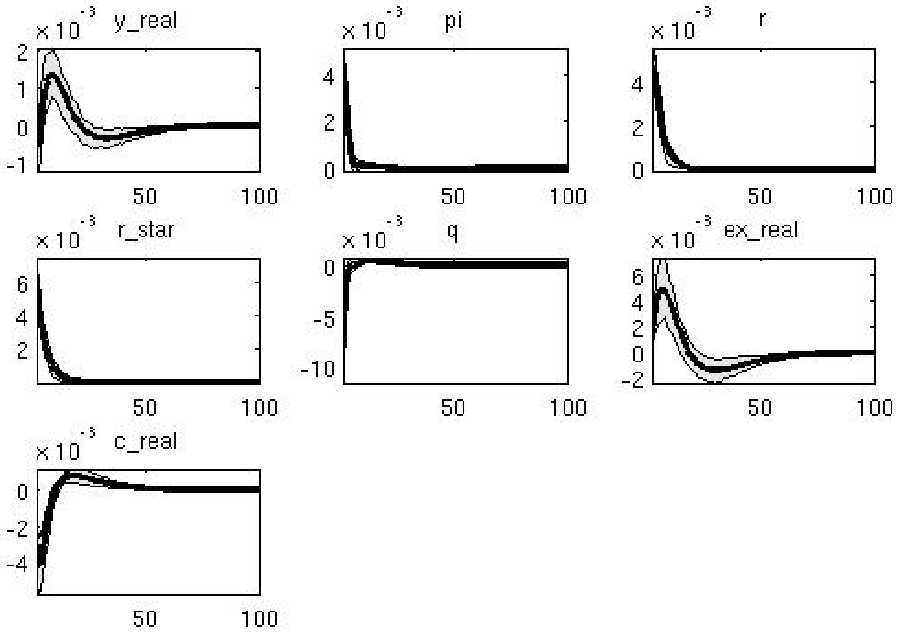

Figure 37: Response functions when a shock, ϵ_{r^*}, is applied to the foreign interest rate, r_t^*. Monetary policy 3 is used here, equation 3.52. Figure 7/9.

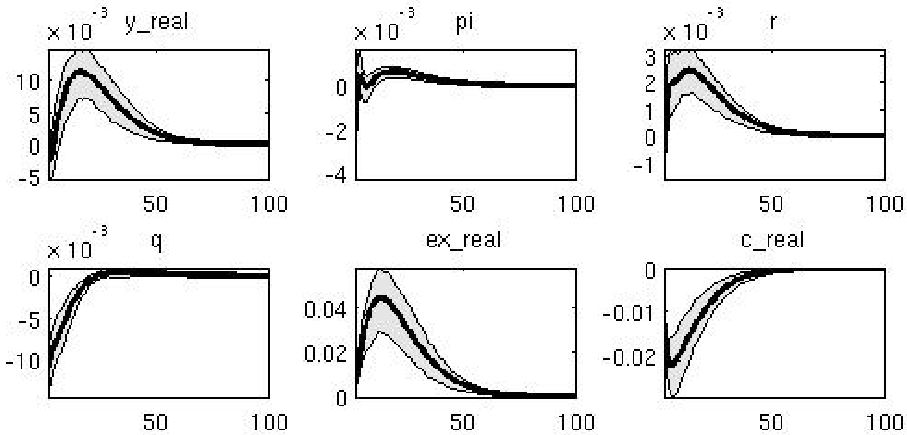

Figure 38: Response functions when a shock, ϵ_f, is applied to the net foreign debt, f_t. Monetary policy 3 is used here, equation 3.52. Figure 8/9.

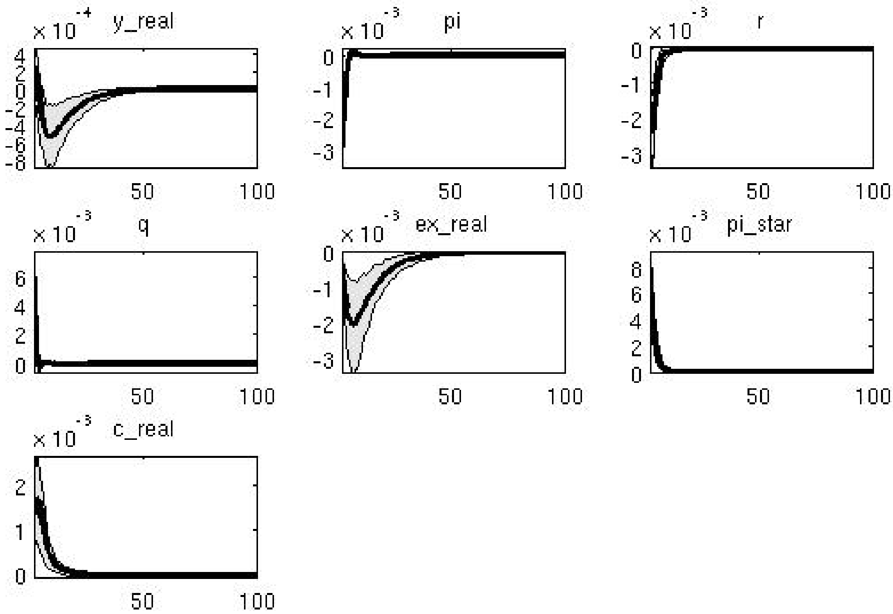

Figure 39: Response functions when a shock, ϵ_{π^*}, is applied to the foreign inflation, π_t^*. Monetary policy 3 is used here, equation 3.52. Figure 9/9.

Appendix E

In this appendix we have the variance of the time series of the observables, for the 100 period simulated response that was calculated in chapter 6. This is the variance of the deviation from the steady state, and can be found in tables 7, 8 and 9.

Parameter	Shock	Variance	Parameter	Shock	Variance
y	ϵ_q	6.2733e-07	y	ϵ_{prem}	2.2284e-07
π	ϵ_q	2.6305e-06	π	ϵ_{prem}	6.7846e-06
r	ϵ_q	3.1325e-06	r	ϵ_{prem}	2.5644e-06
r^*	ϵ_q	0	r^*	ϵ_{prem}	0
q	ϵ_q	6.2390e-06	q	ϵ_{prem}	2.0353e-05
c_H^*	ϵ_q	8.3614e-06	c_H^*	ϵ_{prem}	3.1642e-06
π^*	ϵ_q	0	π^*	ϵ_{prem}	0
c	ϵ_q	2.4283e-06	c	ϵ_{prem}	7.9041e-07
y	ϵ_a	2.3958e-05	y	ϵ_ψ	5.7971e-06
π	ϵ_a	2.7844e-06	π	ϵ_ψ	1.6463e-07
r	ϵ_a	3.6569e-07	r	ϵ_ψ	3.8080e-07
r^*	ϵ_a	0	r^*	ϵ_ψ	0
q	ϵ_a	9.4569e-07	q	ϵ_ψ	1.0847e-05
c_H^*	ϵ_a	4.0604e-05	c_H^*	ϵ_ψ	3.1924e-05
π^*	ϵ_a	0	π^*	ϵ_ψ	0
c	ϵ_a	1.5928e-05	c	ϵ_ψ	2.1841e-06
y	ϵ_{c^*}	6.6656e-07	y	ϵ_f	2.3826e-05
π	ϵ_{c^*}	2.7379e-08	π	ϵ_f	1.6101e-07
r	ϵ_{c^*}	2.4808e-08	r	ϵ_f	1.2290e-06
r^*	ϵ_{c^*}	0	r^*	ϵ_f	0
q	ϵ_{c^*}	5.2257e-08	q	ϵ_f	6.8154e-06
c_H^*	ϵ_{c^*}	4.7649e-06	c_H^*	ϵ_f	3.5099e-04
π^*	ϵ_{c^*}	0	π^*	ϵ_f	0
c	ϵ_{c^*}	4.4833e-09	c	ϵ_f	6.1904e-05
y	ϵ_{r^*}	1.9564e-07	y	ϵ_{π_F}	2.0017e-06
π	ϵ_{r^*}	4.1486e-07	π	ϵ_{π_F}	1.3822e-06
r	ϵ_{r^*}	8.0346e-07	r	ϵ_{π_F}	3.4452e-06
r^*	ϵ_{r^*}	9.9893e-07	r^*	ϵ_{π_F}	0
q	ϵ_{r^*}	9.9399e-07	q	ϵ_{π_F}	8.1554e-05
c_H^*	ϵ_{r^*}	2.5090e-06	c_H^*	ϵ_{π_F}	2.6873e-05
π^*	ϵ_{r^*}	0	π^*	ϵ_{π_F}	0
c	ϵ_{r^*}	8.1292e-07	c	ϵ_{π_F}	7.4681e-06
y	ϵ_{π^*}	3.9139e-08			
π	ϵ_{π^*}	1.5602e-07			
r	ϵ_{π^*}	1.9322e-07			
r^*	ϵ_{π^*}	0			
q	ϵ_{π^*}	3.7201e-07			
c_H^*	ϵ_{π^*}	5.1776e-07			
π^*	ϵ_{π^*}	9.3160e-07			
c	ϵ_{π^*}	1.5442e-07			

Table 7: The variance of the average response functions. The first column shows the observable, the second shows the shock that was applied and the third shows the variance of the response of the observable to the shock. Here monetary policy 1 was used, equation 3.50.

Parameter	Shock	Variance	Parameter	Shock	Variance
y	ϵ_q	2.3827e-07	y	ϵ_{prem}	6.1490e-07
π	ϵ_q	7.5055e-06	π	ϵ_{prem}	2.7611e-06
r	ϵ_q	2.7207e-06	r	ϵ_{prem}	3.1063e-06
r^*	ϵ_q	0	r^*	ϵ_{prem}	0
q	ϵ_q	2.1718e-05	q	ϵ_{prem}	6.5495e-06
c_H^*	ϵ_q	3.3870e-06	c_H^*	ϵ_{prem}	8.2621e-06
π^*	ϵ_q	0	π^*	ϵ_{prem}	0
c	ϵ_q	8.2653e-07	c	ϵ_{prem}	2.3194e-06
y	ϵ_a	2.4878e-05	y	ϵ_ψ	5.6521e-06
π	ϵ_a	2.9405e-06	π	ϵ_ψ	1.6868e-07
r	ϵ_a	3.9898e-07	r	ϵ_ψ	4.0720e-07
r^*	ϵ_a	0	r^*	ϵ_ψ	0
q	ϵ_a	1.0415e-06	q	ϵ_ψ	1.0752e-05
c_H^*	ϵ_a	4.1641e-05	c_H^*	ϵ_ψ	3.3773e-05
π^*	ϵ_a	0	π^*	ϵ_ψ	0
c	ϵ_a	1.6466e-05	c	ϵ_ψ	2.1559e-06
y	ϵ_{c^*}	6.8318e-07	y	ϵ_f	2.2897e-05
π	ϵ_{c^*}	3.4466e-08	π	ϵ_f	1.5074e-07
r	ϵ_{c^*}	1.9465e-08	r	ϵ_f	1.0821e-06
r^*	ϵ_{c^*}	0	r^*	ϵ_f	0
q	ϵ_{c^*}	4.7701e-08	q	ϵ_f	6.8395e-06
c_H^*	ϵ_{c^*}	4.9367e-06	c_H^*	ϵ_f	3.3112e-04
π^*	ϵ_{c^*}	0	π^*	ϵ_f	0
c	ϵ_{c^*}	5.4835e-09	c	ϵ_f	5.6151e-05
y	ϵ_{r^*}	1.9220e-07	y	ϵ_{π_F}	1.9500e-06
π	ϵ_{r^*}	4.1441e-07	π	ϵ_{π_F}	1.6991e-06
r	ϵ_{r^*}	7.7671e-07	r	ϵ_{π_F}	3.6678e-06
r^*	ϵ_{r^*}	1.0020e-06	r^*	ϵ_{π_F}	0
q	ϵ_{r^*}	1.0054e-06	q	ϵ_{π_F}	7.7295e-05
c_H^*	ϵ_{r^*}	2.5154e-06	c_H^*	ϵ_{π_F}	2.6389e-05
π^*	ϵ_{r^*}	0	π^*	ϵ_{π_F}	0
c	ϵ_{r^*}	7.8531e-07	c	ϵ_{π_F}	7.0323e-06
y	ϵ_{π^*}	4.1373e-08			
π	ϵ_{π^*}	1.6278e-07			
r	ϵ_{π^*}	1.9602e-07			
r^*	ϵ_{π^*}	0			
q	ϵ_{π^*}	3.8626e-07			
c_H^*	ϵ_{π^*}	5.5122e-07			
π^*	ϵ_{π^*}	9.4724e-07			
c	ϵ_{π^*}	1.5975e-07			

Table 8: The variance of the average response functions. The first column shows the observable, the second shows the shock that was applied and the third shows the variance of the response of the observable to the shock. Here monetary policy 2 was used, equation 3.51.

Parameter	Shock	Variance	Parameter	Shock	Variance
y	ϵ_q	2.9353e-07	y	ϵ_{prem}	1.0284e-06
π	ϵ_q	7.2481e-06	π	ϵ_{prem}	3.3497e-06
r	ϵ_q	2.9948e-06	r	ϵ_{prem}	4.5822e-06
r^*	ϵ_q	0	r^*	ϵ_{prem}	0
q	ϵ_q	2.8619e-05	q	ϵ_{prem}	1.1050e-05
c_H^*	ϵ_q	4.0503e-06	c_H^*	ϵ_{prem}	1.3743e-05
π^*	ϵ_q	0	π^*	ϵ_{prem}	0
c	ϵ_q	1.2745e-06	c	ϵ_{prem}	4.5767e-06
y	ϵ_a	2.4739e-05	y	ϵ_ψ	6.2846e-06
π	ϵ_a	3.3028e-06	π	ϵ_ψ	4.5613e-07
r	ϵ_a	5.5009e-07	r	ϵ_ψ	1.0766e-06
r^*	ϵ_a	0	r^*	ϵ_ψ	0
q	ϵ_a	1.1861e-06	q	ϵ_ψ	1.0168e-05
c_H^*	ϵ_a	3.9010e-05	c_H^*	ϵ_ψ	3.7178e-05
π^*	ϵ_a	0	π^*	ϵ_ψ	0
c	ϵ_a	1.7189e-05	c	ϵ_ψ	2.7089e-06
y	ϵ_{c^*}	6.2923e-07	y	ϵ_f	1.5662e-05
π	ϵ_{c^*}	5.1434e-08	π	ϵ_f	1.0206e-07
r	ϵ_{c^*}	1.6654e-08	r	ϵ_f	7.3407e-07
r^*	ϵ_{c^*}	0	r^*	ϵ_f	0
q	ϵ_{c^*}	4.0947e-08	q	ϵ_f	5.8437e-06
c_H^*	ϵ_{c^*}	4.7164e-06	c_H^*	ϵ_f	2.3243e-04
π^*	ϵ_{c^*}	0	π^*	ϵ_f	0
c	ϵ_{c^*}	5.9968e-09	c	ϵ_f	4.4747e-05
y	ϵ_{r^*}	1.6452e-07	y	ϵ_{π_F}	2.0069e-06
π	ϵ_{r^*}	3.1423e-07	π	ϵ_{π_F}	3.0522e-06
r	ϵ_{r^*}	6.6104e-07	r	ϵ_{π_F}	6.8757e-06
r^*	ϵ_{r^*}	9.5435e-07	r^*	ϵ_{π_F}	0
q	ϵ_{r^*}	1.0449e-06	q	ϵ_{π_F}	6.9013e-05
c_H^*	ϵ_{r^*}	2.1546e-06	c_H^*	ϵ_{π_F}	2.8103e-05
π^*	ϵ_{r^*}	0	π^*	ϵ_{π_F}	0
c	ϵ_{r^*}	7.7323e-07	c	ϵ_{π_F}	7.4655e-06
y	ϵ_{π^*}	2.3606e-08			
π	ϵ_{π^*}	1.0850e-07			
r	ϵ_{π^*}	1.2722e-07			
r^*	ϵ_{π^*}	0			
q	ϵ_{π^*}	3.5147e-07			
c_H^*	ϵ_{π^*}	3.1777e-07			
π^*	ϵ_{π^*}	9.0516e-07			
c	ϵ_{π^*}	1.0658e-07			

Table 9: The variance of the average response functions. The first column shows the observable, the second shows the shock that was applied and the third shows the variance of the response of the observable to the shock. Here monetary policy 3 was used, equation 3.52.

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