

# A life-cycle hypothesis of research activity

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In this paper the optimal allocation of effort over time is explored in the university setting where professors face a known date of retirement. The setup is simple: individuals dislike effort – at least beyond a certain level – but realise that current effort generates greater human capital. The human capital can be measured in the quality of their curriculum vitae, which generates higher wages and employment opportunities, or more generally in knowledge and understanding of their field of study. There results an inter-temporal trade-off where current sacrifices in the form of higher effort bring future rewards in greater human capital.

We start by laying out a very simplified model of the representative professor's behaviour and then test the model's implications about research activity over the life cycle using data from the University of Iceland. Each year the university allocates points to each academic member of staff that measures his or her research for the year. The data provide us with a measure of effort for each individual in a given year, in addition to information about his age, sex and academic department.

## Literature

Oster and Hamermesh (1998) find that economists' productivity measured by publications in leading journals declines with age. Moreover they find that the median age of authors of articles in leading economics journals was 36 in the 1980s and the 1990s and that a very small minority of authors are over 50 in spite of a substantial percentage of AEA members being over the age of 50. Similar results are reached by Lehman (1953), Diamond (1986), McDowell (1982) and Levin and Stephan (1992) for other disciplines. In a recent paper Jones (2010) analyses the age of individuals at the time of their greatest achievements in science using data on research that leads to the Nobel Prize in physics, chemistry, medicine and economics and also data on research that leads to great technological achievements as shown in the almanacs of the history of technology. He finds that the greatest concentration of innovations in the life of a scientist comes in the 30s but a substantial amount also comes in the 40s, while scientists in their 50s, and even more so in the 60, generate far fewer discoveries.

Chen and Zoega (2010) derive an optimal stopping model of the decision whether to continue doing research where the productivity of a research-active individual follows a geometric Brownian motion. Their model generates a threshold productivity level below which the professor decides to stop doing research. This productivity level depends on his disutility (or utility!) of doing research, the probability of a penalty if he stops doing research and the value of the sacrificed option of continuing research in the future if his productivity were to improve – the danger of missing out on a big discovery that was awaiting him. The last consideration becomes less important as the professor approaches retirement making it more likely that an older professor becomes inactive.

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## The effort decision

Assume that a professor has time T left before retirement. Research output depends on his effort e and human capital H according to the expression eH. The professor maximizes the sum of his utility from now (time zero) until he retires at time equal T with respect to effort at work. His utility is by assumption an increasing function of his human capital and a decreasing function of his effort on the job. The parameter  $\beta$  measures the extent of his disutility of effort:  $u = H - e^{\beta}$  and  $\beta > 1$ . When the worker decides on the level of e he takes into account the effect on the future evolution of his human capital H since there is a relationship between human capital and effort so that the growth of human capital depends on effort exerted, as shown by equation (1) below

$$\dot{H} = \alpha e \tag{1}$$

with  $\alpha > 0$  measuring the effect of effort on human capital growth. The sum of utilities from time zero (which is now) to the date of retirement T can then be written as

$$\int_0^T (H - e^{\beta}) dt \tag{2}$$

where the discount rate is for simplicity assumed to equal zero. Furthermore, assume that the initial level of human capital equals  $H_0$ ;  $H(0)=H_0$ , and that the worker is not constrained in any way in the terminal level of human capitals H(T).

The worker maximizes (2) subject to (1) and the initial and terminal conditions described in the previous paragraph. The Hamiltonian for this problem is

$$H = H - e^{\beta} + \lambda \alpha e \tag{3}$$

which gives the necessary condition

$$H_e = -\beta e^{\beta - 1} + \alpha \lambda = 0 \tag{4}$$

The condition makes the marginal disutility of effort  $\beta e^{\beta-1}$  equal to its marginal benefit where the latter is equal to the value of the human capital generated, which is equal to the multiple of the effect of effort on human capital  $\alpha$  – this is the marginal product of effort in generating human capital – and the shadow price of human capital  $\lambda$ :

$$\alpha\lambda = \beta e^{\beta - 1} \tag{4'}$$

The following two equations show changes in the shadow price of human capital  $\lambda$  and the level of human capital H.

$$\dot{\lambda} = -1 \tag{5}$$

$$\dot{H} = \alpha e \tag{6}$$

Equation (4) can be used to give the optimal level of effort as a function of the shadow price, the productivity of effort in generating human capital  $\alpha$  and the marginal disutility of effort  $\beta$ 

$$e = \left(\frac{\alpha}{\beta}\lambda\right)^{\frac{1}{\beta-1}} \tag{7}$$

Equation (5) gives a solution for the shadow price  $\lambda$  (upon integration of both sides and taking into account the terminal conditions  $\lambda(T) = 0$ ):

$$\lambda(t) = T - t. \tag{8}$$

Combining equations (7) and (8) gives

$$e = \left(\frac{\alpha}{\beta}(T - t)\right)^{\frac{1}{\beta - 1}} \tag{9}$$

This equation implies that effort is decreasing in  $\beta$  and increasing in  $\alpha$  and the time left before retirement. Putting equation (9) into equation (6) gives

$$\dot{H} = \alpha \left(\frac{\alpha}{\beta} (T - t)\right)^{\frac{1}{\beta - 1}} \tag{10}$$

where the growth of human capital H is increasing in time to retirement T-t, increasing in the effect of effort on human capital  $\alpha$ , and decreasing in the marginal disutility of effort  $\beta$ . Note that equation (10) yields a standard Mincer earnings equation (see Mincer, 1958) if wages are made a function of human capital in that wages are increasing in tenure but at a decreasing rate. Moreover, it follows that professors who are productive – have a high value of  $\alpha$  — exert more effort and generate more human capital over their working life while professors who dislike effort – have a high value of  $\beta$  – exert less effort and generate less human capital. The main implication of the model is the age dependency of research effort, which is monotonically decreasing in time t and falls to zero on the last day at work.

## Empirical predictions and testing

An empirical prediction coming from the model is that professors should become less research active as they approach retirement. This implication can be tested by using data from The University of Iceland. The university uses data on research activity to calculate a single number for each member of staff per year. Activities such as publishing papers in academic journals and books, seminar presentations and so forth each give a fixed number of points each which are then summed up to generate one grand total for each member of staff per year (see description of the scheme in an appendix). The same point system is used for all departments which enables us to study research activity for the whole university. Since the point system measures not just the number of articles published in academic journals but also working papers, seminar presentations and so forth, individuals with zero points can be said to be almost completely inactive when it comes to research. Data for the calendar year 2008 is used when 640 members of staff were assessed.

From equation (9) above it follows that research activity e should be linearly dependent on the time left before retirement T-t and hence also age t if  $\beta = 2$ . We estimate a linear function where research output (measured in points in year 2008) is regressed onour measure of effort and the regressors include age and a host of dummy variables for status (professor, associate professor and assistant professor) and academic department. The objective The results are reported in Table 1 below. The regression results are meant to be indicative of the relationships found in the data.

<sup>&</sup>lt;sup>1</sup> However, some members of staff do publish books that take several years to write and choose not to present seminars or show any other activity in between.

However, since equation (9) is derived to express the economic intuition is simple terms, the true empirical relationship between research effort and age is likely to be more complicated, which calls for caution in interpreting the statistical significance of the regression.

The objective of the statistical exercise is to map the age profile of research. The coefficient of age is negative and statistically significantly different from zero for all workers combined and in particular for men while it is negative and insignificantly different from zero for women. For all workers combined, each passing year lowers research effort by 0.61 points; for all men combined it lowers research output by 0.78 points per year; while no statistically significant relationship is found between age and effort for women. The equality of the coefficient of the age variable for men and women can be rejected. When one confines the sample to research active members of staff only in columns (2) of the table – defined as not having zero research points in 2008 – one finds that each year lowers effort by 0.53 points for both sexes combined and 0.74 points for men only while the relationship remains insignificant for women.

In addition, the results indicate that professors tend to be more active than the lower ranking members of staff when controlled for age and academic department. The coefficients of the departmental dummies are reported in Table A1 in an appendix.

 $<sup>^{2}</sup>$  A Wald test gives F=5.99 with probability of null hypothesis being true equal to 0.02.

Table 1. Age and research output

	All		Men		Women	
	(1)	(2)	(3)	(4)	(5)	(6)
	All	Active	All	Active	All	Active
Constant	61.02*	58.8*	71.16*	71.97*	32.69*	26.71*
	(8.99)	(7.78)	(7.93)	(6.95)	(3.22)	(2.53)
Age	-0.61*	-0.53*	-0.78*	-0.74*	-0.15	-0.01
	(5.35)	(4.07)	(5.07)	(4.09)	(0.89)	(0.08)
Professor	15.46*	16.35*	13.10	14.24	27.22*	27.66*
1 10105501	(2.82)	(2.82)	(1.86)	(1.86)	(2.73)	(2.78)
Associate	1.83	2.90	-1.65	0.19	17.21	16.56
professor	(0.34)	(0.50)	(0.23)	(0.02)	(1.75)	(1.68)
Assistant	-7.46	-4.08	-11.54	-4.33	7.71	9.19
professor	(1.36)	(0.70)	(1.58)	(0.53)	(0.79)	(0.95)
R-sq.	0.19	0.16	0.20	0.16	0.30	0.29
Obs.	640	555	411	344	229	211

Least squares estimates, t-ratios in parentheses. \* indicates significance at the 5% level.

We explored different functional forms by adding age squared to the equation in order to capture non-linearities. However, the squared term turned out to be statistically insignificant. Also, White tests indicate the absence of heteroskedasticity for all workers and for men only but not for women. Heteroskedasticity would make the computed standard errors invalid although the estimates remain consistent. The test results also indicate that the linear functional form may be accurate, at least for all worker put together.

In Table 2 the age variable is replaced with age dummies for each half decade without controlling for academic department. Because of a lack of observations, age groups, status groups and departmental variables could not be included simultaneously for women. However, the results for all workers together and for men when departmental controls are included are qualitatively identical to those shown in Table 2 and available from the author.

The results suggest that research output when both sexes are combined is rising until the early forties and then declining. Men slow significantly down in their fifties and sixties, so much that the average number of points (before taking the effect of status into account) drops by more than 50% from around 42 to about 18 per year. Women also peak in their early forties at around 25 points but only decline down to 15 in their late sixties. It follows that men tend to produce more in their thirties and forties but lose their edge in their fifties and sixties. The null hypothesis that research output remains the same throughout life can be rejected for all workers combined as well as for men at the 5% level of significance but not for women.<sup>3</sup>

<sup>&</sup>lt;sup>3</sup> A Wald test for the equality of all the coefficients of all age variables yields F=4.83 (probability=0.00) for all workers; F=4.68 for men (probability=0.00); and F = 0.69 for women (probability=0.68).

Table 2. Age dummies and research output

	All		Men		Women	
	All	Active	All	Active	All	Active
30-34	33.23*	33.30*	37.04*	36.48*	20.71*	20.36*
	(5.92)	(5.77)	(5.23)	(4.94)	(2.27)	(2.23)
35-39	34.44*	36.28*	40.11*	43.44*	20.56*	19.73*
	(7.87)	(7.80)	(6.97)	( <i>6.87</i> )	(3.21)	( <i>3.06</i> )
40-44	37.25*	39.87*	42.53*	47.62*	25.93*	26.12*
	(9.71)	( <i>9.87</i> )	(8.03)	(8.29)	(5.02)	(4.98)
45-49	28.56*	28.98*	32.34*	32.13*	19.15*	20.08*
	(8.24)	(8.08)	(6.91)	(6.56)	( <i>3.96</i> )	(4.08)
50-54	28.77*	30.76*	30.70*	33.57*	21.79*	22.85*
	(8.56)	(8.76)	(6.56)	( <i>6.76</i> )	(4.80)	(4.93)
55-59	23.53*	26.61*	25.26*	28.23*	17.97*	20.84*
	(7.09)	(7.49)	(5.63)	(5.79)	( <i>3.87</i> )	(4.29)
60-64	20.96*	21.96*	20.68*	21.24*	21.20*	21.20*
	(5.82)	(5.81)	(4.14)	( <i>3.93</i> )	(4.46)	(4.44)
65-70	17.87*	23.26*	18.53*	24.04*	15.41*	19.69*
	(4.52)	(5.21)	(3.63)	(4.13)	(2.49)	(2.88)
Professor	8.26*	8.59*	6.19	6.66	15.25*	15.18*
	(2.59)	(2.59)	(1.46)	(1.48)	(3.26)	(3.19)
Associate professor	-5.79	-4.75	-10.02*	-8.50	4.03	3.65
	(1.81)	(1.41)	(2.31)	(1.81)	(0.91)	(0.82)
Assistant professor	-13.37*	-11.21*	-16.47*	-10.62	-5.40	-4.15
	(4.11)	( <i>3.21</i> )	( <i>3.46</i> )	(1.96)*	(1.25)	(0.93)
R-sq.	0.13	0.11	0.14	0.11	0.16	0.14
Obs.	640	555	411	344	229	211

Least squares estimates, t-ratios in parentheses. \* indicates significance at the 5% level.

We should note that while the observation that research output is increasing until the early 40s runs counter to the model's predictions, the difference between the output of those aged 40-44 and those aged 30-34 or 35-39 is not statistically significant, in contrast to the difference between those aged 40-44 and 65-69 years. The figure below plots the research profiles for men and women based on the numbers in the table.

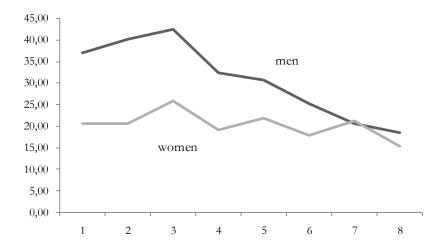


Figure 1. Relationship between age and research effort in Table 2

Only the research active are included in column (2) of Table 2. For the whole sample research activity peaks in the early forties at close to 40 points and then falls down to 23 points in late sixties. For men, the peak occurs in the early forties at 47.6 points, followed by a steep decline to 32 points in the late forties and 21 points in early sixties. For women the peak also occurs in early forties at 26.12 points but this is not followed by a steep decline, which was the case for men, since effort stabilises around 20 points in late forties and no further downward trend is visible. The equality of coefficients can now longer be rejected for men at the 5% level of significance although it can still be rejected at the 10% level.<sup>4</sup>

#### Conclusions

Future rewards justify current effort when human capital is a function of current and past research effort but as the future horizon becomes shorter the incentive to exert effort falls. The empirical prediction of research output falling as a worker gets closer to retirement is confirmed for academic members of staff at the University of Iceland. However, the pattern is much weaker for women. One possible reason for the observed difference between men and women is the fact that men complete their PhD studies earlier than women (average age for female members of staff is 39.6 years while it is 34.1 years for males). Another reason could lie in longer life expectancy of women. However, taking these two possibilities into account would require changing the model of this paper. There is also the possibility that he current group of younger men happen to be more productive – have a higher value of  $\alpha$  in the model – than those who are older while this is not the case for female members of staff. Further work is needed to test for this possibility using panel estimation which may help explain the male-female difference.

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<sup>&</sup>lt;sup>4</sup> The Wald test for the equality of all coefficients yields F= 3.40 (probability =0.00) for all teachers; F= 3.70 (probability=0.00) for men; and F=0.39 (probability=0.91) for women. We can reject equality for all workers combined as well as for men at the 5% level of significance but we cannot reject equality in the case of women.

## References

- Ando, A., & Modigliani, F. (1963). The "life cycle" hypothesis of saving: Aggregate implications and tests. *The American Economic Review*, *53*(1), 55-84.
- Chen, Y., & Zoega, G. (2010). An essay on the generational effect of employment protection. *Mathematical Social Sciences*, 59(3), 349-359.
- Diamond, A. (1986). The life-cycle research productivity of mathematicians and scientists. *Journal of Gerontology*, 41, 520-525.
- Jones, B. F. (2010). Age and great invention. The Review of Economics and Statistics, XCII(1), 1-14.
- Lehman, H. (1953). Age and achievement. Princeton: Princeton University Press.
- Levin, S., & Stephan, P. (1992). Striking the mother lode in science: The importance of age, place and time. New York: Oxford University Press.
- McDowell, J. (1982). Obsolescence of knowledge and career publication profiles. *American Economic Review*, 72(4), 752-768.
- Mincer, J. (1958). Investment in human capital and personal income distribution. *The Journal of Political Economy*, 66(4), 281-302.
- Oster, S. M., & Hamermesh, D. S. (1998). Aging and productivity among economists. *The Review of Economics and Statistics*, 80(1), 154-156.

## Appendix

University of Iceland point system for research (Since 2003)

#### 1. Dissertations

- 1.1 Candidate- or masters thesis (15 points)
- 1.2 Doctoral thesis (30 points)

#### 2. Books

- 2.1 Books, academic (10-60 points)
- 2.2 Books, republications (0-10 points)

#### 3. Academic articles in journals

- 3.1 Article in internationally acknowledged journals cited in ISI Web of Science(15 points)
- 3.2 Article in other refereed journals (10 points)
- 3.3 Other material in refereed journals (0-5 points)
- 3.4 Article in a non-refereed journal (0-5 points)

## 4. Papers in refereed conference proceedings and book chapters

- 4.1 Paper in a refereed conference proceedings (5-10 points)
- 4.2 Book chapter (5-10 points)

## 5. Other academic activity

- 5.1 Scientific report or memorandum (0-5 points)
- 5.2 Book review (1-2 points)
- 5.3 Lectures
- 5.3.1 Lecture at scientific conference (3 points)
- 5.3.2 Lecture for the academic community (1 point)
- 5.3.3 Plenary lecture or keynote address at an international conference (5 points)
- 5.4 Posters
  - 5.4.1 Poster at a scientific conference (2 points)
  - 5.4.2 Poster at other meetings (1 point)
- 5.5 Translations (0-10 points)
- 5.6 Other (software, patents, psychological tests, bills, design projects etc.) (0-10 points)

#### 6. Citations in ISI Web of Science

First 10 citations: 1 point/citation Next 20 citations: 0,5 point/citation Citations exceeding 30: 0,1 point/citation

### 7. Editorial work, academic publications

- 7.1. Editor of an academic journal (2-5 points/year)
- 7.2. Member of editorial board of an academic journal (1-2 points/year)
- 7.3. Editor of an academic book (2-5 points)
- 7.4 Member of editorial board of an academic book (1-2 points)

In the case of multiple author articles or books, the points are calculated using the following formula: 2 authors: 1,5 x points / 2, 3 authors: 1,8 x points / 3, 4 authors or more 2,0 x points / number of authors

Table A1. Departmental effects

	(1)	(2)	(3)	(4)	(5)	(6)
Social and	2.92	1.43	7.68	1.77	-8.69	-5.62
human sciences	(0.37)	(0.17)	(0.68)	(0.15)	(0.73)	(0.47)
Social work	-4.02	-8.55	-10.91	-20.79	-12.60	-14.20
Social Work	(0.38)	(0.78)	(0.41)	(0.75)	(0.98)	(1.10)
Economics	-2.77	2.51	-3.44	1.09	2.29	2.30
Economics	(0.35)	(0.30)	(0.36)	(0.11)	(0.11)	(0.11)
Law	-6.67	-4.99	-6.82	-5.79	-13.08	-11.57
Law	(0.88)	(0.60)	(0.69)	(0.52)	(1.06)	(0.92)
Political science	-1.24	-4.27	-0.36	-4.41	NA	NA
	(0.12)	(0.39)	(0.02)	(0.33)		1 1/1
Business	-17.60	-21.44	-17.69	-23.97	-20.10	-22.18
Administration	(2.24)	(2.54)	(1.81)	(2.23)	(1.35)	(1.49)
Nursing	-12.92	-14.05	-30.35	-28.00	-20.01	-19.12
Ü	(1.73)	(1.75)	(1.82)	(1.38)	(1.80)	(1.71)
Pharmaceutical	-22.77	-25.75	-14.68	-18.53	-35.72	-36.57
sciences	(2.29)	(2.51)	(0.97)	(1.17)	(2.66)	(2.72)
Medicine	-14.91	-14.60	-11.81	-11.96	-28.92	-28.41
	(2.30)	(2.12)	(1.40)	(1.30)	(2.67)	(2.60)
Food science	2.18	-1.18	-6.44	-11.57	14.41	13.52
and nutrition	(0.23)	(0.12)	(0.51)	(0.88)	(0.96)	(0.90)
Psychology	-0.40	-3.98	5.53	0.51	-25.36	-25.50
-)8)	(0.04)	(0.41)	(0.48)	(0.04)	(1.52)	(1.53)
Odontology	-9.45	-5.61	-4.56	0.43	-33.90	-33.66
O,	(1.14)	(0.57)	(0.44)	(0.03)	(2.02)	(2.01)
Languages,	-13.90	-14.11	-14.56	-15.51	-21.65	-20.07
literature and	(1.75)	(1.63)	(1.25)	(1.18)	(1.83)	(1.66)
linguistics	, ,	, ,	, ,	` ′	, ,	, ,
Theology and	-16.61	-19.17	-15.29	-18.79	-20.83	-22.41
religious studies	(1.74)	(1.82)	(1.27)	(1.37)	(1.25)	(1.35)
Icel. and comp.	-2.45	-6.31	4.19	-1.15	-17.93	-19.48
cultural studies	(0.33)	(0.81)	(0.41)	(0.11)	(1.54)	(1.68)
History and	2.76	-0.83	4.56	0.25	-5.05	-6.39
philosophy	(0.38)	(0.11)	(0.48)	(0.03)	(0.43)	(0.54)
Sport, leisure and social	-8.11	-10.46	-7.61	-10.09	-14.35	-15.82
	(0.90)	(1.09)	(0.54)	(0.64)	(1.22)	(1.35)
education	110	-7.95	2.67	-8.45	-15.67	-16.37
Teacher	-4.18 (0.63)		-2.67 (0.30)			
education Educational	(0.63) -1.20	(1.12) -4.54	(0.30) 7.06	(0.85) 7.39	(1.44) -15.08	(1.50) -17.28
Educational studies	(0.16)	-4.54 (0.56)	(0.57)			
Ind, mech. eng.	(0.16)	(0.26)	(0.57)	(0.54)	(1.33)	(1.52)
_	-18.57	-17.76	-17.56	-18.18	-25.88	-26.12
and computer science	(2.32)	(2.04)	(1.76)	(1.64)	(1.55)	(1.56)
	-3.08	-6.93	-1.25	-5.52	-7.00	-8.67
Earth sciences	-3.06 (0.30)	-0.93 (0.65)	(0.10)	-3.32 (0.42)	(0.33)	(0.41)
Life and environ.	-9.73	-11.72	-14.11	-17.21	-9.43	-10.42
Sciences	(1.33)	(1.52)	(1.40)	(1.57)	(0.85)	(0.94)
Electrical and			, ,	, ,	, ,	, ,
computer	-3.10	-1.86	2.07	2.94	-45.35	-46.62
engineering	(0.32)	(0.18)	(0.18)	(0.22)	(2.14)	(2.21)
	-18.33	-16.33	-16.75	-15.74	-38.23	-37.02
Physical sciences			(1.90)		-36.23 (1.41)	
Civil and	(2.57)	(2.09)	` '	(1.61)	, ,	(1.37)
environmental	0.24	-0.73	0.82	-1.08	-0.46	1.27
CirviioiiiiCiitai	(0.03)	(0.09)	(0.09)	(0.11)	(0.02)	(0.06)

t-statistics in parentheses.