Meistaraprófsritgerð

Fjármál fyrirtækja

Modern Portfolio Theory: Does it work?

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Abstract

The main objective of this paper is to discuss and test some of the fundamental elements of the Modern Portfolio Theory in an effort to establish whether the optimal portfolios based on the theory provide the best asset allocation strategies in the real investment world. To answer this question, three of the fundamental elements of the theory are tested statistically. Furthermore, a portfolio is constructed using three different types of models based on the theory and the outcome discussed and evaluated.

The empirical results show that two of the three elements of the theory which were tested did not hold up to statistical tests. The efficiency of the market was the only statistically tested element that held up to the tests. The construction of portfolios showed that the models were inconsistent and one of the models, the efficient portfolio, showed very unrealistic positions. These results led to the conclusion that optimal portfolios based on Modern Portfolio Theory are not the best asset allocation strategies in the real investment world. At the same time, however, it is concluded that despite its flaws the theory is still the only properly formed and implemented theory for portfolio construction.

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1 Introduction

There are many ways to handle risk in financial markets. The oldest one, and at the same time simplest one, is fundamental analysis, which aims to identify causes of changes in stock price by studying the relevant company, the industry behind it or the economy around it. The study is then used to predict the stock's next move. Investors and portfolio managers have gradually realized that it is very hard, indeed almost impossible, to predict the consequences of events on stock prices by studying only the environment surrounding the stock as described above. Other tools have therefore been developed. After fundamental analysis came technical analysis, which included the act of recognizing patterns in pricing, volume and other indicators to look for clues as to whether to buy or sell. This type of analysis is thought to work at times but cannot be a foundation on which to build a global risk management system. What we recognise today as "modern finance" came about as a reaction to the flaws of both fundamental and technical analysis. It emerged from the mathematics of chance and statistics and has been for years the foundation of financial management (Mandelbrot B. , 2004). It is the methods of modern finance, specifically "Modern Portfolio Theory", that will be the issue in this thesis. The theory has been subject to a lot of criticism ever since it emerged in the 1960's and gradually the arguments against it have strengthened. However, it is still the principal foundation for the financial theory being taught all around the world.

For decades, scholars and investors have had their doubts about whether predictions of expected returns, which are the fundamental objective of Modern Portfolio Theory, are accurate or even possible. In fact, the general consensus has been that expected returns are notoriously difficult to predict, and at the same time that the optimization process is very sensitive to differences in expected returns (Chan, Karceski, & Lakonishok, 1999). Other methods have, however, not been able to replace, at least not in the academic environment, the conventional methods that have developed in the spirit of Modern Portfolio Theory.

The aim of this thesis is to examine whether Modern Portfolio Theory is an effective tool for portfolio management. The first part of the thesis, chapter two, will be dedicated to explaining in as simple terms as possible, without mathematical complications, the main elements of the theory and the main highlights in its development. Criticism of the theory will not be much discussed in that part, though some of its disadvantages will be explored. Chapter three of the thesis contains more detailed discussions on the main criticisms of the theory, which are chiefly directed against its main foundations. It will be noted that spokesmen and advocates of the theory all have serious doubts as to whether it actually works in reality. In chapters four and five, before results of the research will be set forward, data, key variables and research design will be described. In chapter six empirical tests will be conducted in order to answer the following question: "Are the optimal portfolios suggested by the Modern Portfolio Theories the best asset allocation strategies in real investment world?"

In an effort to answer that question, the following hypothesis will be tested:

1. Modern portfolio models which use historical returns data for input are realistic in their predictions of future optimal portfolios for subsequent periods.
2. The market is efficient: $\alpha = 0$.
3. Higher volatility means higher average returns: stocks with similar beta coefficients have similar average returns (and vice versa).
4. Expected returns compared to actual returns.

In the last chapter of the thesis conclusions will be set forward which will summarize the main results of the empirical tests and answer the above center question of the thesis.

2 Literature Review

2.1 Modern Portfolio Theory

The concept of investment diversification is an old one and existed long before modern finance theory. It was, however, not until 1952 that Harry Markowitz published a formal model of portfolio selection based on diversification principles. This work contributed to Markowitz receiving the Nobel Prize in Economics in 1990. His model can be regarded a first step in portfolio management, which is the identification of the efficient set of portfolios or *the efficient frontier of risky assets* (Bodie, Kane, & Marcus, 2009).

Actually, the work began in 1900 when the French mathematician, Louis Bachelier, studied financial markets. Based on his studies, Bachelier argued that prices will go up or down with equal probability and that their volatility is measurable. The so-called bell curve was born, whereby the distribution of price movements is thought to be bell-shaped with very large changes assumed to be extremely rare. It was Markowitz who took the first step in applying Bachelier's ideas (Mandelbrot B. , 2004). Others followed in their footsteps, making Modern Portfolio Models simpler and more usable for investors and portfolio managers. In this chapter, an attempt is made to cover the basic development of Modern Portfolio Theory in as simple and neutral a way as possible and without going into mathematical details. The most significant milestones in its development will be described and discussed along with some of the less fundamental criticisms i.e. those which were mostly aimed at the implementation of the theory rather than at the theory itself.

2.1.1 Diversification in a portfolio

As mentioned above, the concept of diversification is far from new. It has long been regarded self-evident that it is unwise for investors to invest their entire wealth in a single asset. This is because most investors have an aversion to risk, which is one of the bases in Modern Portfolio Theory. It is more common for investors to invest in several securities simultaneously, in the hope that a loss in one will be compensated by gains in others. In other words, holding more than one security at a time is an attempt to spread and minimize risk in an investment. The objective of diversification is to reduce total risk without sacrificing portfolio returns. The aim of every rational investor is to maximize his or her returns and minimize the risk. Diversification is the method adopted in order to reduce risk.

It essentially results in the construction of portfolios. The proper goal of portfolio construction would be to generate a portfolio that provides the highest return and the lowest risk. Such a portfolio would be known as the “optimal portfolio” (Kevin, 2001). As an investor diversifies into many securities he or she continues to reduce exposure to firm-specific factors and at the same time the portfolio’s volatility should continue to fall. In the end though, even with a large number of stocks, risk cannot be avoided altogether. All securities are affected by common macroeconomic factors so even extensive diversification cannot eliminate risk. The risk that always remains is called market risk - also referred to as systematic risk or non-diversifiable risk - and is attributable to market-wide risk sources. The risk that can be eliminated by diversification is called firm-specific risk, non-systematic risk or diversifiable risk (Bodie, Kane, & Marcus, 2009).

2.1.2 Markowitz‘s portfolio theory

In the 1950’s the investment community talked about risk but there was no measurable specification for the term. However, investors were eager to quantify their risk variable. Markowitz showed that the variance of the rate of return was an important measure of risk under a reasonable set of assumptions and came forward with the formulas for computing the variance of the portfolio. The use of this formula revealed the importance of diversifying to reduce risk and also provided guidance on how to diversify effectively (Reilly, 1989). When Markowitz first published his ideas of portfolio selection in 1952 he rejected the notion that investors should maximize discounted returns and choose their portfolio accordingly. Markowitz’s view was that this rule failed to imply diversification, no matter how the anticipated returns were formed. The rule he rejected implied that the investor should place all of his or hers funds in the security with the greatest discounted value. He also rejected the law of large numbers in portfolios made up of securities, objecting to the claim that it would result in both maximum expected returns and minimum variance, and pointing out that returns from securities are too intercorrelated for all variance to be eliminated with diversification. Markowitz also pointed out that a portfolio with maximum expected returns is not necessarily the one with the minimum variance. Hence, that there is a rate at which the investor can gain expected returns by accepting more variance, or reduce variance by giving up expected returns. Building on these observations he presented the “expected returns-variance of returns” rule (Markowitz, 1952). Markowitz‘s idea was that investors should hold mean-variance efficient portfolios. While not an entirely new concept, mean-variance optimization was not a widely used strategy at the time. Most investment managers were focusing their efforts on identifying securities with high expected returns (Chan, Karceski, & Lakonishok, 1999).

In his paper, Markowitz formally presented his view that although investors want to maximize returns on securities they also want to minimize uncertainty, or risk. These are conflicting objectives which must be balanced against each other when the investor makes his or her decision. Markowitz asserts that investors should base their portfolio decisions only on expected returns, i.e. the measure of potential rewards in any portfolio, and standard deviation, the measure of risk. The investor should estimate the expected returns and standard deviation of each portfolio and then choose the best one on the grounds of the relative magnitudes of these two parameters (Sharpe, Alexander, & Bailey, 1999).

As previously mentioned, Markowitz rejected the expected returns rule on the grounds that it neither acknowledged nor accounted for the need for diversification, contrary to his "expected return-variance of return" rule. In addition, he concluded that the expected return-variance of return rule not only revealed the benefits of diversification but that it pointed towards the right type of diversification for the right reason. It is not enough to diversify by simply increasing the number of securities held. If, for example, most of the firms in the portfolio are within the same industry they are more likely to do poorly at the same time than firms in seperate industries. In the same way it is not enough to make variance small to invest in large number of securities. It should be avoided to invest in securities with high covariance among themselves and it is obvious that firms in different industries have lower covariance than firms within the same industry (Markowitz, 1952).

Simply put, Markowitz concluded that by mixing stocks that flip tail and those that flip heads you can lower the risk of your overall portfolio. If you spread your investments across unrelated stocks you will maximise your potential profit whether the economy is slowing down or growing. If you then add more and more stock in different combinations you have what Markowitz called an 'efficient' portfolio. An efficient portfolio is the portfolio which gives the highest profit with the least risk. The aim of Markowitz's methods is to construct that kind of portfolio (Mandelbrot B. , 2004).

Until Markowitz suggested this approach to portfolio analysis no full and specific basis existed to justify diversification in portfolio selection. Also the concept of risk had rarely been defined in a thorough manner in portfolio analysis before Markowitz's writings, let alone treated analytically. With his approach these issues, diversification and risk, got a specified framework and a workable algorithm for employing that framework for practical problems was provided. Markowitz did not, however, suggest a preferred technique for security analysis or a suitable method for portfolio selection. He concentrated on providing a general structure for the whole process and providing an algorithm for performing the task of portfolio analysis (Sharpe W. F., Portfolio Analysis, 1967).

Markowitz created a theory of portfolio choice in the uncertain future. He quantified the difference between the risk that was taken on individual assets and the aggregated risk of the portfolio. He showed that the portfolio risk came from covariances of the assets which made up the portfolio. The marginal contribution of a security to the portfolio return variance is therefore measured by the covariance between the return of the security and the return of the portfolio but not by the variance of the security itself. In his writings, Markowitz argues that the risk of a portfolio is less than the risk of each asset in the portfolio taken individually and provides quantitative evidence of the merits of diversification (Amenc & Le Sourd, 2003).

In his model of portfolio management Markowitz indentified the efficient set of portfolios, or the efficient "frontier of risky assets". The principal idea behind the frontier set of risky portfolios is that the investor should only be interested in the portfolio which gives the highest expected return for any given risk level. Also, the frontier is a set of portfolios that minimizes the variance for any target expected return (Bodie, Kane, & Marcus, 2009).

With his work, Markowitz introduced a parametric optimization model that was both sufficiently general to be applicable to a significant range of practical situations and simple enough to be usable for theoretical analysis. Nevertheless, the subject is so complicated that Markowitz's work in the 1950's probably raised more questions than it answered. Indeed, it spurred a tremendous amount of related research (Steinbach, 2001).

2.1.3 Disadvantages of Markowitz's approach and difficulties in portfolio theory

Many analysts point out a number of disadvantages of the Markowitz model. In this chapter we will not look much at the critique of Modern Portfolio Theory as a foundation of portfolio management but concentrate more on the critique concerned with the execution of Markowitz's model. The most significant and most frequently cited critique is that the model requires a huge number of estimates to construct the covariance matrix and, furthermore, that extensive calculations are required to construct the efficient frontier. Also, the Markowitz model does not provide any guideline as to the forecasting of the security risk premiums that are necessary to compute the efficient frontier of risky assets (Bodie, Kane, & Marcus, 2009). To construct efficient portfolios good forecasts of earnings, share prices and volatility for possibly thousands of stocks are needed. Also it is necessary to calculate its covariance with every other stock, which requires extensive calculations. Last but not least, the exercise needs constant repetition because of changes in the price of stocks (Mandelbrot B. , 2004).

A few years after Markowitz wrote about his approach to portfolio selection a number of researchers had rejected one or more of the bases of his theory. For example, that the mean and variance of future assets are not sufficient parameters for decision making. Some scholars writing in the first years after Markowitz published his work also suggested some other approach to some parts of Markowitz theory, for example that the probability distribution of a single security's rate of return was best estimated by distribution with an infinite variance. But even under these conditions many of the conclusions of Markowitz's approach do hold, especially those attained with one of the simplified versions. Indeed, some subsequent research dealt with trying to simplify his approach (Sharpe W. F., Portfolio Analysis, 1967).

Markowitz intended his model to be practical and implementable. In 1976, Elton, Gruber and Padberg wrote that it was ironic that the primary outgrowth of his model had, at that time, been normative and theoretical and that modern portfolio theory had not yet been implemented properly. The main reason for this, according to them, was the difficulty in estimating the type of input data that was necessary, especially in terms of correlation matrices. Other reasons were the time and cost required to compute efficient portfolios and the difficulty of getting portfolio managers to think in terms of the risk-return tradeoffs which become apparent when covariances are considered as well as returns and standard deviations. According to Elton and his co-writers, part of these problems had been solved with the use of the Single Index Model to generate variance-covariance structures. The Single Index Model is one of two approaches especially designed to deal with the difficulty of estimating the type of input data which is necessary, but it can also be employed to deal with other problems mentioned above. The other approach is to assume a simple structure for the variance-covariance matrix (Elton, Gruber, & Padberg, Simple Criteria for Optimal Portfolio Selection, 1976).

Ever since Markowitz published his theory, and before, scholars and investors have tried to answer the fundamental question in portfolio constructing, which is how risk should be measured. The debate is still going on (Steinbach, 2001). But Chan, Karceski and Lakonishok argued in 1999 that decades after Markowitz‘s publication, there was still a shortage of scientific research into the performance of different risk optimization methods. Instead, theoretical research on investments had concentrated on modeling expected returns and empirical research had focused on testing such equilibrium models or finding patterns in stock returns that seemed to be inconsistent with the models. They found that there has been a shortage of scientific evidence evaluating the performance of different risk optimization methods. Specially, the benefits promised by portfolio optimization depend on how accurately the distribution of returns can be predicted. At the same time, they observed trends suggesting that professional investors were rediscovering the importance of portfolio risk management and mounting evidence that superior returns to investment performance were elusive. Risk control in the investment management industry was at that time increasingly becoming the main emphasis (Chan, Karceski, & Lakonishok, 1999).

In the next chapters of this thesis the development of Modern Portfolio Theory will be covered, constituting mostly efforts to make Markowitz’s model more simple and practicable.

2.2 The development in Modern portfolio theory continues

Ever since Markowitz came forward with his mean-variance portfolio selection model numerous researchers have developed algorithms to produce solutions based on the model, as well as introduced simplifying assumptions in attempts to operationalize the model. The main results of these efforts have been the diagonal model and linear programming approximation of Sharpe (1963, 1973), the multi-index models of Cohen and Pogue (1967), Jacob‘s limited diversification model (1974) and the simple criteria of Elton et al. (1976) (Schnabel, 1984).

Earlier it was discussed that Elton, Gruber and Padberg maintained that the main reason for portfolio theory not having been implemented at that time (1976) was the difficulty in the estimation of the type of input data necessary, especially correlation matrices. According to them, there were two main approaches in the literature to try to solve this problem. One of them is the use of a Single Index Model to generate a variance-covariance matrix. The other is to assume a simple structure for a variance-covariance matrix, which is that all pair wise correlations are the same (Elton, Gruber, & Padberg, Simple Criteria for Optimal Portfolio Selection, 1976). We will in the next chapter concentrate on the single index model, which appeared to be developed to simplify Markowitz‘s model.

2.2.1 The Single Index Model

William Sharpe (1963) studied and worked on Markowitz‘s model, trying to simplify the calculations in order to make it more practical for use. His paper describes the advantages of using a particular model of the relationships between securities for practical applications of Markowitz’s portfolio analysis technique. According to Sharpe, evidence showed that using comparatively few parameters could lead to almost the same results as obtained by using much larger sets of relationships among securities. Sharpe was confident that his model enabled analysis at low cost and therefore was an initial practical application of the Markowitz technique (Sharpe W. F., A Simplified Model for Portfolio Analysis, 1963).
It has been mentioned before that the difficulty in computing the variance-covariance matrix was an obstacle to the implementation of the model. Sharpe presumed that the asset returns were made up of a combination of factors that were common to all assets and factors that were unique for each security. Studies showed then, that the best explanatory factor was the market as whole. Sharpe‘s single index model, or empirical market model as it is sometimes called, has no theoretical basis; it only proposes a simplified view (Amenc & Le Sourd, 2003). Markowitz‘s approach for deriving efficient sets was that every security can be viewed as being related to an index unique to itself. Sharpe‘s approach, however, with his single-index model, was that every security is related to the same index. The returns on various securities in Sharpe‘s model are assumed to be related only through common correlations with the market return (Phillips & Seagle, 1975). Later, Cohen and Pogue (1967) came forward with two intermediate models. They discovered, though, that Sharpe‘s single-index model came closer than their own models to approximating the Markowitz model‘s efficient set when empirically tested on a sample of common stocks (Alexander, 1978). In a

report by Frankfurter et al. (1976) comparing the Sharpe portfolio selection model and Markowitz's model, it was found that under conditions of uncertainty the Sharpe approach was more than just a „shortcut computational scheme."The conclusion was that the Sharpe approach appeared less subject to unstable behavior when relevant historical data are limited (although the advantage diminishes when more data is available). Also, the Sharpe approach outperformed the standard Markowitz approach in selecting efficient portfolios, which means that the Sharpe approach has potential advantages over the Markowitz approach. The fact that the Sharpe model uses fewer, and different, estimators to summarize past history wasthought to be one of the main reasons for this (Frankfurter, Phillips, & Seagle, Performance of the Sharpe Portfolio Selection Model: A Comparison, 1976).

Later, Elton and Gruber came forward with even more simplified techniques for devising optimal portfolios which also gave a better understanding of the choice of securities to be included in a portfolio. These techniques were based on the Single Index Model but the calculations in the model were easy to compute and led to very similar results as those Markowitz's model would produce using the complete matrix. According to Elton et al., the one reason security returns are correlated is that there is a common response to market changes; therefore, a useful measure of this correlation might be obtained by relating the return of a single security to the return of a market index. The key assumption of the single index model is that e_i (the random or uncertain element of a_i in stock i) is independent of e_j (same element in stock j) for all values of i and j. This indicates that the only reason stocks vary together, systematically, is their common co-movement with the market. In other words, that no other effects beyond the market explain for co-movements among securities (Elton, Gruber, Brown, & Goetzmann, 2007).

The Single Index Model, therefore, uses the market index as a proxy for the common factor. A market index, like S&P 500 for example, gives us a considerable amount of past data which we can use to estimate systematic risk. Because the index model is linear it is possible to estimate the beta coefficient, or sensitivity, of a security on the index using a single-variable regression. The excess return of a security is regressed on the excess return of the index. In order to estimate the regression, historical samples of paired observations, i.e. the returns of the security and the returns of the market at the same point in time, are collected (Bodie, Kane, & Marcus, 2009).

2.2.2 The Capital Asset Pricing Model

We have already covered Markowitz's portfolio analysis model and the empirical market model developed by Sharpe in order to simplify the calculations required by Markowitz's model, thereby making it more practical. Research in financial economics has in large part been directed towards improving our understanding of how investors make their portfolio decisions and at the same time how asset prices are determined. Over the years, in fact, many capital asset-pricing models have been developed, the main ones being mean-variance analysis and the Capital Asset Pricing Model (CAPM). These are widely considered to be among the major contributions of academic research in the post war period (Sciubba, 2006). The background of CAPM was the study of the influence of investor behavior on asset prices. The result of that study was a theory of asset valuation in an equilibrium situation, drawing together risk and return, which is the CAPM. Several authors have contributed to the model, first and foremost Sharpe, but also Treynor, Mossin, Litner and Black. The CAPM is the first model to introduce the notion of risk into the valuation of assets. It evaluates both asset returns in connection to market returns and the sensitivity of the security to the market (Amenc & Le Sourd, 2003).

CAPM is in principle a method to calculate the rate of return which it is normal to demand of an asset of a certain nature. The search for the normal rate of return is divided into two parts according to CAPM. On the one hand, a risk-free rate is found. On the other hand, the rate of return on a risky asset is found, constituting the risk premium. In CAPM the standard deviation of a single asset does not matter greatly, rather the effect of the asset on the systematic risk of the portfolio to which the asset is added. The main concern is the conjunction between the rate of return of the efficient portfolio and a single asset. If the conclusion of the CAPM is that the correlation between the rate of return of the portfolio and an asset is high, then it is appropriate to demand a high risk premium of that asset. If the correlation is low, on the other hand, only a low risk premium should be demanded. CAPM has a number of presumptions:

1. It is assumed that all investors are price-takers, i.e. that they act as though security prices are unaffected by their own trades (similar to the perfect competition assumption of microeconomics).
2. It is assumed that all investors plan for one identical holding period.

3. Investments are only made in publicly traded financial assets; that is assets which are traded in an open market.
4. Taxes and transaction costs are ignored.
5. All investors are assumed to be rational mean-variance optimizers, that is Markowitz's portfolio selection theory is used by them all.
6. All investors are assumed to analyze expected returns, variance, covariance and other factors relevant to the analysis of securities in the same way. This assumption is often referred to as homogeneous expectations (Magnússon, 2002).

The expected return-beta relationship is the most familiar expression of the CAPM to those who use the model. The relationship is reflected in the following equation

$$E(r_s) = r_f + \beta_s\left[E(r_M) - r_f\right]$$

r_s = return of a single stock, s

r_f = risk free rate

β_s = beta of a single stock, s

r_M = market premium

The reward of an individual asset would be expected to depend on the contribution of the individual asset to the risk of the portfolio. The beta of a single stock measures its contribution to the variance of the market portfolio. The CAPM states that a security's risk premium is directly proportional to both the beta and the risk premium of the market portfolio, that is the risk premium = $\beta\left[E(r_M - r_f\right]$. The capital market line (CML) graphs the risk premiums of efficient portfolios (portfolios assembled of the market) and the risk-free asset, as a function of portfolio standard deviation. The standard deviation is used in CAPM as a measure of risk for efficiently diversified portfolios which the investor is considering for his or her overall portfolio (Bodie, Kane, & Marcus, 2009).

The CAPM model of Sharpe and Lintner marks the beginning of asset pricing theory. Decades later, CAPM is still widely used in applications such as estimating the cost of capital for firms and evaluating the performance of managed portfolios. The strength of CAPM is that it offers a powerful and easy way to measure risk and the relation between expected returns and risk. The drawback, however, is that the empirical record of the model is poor. Its empirical problems may reflect theoretical failings resulting from many simplifying assumptions and/or difficulties in implementing valid tests of the model (Fama & French, The Capital Asset Pricing Model, 2004). The CAPM, being a main foundation of Modern Portfolio Theory and built on Markowitz's portfolio model, has been subject to considerable criticism over the years and increasingly so. This will be discussed in more detail later in the thesis.

2.2.3 The Global Minimum Variance Portfolio

As part of our coverage of the development of Modern Portfolio Theory we will discuss shortly the main characteristics of the Global Minimum Variance portfolio (GMVP) and later introduce examples of such portfolios. It is a widely known and accepted fact that expected returns are difficult to estimate, as will be covered in more detail in subsequent chapters. As an answer to that, several papers suggest that attempts to estimate expected returns should be avoided altogether. Instead, it is assumed that all stocks have equal expected returns and therefore all stock portfolios differ only with respect to their risk. The result is that the only efficient stock portfolio is the one with the smallest risk, which is the GMVP portfolio. The construction of the GMVP portfolio is solely done on the basis of the covariance matrix of stock returns. According to Kempf and Memmel, the estimatet risk to the investor can thus be reduced, since the covariance matrix can be estimated much more precisely than the expected returns. In their research, Kempf and Memmel aim to obtain the conditional distributions of the estimated weights of the GMVP, its estimated expected returns and its estimated returns variance. These factors are very difficult to estimate according to the authors and this is in fact the model's main disadvantage. Their findings are that the conditional distributions of the estimated portfolio weights and the estimated returns parameters can be obtained by applying the OLS (ordinary least squares) methodology (Kempf & Memmel, 2003).

2.2.4 The Black-Litterman Approach

Before the Black-Litterman Approach will be discussed, which is one of the highlights in the development of Modern Portfolio Theory, we will consider various problems that are still being tackled in the theory. It is not an overstatement to say that Markowitz changed the paradigm of investment management. With him, and the ones who followed and added to his work, Modern Portfolio Theory changed the way intelligent investors discussed investments. Nonetheless, the theory has encountered some difficulties and anyone who has tried to implement portfolio optimization using market data realizes the problem. The problem is that actual implementations of portfolio theory often produce very unrealistic portfolios. For example, when investors have no constraints on their portfolio, the models almost always create portfolios with very large short positions in many assets. Another example is that when constraints rule out short positions, the models often produce portfolios with zero weights on many assets and at the same time unreasonably large positions in assets with small capitalizations. Another big problem is that the optimal portfolio asset positions and currency positions in an ordinary model are very sensitive to the returns assumptions used. Because of these problems, there are few global investment managers who in practice and on a regular basis allow quantitative models to play a big role in their asset allocation decisions (Black & Litterman, 1992). Methods have been developed to try to diminish these effects, especially in context of estimating the variance-covariance matrix. Methods such as the ”shrinkage method” may produce more reliable estimates of the covariances, as we have seen before in this paper.

In 1991, the Black-Litterman Approach was published by Fisher Black and Robert Litterman. In their approach, they sought to provide a solution to the two problems mentioned before. Their solution was to combine two established views of Modern Portfolio Theory: the mean-variance optimization framework of Markowitz and the Capital Asset Pricing Model (CAPM) of Sharpe and Lintner (Black & Litterman, 1992). The Black-Litterman Approach deals with many of the problems of portfolio optimization and is in fact a continuation of the GMVP model and an improvement of it. It starts with the assumption that investors choose their optimal portfolio from a given group of assets, such as the S&P 500 or Russell 2000 for example. This group of assets then defines the framework within which the investor chooses his portfolio and at the same time defines a benchmark portfolio. The Black-Litterman Model changes Modern Portfolio Theory in that it assumes that a given portfolio is optimal and acquires from this assumption the expected returns of the benchmark components. The starting point of the Black-Litterman Model is the implied vector of the expected benchmark returns. If the investor is happy about this market assessment he can stop right there. But if not, Black and Litterman show how the investor’s opinion can be incorporated into the

optimization problem to create a portfolio which is better for the investor (Benninga, 2008). The approach produces optimal portfolios that start at a set of neutral weights and then the views of the investor are tilt in. The investor can control how strongly his or hers views influence the portfolio weights and also which views are expressed most strongly in the portfolio (Black & Litterman, 1992).
We will later look at some examples of Black-Litterman portfolios and discuss their model's outcome in more detail.

2.3 Problems continue

Even though many efforts have been made to develop and simplify Modern Portfolio Theory, problems still persist in the basis of the theory. For decades, indeed, ever since Markowitz published his model, many writers have questioned the theory's efficiency. Numerous studies have, over the past few decades, examined the empirical performance of CAPM. These have consistently shown strong evidence of CAPM's inability to explain, and at the same time predict, the behavior of financial markets. Even so, CAPM is still the preferred model for classroom use in all major managerial finance courses (Sciubba, 2006).
As we come closer to the present date, the emphasis of the critique has increasingly been on the error in estimating means, variances and covariances in security returns. Frankfurter, Phillips and Seagle (1971) argued that their experiment on the matter showed that the impact of estimation error was so strong as to bring into question the usefulness of mean-variance approaches to portfolio selection. That was the case even though the literature dealing with portfolio theory had, at this time grown considerably since Markowitz, Tobin, Sharpe and others introduced the mean-variance approach to portfolio selection. The reason, they argued, was that the present models (in 1971) for selecting portfolios according to the mean-variance approach did not account for significant errors in estimating the required parameters. Although their results were not fully conclusive, they strongly suggested that under realistic conditions, portfolios selected using mean-variance criteria would not be more likely to be efficient than randomly selected portfolios (Frankfurter, Phillips, & Seagle, Portfolio Selection: The Effectes of uncertain Means, Variances and Covariances, 1971).
Markowitz and those who followed in his footsteps offered familiar advice, such as to diversify, along with statistical and implementation meaning to the theory, but ultimately

did not successfully address the problems which had been raised. The main problem with respect to the mechanical implementation of portfolio optimization would seem to be the fact that historical asset return data produce bad predictions for future asset returns. The estimation, from historical data, of the covariances between asset returns and the estimation of expected returns, which is fundamental to portfolio theory, often produces incredible results. It seems most writers are trying to find a way around this lack of reliability rather than accept increasingly mounting evidence of its inevitability (Benninga, 2008). Thus, the disadvantages of Modern Portfolio Theory remain very relevant, in spite of great efforts by various scholars to overcome them. In the next chapter, we will discuss a different and more fundamental kind of critique put forward against the theory; one that is driven by the belief that the conventional methods of portfolio constructing do not work at all. This is something which many writers and scholars in the field seem to have suspected for a long time, but for some reason their doubts have been pushed aside.

2.4 Does Modern Portfolio Theory work?

Until now this thesis has concentrated on explaining the main developments of Modern Portfolio Theory and how its pioneers tried to make it simpler and to adapt it to the fast moving financial world. Some important technical critiques of the theory have been discussed, but as we will now explore, the theory has come under ever increasing attack in recent years. It has even been seriously questioned whether the main foundations of the theory hold in reality and thus whether it is usable at all as a tool in real portfolio management. One point being made is that in conventional financial theory, CAPM, under well-known assumptions, arrives from rational behavior. However, lines of literature on evolution and market behavior have recently stressed that rationality is neither a sufficient nor necessary condition for survival. Blume and Easley (1992) developed an evolutionary model of a financial market, identified conditions for survival and proved false the common belief that rational behavior is always selected by market forces and irrational behavior always selected against. They showed that the optimal behavior in a risky securities market is prescribed by a logarithmic utility function, that is: traders who follow the “Kelly criterion” dominate and determine equilibrium prices. . The Kelly criterion was originally a formula used to determine the optimal size of a series of bets, published by J.L. Kelly Jr. in 1956 in the Bell System Technical Journal. Edward O. Thorp demonstrated the practical use of the formula in a 1961 address to the American Mathematical Society and in his books Beat the Dealer (for gambling) and Beat the Market (with Sheen Kassouf, for investing). The Kelly criterion has been studied in both discrete-time in Kelly (1956), Breiman (1961) and Thorp (1969) and in continuous-time in Pestien and Sudderth (1985) and Heath et al. (1987). There is also a Bayesian version of both the discrete and continuous-time Kelly criterion (Browne and Whitt 1996) (Browne, 1997). Emanuela Sciubba also conducted an experiment showing that the assets of a trader using CAPM or mean-variance models will be eliminated when an investor endowed with a logarithmic utility function enters the market. In the experiment, three types of traders were considered: CAPM traders (i.e. traders who use CAPM predictions as a rule), traders who display genuine mean-variance behavior, and traders who are endowed with a logarithmic utility function and therefore use the Kelly criterion for investment. As already mentioned, the result was that logarithmic utility maximisers dominated and determined asset prices in the long run, while the assets of CAPM traders and mean-variance traders were eliminated (Sciubba, 2006).

Many of the criticisms directed against Modern Portfolio Theory are in fact aimed at the assumptions of the efficient market hypothesis, upon which the theory relies, such as that all investors are rational, that they have access to the same information at the same time and that they aim to maximize economic utility. The validity of all these assumptions has been questioned. Another contested claim is that correlations between assets are fixed and constant over time.

It is clear, therefore, that many of the very foundations of Modern Portfolio Theory are being questioned. In the next chapter, an attempt will be made to cover the main points of criticism, which have in fact been known for decades but have enjoyed increasing support and acceptance in recent years.

2.4.1 The Efficient market hypothesis and Modern Portfolio Theory

Since many of the critiques of Modern Portfolio Theory relate to the efficient market hypothesis (EMH) the main assumptions of that hypothesis will be discussed in this chapter, as well as some of the objections put forward against it.

EMH is based on the assumption that investors are rational and risk adverse. Stock prices are expected to follow a random walk, that is, price changes should be random and unpredictable, because they are the result of intelligent investors competing to discover relevant information on which to base their decisions to buy or sell stocks before the rest of the market becomes aware of the same information. Random walk is therefore a natural result of prices that reflect all current knowledge. If changes in stock price were predictable, this would be proof of stock market inefficiency, since the ability to predict prices would indicate that all available information was not already reflected in stock prices. The notion that stocks already reflect all available information is referred to as the efficient market hypothesis (Bodie, Kane, & Marcus, 2009).

If it were possible to use past prices to predict future prices changes, investors could make easy profit. But in competitive markets, easy profits are not sustainable. As investors try to take advantage of the information of past prices, prices adjust immediately until the profit from studying past price movements disappear. The result is that all the information contained in past prices will be reflected in today‘s stock price, not tomorrow‘s price, and the share price will follow a random walk. Economists often define three levels of market efficiency, which are distinguished by the degree of information reflected in security prices. At the first level, a weak form of efficiency, it is assumed that all past price information is already included in the current price. Analysts cannot, according to this, predict future movements because all past information has already been accounted for. The second level of efficiency assumes that prices reflect not just past prices but all other published information. This is called a semistrong form of market efficiency and at this level, prices will adjust immediately to public information such as a new issue of stock or a proposal to merge two companies. Finally, there is the strong form of efficiency, in which prices reflect all the information that can be acquired by thorough analysis of the relevant company and the economy. In such a market we would not find any superior investors who can consistently beat the market, only lucky or unlucky investors. There is evidence for and against the existence of a weak and semi-strong form of efficient markets, but powerful evidence against the existence of a strong form of efficient market (Brealey, R.A., Myers, S.C. and Allen, F., 2008).

Supporters of the efficient market hypothesis believe that active management is, therefore, largely wasted effort and unlikely to justify the expenses incurred. They prefer a passive investment strategy that makes no attempt to outsmart the market. The aim of a passive strategy is merely to establish a well-diversified portfolio of securities without attempting to find under- or overvalued stocks (Bodie, Kane, & Marcus, 2009).

EMH and Modern Portfolio Theory are based on the assumption that risk is determined by volatility and that investors are risk adverse, i.e. willing to accept higher risk for higher returns and to accept lower returns on less risky assets. Risk being determined by volatility means that the greater the volatility of the portfolio, the greater the risk, although some writers have questioned this definition of risk. Various examples can be pointed out that make the trade-off between return and risk very questionable. For example, the conventional definition of volatility as risk regards both upwards and downwards movements of stocks equally. For example, there are stocks, such as speculative stocks (penny stocks are the most common), that do not fit this definition, since they are extremely volatile and give low returns. J.M. Murphy came to the conclusion in his study in 1977 that returns on low-risk securities appeared to be higher than expected given the low risk, and conversely that returns on high-risk securities appeared to be lower than expected given the high risk. He also questioned the existence of any stable relationship between risk and return, and maintained that high volatility was not compensated by greater returns (Murphy, 1977).

Eugene Fama, one of the original researchers who had been at the center of the development of the efficient market hypothesis, published a paper on risk and return in 1992, along with Kenneth French. In their paper, Fama and French concluded that the simple positive relationship between a market's β and average returns disappears during the period 1963-1990. They found only a weak relation between average returns and beta over the period 1941-1990 and no relation over the shorter period of 1963-1990. Their test therefore did not support the central prediction of the Sharpe-Linther-Black model, that average stock returns should be positively related to a market's β. However, Fama and French concluded in their paper that for the period 1963-1990, size and book-to-market equity captured the cross-sectional variation in average stock returns associated with size, E/P, book-to-market equity, and leverage (Fama & French, Cross-section of Expected Stock Returns, 1992). An extensive amount of empirical research over the last decades has provided evidence contradicting the prediction of the Sharpe (1964), Lintner (1965) and Black (1972) capital asset pricing model, which is that the cross-section of expected returns is linear in beta. Kothari, Shanken and Sloan came to the conclusion with their research that average returns do reflect substantial compensation for beta risk, if the betas are measured at the annual interval. They did not believe, however, that this meant that beta alone accounted for all the cross-sectional variation in expected returns, as implied by the capital asset pricing model. They did see evidence of a size effect, but doubted the explanatory power of book-to-market equity (Kothari, Shanken, & Sloan, 1995). S. Basu also came to the conclusion years before, in 1977, that the behavior of security prices over the 14-year period studied was not completely explained by the efficient market hypothesis. That was concluded because that low P/E portfolios did earn superior returns on a risk-adjusted basis. It seemed, therefore, that contrary to the belief that publicly available information is instantaneously impounded in security prices, lags and frictions had an effect in the adjustment process. The result was then, according to Basu, that publicly available P/E ratios seemed to possess relevant information and might warrant an investor's attention at the time of portfolio formation or revision (Basu, 1977).

According to Fama and French, the version of CAPM developed by Sharpe and Lintner has never been an empirical success. The Black (1972) version of the model, however, in which a flatter trade-off of average returns for market beta is possible, has had some success. But since the late 1970s researchers have increasingly turned to variables such as size, various price ratios and momentum to add to the explanation of average returns provided by beta. Fama and French argue that the problems of CAPM are serious enough to invalidate most applications of it. However, Markowitz's portfolio model and its offspring, the CAPM, cannot be ignored and do stand as major theoretical accomplishments, with all their fundamental concepts of portfolio theory and asset pricing which are built into more complicated models (Fama & French, The Capital Asset Pricing Model, 2004).

2.4.2 Problems with the bell curve distribution in Modern Portfolio Theory

One of the main points made by critics of Modern Portfolio Theory is that the distribution of movements of stock prices does not fit with the well-known bell curve, putting into question the alleged random walk of market movements. At the beginning of our coverage of Modern Portfolio Theory we mentioned Louis Bachelier, who in 1900 founded current market orthodoxy. Bachelier suggested that price movements on the French bond market followed a normal probability distribution. He thus argued that 68% of all price movements would be within one standard deviation of the mean, 95% would be within two standard deviations of the mean and 98% would be within three standard deviations of the mean. Bachelier's model reinforced the diversification strategy of Modern Portfolio Theory. Portfolios based on Modern Portfolio Theory use normal distribution to calculate risk (beta) for every investment in the portfolio. The portfolio is then put together in proportions that give the investor the right level of risk (Sanford, 2005). The random walk of market movements or stock price changes means that price changes should be random and unpredictable, at least in the loose sense of the concept; in fact random walk is somewhat more restrictive since it constrains successive stock returns to be independent and identically distributed (Bodie, Kane, & Marcus, 2009).

The so-called bell curve is a symmetrical graph that represents a probability distribution. It is used to great effect to describe errors in astronomical measurement, and was introduced by the 19th century mathematician Carl Friedrich Gauss (therefore also known as the Gaussian model). This model has been at the center of conventional studies of uncertainty in statistics, economics, finance and social science. Sigma, variance, standard deviation, correlation, R-square and the Sharpe ratio are all directly linked to this model. In most writings on portfolio management, information on risk in portfolios is very likely to be represented with one or more of the above mentioned quantitative measures, which are supposed to indicate the level of future uncertainty. The problem is, according to some, that the Gaussian model of probability distribution disregards the possibility of sharp jumps or discontinuities. Mandelbrot and Talbot argue that we cannot dismiss these occasional and unpredictable large deviations as „outliers“, though they are rare, because cumulatively their impact in the long term is so dramatic. The Gaussian model focuses on the ordinary and then deals with exceptions (outliers) as ancillaries. The bell curve has thin tails which means that large events are considered possible but far too rare to be consequential (fat tails representing the opposite, i.e. that large events are sufficiently common to be consequential). This is appropriate in many fields of study; for example, the odds of running into someone several miles tall can be safely disregarded. In other fields, such as finance, even very excessive observations cannot be ruled out. Economic reality has produced numerous examples of so-called wild uncertainty or wild randomness. Some obvious examples would be when, in the 1920s, the German currency moved from three to the dollar to 4 billion to the dollar in a few years and more recently, in the 1990s, when short-term interest rates jumped by several thousand per cent (Mandelbrot & Taleb, View Points of Commodity Trader, 2006). It seems that history has shown us that the seemingly improbable happens all the time in financial markets, even though the probability of it happenning had actually been estimated at something like one in 50 billion (1997 the Dow fell 7,7% in one day) or one in four trillion (July 2002, the Dow index suffered three steep falls within seven trading days). Nonetheless these unbelievable and supposedly almost impossible events do happen and often have tremendous consequences (Mandelbrot B. , 2004).

The Gaussian distribution, to summarize the argument, arises in connection with sums and averages of independent, identically distributed random variables that have finite variance. Typical financial data, on the other hand, does not fit the symmetric and light-tailed Gaussian profile and there is mounting empirical evidence which illustrates the severity of the mismatch (Goldberg, 2008).

It was actually considered an open question whether market fluctuations were normally distributed, even in the face of steadily accumulating evidence for fat tails in the distribution of price changes. According to David Rowe, market fluctuations are approximately normally distributed (and random) as long as individual market decisions are statistically independent, based on circumstances unique to each individual. But the basic assumptions for statistical normality begin to break down when there is a common influential event. In this case, observers around the world are suddenly focused on a common event with obvious directional implications for the market. In addition, everyone knows that everyone else knows as well. As a result, millions of decisions that drive the market are suddenly no longer randomly independent but subject to a common shared perception. The core structural assumptions that reinforce normal distribution have then temporarily been broken down, self-referential behavior becomes dominant and sudden extremeties are observed (Rowe, Jun 2010).

Another problem with modern financial theory is that it ignores timing. According to Mandelbrot, markets have their own sense of time, unlike random walk, and volatility clusters together in bursts of turbulence (Cookson, 2004). Mandelbrot talks about would-be reformers of the random walk model who explain the way volatility tends to cluster by asserting that the market is in some way changing, and that volatility varies because the pricing mechanism varies. This is wrong according to Mandelbrot, whose supporting examples include his analysis of cotton prices over the past century which shows the same broad pattern of price variability at the turn of the last century, when prices were unregulated, as in the 1930s when prices were regulated (Mandelbrot B. , 2004).

2.4.3 The big things matter the most

The center argument of the most influential critics of Modern Portfolio Theory is that in finance it is the big and unusual things, the so-called outliers, which matter the most. In his book The Black Swan, Nassim N. Taleb argues that people underestimate the likelihood of the unbelievable happenning - the things generally thought impossible – such as the likelihood of seeing a black swan as well as the range of shapes it might take. But why do we not acknowledge the possibility of seeing something we have never seen before? According to Taleb we concentrate on things we already know, and again and again we fail to take into consideration what we do not know (Taleb, 2007). Logical error is therefore one of many reasons Taleb gives for this, i.e. our tendency to conclude from an observed sample consisting only of white swans that there is no such thing as a black swan. Also that people recollect selectively from history about big events and mask those things with false positives, exclude failures of statistical analysis and tend to seek patterns and avoid randomness. Taleb offers plenty of explanations for this behavior ranging from arrogance to biochemistry. He argues that the biggest error is the reliance on the Gaussian model of distribution, which is at the heart of the current generation of risk management and portfolio construction tools. Gaussian risk management and portfolio construction face intrinsic limitations since no data set can ever represent all important risk factors and relevant data is inevitably scarce. As a result, any quantitative analysis in finance must be closely coupled with common sense (Goldberg, 2008).

The crash of 2008 reminded everyone that excess returns are only generated by taking on more risk, even if that risk remains hidden for a period of time. From October 2007 until March 2009 global equities fell by almost 60%, taking equity-oriented portfolios down with them, the S&P 500 was down 26% and most real money investors suffered losses in the range of 20%-40%. It is obvious to all investors that the crash of 2008 highlighted flaws in portfolio management models. According to Steven Drobny, author and financial adviser, it is clear after the crash that real money managers need to reorient their thought process and approach towards improving the portfolio construction process. A more forward looking risk-based approach should be the basis of real money portfolios, so that extreme worst-case scenarios are accounted for and dealt with in the investment process. Also, history should not be used as an indicator of the future. Real money portfolios in particular should not be constructed to fit the recent past, no matter how comfortable that may be. First and foremost, it should be the macro environment which is given the greatest weight when constructing portfolios. Events which have a low probability of occuring by definition escape most models, but this does not mean they should be ignored. On the contrary, it is vital for portfolio managers to account for potential scenarios where liquidity can disappear (Drobny & Diamond, 2010).

Mandelbrot argues that extreme price swings are the norm in financial markets and therefore cannot be ignored. Price movements follow a much more violent curve than the bell curve, making the investor's ride much bumpier than conventional theory assumes. How much depends on the appetite of the investor and his or her resources and talent. The bottom line is that the market is risky, much more risky than the conventional portfolio theory and models have implied (Mandelbrot B. , 2004). Mandelbrot's intellectual contribution is that we realize that volatility and irregularity are not just a small departure from some perfect shape, as scientists have tended to treat them, but rather the very essence of many phenomenon, including economic ones (Cookson, 2004).

2.5 What should investors use then?

In one of the previous chapters we cited Sciubba as pointing out that even though strong evidence exists of the inability of CAPM to explain and predict – and has indeed existed for decades – it is still the preferred model for thousands upon thousands of students of finance. Mandelbrot and Taleb echo this concern, observing that the inability of the bell curve to explain financial movements has long been established and yet close to 100.000 MBA students a year in the USA alone are taught to use it to understand financial markets. To quote these authors, it seems that to financial teachers some number is better than no number, even if it is the wrong number. According to Mandelbrot and Taleb a better theory is needed and does exist, namely the fractal theory of risk, ruin and return. In this approach, random jumps and concentration are the point of departure. The term fractal, coined in the 1970s by Mandelbrot, was originally used to describe the many phenomena of nature in which small parts resemble the whole, such as the veins in leaves which look like branches and rocks which look like miniature mountains. Mandelbrot and Taleb argue that similar patterns can be found in economic data: the parts often relate to the whole according to the so-called power law. A power law distribution, in market terms, means that the likelihood of a daily or weekly drop exceeding 20% can be predicted from the frequency of drops exceeding 10%; the same ratio applies to a 10% vs. a 5% drop. According to the bell curve, the likelyhood of a big drop would be so small that it would probably be ignored. However, in power law finance, big drops are not ignored but remain a real possibility, although they are naturally considered less likely than small ones. Another advantage of fractal finance, according to Mandelbrot and Taleb, is that markets have a memory of past moves, especially of volatile times, and act according to such memory. According to fractal finance, volatility causes volatility and thus it occurs in clusters and lumps. The authors maintain that this is not a difficult way to understand markets, on the contrary, it corresponds much better with reality than the bell curve approach. Even so, fractal finance has not gained much ground among academics and market players. The advice Mandelbrot and Taleb give to investors is to diversify as broadly as possible and appreciate that conventional measures of risk severely underestimate the risk; the exposure is almost always greater than people think (Mandelbrot & Taleb, How the Finance Guru Get Risk All Wrong, 2005).

In the previous chapter we noted that Sciubba, with her experiment, questioned the usefulness and efficiency of the CAPM model for investors. Her conclusion led her to a discussion of the debate on the normative appeal and descriptive appeal of the logarithmic utility approach, as opposed to the mean-variance approach in finance. Since the Kelly criterion (Kelly 1956) was published, several financial economists and mathematicians have debated whether maximizing a logarithmic utility function is a more rational method for the rational investor. According to Sciubba, the debate is due to the dissatisfaction with the mean-variance approach and its failure to single out a unique optimal portfolio. The Kelly criterion argues that a rational long-run investor should maximize the expected growth rate of his wealth share and, therefore, should behave as if he or she were endowed with a logarithmic utility function. This will lead to a unique solution to the optimal portfolio problem. Critiques of the criterion have stressed the contradiction in arguing that rational investors should maximize a utility function which is different from their own. Sciubba argues that though the approach is not perfect, it is more likely to keep you afloat in the financial markets than the conventional mean-variance approach (Sciubba, 2006). In recent years, Kelly has become a part of mainstream investment theory and the claim has been made that well-known successful investors, including Warren Buffet and Bill Gross, use Kelly's methods.

The enormous losses of 2008 crash have, according to Drobny, hopefully shed light on the true risks being assumed in conventional portfolios. He hopes, therefore, that all investors will be compelled to rethink their approach to portfolio management. His interviews with several risk managers in his book, Invisible hands, indicate that there is an increasing emphasis among real money investors on taking a more carefully risk-adjusted absolute return approach. In Drobny’s opinion, it is the duty of real money managers to at least be open to rethinking their approach if it can lead to more effective risk management, better risk-adjusted returns and smaller losses. According to Drobny, the fund managers interviewed in his book have taken the first step toward what he hopes will become an active exploration of current methods and potential solutions for real money (Drobny & Diamond, 2010).

3 Data and variable descriptions

3.1 The Data

In an attempt to conduct Modern Portfolio Models and analyze them, stocks of 17 companies in the US market, from the S&P 500 index, were collected from Yahoo finance for a period of eight-year, from the end of 2001 until the end of February this year, and subdivided into two time periods. Period 1 is from December 31, 2001 to May 30, 2006. The second period is from June 30, 2006 to February 28, 2010. Sector diversification was used as a base in selecting the companies.

To further strengthen the tests of the 2nd hypothesis (the efficiency of the market) and the 4th (expected returns vs. actual returns), historical data was also collected for 17 companies in the UK market, from the FTSE-All share index, collected from UK Yahoo finance. Same time periods were of course used and the same sector diversification was used as a base in the selection of these companies. Also, to test whether the efficiency of the market was evident in small markets as well as in large, stocks were collected from the Nordic stock exchange in Copenhagen, or from the OMX Copenhagen index that were collected from OMX Nordic stock exchange side.

It has to be stressed that this research is, of course, very small concerning the number of stocks included. Most researchers in the portfolio field use much larger quantity of stocks, even hundreds. However, this is just a small example to show the outcome of Modern Portfolio Models and try to decide, in various ways, if their results are the best asset allocation strategies in the real investment world.

3.2 Definitions and description of the main variables of the research

Before we go into the findings of the research it is essential to cover some of the main „tools“ that are used in the computation of the portfolios, expected returns and etc. While few equations will be presented, detailed mathematical descriptions will not be provided, although general definitions and some discussions will be necessary to understand some of the main points in the research.

3.2.1 The Variance-Covariance Matrix

First we should look briefly at the definition of the variance-covariance matrix and the main methods in its implementation.

As has been mentioned earlier, the covariance is a measure of how returns of two securities move together. The variance-covariance matrix is a compact way to represent the covariance of the returns of a set of securities in a portfolio. If returns of two securities move in the same direction consistently the covariance would be positive but if they move in the opposite direction consistently the covariance would be negative. Covariance would be close to zero if the movements of return are independent of each other. Covariance is a measure of interactive risk between two securities. The variance, or risk, of a portfolio is not simply a weighted average of the variances of the individual securities in the portfolio. It is the relationship between each security in the portfolio with every other security as measured by the covariance of return that has to be considered too. The return and risk of a portfolio depends on two set of factors. The returns and risk of individual securities and the covariance between securities in the portfolio, and the proportion of investment in each security. The investor has no control over the returns, risks and the covariance of individual securities but he/she can choose the proportions of each security in the portfolio (Kevin, 2001). In order to calculate efficient portfolios, it is necessary to compute the variance-covariance matrix from return data for securities. The most distinct calculation of the variance-covariance matrix is the *sample variance-covariance matrix*, which is computed directly from the historic returns. There are few alternatives in computing the sample matrix, which we will not go into here. While the sample matrix is the most straightforward method, it is recognized that it may not be the best estimate of variances and covariances. The reason for this is its often unrealistic parameters and its inability to predict. Therefore alternatives have been formed and developed, like the single index model and the constant-correlation model. All of those methods have two common factors. First, they leave the variance alone and compute them from the sample variance. Second, they change the covariance elements of the variance-covariance matrix (the off diagonal elements).

The shrinkage method is the third class of methods of calculating the variance-covariance matrix. It has in the last few years been gaining popularity. The method assumes that the variance-covariance matrix is a convex combination of the sample covariance matrix and some other matrix (Benninga, 2008).

3.2.2 Alpha

Alpha, or Jensen‘s alpha (1968), in connection to constructing optimal portfolio is a risk-adjusted performance measure that adjusts expected or average returns for beta risk. (Nielsen & Vassalou, 2004) Alpha in the regression equation is, put in simple terms, a return a portfolio is attaining over a comparing investment, an index, taking risk also into consideration. Alpha is the active components of an investment and typically represents either market timing or security selection (Scott, 2009). Alpha of a security is therefore the component of a securities return that is independent of the market‘s performance, or a random variable. In other words it represents that component of return insensitive to the return on the market (Elton, Gruber, Brown, & Goetzmann, 2007).

Investor’s goal is getting better return on their investment selection than they would get investing in an index, also taking risk into consideration. Getting positive alpha means that you are beating the market (Gupta & Straatman, 2005). Alpha is therefore a measure of whether or not an asset beats the market on risk adjusted basis (Gorman & Weigand, 2007). Alpha is the return associated with an asset for exposure to non-systematic (idiosyncratic) risk. When looking at alpha we are interested in the average value of the firms return net of the impact of market movements (Bodie, Kane, & Marcus, 2009).

Alpha of a portfolio, relative to an index or benchmark x, is defined as $\alpha = Er_p - r_f - \beta(Er_x - r_f)$, where r_f is the riskless rate, Er_p and Er_x is the expected rate of return on the portfolio p and the index x, and $\beta = \mathrm{cov}(r_p, r_x)/\mathrm{var}(r_x)$ is the beta of the portfolio with respect to the index. If the index x is efficient, then the true alpha of every security and every portfolio will be zero, though estimated alpha may be different from zero because of estimation error. Alpha can, however, be calculated and given a precise interpretation in terms of portfolio optimization even if the index is not efficient (Nielsen & Vassalou, 2004).

The nonmarket return component of the return of the portfolio, R_P, that is the alpha of the portfolio, α_P, is the average of the individual alphas of the stocks included in the portfolio. That is

$$\alpha_P = \frac{1}{n}\sum_{i=1}^{n}\alpha_i$$

The regression statistics for the SCL of the firm (or stock in question) shows the intercept, which is the estimate of the stocks alpha for the sample period. It has to be considered whether the alpha coefficient is statistically insignificant or not. This can be seen from the three statistics which are next to the estimated intercept coefficient in the regression statistics table for the SCL of the stock. First is the standard error of the estimate, which is the measure of the imprecision of the estimate. If it is large, the range of likely estimation error is equivalently large. Second is the t-statistic, which is the ratio of the regression parameter to its standard error. The t-statistic equals the number of standard errors by which our estimate exceeds zero. It can therefore be used to assess the probability that the true value might actually equal zero. The idea is that if the true value were zero, it would be unlikely that we observe estimated values far away from zero. Large t-statistics, therefore, imply low probabilities that the true value is zero. The third value is the p-value, which is the level of significance. The conventional cut-off for statistical significance is a probability of less than 5% (which requires a t-statistic of about 2,0) (Bodie, Kane, & Marcus, 2009). We will see a practical example of the use of regression statistic in connection to alpha estimate later, or when one of the hypotheses of this paper will be tested.

It has to be stressed that alpha alone does not say how much the investor should optimally invest in a stock or fund, for example. The variance that is distinctive to the stock or fund also matters. There the connection between alpha, beta and Sharpe Ratio for example, can be seen, because if the investor puts too large fraction of his wealth into particular stock then the distinctive risk may result in a Sharpe Ratio and a lower expected utility (Nielsen & Vassalou, 2004).

3.2.3 Beta

Beta in the return of the security equation, that is beta of the security, is a constant that measures the expected change in the security given the change in the return of the market index, measured in standard deviations. In other words the beta of the security measures how sensitive a security‘s return is to the return of the market (Elton, Gruber, Brown, & Goetzmann, 2007). Beta is a statistical coefficient estimated via linear regression that describes how a particular assets return is influenced by the return associated with a systematic risk factor. Beta is a scaled measure of the correlation of returns between the asset and the systematic risk factor (Gorman & Weigand, 2007).

Formally, beta of a asset or stock is defined as $\beta_i = Cov\ (r_i,\ r_m)/\sigma^2_M$. Where the r_i is the rate of return of the stock, r_m is the rate of return of the market index (like S&P 500) and σ^2_M is the variance of the market index. The return of the portfolio, R_P, has the sensitivity to the market, that is β_P, the same as the average of the individual betas of the stocks included in the portfolio. That is

$$\beta_P = \frac{1}{n}\sum_{i=1}^{n}\beta_i$$

The use of a Single Index Model demands an estimate of the beta of each stock which is under consideration to be included in a portfolio. Therefore, the beta estimates for individual securities determine, in part at least, which securities will be selected for inclusion in investment portfolios when an optimization algorithm such as the Sharpe approach is used (Phillips & Seagle, 1975). There is evidence, according to Elton and co-writers, that those historical betas provide useful information about future betas. Firms should therefore, at least to start with, use the best estimates of beta available from historical data and various techniques are available to do that (Elton, Gruber, Brown, & Goetzmann, 2007).
The regression statistics for the SCL of a stock in question shows the beta estimate for a stock compared to a market index (like S&P 500). If, for example, this beta estimate is 2,0 then the beta estimate for that particular stock is twice that of the market index used. In the same way as was described before in the matter of alpha, the statistical significance of the estimate can be seen with the three statistics next to the beta coefficient, which is standard error, t-statistics and p-value (Bodie, Kane, & Marcus, 2009).
Like with the alpha, we will see practical use of the regression statistics table in connection to beta estimate when one of the hypotheses of this paper will be tested.

To summarize the alpha and beta coverage, the alpha is unrelated to the market movements and positive alpha is a sign of a good choice of investment. Beta, on the other hand, is related to market movements and the higher the beta is the more volatile the stock is to market movements. However, beta and alpha are related since the estimation of the beta will have an effect on the value of alpha, for example, overestimating alpha will underestimate beta (Tofallis, 2008). Because of this it can be speculated that market movements are indirectly related to alpha through beta.

3.2.4 Sharpe Ratio

The Sharpe Ratio, or reward-to-variability ratio, is a measure of risk-adjusted performance that uses a benchmark based on the (*ex post*) capital market line (CML). The ratio measures return relative to the total risk of the portfolio, where the total risk is the standard deviation of portfolio returns. It is necessary to determine the location of the CML in order to use the Sharpe ratio. There are two points on the CML graph that measures average return (a_M) on the vertical axis and standard deviation (σ_M) on the horizontal axis. Because the CML goes through these points, its slope can be calculated as the vertical distance between the two points divided by the horizontal distance between the two points. Once the location of the CML has been found, the average return and standard deviation of the portfolio can be calculated. When those values are known the portfolio can be located on the graph as the CML. The calculation of the Sharpe ratio for the portfolio being examined involves dividing the portfolio's average excess return by its standard deviation, that is

$$SR_P = \frac{ar_P - ar_f}{\sigma_P}$$

ar_P = average return of the portfolio

σ_P = standard deviation of the portfolio

ar_f = average return of a risk-free rate

Because the CML stands for various combinations of risk-free lending or borrowing with investing in the market portfolio, it can be used as a benchmark for the Sharpe Ratio. If the SR_P is greater than the value of the slope of the CML, the portfolio being examined lies above the CML, which means that it has outperformed the market. On the other hand if it is less than the value of the slope of the CML, the portfolio lies below the CML, indicating that it has not performed as well as the market (Sharpe, Alexander, & Bailey, 1999).

Thus, according to mean-variance portfolio theory, if investors face an exclusive choice among a number of funds, then they can rank them on the basis of their Sharpe ratios. A stock, for example, with higher Sharpe ratio will enable investors to achieve a higher expected utility (Nielsen & Vassalou, 2004).

4 Research Design

The main objective of this thesis is to answer this question: Are the optimal portfolios suggested by the Modern Portfolio Theories the best asset allocation strategies in real investment world?

The first part of the research concerns the constructing of portfolios with methods that are based upon Modern Portfolio Theory. Selection of the companies (stocks) that will be used to form the portfolio will first be shown and explained. Historical prices from these stocks will be collected and returns calculated. Three models will be used in the portfolio construction, the GMVP model, the Black-Litterman approach and the efficient portfolio. The Black-Litterman approach is based on the efficient portfolio model and is supposed to be an improved version of it, as has been mentioned before. In all of the models the Single-Index Model will be used to estimate the variance-covariance matrix. Then the portfolios are constructed according to each model's method and the outcome of the portfolio construction will be displayed and discussed. A comparison will be made between the three models and also between the two time periods. Statistics like mean average, standard deviation and Sharpe ratio concerning the portfolios will also be examined, compared and discussed. The main aim of this portfolio construction is to show the outcomes of these models, which are based on Modern Portfolio Theory, and try to decide whether their positions and other various statistics are realistic numbers. That is, to test the first hypothesis whether Modern Portfolio Models based on historical return data are realistic in their prediction of future optimal portfolios for the subsequent periods.

In effort to seek an answer to the main research question in a more statistical way, the remainder of the hypotheses, that were stated in the introduction of this paper, will be tested and discussed. Methods used in each analysis will be explained along with the results. All calculations, including regression analysis, were conducted in Excel. For guidelines in the construction of the portfolios with the three models mentioned above, the book Financial Modeling by Simon Benninga (2008) is used.

5 Empirical results

5.1 Selection of companies

The selection of stocks in this research was based at diversifying between different market sectors. To diversify a portfolio by looking at stocks in different market sectors is a more hands-on approach. It is a fact that money does not typically flow into particular stocks alone but to the entire market sectors. For example, if the healthcare sector is growing, money will typically flow into the entire sector to take advantage of the strong performance and growth that the entire sector is enjoying. Some stocks will attract more attention than others but the sector as a whole will benefit from the flow of money into it. By owning stocks in different sectors the investor is hedged at some level if one sector goes down, he/she has investments in many other sectors that may be doing well at the same time (Hansen). In their study, Chan, Karceski and Lakonishok, came to the conclusion that the correlation between two stocks is on average larger when they are from the same industry than when they belong to different industries. The difference, they concluded, was however not the same between industries, which suggests that some industries are more homogeneous groupings than others. Also they discovered that differences between the within-industry and across-industry correlations stand out even more strongly for larger firms. The correlations between small firms are lower, even when they share the same industry, and less sharp average correlation between the within-industry and across-industry (Chan, Karceski, & Lakonishok, 1999).

Below is a table of the stocks that were selected for the construction of the portfolio part of this analysis, that is to test the first hypothesis stated in the introduction (see page 6). The stocks selected for this test are all US market stocks (the S&P 500 index). In the table it can be seen which sector the selected companies belong to, their market cap, PE ratio, beta and earnings per share at the time when the research was conducted (collected from Yahoo Finance, February 2010).

Table 1: Stocks selected from the S&P 500 and key statistics.

Companies	Sector	Market Cap (B$)	Trailing P/E	Beta	EPS
NIKE	Consumer Goods	31,5	21,28	0,88	3,04
Apple	Technology	184,28	32,53	1,58	6,29
ITT	Industrial Goods	9,39	14,93	1,12	3,44
JP Morgan	Financial	170,91	27,06	1,1	1,20
Life Tec	Healthcare	8,89	N/A	0,83	-0,36
IBM	Technology	167,89	13,17	0,81	9,71
Moody's Corporation	Financial	5,57	14,4	1,3	1,64
Humana	Healthcare	7,1	7,33	1,32	5,71
Boeing	Industrial Goods	37,75	N/A	1,26	-0,07
Exxon	Basic Materials	360,28	17,68	3,29	0,18
Public Service Ent. Group	Utilities	15,92	10,8	0,57	2,92
Starbuck	Service	16,17	41,53	1,35	0,52
Cooper Industry	Conglomerates	7,1	18,16	1,65	2,34
Xcel Energy	Utilities	9,25	13,73	0,44	1,48
Clorox	Consumer Goods	8,45	15,06	0,38	4,00
Safeway	Service	9,25	11,32	0,69	2,00
Tesoro Petroleum Corp.	Basic Materials	1,84	15,85	1,31	0,98

As is evident from the table, the companies selected come from various market sectors, as sector diversification was the basis of how the stocks that should form the portfolio were selected. At the same time the companies selected have very different market capitalization, betas and earnings per share. It seems, therefore, that the companies are of various sizes and values, have different volatilities and position in the stock market.

5.2 Constructing a portfolio based on Modern portfolio theory

The next step in the research is to construct a portfolio with two of the models that were covered in the previous chapters, the Global Minimum Variance Portfolio (GMPV) and the Black-Litterman approach, along with the ordinary efficient portfolio for comparison. The objective is to demonstrate in black and white what positions these portfolios come forward with and try to conclude if they are realistic or not. Also, since they are in one way or the other based on historical stock returns, it is interesting to see if there is some similarity between them. At least they are all developed to do the same, which is to construct the optimal portfolio.

Like we saw in the previous chapters, the GMPV and the Black-Litterman are models that have been developed to answer the critiques of Modern Portfolio Theory, while the efficient portfolio is more of the original mean-variance approach. The GMPV and the Black-Litterman have been thought to be more realistic in proportions than other models with their huge short and long positions and more problems with the estimation of the expected return. In connection to that it has to be stated that the Black-Litterman is a part of the efficient portfolio approach, although an advantaged edition of it. As has been mentioned before, the periods have been divided into two parts and we will examine the outcome and compare them between the two periods and between the three models. In the table below are the positions of the GMVP model, the Black-Litterman model and the efficient portfolio. All are conducted on the base of the same historical returns of stocks above from the 31st of December 2001 until the 31st of May 2006, or period 1 as we will call it here.

Table 2: The GMVP, Black-Litterman and efficient portfolios for period 1.

Companies	GMVP Period 1	Black-Litterman Period 1	Efficient portfolio Period 1
NIKE	14,17%	14,56%	-495,93%
Apple	-0,35%	17,14%	242,89%
ITT	15,78%	0,81%	-322,13%
JP Morgan	-6,60%	14,46%	-175,94%
Life tec	4,68%	0,81%	-34,43%
IBM	-7,00%	14,10%	-204,35%
Moody's Corp.	0,05%	0,57%	385,44%
Humana	4,56%	0,68%	50,65%
Boeing	8,90%	4,27%	601,25%
Exxon	15,17%	26,53%	-258,67%
Public service Ent. Group	11,63%	1,32%	140,99%
Starbuck	8,13%	1,54%	315,10%
Cooper industry	3,78%	0,67%	-98,93%
Xcel energy	3,65%	0,82%	-137,17%
Clorox	22,39%	0,75%	353,71%
Safeway	2,65%	0,82%	-413,17%
Tesoro Petroleum Corp.	-1,58%	0,16%	150,70%

As is evident from the table, in some cases among the GMVP and Black-Litterman the positions are similar but totally different in others. The good thing about these two portfolios, GMVP and Black-Litterman, is that they do not conduct enormous positions like many models do, unrealistic positions both long and short of hundreds of percentages. However that seems to be the case with the efficient portfolio. For the individual investor, these kind of positions do not make any sense and even for a large investment company it does not either. Raising capital for such large positions would be almost impossible, apart from the fact that they make no sense. The efficient portfolio gives so absurd positions that it is obvious that it is of no use for anybody, at least not in this case. The other portfolios look much more believable. But do they work in reality? That depends on the accuracy of the prediction of expected returns, which the portfolios are based on. Like we talked about before, the GMVP does not estimate expected returns like most other portfolio models. Instead it assumes that all stocks have equal expected returns. But it relies on the covariance matrix in its construction and that is based on historical data and therefore all the disadvantages of that are likely to come along. Also the distribution of the estimated portfolio weights and estimated return parameters are based on the normal distribution, which we have discussed, is one of the main points in the critiques against Modern Portfolio Theory. In the Black-Litterman Model the investors' opinion can be implemented into the construction of the portfolio. That is, the investor's opinion whether an asset in the portfolio differ from the market return. This opinion, if implemented, will translate to an opinion about all other asset returns, because the asset returns are correlated. In this particular portfolio the estimation that Nike will have monthly return that is 0,8%, or 0,1073% higher than the market return of 0,6927%. This has an effect on the returns of the other stocks in the portfolio as well because of the covariance between them. The comparison between the GMVP and Black-Litterman is a little defective because there are no short positions in Black-Litterman. Nevertheless, there is a huge difference in the positions in at least 7 cases, for example Apple, with a short position in GMVP but the second largest long position in Black-Litterman. The only positions which is almost exactly the same in both models is Nike, which would probably been with a lower position in Black-Litterman if it was not for the investor's opinion of a higher return for that company.

If the second period, from the 30th of July 2006 until the 28th of February 2010, is used to predict and construct a portfolio a new positions are computed by the models, with some proportions looking similar to the former period but others totally different. The portfolio based on historical stock prices of the second period can be seen in the table below, constructed with the three models like before.

Table 3: The GMVP, Black-Litterman and efficient portfolios for period 2.

Companies	GMVP Period 2	Black-Litterman Period 2	Efficient portfolio Period 2
NIKE	6,48%	12,66%	-53,35%
Apple	-2,01%	17,52%	-60,85%
ITT	1,14%	0,83%	55,27%
JP Morgan	-0,58%	14,79%	5,44%
Life tec	3,81%	0,83%	21,65%
IBM	10,83%	14,42%	-99,39%
Moody's Corp.	-0,03%	0,58%	26,16%
Humana	-1,90%	0,70%	-38,20%
Boeing	-2,32%	4,37%	15,27%
Exxon	27,74%	27,12%	84,09%
Public service Ent. Group	8,92%	1,35%	23,75%
Starbuck	-2,05%	1,57%	66,91%
Cooper industry	-13,50%	0,68%	-28,51%
Xcel energy	27,25%	0,84%	-20,58%
Clorox	26,70%	0,76%	57,95%
Safeway	10,60%	0,84%	18,59%
Tesoro Petroleum Corp.	-1,10%	0,17%	25,79%

The difference between models is similar as before. The short positions in GMVP are much more frequent in the second period, 8 out of 17, almost half of the portfolio which is not very realistic. Especially when short positions are not that common in reality. Models can though be, of course, restricted so that short position are not possible. Of course it seems realistic that with new data as a base different positions in the portfolio will be predicted, especially in the light of the crisis that started in 2008 and the economic depression resulting from that. The biggest changes in GMVP between periods with much larger long positions in IBM (which was recommended as short sale in the former period, by that it was expected that the stocks of IBM will fall). The positions for at least half of the companies have also changed significanly and are either much smaller (even from long to short) or larger than before. There is no pattern in which market sector the companies with largest changes belong to.

The positions in the efficient portfolio have also changed, but mostly so that they are more realistic now in period 2 than in period 1. At least now the positions do not go over one hundred percent. But even so they are too large in both directions and are, therefore, unrealistic. The Black-Litterman positions are, however, almost the same compared to the former period.

So the results are that the models are very different in their construction of portfolios even though they are based on the same historical stock returns in some way. Especially is it interesting that while the other two models change a lot between periods the Black-Litterman does hardly change at all. Also, it seems obvious that the efficient portfolio appears to be totally useless. The others seem to be more realistic in their positions, but very different from each other. It sounds realistic that during the second period that stock prices of many companies have changed and because the portfolio is based on these historical stock prices the positions have changed. The second portfolio was constructed on the historical stock prices from the financial crisis time period from 2008 until current day, with all the falls in stock prices etc. And even so it is not nearly certain that the mean-variance approach did take the financial crash into account, like the critics of Modern Portfolio Theory have been stressing. It is difficult for the investors to assess whether that is so which makes it difficult to avoid loss. Also, what if an investor would construct his or hers portfolio based on the historical data of stock prices from the years just before the financial crash of 2008? Is it likely that the historical data of that period would predict accordingly to the situation that developed in 2008 and is still emerging?

The base of the Modern Portfolio Theory is to predict from previous movement of stock prices the future stock prices and acceding to that construct the best mixture of stocks in a portfolio. We have examined the construction of a portfolio with three types of models in two adjoining periods and tried to discuss whether the model‘s portfolio construction could be near reality and effective for the investor. The bottom line seems to be that so many writers and investors have serious doubts that the prediction of Modern portfolio theory is accurate and, first and foremost, if it can serve the investors that use these models effectively. It is hard to tell from mere portfolio construction if that is so, at least concerning the prediction ability of the models, which is the main base of their findings. It is though obvious from the portfolio models before that there is no consistence among them, even though their base is similar, which makes their usefulness very questionable.

In next section, before the rest of the hypothesis of will be tested, the portfolios constructed before will be compared to the market index, S&P 500, in both time periods. Also some basic statistics for both periods will be computed. That kind of calculations can tell us more about the quality of the portfolios, at least according to the Modern portfolio theory which these calculations are all part of. Below are the results of these calculations for period 1.

Table 4: Basic statistics and Sharpe Ratio of the portfolios for period 1.

Period 1	GMVP	Black-Litterm.	Efficient
Mean return	1,1708%	0,9585%	36,4811%
Variance	0,00075	0,00240	0,27883
Standard dev.	2,7406%	4,9006%	52,8044%
Beta	0,37894	1,07040	-4,42880
Risk-free rate	0,411%	0,411%	0,411%
Sharpe ratio	0,2772	0,1117	0,6831
Sharpe ratio S&P	-0,0628	-0,0628	-0,0628

In period 1 the GMVP and Black-Litterman are not so different from each other, but like before the efficient portfolio is totally different in the general statistics. The mean returns are similar, but the efficient portfolio gives an absurd mean return of over 36%, which is very unlikely. Even though GMVP and Black-Litterman have similar mean return, the Black-Litterman portfolio has much higher standard deviation. That means its expected returns have greater risk or uncertainty. Not to mention the standard deviation of the efficient portfolio of almost 53%. The betas of the portfolios are somewhat different as well. The beta of a portfolio, like we discussed before, measures how sensitive the portfolio's return is to the return of the market. If the beta of a portfolio is 0 then the price (returns) are not at all correlated with the market. A negative beta implies that the portfolio is moving in an opposite direction compared to the market. A positive beta means that it follows generally the market, and the market beta is always considered to be 1. The Black-Litterman Portfolio does, therefore, follow the market quite well, with beta just abit over 1. The GMVP does not follow the market nearly as well, although it is positive and therefore not completely uncorrelated to the market index. The efficient portfolio has a negative beta, which means that it inversely follows the market in its prices.

The Sharpe Ratio of all the portfolios is higher than the Sharpe Ratio of the S&P 500 index. That indicates, at least according to the numbers conducted by the Sharpe measure that these portfolios perform better than a riskless portfolio. So in all cases the portfolios are outperforming the market index according to the Sharpe Ratio, and like before the efficient portfolio is more extreme than the others. Here below we see the same statistics and the Sharpe Ratio for the second period.

Table 5: Basic statistics and Sharpe Ratio of the portfolios for period 2.

Period 2	GMVP	Black-Litterm.	Efficient
Mean return	0,0405%	0,4711%	-4,7911%
Variance	0,00082	0,00285	0,00630
Sigma	2,8683%	5,3427%	7,9384%
Beta	0,21855	0,85739	0,11715
Risk-free rate	0,373%	0,373%	0,373%
Sharpe ratio	-0,1159	0,0184	-0,6505
Sharpe ratio S&P	-0,1608	-0,1608	-0,1608

As can be seen from the two tables the numbers in the second period is much lower than in the former. The mean return is a bit lower, and even negative in the efficient portfolio. Standard deviation is pretty high in Black-Litterman and the efficient portfolios, but almost the same as before in the GMVP. The Sharpe Ratios of the portfolios are now not all higher than of the S&P 500. The efficient portfolio has now a lower Sharpe Ratio, which means that it is no longer outperforming the risk-free assets. The others are still higher and are therefore still beating the market in the second period.

To summarize the portfolio constructions and the calculations connected to them, the portfolios seem to be giving realistic positions, apart from the efficient portfolio. The statistics indicate that the GMVP and the Black-Litterman have similar mean returns in both periods but the Black-Litterman has a considerably higher standard deviation in both cases.

The efficient portfolio is again totally different from the others, with an absurd mean return and standard deviation in the first period and a negative mean return in the second.
It is interesting that the models give such different positions when they are all based on the same historical data and come from the same basic theory, even though they are constructed in a different way. That can be an indication of the various flaws in the whole theory when the results of the models are so inconsistent. Of course the models are different in various ways, as mentioned before, so totally identical outcome could naturally not been expected, but some consistency would be expected if it is really working to use historical data and the Modern Portfolio Theory to construct a future portfolio. That will be the task of the next chapters, to try to shed some light on whether the portfolios contain the fundamental elements that the theory is based on.

5.3 The efficiency of the market

The question that will be attempted to answer in this chapter is if there are any abnormal returns in each individual stock in the portfolios that were constructed, compared to the market index (like the S&P 500, the benchmark). This will indicate whether the market is efficient or not, that is if stocks are overpriced or underpriced. This can be examined by looking at the alpha of each of the stocks in the portfolio. The excess return of a stock relative to the return of a benchmark index (like S&P 500) is the stock's alpha. Statistical tests will be done to see if the alpha of each stock is equal to zero or not, which will tell us whether the stocks used in constructing the portfolio are making abnormal excess returns compared to the market index. To cover this the following hypothesis will be tested:

H0: True value of alpha (α) = 0

H1: True value of alpha (α) ≠ 0

In this test it is assumed that the null hypothesis holds, because if it didn't the market wouldn't be efficient. For a short time interval the alpha value could be something other than zero (positive or negative), but in the long run in efficient markets there should not be significant alpha values.

The alpha value of the stocks was estimated by computing a regression analysis on excess returns for each stock that were used in constructing the portfolios and the S&P 500 in period 1. It was decided to test only the alphas in period 1, mostly because that was the period that was used to predict returns and test that estimation statistically, which will be presented later in the thesis. As was stated earlier, the values for each individual stock was calculated and then the t-statistics in ANOVA results was used to see whether the alpha is significantly different from zero. The results of this test can be seen in the table below:

Table 6: Alpha coefficients and statistical results of the S&P 500 stocks.

	Test of Alpha				
Companies	**Coefficient**	**St. Dev**	**T - stat**	**P value**	**Significant**
NIKE	0,0035	0,0083	0,4248	0,6728	No
Apple	0,0313	0,0139	2,248	0,0289	Yes
ITT	0,0101	0,0074	1,364	0,1786	No
JP Morgan	0,0042	0,0088	0,4771	0,6353	No
Life tec	-0,0044	0,0142	-0,3066	0,7604	No
IBM	-0,0077	0,0075	-1,0245	0,3105	No
Moody's corp	0,0122	0,1888	0,0647	0,9486	No
Humana	0,0252	0,0138	1,8246	0,0739	No
Boeing	0,0111	0,0093	1,1962	0,2372	No
Exxon	0,007	0,0067	1,0424	0,3022	No
Public Service Ent. Group	0,0045	0,008	0,5566	0,5803	No
Starbuck	0,0222	0,0107	2,0658	0,0439	Yes
Cooper industry	0,0142	0,0085	1,6762	0,0998	No
Xcel energy	-0,0084	0,0197	-0,4307	0,6685	No
Clorox	0,003	0,0071	0,427	0,6712	No
Safeway	-0,0095	0,0111	-0,8576	0,3951	No
Tesoro Petroleum Corp.	0,0287	0,0244	1,2782	0,2069	No

These are the alpha coefficients for each stock in period 1. The 5% significance level was used and, therefore, in order to significantly deviate from zero the P-value of a stock has to be under 0,05. Then the stock has an alpha that is different from zero. However, as can be seen from the table, most of the stock's P-value is over the 0,05 mark and therefore it can be concluded that the majority of the stocks were not statistically significant. Only Apple and Starbucks were significantly different from zero which means that, for those two stocks particularly, the true value of alpha is not zero and they have abnormal returns. But because the overwhelming majority of the alphas are not significantly different from zero the null hypothesis (H_0 hypothesis) that the true value of alpha is zero for the portfolio cannot be rejected. This means that the portfolio has almost no abnormal returns and the market is efficient. Of course in this research the number of stocks is very small and, as has been mentioned before, it is questionable whether it is possible to conclude about the efficiency of the market with such few cases. At least for this portfolio, with only 17 stocks, the market seems to be efficient.

To strengthen the analysis of alpha and the efficiency of the market the same test was made on 17 stocks that are a part of two other markets, The London Stock Exchange (LSE), or the FTSE All-shares index, and the Nordic stock exchange in Copenhagen, or the OMX Copenhagen (Cap GI). Section diversification was also the basis of selection of stocks in both cases but otherwise the stocks were selected randomly. The same period was analyzed as before with the US stocks from the S&P 500 index or from December 2001 until May 2006. The selection of the stocks in these markets was of course restrained in a way that the historical data available for the stocks had to reach back to the beginning of that period. The results of the alphas for FTSE-All shares stocks can be seen in the table below:

Table 7: Alpha coefficients and statistical results of the FTSE-All shares stocks.

	Test of Alpha in FTSE- All Shares				
Companies	**Coefficient**	**St. Dev**	**T - stat**	**P value**	**Significant**
BP plc	0,0048	0,0065	0,7353	0,4655	No
BT Group plc	-0,0039	0,0080	-0,4881	0,6276	No
Diageo plc	0,0031	0,0063	0,4933	0,6239	No
HSBC holdings plc	0,0000	0,0043	-0,0155	0,9877	No
Lloyds Banking group	-0,0097	0,0083	-1,1741	0,2458	No
Pearson plc	-0,0004	0,0084	-0,0480	0,9619	No
Signet Jewelers	-0,0023	0,0093	-0,3200	0,7503	No
Smith & Nephew	-0,0024	0,0091	-0,2624	0,7941	No
Electrocomponents	-0,0163	0,0100	-1,6359	0,1080	No
Scot.&Sth.Energy	0,0080	0,0056	1,4400	0,1560	No
Sage GRP.	-0,0017	0,0091	-0,1821	0,8562	No
Reed Elsevier	-0,0047	0,0053	-0,8913	0,3769	No
Imperial tobacco GRP	0,0078	0,0091	0,8666	0,3902	No
GlaxoSmithKline	-0,0062	0,0068	-0,9210	0,3614	No
Compass Group	-0,0173	0,0104	-1,6595	0,1032	No
Dairy Crest Group	-0,0012	0,0109	-0,1126	0,9108	No
BHP Billiton	0,0183	0,0090	2,0276	0,0478	Yes

The alpha coefficients of FTSE-All shares stocks can be seen here above in the first column. The 5% significance level was used and like before with the S&P 500 stocks the overwhelming majority of the alphas of the stocks are not statistically significant, i.e. are not significantly different from zero. Almost no abnormal returns existed in these stocks compared to the market index, with the exception of only one stock, BHP Billiton, which is barely significantly different from zero. This means that almost all of the stocks' alpha have a true value of alpha equal to zero and that the market is therefore efficient in this case as well.

In the case of OMX Copenhagen the purpose of looking at that market as well was to see if a smaller market would yield different results concerning efficiency. Below are the results of that test.

Table 8: Alpha coefficients and statistical results of the OMX Copenhagen stocks.

	Test of Alpha in OMX Copenhagen				
Companies	**Coefficient**	**St. Dev**	**T - stat**	**P value**	**Significant**
Bang & Olufsen	0,0029	0,0107	0,2685	0,7894	**No**
Genmab	-0,0309	0,0248	-1,2444	0,2189	**No**
IC Company	0,0224	0,0159	1,4098	0,1646	**No**
Novozymes B	0,0041	0,0068	0,6073	0,5463	**No**
SimCorp	-0,0018	0,0141	-0,1268	0,8996	**No**
Torm	0,0145	0,0197	0,7362	0,4649	**No**
TDC	-0,0202	0,0169	-1,1943	0,2378	**No**
Kobenhavns Lufthavne	0,0106	0,0097	1,0961	0,2781	**No**
Danisco	-0,0044	0,0068	-0,6562	0,5146	**No**
Danske Bank	-0,0002	0,0054	-0,0422	0,9665	**No**
Novo Nordisk B	-0,0150	0,0113	-1,3341	0,1880	**No**
Aarhus Lokalbank	0,0220	0,0062	3,5622	0,0008	**Yes**
D/S Norden	0,0312	0,0185	1,6896	0,0971	**No**
Greentech Energy Systems	0,0289	0,0201	1,4404	0,1557	**No**
Flugger B	0,0218	0,0078	2,8059	0,0070	**Yes**
Andersen & Martin B	0,0232	0,0133	1,7464	0,0866	**No**
RTX telecome	-0,0204	0,0170	-1,1949	0,2376	**No**

Similar to the results in the other two markets, almost all of the stocks selected have an alpha that is not significantly different from zero with a 5% significance level. Two exceptions are here, that is the Aarhus Lokalbank and Flugger B, which are the same results as in the S&P 500 stocks in the US market. But as the overwhelming majority of the OMX Copenhagen stock's, they have a true value of alpha equal to zero. This means that this market as well must be considered efficient.

5.4 Connection between volatility and average return

The third hypothesis tested in this research is a test of CAPM, which states that higher volatility means higher average returns, which means that stocks with similar beta have similar average return and vice versa. A simple comparison of the beta coefficients (β) of the stocks and their average return will be conducted followed with more formal CAPM test of the matter. In the table below the beta of the S&P 500 stocks and their average monthly return can be seen.

Table 9: Average monthly returns and beta of the S&P 500 stocks.

Companies	Average monthly return	Beta
NIKE	0,672%	0,44
Apple	3,202%	1,40
ITT	1,368%	0,56
JP Morgan	0,301%	1,80
Life tec	0,054%	0,44
IBM	-0,782%	1,69
Moody's corp	1,820%	0,29
Humana	2,750%	0,54
Boeing	1,441%	0,63
Exxon	0,827%	0,73
Public Service Ent. Group	0,778%	0,64
Starbuck	2,490%	0,48
Cooper industry	1,767%	1,09
Xcel energy	-0,727%	0,10
Clorox	0,884%	0,43
Safeway	-1,078%	1,10
Tesoro Petroleum Corp.	3,109%	2,03

As can be seen from the table many of the stocks have a beta of around 0,3-0,7. If the CAPM assumption holds it should be evident that these stocks, with similar betas, should have similar average returns. It is obvious from the table that this is not the case. For example, Nike and Starbuck have very similar betas of 0,44 and 0,48 respectively. However their average monthly return is not as similar. Nike has an average monthly return of 0,672% but Starbucks' average return is 2,49%. Also, JP Morgan has, relatively to the others, high beta of 1,8 (more volatility) and should therefore, according to CAPM, have higher average return compared to the other stocks. That is not the case because it's average return is only 0,3%, while Humana has an average return of 2,75% and beta of only 0,54. It almost does not matter in what way the betas and the average returns are compared it is not consistent with the above assumption (the 3rd hypothesis). Tesoro Petroleum corp. seems to be one of the very few exceptions, with a high beta of 2,03 and the second highest average return of 3,109%. But at the same time Apple has the highest average return of 3,202% but considerable lower beta than Tesoro or 1,4.

To do a more formal test of the connection between betas of stocks and their average returns, the average monthly returns of the analyzed stocks will be regressed on their betas. The results of this test can be seen below:

Table 10: Test of the betas and the average monthly returns of the S&P 500 stocks.

Test of Beta and average return	
Intercept	0,00886
Slope	0,00265
R-squared	0,01312
t-statistics, intercept	1,47438
t-statistics, slope	0,44651

The intercept in the table should correspond to the risk-free rate over the period. The average monthly risk-free rate for the period in question is 0,411%, but the intercept here is 0,886%, more than twice the risk-free rate.

The slope in the table should, however, correspond to the market premium. The average monthly return of the S&P 500 index for the period was 0,169% and, as is mentioned above, the average monthly risk-free rate for the period was 0,411%. That makes the market premium about -0,242%, which obviously does not correspond to the slope in the table of about 0,265%.

The t-statistics for the intercept and the slope, with values under 2,0, indicate that they are not statistically different from zero. That means that there is no statistical evidence of the connection between average returns and the betas in the stock examined.

5.5 Estimation of expected returns

In this chapter one of the main question of this thesis, and the main critique of Modern portfolio theory, will be examined and tested for both the S&P 500 stocks and the FTSE-All shares stocks. Before the expected returns for period 2 can be tested and compared to the actual returns the expected return has to be conducted for each stock using the alpha, beta and residuals from period 1 and the return of the market index and the risk free rate of period 2. That is,

$$E(r)_i = \alpha_i + \beta_i (r_m - r_f) + e_i$$

The alpha and the beta coefficients have already been conducted in previous chapters for each of the stocks of S&P 500 index. The r_m is the average market return of the second period, or average Rm_{t+1}, and the r_f is the average risk-free rate of the second period, or average Rf_{t+1}. The residuals, e_i, is gained in the same way that the alpha and beta coefficients were gained, from the regressions of excess returns of each stock against the excess returns of the market index, S&P 500. The residuals stand for the variance of the unexplained portion of the stock‘s return, that is, the portion of return that is independent of the market index. Below are the results of the residuals for each stock and the average market return and the risk-free rate used in computing the expected returns:

Table 11: Residuals of the S&P 500 stocks in period 1, average market return and risk-free rate in period 2.

Companies	Residual (e)
NIKE	0,00362
Apple	0,01023
ITT	0,00287
JP Morgan	0,00406
Life tec	0,01068
IBM	0,00294
Moody's corp	0,00314
Humana	0,01042
Boeing	0,00457
Exxon	0,00235
Public Service Ent. Group	0,00338
Starbuck	0,00610
Cooper industry	0,00381
Xcel energy	0,02049
Clorox	0,00269
Safeway	0,00645
Tesoro Petr. Corp	0,02660

Rm (t+1) average	-0,553%

Risk free (t+1) average	0,371%

From those results the expected return, based on the former period‘s alpha, beta and residuals, and the average return of the market and risk free rate of the second period, were conducted and compared to the actual returns of the second period. Below is a table with the expected returns, conducted from historical prices of the stocks involved, and the actual average returns for each stock in the second period:

Table 12: The expected returns and the average actual returns of the S&P 500 stocks.

Companies	Expected returns	Average actual returns
NIKE	0,304%	0,966%
Apple	2,856%	2,314%
ITT	0,783%	-0,150%
JP Morgan	-0,838%	-0,306%
Life tec	0,223%	1,399%
IBM	-2,042%	0,800%
Moody's corp	1,269%	-2,281%
Humana	3,060%	-0,593%
Boeing	0,983%	-0,587%
Exxon	0,262%	-0,235%
Public Service Ent. Group	0,192%	-0,067%
Starbuck	2,385%	-1,249%
Cooper industry	0,790%	0,035%
Xcel energy	1,116%	-0,147%
Clorox	0,170%	-0,129%
Safeway	-1,318%	-0,410%
Tesoro Petr. Corp	3,658%	-2,466%
Mean expected return	0,815%	-0,183%
Standard deviation	1,53%	1,18%

In the table it is obvious that the expected returns are not the same as the actual returns. But by implementing a statistical t-test on expected and actual returns of period 2 it can be stated with statistical certainty if the mean difference between the predicted returns and the actual returns is zero or not. It will then be possible to either reject or accept the following hypothesis:

H0: Difference in mean return = 0

H1: Difference in mean return ≠ 0

Below are the results from the t-test on expected and actual returns:

Table 13: T-test on expected and actual returns of the S&P 500 stocks.

	Expected returns	Average returns
Mean	0,008147743	-0,001827019
Variance	0,000232814	0,000139022
Observations	17	17
Pearson Correlation	-0,303187326	
Hypothesized Mean Difference	0	
Df	16	
t Stat	1,875372727	
P(T<=t) one-tail	0,0395557	
t Critical one-tail	1,745883669	
P(T<=t) two-tail	0,079111401	
t Critical two-tail	2,119905285	

The results for the test can be seen in the highlighted line. Two-tailed P-value is used because the hypothesis that is tested has not a definite value, that is, it only states that either the difference in mean return is equal to zero or that it is not equal to zero. The P-value is 0,0791, higher than 5% limit, and therefore it can be stated by 5% statistical significance level that the mean difference of zero between expected and actual returns is not statistically significant. The H0 hypothesis is therefore rejected and the H1 hypothesis accepted that there is a difference between the mean actual returns and expected returns. In other words, this indicates that for this time period of approximately four years it is inaccurate to use historical returns to predict expected returns.

The same test of the hypothesis of expected return versus actual returns were conducted on the same group of stocks from the FTSE-All shares index that was used in the analysis of the efficiency of the market. This is done to strengthen further the results of this part of the research. Like before the alpha coefficients from the former period that have already been shown here above were used to conduct the expected returns, along with the beta coefficients and residuals from the same period. Again, like with S&P 500 stocks, the risk-free rate and the average market return of the second period was used in the estimation of the expected returns. Here below these parameters used in the estimation of expected returns for the FTSE-All shares stocks are shown:

Table 14: Betas and residuals of the FTSE-All shares stocks in period 1, average market return and risk-free rate in period 2.

Companies	Beta	Residual (e)
BP plc	0,60319	0,00221
BT Group plc	1,28909	0,00335
Diageo plc	0,10744	0,00208
HSBC holdings plc	0,97499	0,00098
Lloyds Banking group	1,43563	0,00361
Pearson plc	1,38273	0,00373
Signet Jewelers	0,96595	0,00455
Smith & Nephew	0,42035	0,00438
Electro-components	1,52231	0,00528
Scot.&Sth.Energy	0,18128	0,00165
Sage GRP.	1,76747	0,00440
Reed Elsevier	0,73563	0,00150
Imperial tobacco GRP	0,46410	0,00434
GlaxoSmith-Kline	0,38641	0,00243
Compass Group	1,08341	0,00577
Dairy Crest Group	1,13057	0,00631
BHP Billiton	1,27035	0,00433

Rm (t+1) average	-0,184%

Risk free (t+1) average	0,365%

Like before with the S&P 500 stocks the expected returns for the FTSE-All shares stock were conducted in previous described manner and compared to the actual return of the second period. In the table below the results of the expected returns formed can been seen along with the actual average returns of each stock in the second period:

Table 15: The expected returns and the average actual returns of the FTSE-All shares stocks.

Companies	Expected returns	Average actual returns
BP plc	0,370%	1,475%
BT Group plc	-0,763%	-1,667%
Diageo plc	0,459%	1,410%
HSBC holdings plc	-0,437%	-0,759%
Lloyds Banking group	-1,398%	-0,507%
Pearson plc	-0,427%	-0,497%
Signet Jewelers	-0,306%	-0,635%
Smith & Nephew	-0,033%	0,463%
Electro-components	-1,938%	-5,261%
Scot.&Sth.Energy	0,866%	-0,565%
Sage GRP.	-0,701%	0,059%
Reed Elsevier	-0,724%	1,092%
Imperial tobacco GRP	0,959%	-0,060%
GlaxoSmith-Kline	-0,589%	-0,611%
Compass Group	-1,748%	0,066%
Dairy Crest Group	-0,110%	6,770%
BHP Billiton	1,565%	-0,078%
Mean return	-0,291%	0,041%
Standard dev.	0,943%	2,292%

It can been seen from the table that an obvious difference is between the expected returns and the actual average returns of the stocks in question, like became evident with the S&P 500 stocks. To be able to state with statistical certainty that the mean difference between the two is zero or not and answer the hypothesis of expected returns versus actual returns a statistical t-test was done in the same manner as before. Below are the results of that test concerning the FTSE-All shares stocks:

Table 16: T-test on expected and actual returns of the FTSE-All shares stocks.

	Expected returns	Average actual returns
Mean	-0,002914786	0,000408487
Variance	8,88385E-05	0,000525174
Observations	17	17
Pearson Correlation	0,355791119	
Hypothesized Mean Difference	0	
df	16	
t Stat	-0,638652676	
P(T<=t) one-tail	0,266042627	
t Critical one-tail	1,745883669	
P(T<=t) two-tail	0,532085254	
t Critical two-tail	2,119905285	

The P-value for the two-tailed hypothesis gives approximately 0,5321. Therefore it can be stated in this case that the mean difference of zero, between expected and actual returns is, not statistically significant at the 5% statistical significance level . Again with the FTSE-All shares stocks, the H0 hypothesis is rejected and the H1 hypothesis accepted. There is, here as before, a difference between the mean returns of actual and expected returns of these stocks and therefore it is inaccurate to use historical data to predict expected returns. Therefore, for this set of stocks, listed in another market, the LSE in UK, the same results are found and even stronger than before as the P-value is higher and therefore even further from being statistically significant than before.

6 Conclusions

The objective of this thesis was to answer the question whether the optimal portfolios suggested by the Modern Portfolio Theories are the best asset allocation strategy in the real investment world.

Four hypotheses were tested in an effort to answer that question. The first test addressed the matter in a more general way without giving a direct positive or negative statistical answer to the question. The aim of it was to see whether the Modern Portfolio Models based on historical returns data were realistic in their prediction of future optimal portfolios for the subsequent periods. Three Modern Portfolio Models were used to construct a portfolio of 17 companies in the US market: the GMPV, the Black-Litterman and efficient portfolio, in two contiguous periods. The results were that the efficient portfolio model was completely unrealistic in its positions and statistics in both periods in fact. The other two models were more realistic in their positions and statistics but were totally different in their construction, in particular between the two periods. The Black-Litterman did not change its positions much between periods while the other two changed them considerably. The main findings of the test were that there seems to be little consistency between the models despite the fact that they are all based on the same theory and build their positions from the same set of historical data. This leaves the observer confused about the usefulness and credibility of the models and could be an indication of the various alleged flaws in the theory on which the models are based. The question whether Modern Portfolio Models based on historical returns data are realistic in their predictions of future optimal portfolios for the subsequent periods is difficult to answer directly by merely constructing portfolios and calculating their statistics. But the findings seem to indicate that they are not.

The remainder of the hypothesis tested produced more precise negative or positive answers and thus moved us closer towards an answer to the central question of this thesis. They were designed to analyze a few of the main foundations on which Modern Portfolio Theory is based, in order to make it possible to conclude whether Modern Portfolio Theory is the best asset allocation strategy in the real investment world.

The second hypothesis concerns the efficiency of the market. Stocks from three different markets in the US, UK and Denmark were examined to see whether their alpha coefficients would support the assumption of the efficiency of the market or not, i.e. whether their true value of alpha was zero or not. The OMX Copenhagen was tested to see if a market much smaller than the other two would produce different results. This was not the case. The results were almost identical in all three markets. With the exceptions of 1-2 stocks in each case, none of the stocks were significantly different from zero (with a 5% significance level). The results of this test are therefore that all of the three markets examined seem to be efficient.

The third hypothesis was to test another central assumption of Modern Portfolio Theory, or the CAPM, namely that higher volatility in securities means that they have higher average returns. Average monthly returns and betas of the S&P 500 were compared and a formal test conducted as well. When the average monthly returns and the betas of the stocks were compared it was obvious that the assumption that higher volatility translates into higher average returns did not hold. Stocks with similar betas did not have similar average returns, and vice versa, and almost no connection was seen indicating that relatively high betas among the selected stocks meant relatively high average returns. In the formal test, the assumption failed completely as well: no statistical evidence of a connection between the average returns and the betas examined could be established.

Last, but certainly not least, was a test of the hypothesis that the expected returns of the second period, estimated from historical data, would be the same as the actual average returns of that period. In order to increase the robustness of the test, it was performed using historical data from stocks in both the US market and the UK market. In both cases the mean difference of zero between the expected and actual returns was not statistically significant. This analysis shows that an estimation of future returns based on historical data is inaccurate. Therefore, this small experiment supports the critique of Modern Portfolio Theory that the predictive ability of models based on the theory is heavily questionable.

In summary, the results support some of the main arguments of critics of Modern Portfolio Theory with respect to all of the elements that were tested in this thesis, with the exception of the efficiency of the market. Testing of the above hypotheses was an attempt to answer the central question of this thesis: whether the optimal portfolio suggested by Modern Portfolio Theories is the best asset allocation strategy in the real investment world. As we have seen, the outcome of most of the tests was not if favour of Modern Portfolio Theory. That means that some of the main assumptions of the theory do not hold. The most serious failures, which incidentally also produced the most conclusive results, were those suffered by hypotheses three and four, that is whether higher volatility translates into higher average returns and whether expected returns were close to actual returns. The fact that these two presumptions of Modern Portfolio Theory do not hold up to the testing conducted here obviously leads to doubts as to whether models based on this theory work as they are expected to, that is by presenting to the investor the true optimal portfolio. Therefore, it has to be concluded that Modern Portfolio Theory are not the best asset allocation strategies in the real investment world. The fact remains, however, that effective methods for constructing portfolios are in short supply. The main critics of Modern Portfolio Theory have not really structured other and/or better ways for investors to construct portfolios. Mandelbrot and some of his followers have set forward the fractal theory to manage risk and return. The Kelly Criterion has also been suggested by scholars as an alternative, and others have stressed the importance for investors to rethink their approach to portfolio management. But none of these have really formed models that have made any substantial difference in terms of changing the preferred approach to portfolio management. Therefore, even though Modern Portfolio Theory suffers from serious flaws in many of its presumptions, as this research and numerous much more extensive ones have shown, it is still the only properly formed and implemented theory for portfolio constructing. The fact that it has not yet been displaced is all the more remarkable when we consider that it has been challenged ever since its basis was published by Markowitz. The arguments against its merits, however, only seem to get stronger.

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