

A LUMPED PARAMETER MODELLING METHOD FOR HIGH-TEMPERATURE GEOTHERMAL RESERVOIRS

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A LUMPED PARAMETER MODELLING METHOD FOR HIGH-TEMPERATURE GEOTHERMAL RESERVOIRS

A numerical modelling study

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A 30 ECTS credit units Master's thesis

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ABSTRACT

High-temperature geothermal resources, which are those that are exploited for producing electricity, have been thoroughly studied using a variety of techniques. They have been modeled conceptually and numerically, using programs such as *iTOUGH2*. There has not been much effort placed on using more simple techniques, such as lumped parameter methods. This is due to the necessity of dealing with temperature effects, and up to date, lumped parameter methods have been mostly limited to models incorporating only pressure changes. Here a method has been developed which accommodates both pressure and temperature/enthalpy data. A one and two-tank model has been elaborated. The model has been tested against user generated data as well as data from Krafla and Bjarnarflag geothermal power stations. The model was found to match the data in some cases, with the 1-tank model matching the data more closely. The results show the possibility of using the model as a part of the management of high temperature geothermal reservoirs.

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1 INTRODUCTION

Geothermal energy has been harnessed to produce electricity for over 100 years, beginning in Lardarello, Italy in 1904. At first, the development of geothermal energy was slow, as the competing forms of energy were less expensive (Grant et al., 1982). The oil crisis in the 1970s spurred the rapid development of geothermal energy around the globe. The same technologies that had been used to find and develop petroleum reservoirs were put to use to exploit geothermal reservoirs. The geothermal energy capacity increased greatly in countries as different as Kenya and Iceland; what they share is the presence of geologically active areas that contain high temperature geothermal systems (Dipippo, 2008)

A geothermal system is based on the geology of the geographic area and will not change due to exploitation. This is in contrast to a geothermal reservoir, which according to Grant et al. (1982), is the subsurface space likely to be affected by the development of a specific well configuration. The geothermal reservoir size will depend on the location, quantity, and depth of wells drilled, in addition to characteristics such as permeability and porosity of the area. Thus the size of a geothermal reservoir will change as more wells are drilled over the lifetime of the power plant it supplies.

At first, much of geothermal reservoir development was based on techniques adapted from groundwater and petroleum industries, as these industries had developed prior and came from the same source, the earth's crust. There are many differences, however, that necessitated the development of specialized knowledge in geothermal reservoir engineering. For instance, a geothermal reservoir typically involves different, more complex geology than a petroleum reservoir.

Modelling of geothermal fields is often undertaken to predict their future behaviour. While the quality of geophysical and geochemical measurements, undertaken prior to development of geothermal reservoir, has improved greatly in recent years, it is still with great uncertainty that an untapped geothermal reservoir is exploited. After it has been producing for a number of years, the production data obtained will give a much clearer idea of the potential for further development of the reservoir. Thus, utilization of geothermal reservoirs often happens in step-wise increments. Several papers have been published regarding this topic, for example Stefánsson (2002), that show that steps of approximately 30 MW are the most economically feasible.

While each modelling method also has specific steps that are unique from the others, they all have a history-matching step, where the model is matched to existing data. The purpose of matching the historical data is to validate the model. It does not guarantee that the model will be able to predict the future behaviour, but if it is not able to satisfactorily match the existing data, the model must be modified or discarded.

Here are described in detail three methods that have been developed for geothermal reservoirs. The decline curve and trend analyses method is perhaps the simplest and does not necessarily involve the use of computers. Complex numerical models became possible with the advent of powerful computer technology. Lumped parameter models use computers, but do not require the computing power used in complex numerical models.

The volumetric method of estimating the potential of a geothermal reservoir has not been considered here, as it cannot predict the change over time of the reservoir parameters.

1.1 Decline Curve and Trend Analyses

The simplest techniques involve fitting a particular curve to existing data, called decline curve and trend analyses. These techniques involve fitting an equation to a given data set, such as pressure, enthalpy, or mass flow, but are not based on any conceptual model of the reservoir. The use logarithmic, harmonic, and exponential functions to curve fit data have been suggested (Arps, 1945). The data set could be change in world population or the gravitational flux of a black hole, there is no specific basis in the characteristics of the geothermal reservoir. Thus the technique cannot predict changes in a variable, say the pressure of the geofluid at the wellhead, if the management of the field changes, for example if mass flow rate from the well is changed. Some methods have been developed which plot data against cumulative mass extraction rather than time to get around this particular problem (Grant et al., 1982). If a reservoir is closed, constant compressibility the geothermal reservoir pressure will fall linearly with cumulative withdrawal. However, if the geothermal reservoir is relatively open and thus has a high rate of recharge, there is not a fixed amount of geofluid in the reservoir and plotting against cumulative mass extraction will yield indeterminate results.

The methods used have been largely developed from an article by Arps (1945) on decline curve analysis (Reyes et al., 2004), and further developed by Chierici (1964), who found that wells at Larderello, Italy, could be fit to an exponential formula. As long as no changes are made in a geothermal reservoir, good results have been obtained, as shown in figure 1.1 from Wairakei, New Zealand (Wainwright, 1970), where the pressure projections proved to be accurate within 1%.

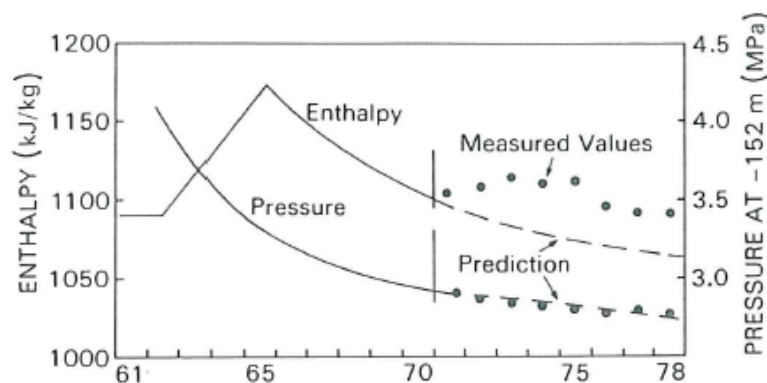


Figure 1.1 Decline curve for Wairakei, with the year on the x-axis (Grant et al., 1982)

1.2 Numerical Modelling

For larger, high temperature geothermal fields, the dominant method of modelling the reservoirs has been using complex numerical techniques. They are based on conceptual models, which means that changes in the utilization of the geothermal reservoir can be accounted for. The models involve setting up a grid of cells for the reservoir. These

models can be quite large (O’Sullivan et al., 2001), and can be extremely expensive to run computationally, requiring clusters of dedicated high-speed computers. In addition, the behaviour of the reservoir can be difficult to predict due to the dynamic nature of many of the properties. *TOUGH2* is a popular program used for numerical modelling of geothermal systems. Figure 1.2 shows an *TOUGH2* model that has been developed for Bjarnarflag. This grid has 3133 cells, with the finest mesh close to Bjarnarflag. Vertically the grid has 9 sections from the surface to a depth of 2750 meters.

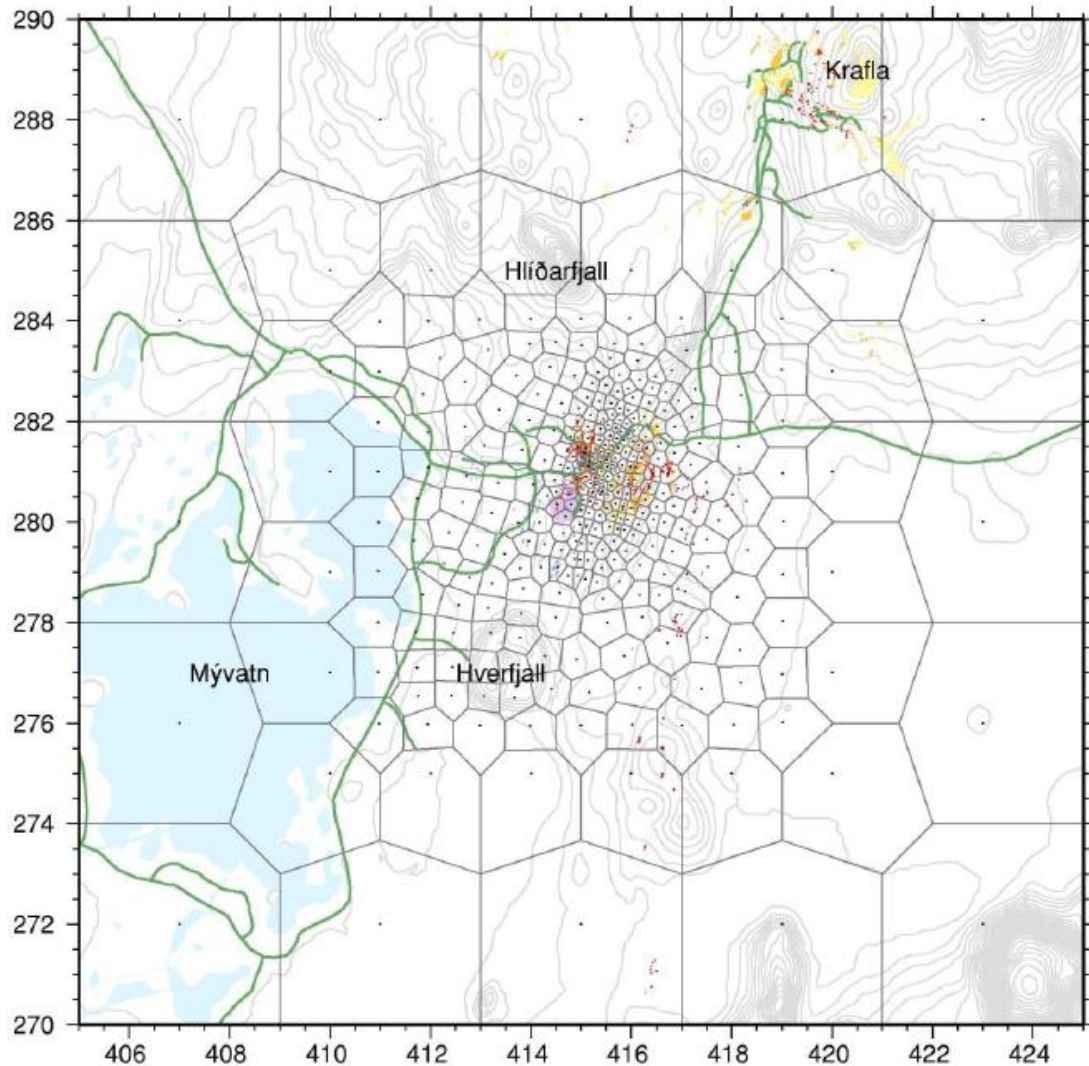


Figure 1.2: Horizontal grid representation of a model for Bjarnarflag (Hjartarsson et al., 2005)

For each of the cells the bulk properties are determined, and for the outer cells boundary conditions are applied. The cells, while large in number, still provide very limited spatial resolution. Typically, the minimum dimensions of cells are much larger than important reservoir features such as fractures, such that they are missed in the model. Also, small surface geothermal features, such as hot springs and steaming ground cannot be represented accurately. Accurate modelling around wells is impossible.

The boundary conditions for a model can seem straightforward at first, but complications are encountered when the model is run. For example, at the surface, a boundary condition of constant atmospheric conditions would seem to be appropriate to apply. The

complications are that when applied to the model, it can allow for unlimited inflow of cold water or unlimited outflow of warm fluids (O’Sullivan et al., 2001). In a real geothermal system that has a finite permeability, this would lead to the lowering of the water table, which is challenging to incorporate into a model. Some modelers have tried to more accurately represent the shallow zone of a geothermal reservoir by including the unsaturated zone above the water table, representing the uppermost cells as having a mixture of water and air. There will still be inaccuracies however, unless the uppermost layers are made on a scale that can accurately model the changes in the water table.

The first step in calibrating a numerical model consists of natural state modelling of the geothermal reservoir. Natural state refers to the geothermal reservoir before wells have been drilled and geothermal fluids have been extracted. Natural state modelling consists of running the model for a long time in a simulation of the geothermal field over geological time scales, i.e. over thousands of years rather than only a few years. This step is required because the mathematical model is based on a conceptual model and is not simply matching one particular set of data. Data is required to confirm that temperature and surface outflows match the model, with the permeability structure of the model changed to arrive at a good match.

Once a good fit for the natural state has been found, the model should be further calibrated by history matching the data, if they are available. Sometimes a model is developed before a geothermal reservoir is producing, with the aim of estimating the electricity generating potential of the field. With producing fields, the purpose of history matching is to conform the simulated behaviour of the reservoir to the measured data. Parameters of the model, such as porosity and permeability, are adjusted to obtain a better fit. In addition to pressure and enthalpy data, modellers have used tracer test results, geochemical measurements, and geophysical data such as gravity measurements.

Initially there were problems in deciding which reservoir properties to adjust so that the model would conform better to the measured data. A subjective bias in the adjustments necessary to achieve a better fit require an ‘expert’, thus making the process an ‘art’. Advances have been made such that automated processes have been developed for finding the optimal adjustments in parameter values based on the data (O’Sullivan et al., 2001).

1.3 Lumped Parameter Models

A method that has been applied with considerable success to low temperature fields as well as some high temperature fields is the lumped parameter technique. Considerable effort is often taken to back calculate field parameters such as reservoir size, transmissivity, storativity etc. to make sure that the model is forecasting a logical result. The primary goal of these models, however, is to predict future response of a geothermal field based on past performance and thus predict the future power production.

Lumped parameter modelling has been used extensively in Iceland for low temperature fields where costly numerical modelling of the geothermal reservoirs was not economical. These lumped parameter modelling studies were often performed for small local heating services and have involved an isothermal model as discussed previously (Axelsson, 1989). Since that time, there has been interest in applying stochastic models, with large number of runs to determine the sensitivity of some of the variables. This is not feasible using complex numerical techniques due to the time necessary to run a model.

There has been very little research done applying non-isothermal lumped parameter models to higher temperature geothermal reservoirs. Lumped parameter models that integrate temperature or enthalpy have only recently been considered (Onur, 2008). They have focused on predicting the pressure response of reservoirs within low temperature geothermal fields. In such fields, where the geothermal fluid can be safely assumed to be of a single phase, the enthalpy (and temperature) can be assumed to have a negligible influence on the pressure response. Thus, they have been used mainly for application to lower temperature reservoirs.

Whenever changes are made to the wells in a given geothermal reservoir, such as when a new well is put into operation, or an old well is shut in or is used for reinjection, changes can occur in the behaviour of other wells, which continue in operation. Thus models have to make sure that these changes are represented, or else limitations have to be placed on their application. Otherwise, taking data from before such changes occurred and using this to predict future performance can result in erroneous results.

2 MEASUREMENT TECHNIQUES

The basis for any predictive model of a geothermal reservoir is the data available from the geothermal wells, such as the mass flow, pressure, and enthalpy measured over time. It is imperative to have knowledge about the methods used in obtaining this data and their reliability and accuracy. Measurement methods existed for mass flow, pressure, and enthalpy of water prior to the development of geothermal power and have been used for many years with high degrees of accuracy. Many of these techniques were developed by the petroleum industry; unfortunately, they are often not suitable for measuring typical geothermal well flows, which are usually higher in temperature.

High temperature geothermal wells can be divided into two types, dry steam and two-phase (water and steam). A few reservoirs, such as those at Larderello, Italy and the Geysers, United States produce dry steam. A great majority, however, can be classified as two-phase flows, such as those in New Zealand and Iceland. These are typically producing from a liquid single-phase underground source (Grant et al., 1982). The reservoirs may begin flashing to steam in the rock formation itself, which a condition that is often called a 'steam pillow'. Alternatively, the geothermal fluid may start flashing at a position in the well, when the geothermal fluid pressure decreases and the saturation curve is intersected.

2.1 James' method

Measurement of the mass flow, pressure, and enthalpy of dry steam or pure liquid can be performed using methods that have previously been developed in other industries. Measurement of the enthalpy and mass flow of a steam-water mixture is more difficult, and initially was performed using calorimeters or phase separators. It was found that the calorimeter, along with the large amounts of cooling water required, was too large to be practical (James, 1970). It is difficult to make accurate measurements with phase separators, due to a persistent small percentage of steam carryover. Steam carryover of 0.5% can result in over-estimation of water flow by 30%, and this carryover has proven difficult to prevent. When a well has a dedicated separator, this method can be applied with minimal additional equipment. More frequently, however, a separator serves several wells, or the separation process takes place at the power-house to minimize the cost. In this case, a separator needs to be brought to the well, which can be prohibitively expensive. Thus, considerable work has been conducted by the geothermal industry in finding an accurate technique that does not require overly cumbersome testing equipment or setup procedures.

In the 1960s, Russell James developed a method of measurement of the flow rate and enthalpy using the so-called lip pressure. This method is applicable to two-phase- as well as dry-steam flows. It is based on a property of compressible fluid flows in pipes to regions of low pressure. When the flow in the pipe reaches sonic velocity, the flow rate cannot increase further. Thus choked conditions are reached, and the pressure will not decrease further. Thus the pressure in the pipe at the discharge point will be above the ambient pressure at the exit of the pipe, in this case atmospheric pressure. In Figure 2.1, James' arrangement can be seen. Downstream from the discharge of the pipe, the liquid portion of the flow is measured over a weir.

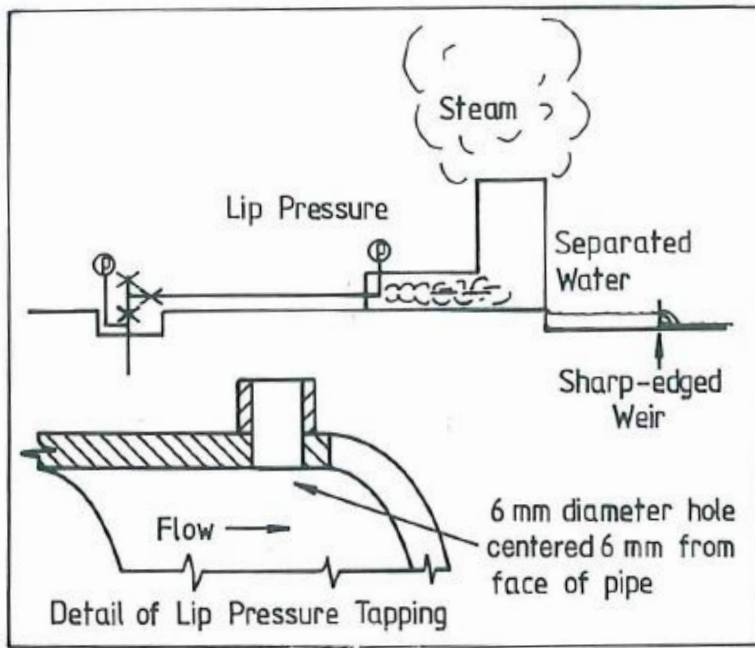


Figure 2.1 Measurement arrangement for James Method (Grant et al., 1982)

After testing with 3, 6, and 8 inch diameter pipes, James was able to find a correlation that related the stagnation enthalpy to the flow rate and lip pressure according to the following equation.

$$\frac{GH_t^{1.102}}{P_{lip}^{0.96}} = 1680 \quad (1)$$

The units in this equation are not in SI, the flow rate (G) is in $\text{kg}/(\text{cm}^2 \cdot \text{s}^{-1})$, the enthalpy (H) is in kJ/kg , and pressure (P) is in MPa . In addition, the value on the right hand side is not dimensionless. However, it has been found with experience in New Zealand wells to be accurate within 8 percent, based on comparison with the measured power plant flows. It has proven to be reliable over a wide range of enthalpies and flow rates (Grant 1982).

In addition, theoretical studies have shown that the James' method gives results within 8% of those predicted by 3 analytical models for two-phase one-component choked flow (Karamarakar et al., 1980). When dissolved gases or salts are present in the flow in significant quantities however, the studies have shown that much larger errors can be expected.

2.2 Tracer flow testing (TFT)

The James' method has now been superseded by tracer flow testing (TFT), which came into use in the 1990s (Hirtz et al., 2001). This method has several advantages, perhaps most important, the method does not require the diversion of flow from the power plant, allowing the plant to stay in operation. This allows the wells to be monitored on a regular

basis without having to rely on well/plant shutdown periods. Some flows contain a significant amount of hydrogen sulfide, which is hazardous and must be carefully monitored and can be released by testing with the James' method. The disposal of brine can be another environmental problem when using the James' method, with the need for equipment that can greatly increase the cost of testing.

TFT testing uses two tracers that are introduced into the flow stream, one that is suitable for the vapour phase and another for the liquid phase. The amount of tracer injected is monitored using very accurate equipment. The concentration of the tracers are then measured at a point far enough downstream that the tracers are fully mixed in their respective phases. The mass flow rate of each phase is then found by dividing the upstream tracer injection mass flow rate by the downstream measured tracer concentration. The downstream measured tracer concentration is dimensionless, as it is kilogram of tracer divided by kilogram of total flow. The pressure in the pipe is simply measured with a gauge from a tap. Thus knowing the pressure and the ratio of the liquid and steam flow rates, the enthalpy can be determined.

Testing using the TFT method has been found to be more accurate than the James method. Some existing geothermal fields have converted from using the James method to the TFT method. High temperature geothermal fields that have been recently commissioned, such as those at Hellisheidi, Iceland, are measured using tracer flow testing.

2.3 Measurement methods used for the data in this study

In this study, the data for Bjarnarflag are all acquired using James' method (Hauksson, pers. comm.) and this technique will continue to be used well into the future for geothermal fields that had been developed prior to 1992, to maintain consistent measurement techniques. These data are measured once a year when the wells are taken offline for plant maintenance. The shut-in time before well testing is minimal according to Hauksson, who has taken site measurements every year since the start-up of the power-plant. There has been considerable consideration placed on having the enthalpy measured with the wells while in production, and thus at a more frequent rate, but this not been implemented to date.

P_0 is the wellhead pressure. A measurement reading is taken once the pressure stabilizes, which can take anywhere up to 24 hours. H_0 is the wellhead enthalpy. It is measured using the James' method as described above. P_s is the separator pressure. It can be lower than the wellhead pressure because it is not measured simultaneously with P_0 and the pressure can vary rapidly with time. It is used to properly evaluate geochemical samples that have been taken. The mass flow rate used is determined by taking the cumulative flow for the year, and dividing it by the time it was in operation (Hauksson and Benjaminsson).

3 RESEARCH METHODS

In this thesis, a lumped parameter model has been developed which couples pressure and enthalpy calculations. MATLAB software was used to perform all numerical modelling of the equations. Specifically, a set of differential equations was fitted to the given data for pressure and discharge enthalpy (or downhole temperature), with the mass withdrawal rate as an input function. The given data set contained, on average, one data point per year for the data. The data used in this study is used with permission from Landsvirkjun, for the Bjarnarflag and Krafla high temperature geothermal fields in northern Iceland. In Figure 3.1, the existing power plants in Iceland can be seen, with the geothermal plants supplied by the fields that are used in the study seen.

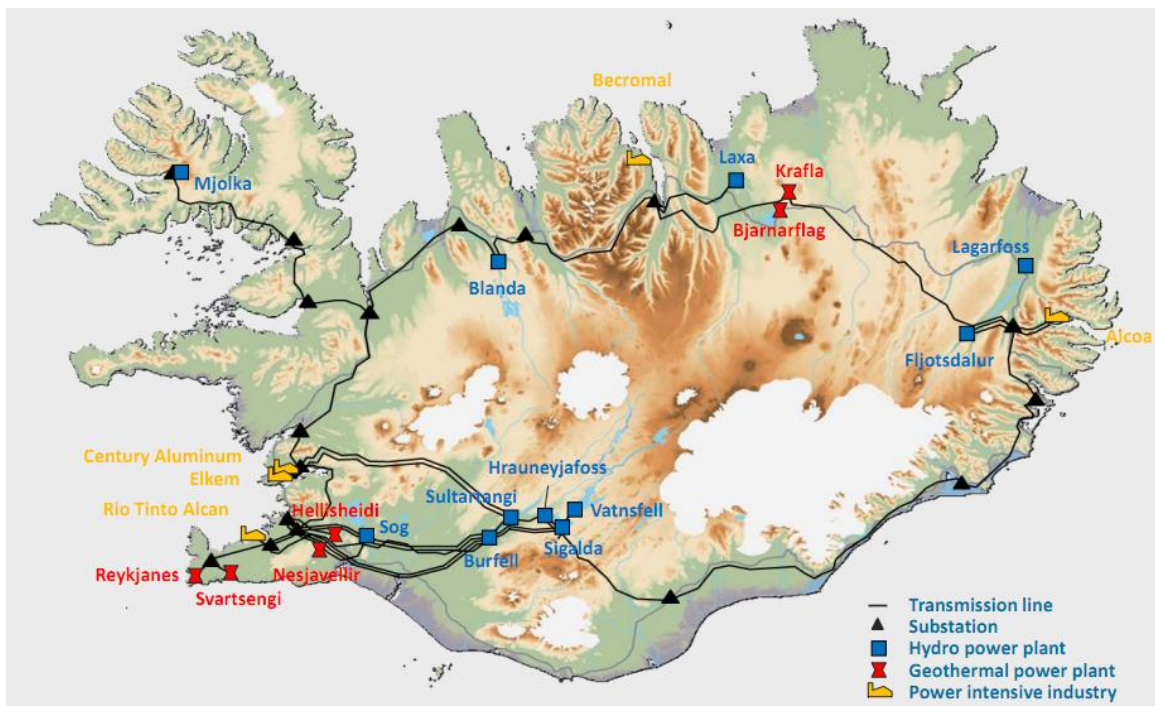


Figure 3.1: Existing power plants in Iceland (Landsvirkjun)

Bjarnarflag and Krafla are located in geologically one of the younger parts of Iceland. The European and North American tectonic plates meet in Iceland. In the region of Krafla there are fissures running north-south for 100 kilometres that are separating the tectonic plates at a rate of approximately 2 cm a year. This is a volcanically active region that has seen eruptions during the lifetime of the Bjarnarflag and Krafla power plants, resulting in their shutdown. Remarkably, little damage to the power plants resulted from these eruptions, although the behaviour of many of the existing geothermal wells changed. Currently, the rated capacity of Bjarnarflag is 3 MW and that of Krafla is 60 MW.

For Bjarnarflag, data was used for wells that are in production, some of which have been in continuous operation for over 30 years. The discharge enthalpy and pressure data, as well as the annual mass withdrawal, were used in the model.

For Krafla, there is data for up to 25 years, depending on the region studied. Downhole measurements of wells that are not in production were used to try to obtain the best

approximation of the undisturbed rock temperature and pressure. This is thought to be a better indication of changes in the reservoir than discharge conditions of wells (Hauksson 2010). The cumulative annual mass withdrawal from the producing wells surrounding the monitor well was used.

In addition to the data from Landsvirkjun, user generated data was used to test the model. This data was generated with a user defined mass flow starting at 10 kg/s reducing to 0 kg/s over 100 years, then increasing to 10 kg/s over the next 100 years. The pressure and temperature were derived from the mass flow. Both pressure and temperature reduced at a rate proportional to the mass flow.

3.1 Brief mathematical description

The equations developed were based on the lumped parameter technique. This does not divide a geothermal reservoir into many geometrically defined cells like finite element and difference methods do. Rather, the technique ‘lumps’ the parameters of the reservoir together in one or more ‘tanks’, and then matches and predicts the overall behaviour of the ‘tanks’. The size of the ‘tanks’ is not the primary interest, but once a model is found to be accurate enough, the size of the reservoir can be calculated knowing other parameters of the reservoir, such as the mean porosity of the reservoir.

The technique is normally only applied such that the mass withdrawal is linked to changes in the production zone of the reservoir. Thus if changes happen outside of this zone, they will not be reflected in the model and the result will not adequately reflect the system (Grant et al., 1982).

Often the models developed only match the pressure history, and changes in temperature or discharge enthalpy are ignored, therefore the model is not able to make any predictions for them. Recently, there has been some study of non-isothermal lumped parameter modelling (Onur, 2005), where both pressure and temperature are considered. Here a model is developed which incorporates discharge enthalpy (or alternatively temperature). In much the same way as the models that match only pressure data, the mass withdrawal is tied to the change in enthalpy.

The 2-equation (1-tank) model of the reservoir is shown in equation (2):

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} P \\ H \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \dot{m} = \begin{bmatrix} dP/dt \\ dH/dt \end{bmatrix} \quad (2)$$

The value \dot{m} , represents the mass flow rate. The values of P and H represent pressure and enthalpy respectively. The coefficients a_{mn} and b_n where determined to best fit the values for the change in pressure and enthalpy over time, dP/dt and dH/dt . In the case of the user generated data and Krafla, temperature was used in place of enthalpy.

By specifying an initial guess for the coefficients and initial conditions of P and H , the system of differential equations can be calculated, using the algorithm *ODE23T* available in MATLAB. This algorithm is best suited for stiff differential equations, those that have a large change of rate between the non-transient and transient portions of a data set.

Algorithm solvers for stiff differential equations generally maintain stability in finding a solution by reducing the step size between guesses, in comparison to solvers for nonstiff equations. The disadvantage of stiff solvers such as *ODE23T* is that they take longer to converge, they require more iterations, and thus are mathematically more expensive.

The error between the guessed differential equations and the actual data for pressure and enthalpy is then found, by calculating the difference between the data and the equation. This is calculated for every time step there is data for, and the squared sum of the errors is calculated, known mathematically as the norm. The non-linear curve fitting algorithm *LSQNONLIN* is used to calculate the values for the coefficients and initial conditions. A number of iterations are performed, until there is little change in the guesses between rounds. The algorithm *LSQNONLIN* uses a trust-region-reflective algorithm to calculate the next guess and is based on the interior reflective Newton method (www.mathworks.com).

To try and obtain a more precise fit of the enthalpy and pressure data, a 4-equation (2-tank) model was also developed. This also increases number of coefficients and initial conditions for the *LSQNONLIN* algorithm to optimize from 8 to 22.

The 4-equation (2-tank) model of the reservoir is shown in equation (3):

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ H_1 \\ H_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \overset{\circ}{m} = \begin{bmatrix} dP_1/dt \\ dP_2/dt \\ dH_1/dt \\ dH_2/dt \end{bmatrix} \quad (3)$$

Similarly to the 1-tank model, the value $\overset{\circ}{m}$, represents mass flow rate and the values of P_1 and H_1 represent the pressure and enthalpy of the inner reservoir, respectively. Meanwhile, the values of P_2 and H_2 represent the pressure and enthalpy of the outer reservoir. The coefficients a_{mn} and b_n where determined to best fit the values for the change in pressure and enthalpy over time, dP_n/dt and dH_n/dt .

The equations are implemented in the MATLAB code as shown in Appendix A. For each data set, the data were processed with MATLAB using the sequence described in figure 3.2. The sequence is driven by the *run_model.m* file. The file contains script that will load a data file, *.mat into memory, run the *lsq_enth.m* file, and will run some post-processing to create graphs. The *lsq_enth.m* file sets the initial guesses for the coefficients and initial conditions of the pressure and enthalpy/temperature, as well as defining upper and lower bounds on the allowable change in these parameters. Finally, the file runs the *LSQNONLIN* algorithm in MATLAB to optimize the error between the model and data, which is defined in the *resp_err.m* file. This file first non-dimensionalizes the pressure and enthalpy/temperature data and then calculates the error between the model and data. The model is called from *resp_func.m*, and is calculated using the *ODE23T* algorithm in MATLAB. This is based on an equation, as well as mass and time parameters, defined in *resp_ode.m*

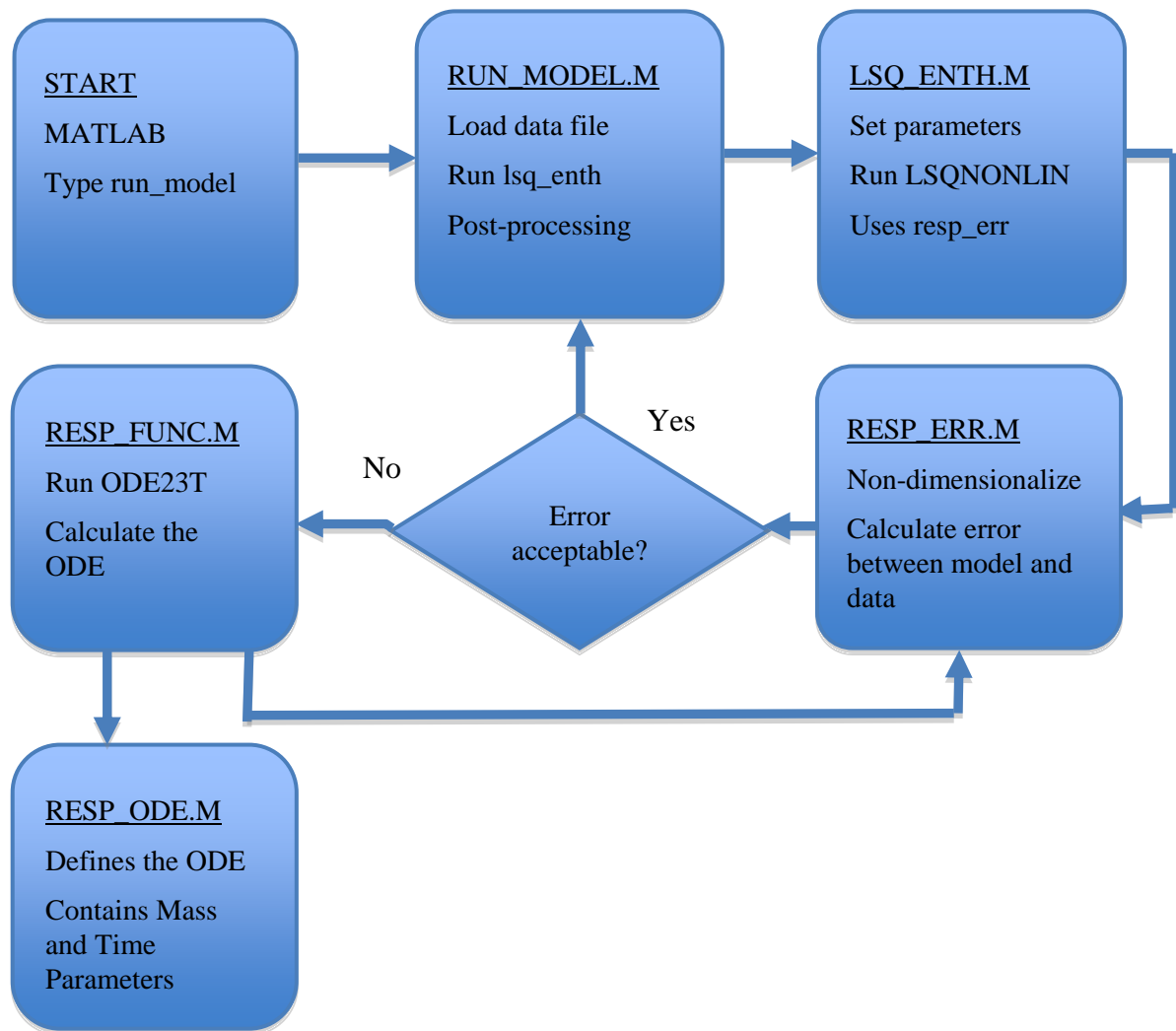


Figure 3.2: MATLAB code graphical representation

3.2 Normalization of the data

In order to properly compare parameters, they were normalized so that one would not have more weight than another. The pressure data were divided by the first data point for pressure. Similarly, the enthalpy/temperature data were divided by the first data point for enthalpy/temperature. The calculation of the optimal fit for the model involved both values for enthalpy/temperature and pressure, thus normalization was required, as can be seen in the MATLAB code in Appendix A. The time and mass flow rate data were not used in the calculation of the optimal fit, thus they did not need to be normalized.

3.3 Interpretation of the data

In order to quantify the quality of the fit of the model to the data, different measures are available. One of the most common is the standard deviation of the data. The equation used for the standard deviation is shown in equation (4):

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (Data_i - Model_i)^2} \quad (4)$$

In this equation N is the number of timesteps, with the difference between the data and model points for each timestep squared and then summed with the values for the other timesteps. The summed value is then divided by the number of timesteps, and the square root is taken. This value is taken as the standard deviation, σ , for the remainder of this thesis.

4 RESULTS

Before real data was used, the 1 and 2-tank models were tested against user generated data. After the model was validated with the user generated data, it was tested with the data from Bjarnarflag and Krafla that was obtained from Landsvirkjun (Hjartarsson et al., 2005).

4.1 User generated data

The 1-tank model was compared against user-generated data to test whether the 1-tank model was capable of matching the data. The model fit the data closely, as can be seen in Figure 4.1. The standard deviation for this model is $1.784e-04$, indicating a very good fit. The smaller the value of the standard deviation is, the closer the fit is to the data.

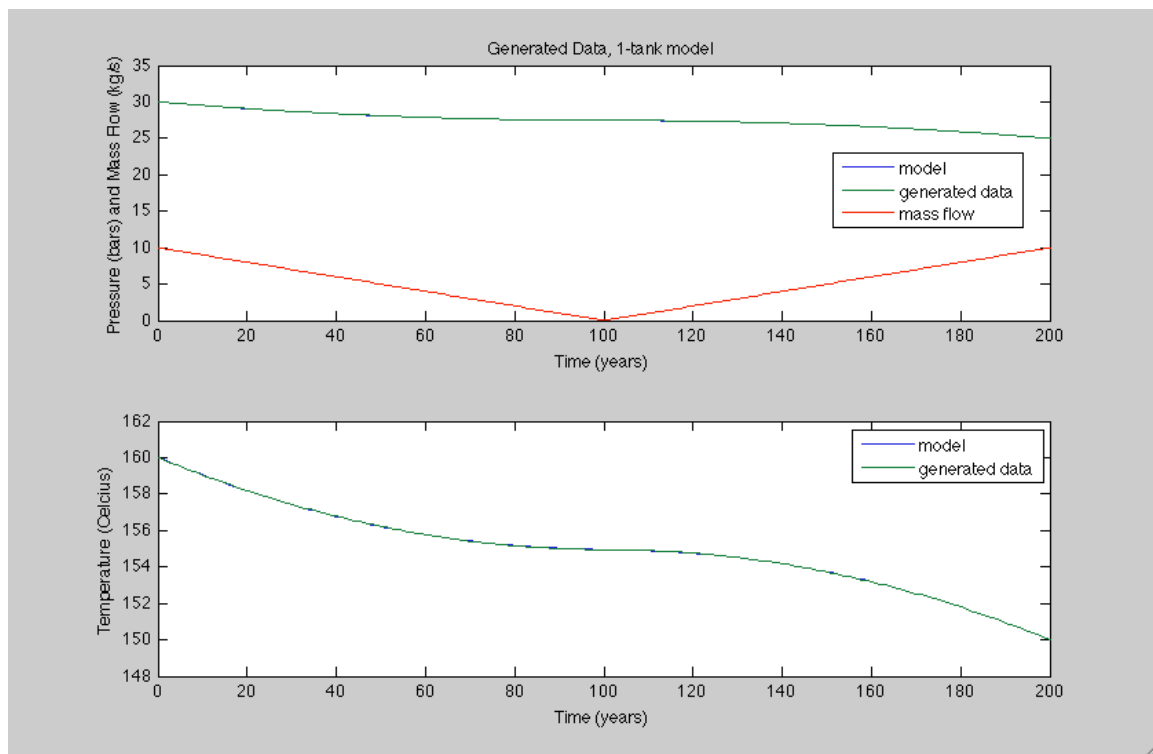


Figure 4.1: User generated data, 1-tank model

Afterwards, random noise of $\pm 5\%$ of the variable value was added to the data to test the stability of the model. The model performed well, as can be seen in Figure 4.2, which means that the 1-tank model is accurate and robust. The standard deviation for this case is 0.0259. The standard deviation was larger than for the data without the noise because of the differences of the model from the actual data due to the added noise. While the model was close to the average, it did not match the actual data precisely during all the short-term random fluctuations, so the value of the standard deviation was 2 orders of magnitude larger than the data without the noise.

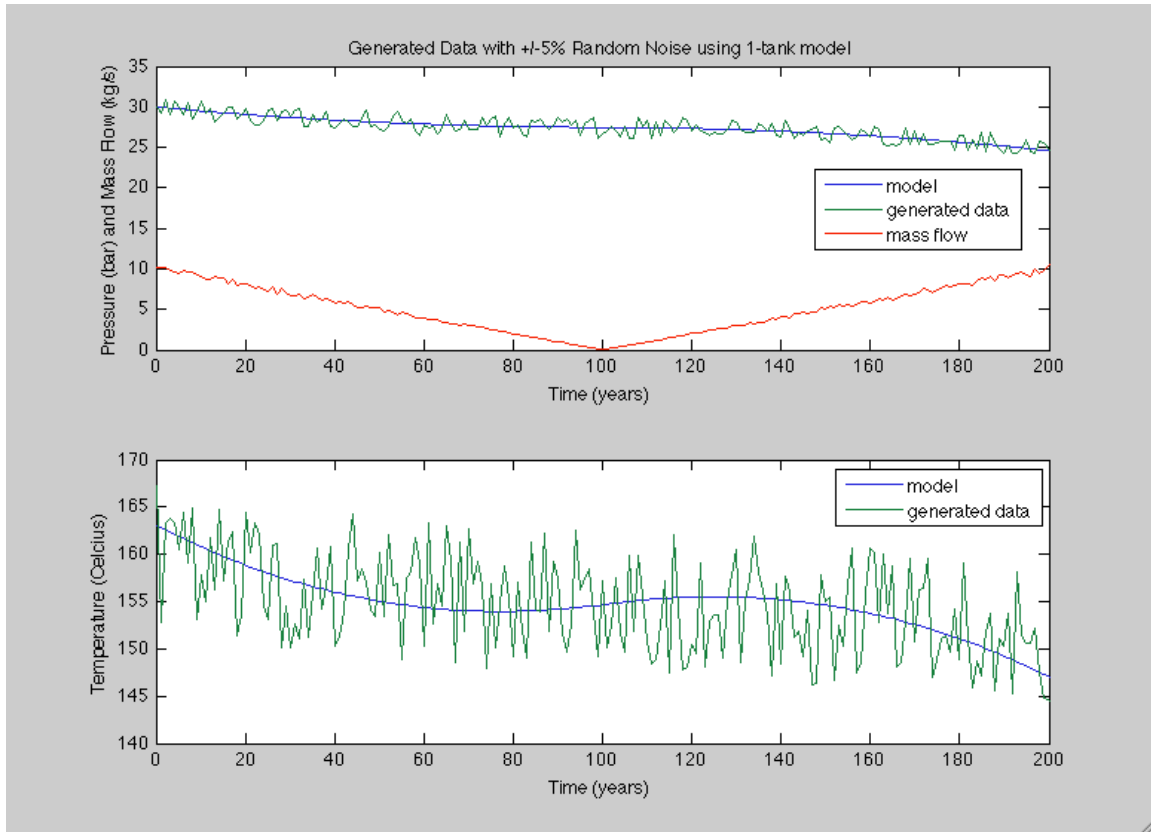


Figure 4.2: User generated data, 1-tank model with +/- 5% random noise

Next, the user generated data was tested with the 2-tank model. The results were not as good as for the 1-tank model, as can be seen in figure 4.3. The standard deviation for the data is 0.0485, which is 2 orders of magnitude larger than that for the 1-tank model without noise.

The 2-tank model with noise also did not match the temperature data, but performed surprisingly well with the pressure data. The norm for the data is 0.0674, which is more than twice that of the 1-tank model with noise.

The 2-tank model could have problems matching the data as the result of a much greater number of coefficients and initial conditions to adjust. Some of the coefficients can be correlated, such that the *LSQNONLIN* algorithm may adjust some of the coefficients in a way that changes in two or more coefficient cancel each other out.

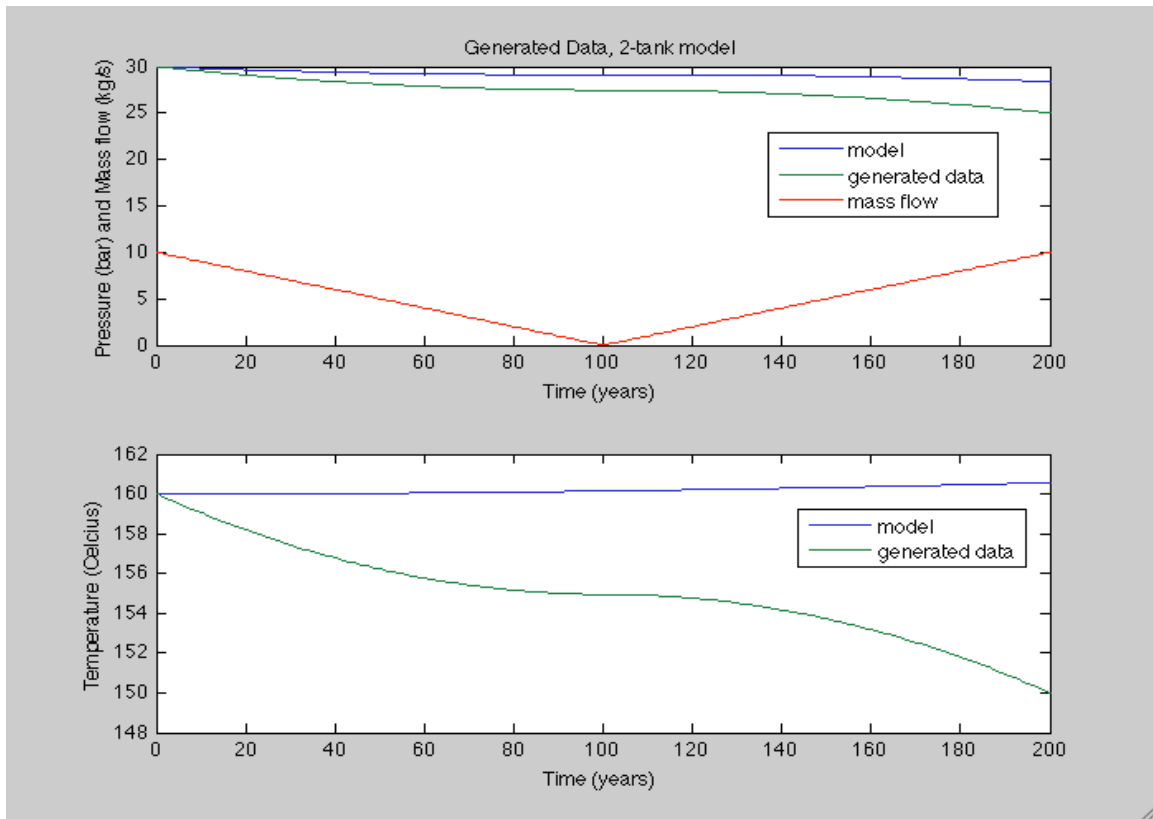


Figure 4.3: User generated data, 2-tank model

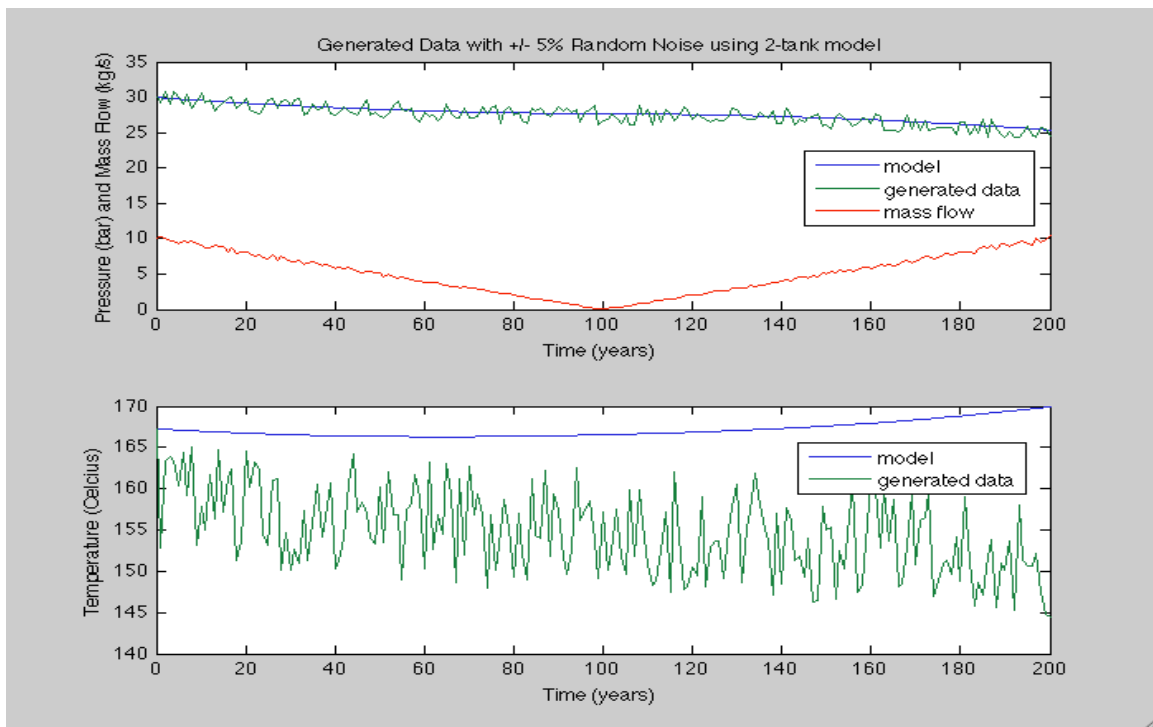


Figure 4.4: User generated data, 2-tank model with +/- 5% random noise

4.2 Bjarnarflag

For Bjarnarflag, hole BJ-11 was initially used in this study. Other holes were tested as well, but BJ-11 is representative of the results from the others. It has been continuously producing for over 25 years. The average mass flow for the year, and the enthalpy and pressure at the wellhead are used for modelling, as described in section 2.3. In figure 4.5, the 1-tank model can be seen to match the average behaviour of the data, but misses the yearly fluctuations. It is important to note that the data is taken once a year in large part, which will not show annual fluctuations. The standard deviation for the 1-tank model is 0.2555.

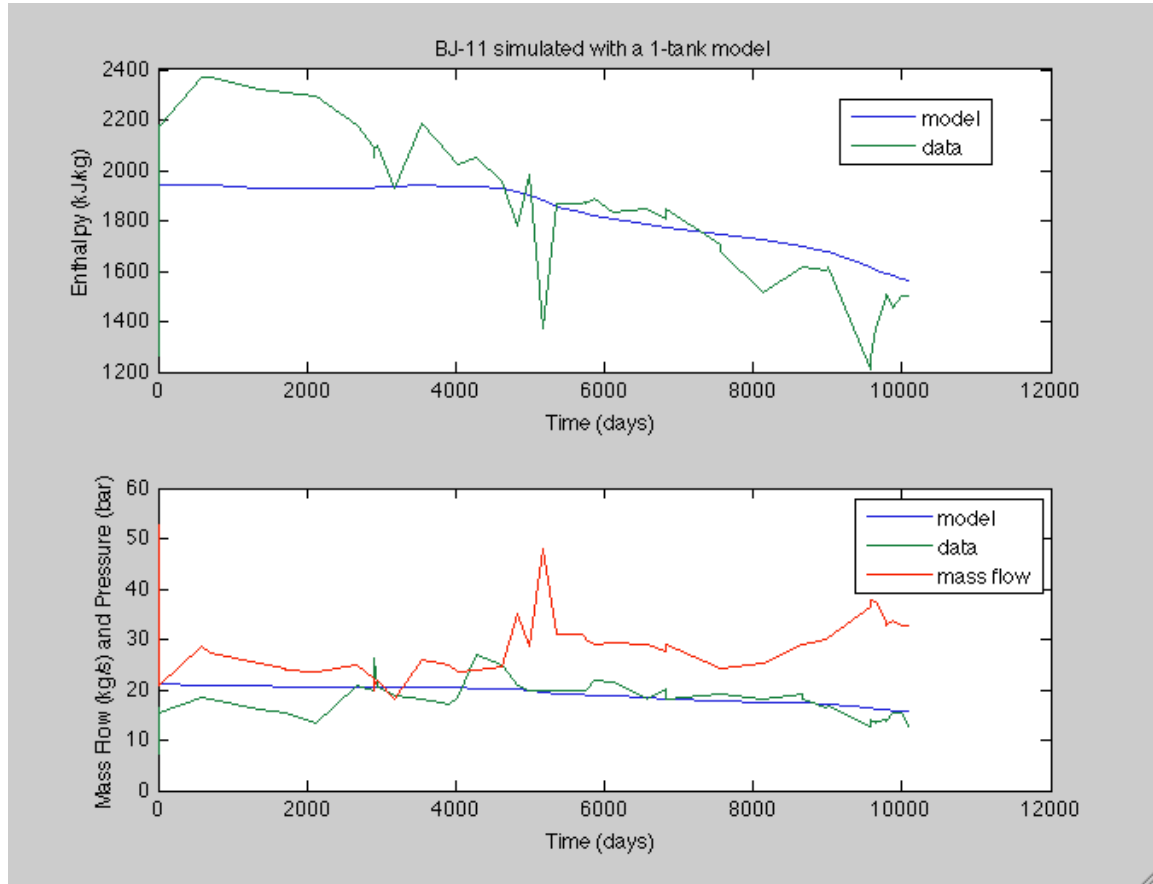


Figure 4.5: BJ-11 simulated with a 1-tank model

The 2-tank model had generally a poorer fit than the 1-tank model. While the 1-tank model enthalpy was close to the average, the 2-tank model pressure was way off the average and did not trend in the same direction as the data. The standard deviation for this is 1.3523, which is much larger than that of the 1-tank model.

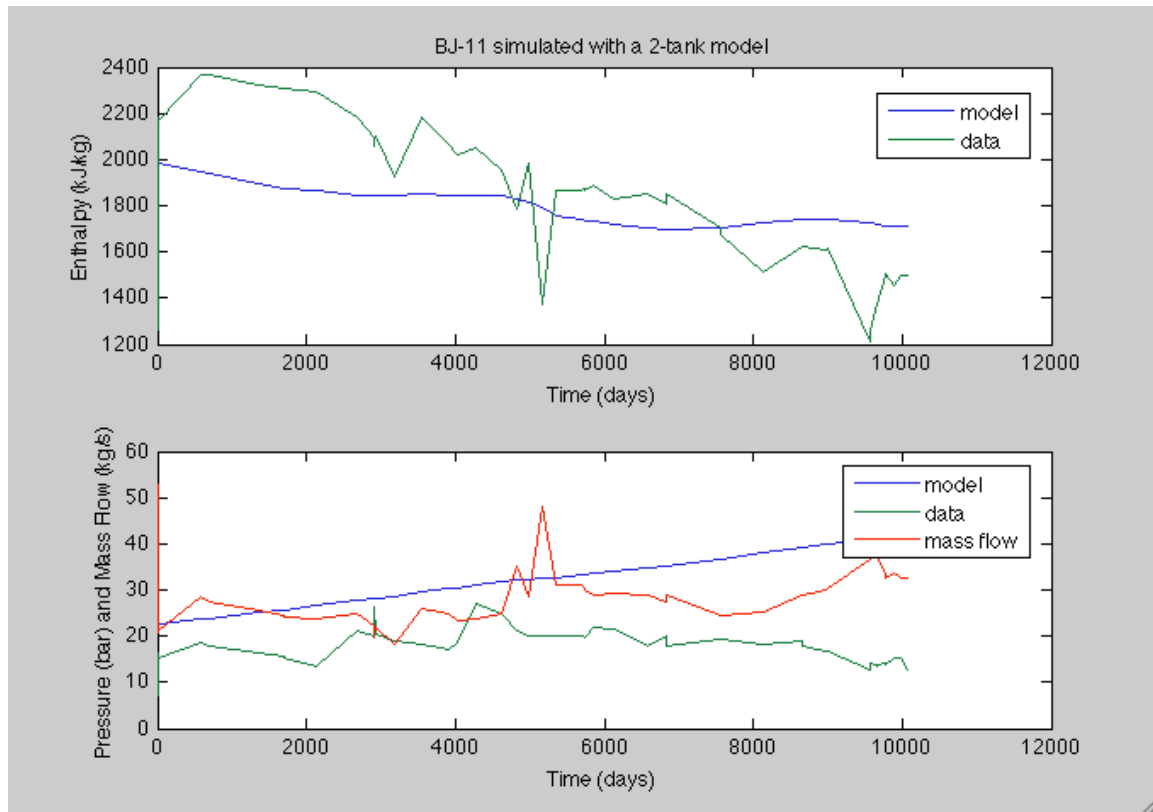


Figure 4.6: BJ-11 simulated with a 2-tank model

4.3 Krafla

The KJ-6 hole at Krafla in the Leirbotn region was used for this study. There were other holes that have been used for monitoring in the Krafla area, but the KJ-6 was selected as it was in an region that had continuous mass flow. This hole was initially drilled to a depth of 2000 meters in 1976. It was used for production until the year 1985, when it was capped and has since has been used as a monitoring well. The downhole temperatures and pressures at a depth of 400 meters were used. They were decided to be the most suitable for this study because unaccountable fluctuations in the parameters were less than for other depths.

In Figure 4.7 can be seen the best fit the model was able to achieve using the 1-tank model. While the overall fit of the model is poor, the model finds the average of the data quite well. The standard deviation for this model is equal to 0.1084.

Next, the model was simulated with a 2-tank model and similar results were obtained, as shown in Figure 4.8. The result is actually slightly further off the average than that of the 1-tank model, which is to be expected considering the results from Bjarnarflag and the user generated data. The standard deviation is equal to 0.2302

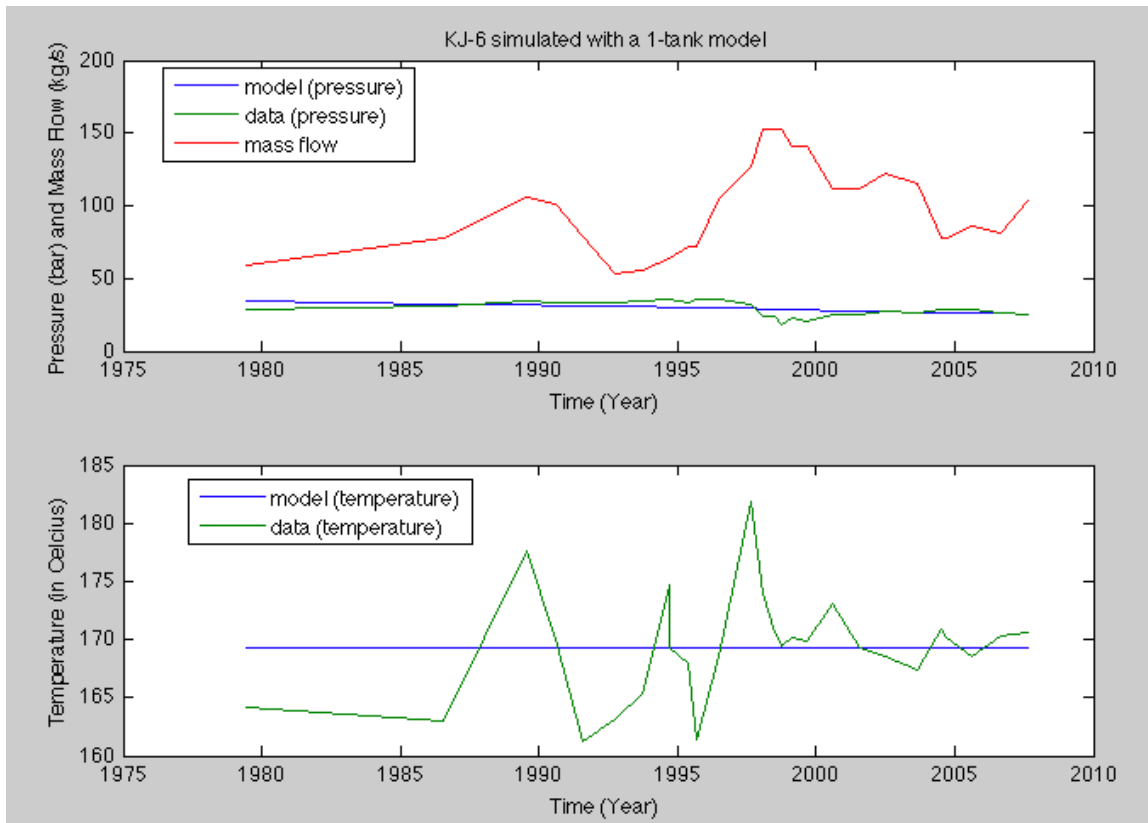


Figure 4.7: 1-Tank Model of well KJ-6 at the Leirbotn area of Krafla.

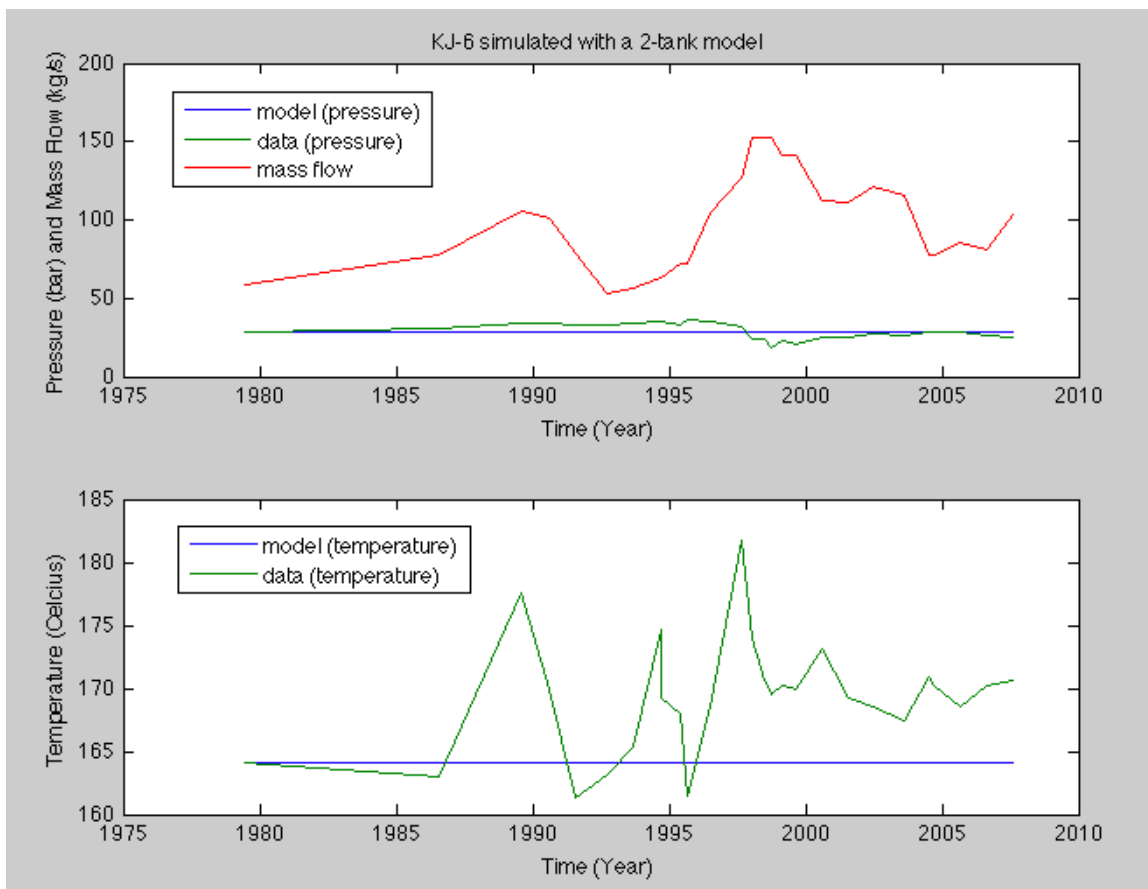


Figure 4.8: 2-Tank Model of well KJ-6 at the Leirbotn area of Krafla.

A correlation study of the data for KJ-6 was performed showing a poor relation between mass flow, pressure and temperature. The mathematical model is built on the assumption that a greater mass flow will result in lower temperature and pressure. Without a good correlation of mass flow with temperature and pressure, the model will not be able to achieve satisfactory results.

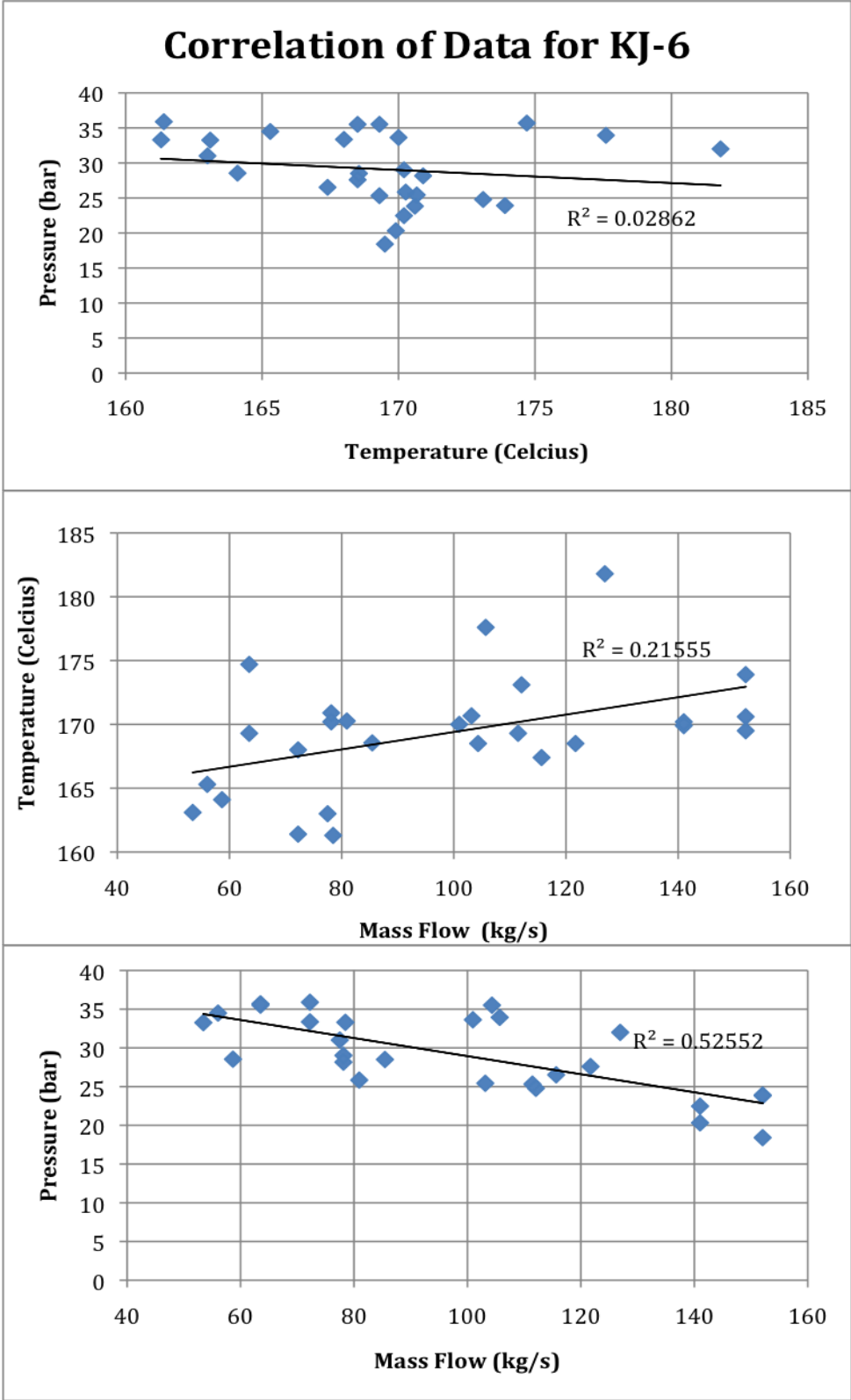


Figure 4.9: Correlation of data for KJ-6

The mass flow data was offset from the pressure and temperature data for KJ-6, with the theory that changes in mass flow are reflected in changes in pressure and temperature at a later time. The data correlation of the pressure with the mass flow was in fact improved from a R^2 value of 0.5255 to 0.6680, as can be seen in Figures 4.9 and 4.10.

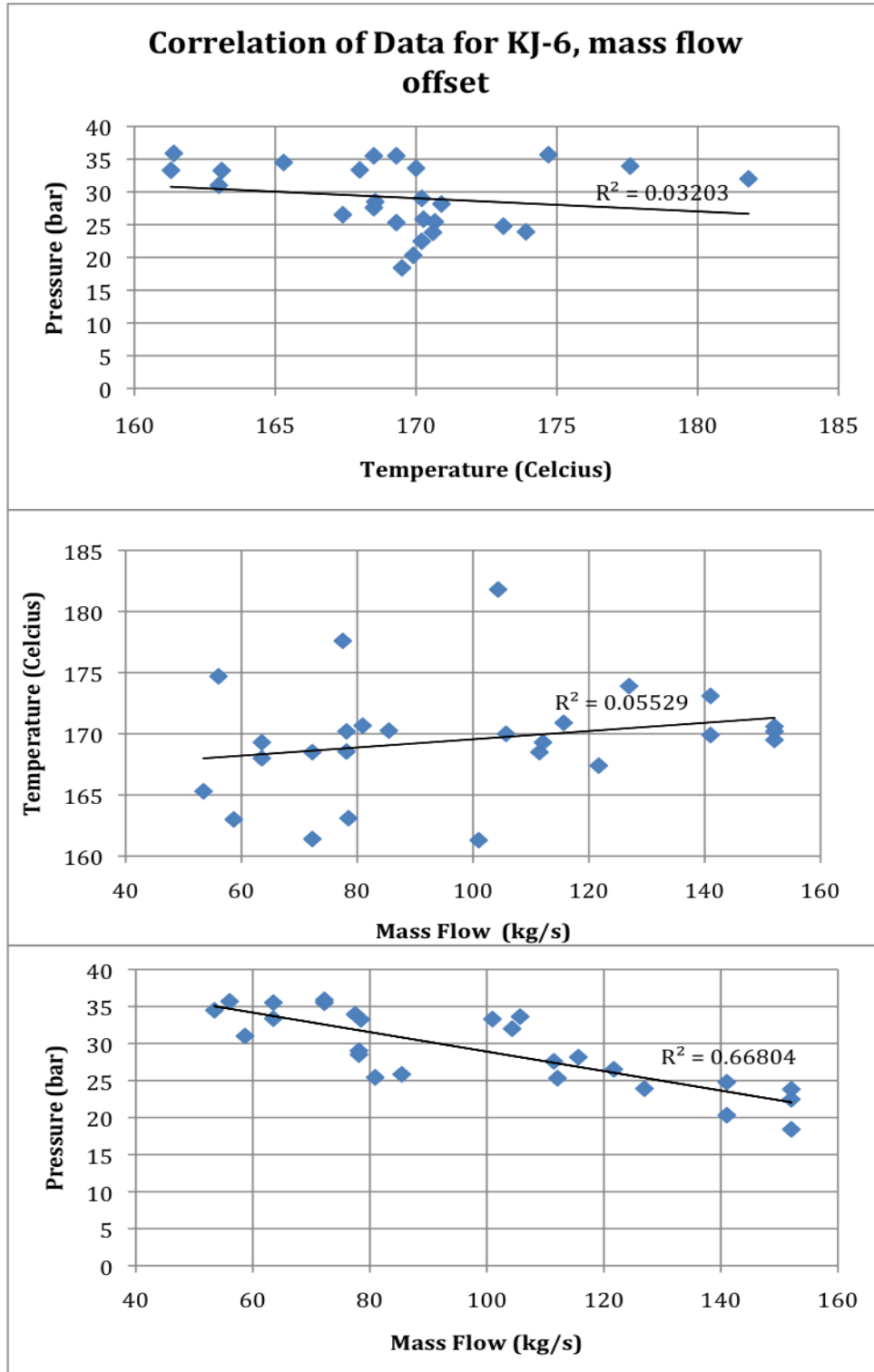


Figure 4.10: Correlation of data for KJ-6 with mass flow offset

This improved correlation did not result in a better fit of the model to the data, however, as can be seen in Figure 4.11. The standard deviation is equal to 0.0985, which is a slight improvement from before. This was a relatively rudimentary approach that did not take into account various parameters that could affect the time response of pressure and temperature to mass withdrawal such as the distance of the extraction wells to observation well.

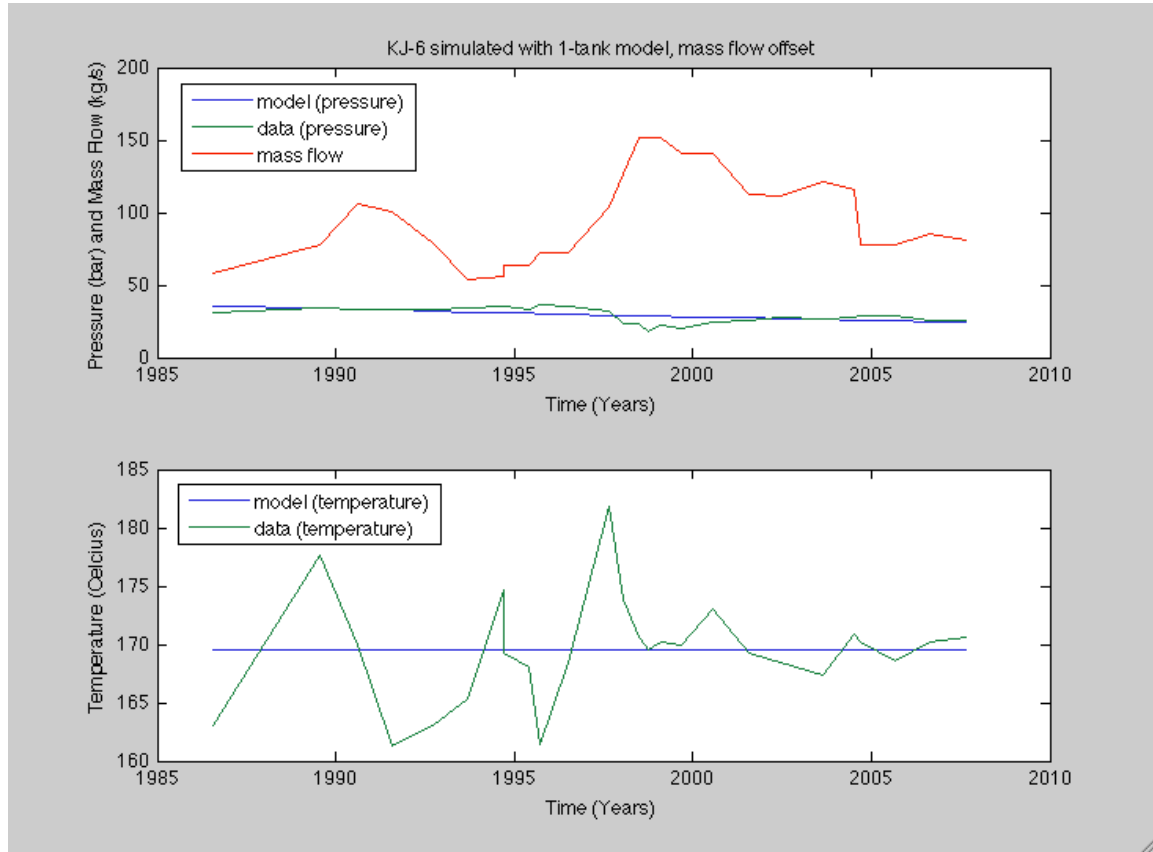


Figure 4.11: 1-tank model of well KJ-6 at the Leirbotn area of Krafla, mass flow offset

5 CONCLUSIONS

The 1-tank model was able to fit the user generated curve well, and generally provided a better fit to the data from Bjarnarflag and Krafla than the 2-tank model. This seems to be the result of the 1-tank model optimizing only 8 rather than 22 values. In general, both the 1 and 2-tank models represented the average of the data well, but did not follow changes very well.

The model has been history matched, but has not been used to predict the future performance of the reservoir under various utilizations schemes, using several mass withdrawal rates, similar to other lumped parameter models. This is relatively straightforward, but was not completed due to time constraints.

The 2-tank model could be analyzed to determine if some of the coefficients could be fixed, such that fewer values would have to be optimized in the model. The current model uses 'brute force,' in that all coefficients are adjustable. In a typical lumped parameter model, some coefficients are set equal to zero or set equal to each other, and thus are simpler to optimize. A relation to physical properties of the geothermal reservoir could also be possible.

The correlation of the mass flow, down-hole temperature and pressure was poor. With an offset between the cumulative mass flow for a region, and the down-hole temperature and pressure, the correlation between pressure and mass flow was somewhat better. A 1-tank model was tested with the data offset showing a poor correlation.

An improved method of correlation between pressure and temperature with the mass flow could be developed. For instance, the pressure will usually change in response to mass flow withdrawal more quickly than temperature. In addition, the time will vary depending on the distance of the monitoring well to the production wells. Adjusting the pressure and temperature offsets with the mass flow rate based on these two factors could improve the model fit.

The data for enthalpy and down-hole temperature measurements is taken once a year, when the power plants are down for maintenance. This is a very sparse data set to accurately model the behaviours of the geothermal reservoir. Comparatively, pressure/water level data for low temperature geothermal resources are measured several times a year, so fluctuations throughout the year can be seen more clearly. Tracer flow testing technology allows companies to test enthalpy throughout the year and not just during outages. Companies are now implementing this technology and have begun testing their wells more often as a result.

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APPENDIX A - MATLAB CODE

Below is the MATLAB code written for the 1-tank model.

Run_model.m

```
% Post-processing to show the new differential equation vs. the data  
% Calculate the new differential equation
```

```
load bj11a.mat  
lsq_enth  
a11 = x(1);  
a12 = x(2);  
a21 = x(3);  
a22 = x(4);  
b1 = x(5);  
b2 = x(6);  
p0 = x(7);  
e0 = x(8);  
[t,R]=resp_func(a11,a12,a21,a22,b1,b2,p0,e0);
```

```
% Redimensionalize the model before comparing to the dimensional model
```

```
pdat = bj11a(:,3);  
edat = bj11a(:,2);
```

```
R_rd1 = R(:,1)*pdat(1);  
R_rd2 = R(:,2)*edat(1);
```

```
% plot Pressure and enthalpy data
```

```
subplot(2,1,1)  
plot (bj11a(:,1),R_rd1,bj11a(:,1),bj11a(:,3))
```

```
subplot(2,1,2)  
plot (bj11a(:,1),R_rd2,bj11a(:,1),bj11a(:,2),bj11a(:,1),bj11a(:,4))
```

Lsq_enth.m

```
% Least squares approximation of the coefficients for the ODE, a11, a12,  
% a21, a22, b1, and b2. For a given time series of pressure and enthalpy  
% using a single tank model.
```

```

% Bjorn Sveinbjornsson 2010
a11 = 0;
a12 = 0;
a21 = 0;
a22 = 0;
b1 = 0; %0.0005;
b2 = 0; %0.005;
p0 = 1.5;
e0 = 1.5;

x0 = [a11 a12 a21 a22 b1 b2 p0 e0];

lb = x0/2-0.02;
ub = 2*x0+0.02;

% Least squares nonlinear fit
options = optimset('MaxFunEvals',600,'disp','iter','TolFun',1e-08,'TolX',1e-10)
[x,resnorm] = lsqnonlin(@resp_err,x0,lb,ub,options);

```

Resp_err.m

```

% This function calculates the error between the model and the actual data
% for pressure and enthalpy

```

```

function err = resp_err(x)

```

```

global xdat pdat edat pdatnd edatnd xvec

```

```

a11 = x(1);
a12 = x(2);
a21 = x(3);
a22 = x(4);
b1 = x(5);
b2 = x(6);
p0 = x(7);
e0 = x(8);

```

```

[t,R] = resp_func(a11,a12,a21,a22,b1,b2,p0,e0);

```

```

% Pressure and Enthalpy data is loaded from a data file

```

```

load bj11a.mat ;

```

```

pdat = bj11a(:,3);
edat = bj11a(:,2);
xdat = [pdat; edat];

```

```

% Pressure and enthalpy formed into a vector for comparing to the measured

```



```

% data
xvec = [R(:,1); R(:,2)];

% Pressure and Enthalpy measurements non-dimensionalized
% These values need to be non-dimensionalized to give the enthalpy and
% pressure measurements equal weight

pdatnd = pdat/pdat(1);
edatnd = edat/edat(1);

xdatnd = [pdatnd; edatnd];

err = xvec-xdatnd;

```

resp_func.m

```

% This function calculates the ODE, using the given parameters

```

```

function [t,R]=resp_func(a11,a12,a21,a22,b1,b2,p0,e0)

```

```

t0
=[0;0.0173611110000000;0.479166667000000;0.718750000000000;0.763888889000000;
0.947916667000000;1.447916667000000;1.614583333000000;1.739583333000000;2.552083
333000000;2.718750000000000;3.406250000000000;3.843750000000000;4.406250000000000;
4.777777778000000;5.406250000000000;6.406250000000000;10.781250000000000;14.468750
0000000;19.468750000000000;21.010416670000000;22.500000000000000;581.6180556000000;7
01.5798611000000;1357.010417000000;1706.802083000000;1707.350694000000;2123.01041
7000000;2679.010417000000;2904.468750000000;2904.673611000000;2904.718750000000;29
04.753472000000;2905.416667000000;2938.010417000000;3176.468750000000;3555.427083
000000;3917.350694000000;4032.593750000000;4283.444444000000;4621.406250000000;482
8.475694000000;4829.597222000000;4989.434028000000;5164.434028000000;5360.6145830
0000;5695.614583000000;5738.635417000000;5744.604167000000;5862.670139000000;6117
.427083000000;6117.618056000000;6577.555556000000;6823.427083000000;6823.43055600
000;7552.503472000000;7552.548611000000;8125.427083000000;8651.427083000000;8651.
440972000000;8987.583333000000;8996.430556000000;9574.586806000000;9574.663194000
00;9575.416667000000;9575.417361000000;9658.447917000000;9658.666667000000;9776.5
79861000000;9778.545139000000;9888.420139000000;9981.697917000000;10078.552080000
0;];

```

```

%ODE solved

```

```

[t,R]=ode23t(@(t,r) resp_ode(t,r,a11,a12,a21,a22,b1,b2),t0,[p0;e0]);

```

resp_ode.m

% This function describes the differential equations used for the model

function dr = resp_ode(t,r,a11,a12,a21,a22,b1,b2)

% global dm a11 a12 a21 a22 b1 b2

%variable flow rate

t0 =

[0;0.0173611110000000;0.4791666670000000;0.7187500000000000;0.7638888890000000;0.9479166670000000;1.4479166670000000;1.6145833330000000;1.7395833330000000;2.5520833330000000;2.7187500000000000;3.4062500000000000;3.8437500000000000;4.4062500000000000;4.7777777780000000;5.4062500000000000;6.4062500000000000;10.7812500000000000;14.4687500000000000;19.4687500000000000;21.0104166700000000;22.5000000000000000;581.61805560000000;701.57986110000000;1357.01041700000000;1706.80208300000000;1707.35069400000000;2123.01041700000000;2679.01041700000000;2904.46875000000000;2904.67361100000000;2904.71875000000000;2904.75347200000000;2905.41666700000000;2938.01041700000000;3176.46875000000000;3555.42708300000000;3917.35069400000000;4032.59375000000000;4283.44444400000000;4621.40625000000000;4828.47569400000000;4829.59722200000000;4989.43402800000000;5164.43402800000000;5360.61458300000000;5695.61458300000000;5738.63541700000000;5744.60416700000000;5862.67013900000000;6117.42708300000000;6117.61805600000000;6577.55555600000000;6823.42708300000000;6823.43055600000000;7552.50347200000000;7552.54861100000000;8125.42708300000000;8651.42708300000000;8651.44097200000000;8987.58333300000000;8996.43055600000000;9574.58680600000000;9574.66319400000000;9575.41666700000000;9575.41736100000000;9658.44791700000000;9658.66666700000000;9776.57986100000000;9778.54513900000000;9888.42013900000000;9981.69791700000000;10078.55208000000000;];

dm0 =

[50;52.7000000000000000;47.9000000000000000;38.1000000000000000;37.1000000000000000;35.9000000000000000;33.1000000000000000;32;33.6000000000000000;27.5000000000000000;27.5000000000000000;26.7000000000000000;25.8000000000000000;25;24.8000000000000000;24;23.3000000000000000;20.8000000000000000;21.4000000000000000;21.9000000000000000;22.5000000000000000;21;28.7000000000000000;27.4000000000000000;25.4000000000000000;24.3000000000000000;24;23.7000000000000000;24.8000000000000000;22.2000000000000000;19.8000000000000000;20.3000000000000000;20.7000000000000000;20.9000000000000000;21.8000000000000000;18.3000000000000000;25.8000000000000000;25;23.4000000000000000;23.8000000000000000;24.7000000000000000;35;35;28.5000000000000000;48.1000000000000000;31;31;30.3000000000000000;29.9000000000000000;28.9000000000000000;29.4000000000000000;29.4000000000000000;29;27.5000000000000000;29;24.3000000000000000;24.4000000000000000;25.2000000000000000;29;29;30;30.4000000000000000;36.5000000000000000;36.5000000000000000;37.7000000000000000;37.7000000000000000;37.5000000000000000;37.5000000000000000;33.4000000000000000;32.7000000000000000;33.6000000000000000;32.6000000000000000;32.6000000000000000;];

dm = interp1(t0,dm0,t);

p = r(1);

h = r(2);

dp = -a11*p-a12*h-b1*dm;

dh = -a21*p-a22*h-b2*dm;

$$dr = [dp ; dh]$$