Estimating Potential Merger Gains with Bilevel Programming DEA

by
Haofei Wang

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Master of Science

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Supervisor:

Desheng Wu
Assistant Professor, School of Science and Engineering,
Reykjavík University, Iceland.

Examiner:

Eyjólfur Ingi Ásgeirsson
Assistant Professor, School of Science and Engineering,
Reykjavík University, Iceland.

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Abstract

In today's economy and society, merger has a very important role in the restructuring of many sectors. In this thesis, we introduce an innovative Bilevel Programming Data Envelopment Analysis approach to evaluate the profit efficiency of the hierarchical system both pre-merger and post-merger. This hierarchical system consists of two levels, the Leader and the Follower, and the Leader is at the dominant level. A α -Strategy is proposed to stimulate the Follower to actively participate and stabilize the hierarchical structure. The potential gains from the merger are decomposed into harmony effect and scale effect. Two case studies are used to illustrate our proposed approach. The results show considerable potential gains from the promising mergers. The concept of coordinated effective merger is also discussed, and it is very important since every member in the system benefits from the merger.

Keywords: Data Envelopment Analysis, Bilevel Programming, Merger, Profit Efficiency

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Chapter 1

Introduction

Over the course of the past 30 years, attempts of consolidations in the form of merger and acquisition have greatly increased in competitive business environments and have attracted the attention of a growing number of financial economists. A lot of mergers and takeovers are reported in the business press. The shareholder value and operating performance can be improved by a merger in operating benefits such as economies of scale, asset restructuring, and technical and managerial skill transfer, financial benefits such as risk reduction, increased debt capacity and lower interest rates and tax savings (Rappaport, 1986).

From many empirical findings, the advantages of mergers have been verified. Hoffman and Weinberg (1998) report a case study that the Chemical Banking Corp and the Chase Manhattan Bank NA gained $1.5 billion from cost saving after three years of a merger. Similarly, Murray (1997) states that owing to the merger First Union Core States bank achieved a $50 million cost saving in updating banking information systems.

Gugler et al. (2003) identify that for large companies, mergers increase profits by increasing market power, whereas for small firms, mergers increase profits by increasing efficiency.

The basic analytical framework to investigate the merger is examining efficiency effect, financial ratios, econometric cost measures and the effect of the merger announcement on the stock of the acquiring and acquired firms. A number of studies evaluate the effects of the actual mergers.

A recent contribution is Bogetoft and Wang (2005). They estimate the potential gains from mergers and decompose the gains into two parts, one is called the harmony effect, the gains associated with reallocation among similarly sized firms, and the other is called the scale effect, the gains that are available by changing the scale of the firms.

With the development of human society and the intensification of economic globalization, the scale of practical decision problems increases, meanwhile the structure of them becomes more and more complex. As one of the characters of system, hierarchy is very important when there are multiple players involved in the system, such as productive plans, resource distributions, engineering design problems etc.

Hierarchical decision problems have more than one decision maker, which have their own decision variables and objectives to optimize, also called multilevel programming problems. The Bilevel programming problem (BLP) is a special case of the multilevel programming problems with two levels in a hierarchy, the upper level and lower level decision makers. The decision maker at the upper level, which is also termed as the Leader, makes the choice first to optimize his objective. Knowing the decision of the Leader, the Follower makes his response which in turn affects the Leader’s outcome.

Since the formal formulation of the linear BLP proposed by Candler and Townsley in 1982, many authors studied BLP intensively and contributed to this field. A lot of potential applications of BLP are presented by Dempe (2003), such as in the field of economics, engineering, ecology, transportation, game theory and so on.

Due to the hierarchical structure, the BLP is generically non-convex and non-differentiable and intrinsically hard to solve, even if the objective functions of the both levels and the constraints are all linear. The main existing methods for solving this problem are methods based on vertex enumeration, methods based on Kuhn-Tucker conditions, fuzzy approach and methods based on meta heuristics. The most popular among them is Kuhn-Tucker approach, and in this method, a one-level optimization problem is obtained by replacing the Follower’s problem by the Kuhn-Tucker conditions. However, this method is proved to be deficient when the constraint functions of upper-level are in an arbitrary linear form and an extended Kuhn-Tucker approach was proposed by Shi et al. (2005) to overcome the deficiency of the original Kuhn-Tucker approach.

Many papers have been published on evaluating the operating efficiency using various approaches. To distinguish the best practice group among a set of observed units based on their inputs and outputs, and to indicate the differences between the inefficient units and the best practice group and improvements possible for the inefficient units, Data Envelopment Analysis (DEA) is by far the most used technique

(Wu, 2009). This technique is flexible and powerful, and widely used in numerous empirical studies (Cooper et al.2000).

Based on the DEA technique, three types of the allocative efficiency can be identified when information on prices and costs are known exactly, cost efficiency, revenue efficiency and profit efficiency. Since considering the effects of the choice of vector of production on both costs and revenues, profit efficiency is a broader concept than the other two (Ariff and Can, 2007). In this thesis, we use the profit efficiency as the criteria to evaluate the operating performance.

Three existing studies, Wu (2010), Wu and Birge (2011), and Wu, Zhou and Birge (2010) have attributed much to this thesis. Wu (2010) creates innovative Bilevel programming DEA models to solve performance evaluation problem with the hierarchical structure using a cost-effective way. Based on this approach, the performance of both the system and the subsystem are exposed in details. Wu and Birge (2011) develop multi-stage series-chain merger DEA to evaluate merger efficiency and define merger efficiency concepts. A case illustration in a mortgage banking merger is given and significant gains from promising mergers of the mortgage banking chains are reported. In Wu, Zhou and Birge (2010), the merger efficiency is evaluated by a merger dynamic DEA model and using a comparison to stochastic frontier analysis, the utilization of DEA is validated. Our thesis is related to these three papers, but we apply the Bilevel programming DEA approach in the merger event.

In this thesis, we follow the DEA non-parametric approach and develop a DEA Bilevel programming model to evaluate the profit efficiency of above-mentioned hierarchical decision network structures before the merger and also after the merger. To encourage the Follower to participate in this hierarchical system, a α -Strategy is suggested to strengthen the structure. With the completion of the merger, merger effect, harmony effect, and scale effect are derived and also used to measure the gain from the merger. The concept of coordinated effective merger is also discussed.

This thesis is outlined as follows. Chapter 2 gives a brief review of the Bilevel programming problem and DEA profit efficiency model. Chapter 3 develops Bilevel programming DEA to evaluate the profit efficiency. Chapter 4 proposes a methodology to evaluate the merger efficiency and decompose it into harmony effect and scale effect. Chapter 5 reports the empirical results from two numerical examples. Finally, conclusions, limitations and future research are presented in Chapter 6.

Chapter 2

Review of Bilevel programming problem and DEA profit efficiency

2.1 Bilevel programming problem

Bilevel programming problem is a hierarchical optimization problem consisting of two levels when the constraints of an optimization problem are also determined by the other optimization problem. The upper level, which is also termed as the Leader's level, is dominant over the lower level which is also considered as the Follower's level. The Leader makes the choice first to optimize his objective function. Observing the selection of the Leader, the Follower makes response which in turn affects the leader's outcome.

A Bilevel Linear Programming (BLP) given by Bard (1998) is formulated as follows:

$$\begin{aligned} &\min_{x} \quad F(x,y) = p_1^T x + q_1^T y \\ &s.t. \quad A_1 x + B_1 y \le b_1 \\ &\min_{y} \quad f(x,y) = p_2^T x + q_2^T y \\ &s.t. \quad A_2 x + B_2 y \le b_2 \end{aligned} \tag{1}$$

where $x \in R^n$, $y \in R^m$ refer to the decision variables corresponding to the upper and lower level respectively, $p_1, p_2 \in R^n, q_1, q_2 \in R^m, b_1 \in R^c, b_2 \in R^d, A_1 \in R^{c\times n}, B_1 \in R^{c\times m}$, $A_2 \in R^{d\times n}, B_2 \in R^{d\times m}$ and T denotes transpose.

Let $u \in R^p, v \in R^q$ and $w \in R^m$ be the dual variables associated with the constraints $A_1 x + B_1 y \le b_1$, $A_2 x + B_2 y \le b_2$ and $y \ge 0$, respectively. The following theorem is presented and proved by Shi et al. (2005) using an extended Kuhn-Tucker approach to reformulate Bilevel linear programming.

Theorem 1 (Shi et al., 2005) A necessary and sufficient condition that $\left(x^*, y^*\right)$ solves the BLP problem (1) is the existence of (row) vectors u^*, v^* and w^* such that $\left(x^*, y^*, u^*, v^*, w^*\right)$ solves the following Single level programming problem (2):

$$\begin{aligned}
\min_{x,y,u,v,w} \quad & F(x,y) = p_1^T x + q_1^T y \\
s.t. \quad & A_1 x + B_1 y \le b_1, \\
& A_2 x + B_2 y \le b_2, \\
& u B_1^T + v B_2^T - w = -q_2, \\
& u^T (b_1 - A_1 x - B_1 y) + v^T (b_2 - A_2 x - B_2 y) + w^T y = 0, \\
& x, y, u, v, w \ge 0.
\end{aligned} \tag{2}$$

Theorem 1 provides a method to transform the linear Bilevel programming problem into a single level programming problem which is a standard mathematical program and relatively easy to solve since all but one of the constraints are linear. And an extended branch and bound algorithm is proposed by Shi et al. (2006) to solve this model.

2.2 DEA profit efficiency

Data envelopment analysis (DEA) is a linear programming methodology to measure the efficiency of multiple organizations and indicate the differences between the inefficient ones and the best practice ones. DEA is a widely used technique to evaluate the performance of various organizations in public and private sectors.

In DEA, the organization is also called a decision making unit (DMU). Generically, a DMU is regarded as the entity responsible for converting inputs into outputs. For example, banks, supermarkets, car makers, hospitals etc can all be seen as DMUs.

Consider n DMUs that use a vector of p inputs: $x_i = \left(x_{i1}, ..., x_{ip}\right)$ to produce a vector of q outputs $y_i = \left(y_{i1},...,y_{iq}\right)$. The profit efficiency for DMU j can be evaluated based on a linear programming model proposed by Cooper et al. (2000).

$$\begin{aligned} \max \quad & \sum_{r=1}^{q} d_r{}^T \tilde{y}_{jr} - \sum_{s=1}^{p} c_s{}^T \tilde{x}_{js} \\ s.t. \quad & \sum_{i=1}^{n} \lambda_i x_{ir} \le \tilde{x}_{jr} \qquad (r=1,...,p), \\ & \sum_{i=1}^{n} \lambda_i y_{is} \ge \tilde{y}_{js} \qquad (s=1,...,q), \\ & \lambda_i \ge 0 \qquad (i=1,...,n), \end{aligned} \tag{3}$$

where $\left(\tilde{x}_{j1},...,\tilde{x}_{jp},\tilde{y}_{j1},...,\tilde{y}_{jq}\right)$ are decision variables and $c=(c_1,...,c_p)$ and $d=\left(d_1,...,d_q\right)$ are the unit price vectors correlating to the input $\tilde{x}_j=(\tilde{x}_{j1},...,\tilde{x}_{jp})$ and output $\tilde{y}_{j=}(\tilde{y}_{j1},...,\tilde{y}_{jq})$ vectors respectively, $\lambda=(\lambda_1,...,\lambda_n)$ is a nonnegative multiplier used to aggregate existing activities. The objective of Model (3) is to maximize the profit with the given prices of outputs d and of inputs c. Based on an optimal solution $\left(x^*_{j1},...,x^*_{jp},y^*_{j1},...,y^*_{jq}\right)$ of the above model, the profit efficiency of DMU j $\left(PE_j\right)$ is defined as follows:

$$PE_j = \frac{\sum_{r=1}^{q} d_r y_{jr} - \sum_{s=1}^{p} c_s x_{js}}{\sum_{r=1}^{q} d_r y^*_{jr} - \sum_{s=1}^{p} c_s x^*_{js}} \tag{4}$$

where $y_j=(y_{j1},...,y_{jq}), x_j=(x_{j1},...,x_{jp})$ are the vectors of observed values for DMU j.

Combining the demand and production of DMU j, $\left(PE_j\right)$ indicates the ratio between the observed profit and the optimized profit.

Under the assumption $\sum_{r=1}^{q} d_r{}^T y_{jr} - \sum_{s=1}^{p} c_s{}^T x_{js} > 0$, we have $0 < PE_j \le 1$, i.e. the profit efficiency score is within the range of 0 and 1, and DMU j $\left(x_j, y_j\right)$ is profit efficient if and only if $PE_j = 1$.

Chapter 3

Bilevel programming DEA model

In this chapter, we combine the Bilevel programming and the DEA theory together to create Bilevel programming DEA models to evaluate the performance of the hierarchical system and the sub-levels based on the profit efficiency under two situations. The models are further reformulated in the standard linear Bilevel programming forms which can be easily transformed to the Single level programming problems according to Theorem 1 and solved by the extended branch and bound algorithms in Shi et al. (2006).

Suppose n Bilevel decision systems (DMUs) under evaluation, each indexed by j *(j = 1, 2,…, n)* and each system (DMU) includes two decentralized subsystems: a Leader and a Follower. The Leader utilizes two types of inputs, i.e., the shared input X^1 and the possible direct input X^{D1}, to produce two different types of outputs: the intermediate output Y and the direct output Z^1. To produce the direct output Z^2, the Follower consumes three types of inputs, i.e., the shared input X^2 and the possible direct input X^{D2} and the intermediate input Y from the Leader. The input is mostly constricted due to the limited resource in reality. In this case, the amount of the direct input and the total amount of the shared input are both upper bounded in this thesis. Fig. 1 depicts the framework of this Bilevel programming DEA model with limited resource.

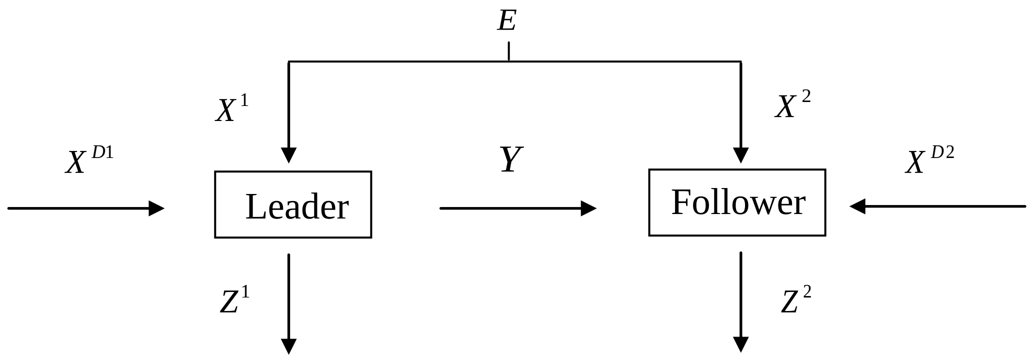

Fig. 1. Bilevel programming DEA model with limited resource

Based on this hierarchical structure, we propose two situations.

In the first situation, we suppose that the Leader makes the decision based on his inputs (shared input X^1 and direct input X^{D1}) and outputs (direct output Z^1 and intermediate output Y) first to maximize his own profit. Depending on the Leader's decision, the maximum resource available for the follower will be X^2 (or $E - X^1$) and Y intermediate input. The Follower, based on his inputs (shared input X^2, direct input X^{D2} and intermediate input Y), determines the outputs (direct output Z^2) to maximize his profit.

The following model represents the case discussed above.

Model (5)

$$
\begin{aligned}
(P1)\ &\max_{\tilde{X}_J^1,\tilde{X}_J^{D1},\tilde{Y}_J^1,\tilde{Z}_J^1,\lambda} (Q^{1^T}\tilde{Z}_J^1 + Q^{2^T}\tilde{Y}_J^1) - (P^{1^T}\tilde{X}_J^1 + P^{2^T}\tilde{X}_J^{D1}) \\
&s.t \quad \tilde{X}_J^1 + \tilde{X}_J^2 \geq \sum_{j=1}^{n} X_j^1\lambda_j + \sum_{j=1}^{n} X_j^2\pi_j, \\
&\quad \tilde{X}_J^{D1} \geq \sum_{j=1}^{n} X_j^{D1}\lambda_j, \\
&\quad \tilde{Z}_J^1 \leq \sum_{j=1}^{n} Z_j^1\lambda_j, \\
&\quad \tilde{Y}_J^1 \leq \sum_{j=1}^{n} Y_j\lambda_j, \\
&\quad \tilde{X}_J^1 + \tilde{X}_J^2 \leq E(const.), \\
&\quad \tilde{X}_J^{D1} \leq X_J^{D1}, \\
&\quad (Q^{1^T}\tilde{Z}_J^1 + Q^{2^T}\tilde{Y}_J^1) - (P^{1^T}\tilde{X}_J^1 + P^{2^T}\tilde{X}_J^{D1}) \geq (Q^{1^T}Z_J^1 + Q^{2^T}Y_J^1) - (P^{1^T}X_J^1 + P^{2^T}X_J^{D1})
\end{aligned}
$$

$$(P2)\ \max_{\tilde{X}_J^2,\tilde{X}_J^{D2},\tilde{Y}_J^2,\tilde{Z}_J^2,\pi} Q^{3^T}\tilde{Z}_J^2-(P^{1^T}\tilde{X}_J^2+P^{3^T}\tilde{X}_J^{D2}+Q^{2^T}\tilde{Y}_J^2) \tag{5}$$

$$\begin{aligned} s.t\quad & \tilde{X}_J^{D2}\ge\sum_{j=1}^{n}X_j^{D2}\pi_j,\\ & \tilde{Y}_J^2\ge\sum_{j=1}^{n}Y_j\pi_j,\\ & \tilde{Z}_J^2\le\sum_{j=1}^{n}Z_j^2\pi_j,\\ & \tilde{X}_J^{D2}\le X_J^{D2},\\ & \tilde{Y}_J^2\le\tilde{Y}_J^1,\\ & Q^{3^T}\tilde{Z}_J^2-(P^{1^T}\tilde{X}_J^2+P^{3^T}\tilde{X}_J^{D2}+Q^{2^T}\tilde{Y}_J^2)\ge Q^{3^T}Z_J^2-(P^{1^T}X_J^2+P^{3^T}X_J^{D2}+Q^{2^T}Y_J^2)\\ & \tilde{X}_J^1,\tilde{X}_J^2,\tilde{X}_J^{D1},\tilde{X}_J^{D2},\tilde{Y}_J^1,\tilde{Y}_J^2,\tilde{Z}_J^1,\tilde{Z}_J^2,\lambda,\pi\ge 0 \end{aligned}$$

where T denotes transpose, Y^1 denotes the intermediate output from the Leader, Y^2 denotes the intermediate input consumed by the Follower, P^1 is the unit cost vector of the shared inputs X^1, X^2 to the Leader and the Follower, P^2, P^3 are the unit cost vectors correlating to the direct inputs X^{D1}, X^{D2} to the Leader and the Follower. Q^1, Q^3 are the unit price vectors of the Leader's direct output Z^1 and the Follower's direct output Z^2, respectively. Q^2 is the unit price vector both of the Leader's intermediate output Y^1 and the Follower's intermediate input Y^2.

Model (5) is characterized as a constant returns-to-scale (CRS) DEA model, which is noted in Cooper et al. (2000). Returns-to-scale refers to changes in output resulting from a proportional change in all inputs. CRS reflects the fact that outputs will change by the same proportion as inputs are changed. If we add the constraints $\sum_{i=1}^{n}\lambda_i=1$, $\sum_{i=1}^{n}\pi_i=1$ respectively, in the upper level and lower level in Model (5), we will have the variable returns-to-scale (VRS) DEA model. VRS reflects the fact that production technology may exhibit increasing, constant and decreasing returns-to-scale. Increasing returns-to-scale means outputs increase by more than that proportional change in all inputs, however decreasing returns-to-scale means outputs increase by less than that proportional change in all inputs.

Model (5) can be reformulated in the standard linear Bilevel programming form as shown in Appendix I.

According to Theorem 1, the corresponding Single level problem can be obtained. The variables and coefficient matrixes in the Single level problem are shown in Appendix I.

In the second situation, we swap the position of the Leader and the Follower. The new Leader, who is the Follower in the first situation, maximizes his profit by choosing the inputs (shared input X^2, direct input X^{D2} and intermediate input Y) and outputs (direct output Z^2). After the intermediate input Y is determined by the new Leader, the minimum intermediate output required from the new Follower will be Y. The new Follower determines on his inputs (shared input X^1 and direct input X^{D1}) and outputs (direct output Z^1 and intermediate output Y) to maximize his profit then. Fig. 2 and the Model (6) depict the case discussed above.

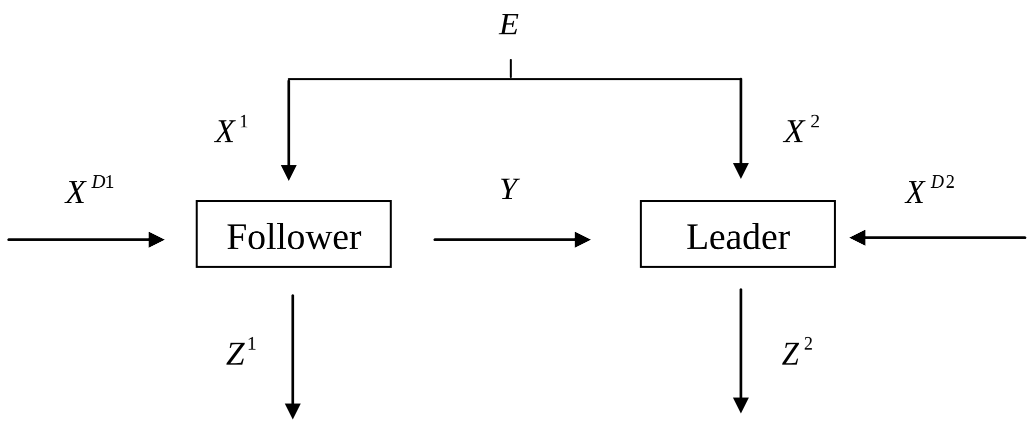

Fig. 2. Bilevel programming DEA model with limited resource in the 2nd situation

Model (6)

$$(P1)\ \max_{\tilde{X}_J^2,\tilde{X}_J^{D2},\tilde{Z}_J^2,\tilde{Y}_J^2,\pi} P^{2^T}\tilde{Z}_J^2-(C^{1^T}\tilde{X}_J^2+C^{3^T}\tilde{X}_J^{D2}+Q^T\tilde{Y}_J^2)$$

$$s.t\quad \tilde{X}_J^2+\tilde{X}_J^1\geq\sum_{j=1}^n X_j^2\pi_j+\sum_{j=1}^n X_j^1\lambda_j,$$

$$\tilde{X}_J^{D2}\geq\sum_{j=1}^n X_j^{D2}\pi_j,$$

$$\tilde{Y}_J^2\geq\sum_{j=1}^n Y_j\pi_j,$$

$$\tilde{Z}_J^2\leq\sum_{j=1}^n Z_j^2\pi_j,$$

$$\tilde{X}_J^{D2}\leq X_J^{D2},$$

$$\tilde{X}_J^1+\tilde{X}_J^2\leq E(const.),$$

$$P^{2^T}\tilde{Z}_J^2-(C^{1^T}\tilde{X}_J^2+C^{3^T}\tilde{X}_J^{D2}+Q^T\tilde{Y}_J^2)\geq P^{2^T}Z_J^2-(C^{1^T}X_J^2+C^{3^T}X_J^{D2}+Q^TY_J^2),$$

$$(P2)\ \max_{\tilde{X}_J^1,\tilde{X}_J^{D1},\tilde{Y}_J^1,\tilde{Z}_J^1,\lambda} (P^{1^T}\tilde{Z}_J^1 + Q^T\tilde{Y}_J^1) - (C^{1^T}\tilde{X}_J^1 + C^{2^T}\tilde{X}_J^{D1}) \tag{6}$$

$$\begin{aligned}
s.t\quad & \tilde{X}_J^{D1} \ge \sum_{j=1}^{n} X_j^{D1}\lambda_j, \\
& \tilde{Z}_J^1 \le \sum_{j=1}^{n} Z_j^1\lambda_j, \\
& \tilde{Y}_J^1 \le \sum_{j=1}^{n} Y_j\lambda_j, \\
& \tilde{X}_J^{D1} \le X_J^{D1}, \\
& \tilde{Y}_J^1 \ge \tilde{Y}_J^2, \\
& (P^{1^T}\tilde{Z}_J^1 + Q^T\tilde{Y}_J^1) - (C^{1^T}\tilde{X}_J^1 + C^{2^T}\tilde{X}_J^{D1}) \ge (P^{1^T}Z_J^1 + Q^TY_J^1) - (C^{1^T}X_J^1 + C^{2^T}X_J^{D1}) \\
& \tilde{X}_J^1, \tilde{X}_J^2, \tilde{X}_J^{D1}, \tilde{X}_J^{D2}, \tilde{Y}_J^1, \tilde{Y}_J^2, \tilde{Z}_J^1, \tilde{Z}_J^2, \lambda, \pi \ge 0
\end{aligned}$$

where the notations are also the same with those in Model (5).

The standard linear Bilevel programming form of Model (6) is shown in Appendix I.

Similarly to the first two situations, the corresponding Single level problem can be obtained according to Theorem 1. The variables and coefficient matrixes in the Single level problem are shown in Appendix I.

The following definitions about the profit efficiency are used to evaluate the performance of the two levels as well as the whole system.

Definition 1(Profit efficiency of the I^{th} Leader)

The profit efficiency of the I^{th} Leader in the first situation is defined as

$$PE_I^L = \frac{(Q^{1^T}Z_I^1 + Q^{2^T}Y_I^1) - (P^{1^T}X_I^1 + P^{2^T}X_I^{D1})}{(Q^{1^T}Z_I^{1*} + Q^{2^T}Y_I^{1*}) - (P^{1^T}X_I^{1*} + P^{2^T}X_I^{D1*})}, \tag{7}$$

where $X_I^{1*}, X_I^{D1*}, Y_I^*, Z_I^{1*}$ are the optimal solutions to Model (5).

The I^{th} Leader is termed profit efficient if and only if the profit efficiency of the I^{th} Leader is 1, i.e. $PE_I^L = 1$.

Definition 2(Profit efficiency of the I^{th} Follower)

The profit efficiency of the I^{th} Follower in the first situation is defined as

$$PE_I^F = \frac{Q^{3^T} Z_I^2 - (P^{1^T} X_I^2 + P^{3^T} X_I^{D2} + Q^{2^T} Y_I^2)}{Q^{3^T} Z_I^{2*} - (P^{1^T} X_I^{2*} + P^{3^T} X_I^{D2*} + Q^{2^T} Y_I^{2*})}, \tag{8}$$

where $Z_I^{2*}, X_I^{2*}, X_I^{D2*}, Y_I^*$ are the optimal solutions to Model (5).

The I^th Follower is termed profit efficient if and only if the profit efficiency of the I^th Follower is 1, i.e. $PE_I^F = 1$.

Definition 3(Profit efficiency of the I^th system)

The profit efficiency of the I^th system in the first situation is defined as

$$PE_I^S = \frac{(Q^{1^T} Z_I^1 + Q^{2^T} Y_I^1 + Q^{3^T} Z_I^2) - (P^{1^T} X_I^1 + P^{2^T} X_I^{D1} + P^{1^T} X_I^2 + P^{3^T} X_I^{D2} + Q^{2^T} Y_I^2)}{(Q^{1^T} Z_I^{1*} + Q^{2^T} Y_I^{1*} + Q^{3^T} Z_I^{2*}) - (P^{1^T} X_I^{1*} + P^{2^T} X_I^{D1*} + P^{1^T} X_I^{2*} + P^{3^T} X_I^{D2*} + Q^{2^T} Y_I^{2*})} \tag{9}$$

where $X_I^{1*}, X_I^{2*}, X_I^{D1*}, X_I^{D2*}, Y_I^{1*}, Y_I^{2*}, Z_I^{1*}, Z_I^{2*}$ are the optimal solutions to Model (5).

The I^th system is termed profit efficient if and only if the profit efficiency of the I^th system is 1, i.e. $PE_I^S = 1$.

Similarly, the profit efficiency of the Leader, the Follower and the whole system can be defined in the second situation.

Under the assumption that the actual profit is positive, we present the following three propositions about the profit efficiency of the Leader, the Follower and the system under both of these two situations. The first and the third propositions are similar to the propositions about the cost efficiency in Wu (2010).

Proposition 1

In the I^th system, if the profit efficiency of the Leader PE_I^L is greater than that of the Follower PE_I^F, then $PE_I^L > PE_I^S > PE_I^F$; if the profit efficiency of the Leader PE_I^L equals that of the Follower PE_I^F, then $PE_I^L = PE_I^S = PE_I^F$; if the profit efficiency of the Leader PE_I^L is less than that of the Follower PE_I^F, then $PE_I^L < PE_I^S < PE_I^F$.

Proof

Similar to the proof of Proposition 1 in Wu (2010). □

Proposition 2

In the I[th] system, the profit efficiency of the whole system PE_I^S, which is the system performance measure, is a convex combination of the profit efficiency of the Leader PE_I^L and profit efficiency of the Follower PE_I^F.

Proof

Let $\omega_1 = \dfrac{(P^{1^T} Z_I^{1*} + Q^T Y_I^{1*}) - (C^{1^T} X_I^{1*} + C^{2^T} X_I^{D1*})}{(P^{1^T} Z_I^{1*} + Q^T Y_I^{1*} + P^{2^T} Z_I^{2*}) - (C^{1^T} X_I^{1*} + C^{2^T} X_I^{D1*} + C^{1^T} X_I^{2*} + C^{3^T} X_I^{D2*} + Q^T Y_I^{2*})}$

and $\omega_2 = \dfrac{P^{2^T} Z_I^{2*} - (C^{1^T} X_I^{2*} + C^{3^T} X_I^{D2*} + Q^T Y_I^{2*})}{(P^{1^T} Z_I^{1*} + Q^T Y_I^{1*} + P^{2^T} Z_I^{2*}) - (C^{1^T} X_I^{1*} + C^{2^T} X_I^{D1*} + C^{1^T} X_I^{2*} + C^{3^T} X_I^{D2*} + Q^T Y_I^{2*})}$;

then, it is easy to see $\omega_1 + \omega_2 = 1$ and $PE_I^S = \omega_1 PE_I^L + \omega_2 PE_I^F$, which complete the proof. □

Proposition 3

In the I[th] system, the system is efficient, i.e. $PE_I^S = 1$, only if both of the Leader and the Follower are efficient, i.e. $PE_I^L = 1$ and $PE_I^F = 1$.

Proof

Similar to the proof of Proposition 2 in Wu (2010). □

Chapter 4

The evaluation of the merger efficiency

In order to analyze the potential gains from the merger of N Bilevel systems, and evaluate the merger efficiency, the following steps are proposed (Wu and Birge, 2011) and the first situation is used to set an example.

Step 1: First solve the Bilevel programming problem for each DMU using Model (5). According to Theorem 1 and the branch and bound algorithm, we can obtain the optimal solution of $(X_J^{1*}, X_J^{2*}, X_J^{D1*}, X_J^{D2*}, Y_J^{1*}, Y_J^{2*}, Z_J^{1*}, Z_J^{2*}, \lambda^*, \pi^*)$, and it is an optimal production decision for each DMU to be efficient. We then construct the efficient input-output combination ($X_J^{1*}, X_J^{2*}, X_J^{D1*}, X_J^{D2*}, Y_J^{1*}, Y_J^{2*}, Z_J^{1*}, Z_J^{2*}$) for each Bilevel system.

Step 2: Compute the average input bundle, intermediate output/input bundle and out bundle

$$\bar{X}^1 = \frac{1}{n}\sum_{j=1}^{n} X_j^{1*},$$

$$\bar{X}^2 = \frac{1}{n}\sum_{j=1}^{n} X_j^{2*},$$

$$\bar{X}^{D1} = \frac{1}{n}\sum_{j=1}^{n} X_j^{D1*},$$

$$\bar{X}^{D2} = \frac{1}{n}\sum_{j=1}^{n} X_j^{D2*},$$

$$\bar{Y}^1 = \frac{1}{n}\sum_{j=1}^{n} Y_j^{1*},$$

$$\bar{Y}^2 = \frac{1}{n}\sum_{j=1}^{n} Y_j^{2*},$$

$$\bar{Z}^1 = \frac{1}{n}\sum_{j=1}^{n} Z_j^{1*},$$

$$\bar{Z}^2 = \frac{1}{n}\sum_{j=1}^{n} Z_j^{2*},$$

Step 3: Solve the Bilevel programming DEA problem with the average input bundle

Model (10)

$$
\begin{aligned}
&(P1) \max_{X_H^1, X_H^{D1}, Z_H^1, Y_H^1, \lambda} (Q^{1^T} Z_H^1 + Q^{2^T} Y_H^1) - (P^{1^T} X_H^1 + P^{2^T} X_H^{D1}) \\
&\quad s.t \quad X_H^1 + X_H^2 \ge \sum_{j=1}^{n} X_j^1 \lambda_j + \sum_{j=1}^{n} X_j^2 \pi_j, \\
&\qquad X_H^{D1} \ge \sum_{j=1}^{n} X_j^{D1} \lambda_j, \\
&\qquad Z_H^1 \le \sum_{j=1}^{n} Z_j^1 \lambda_j, \\
&\qquad Y_H^1 \le \sum_{j=1}^{n} Y_j \lambda_j, \\
&\qquad X_H^1 + X_H^2 \le E(const.), \\
&\qquad X_H^{D1} \le \bar{X}^{D1}, \\
&(P2) \max_{X_H^2, X_H^{D2}, Y_H^2, Z_H^2, \pi} Q^{3^T} Z_H^2 - (P^{1^T} X_H^2 + P^{3^T} X_H^{D2} + Q^{2^T} Y_H^2) \\
&\quad s.t \quad X_H^{D2} \ge \sum_{j=1}^{n} X_j^{D2} \pi_j, \\
&\qquad Y_H^2 \ge \sum_{j=1}^{n} Y_j \pi_j, \\
&\qquad Z_H^2 \le \sum_{j=1}^{n} Z_j^2 \pi_j, \\
&\qquad X_H^{D2} \le \bar{X}^{D2}, \\
&\qquad Y_H^2 \le Y_H^1, \\
&\qquad X_H^1, X_H^2, X_H^{D1}, X_H^{D2}, Y_H^1, Y_H^2, Z_H^1, Z_H^2, \lambda_H, \pi_H \ge 0
\end{aligned}
\tag{10}
$$

Model (10) can be reformulated in a standard Bilevel programming form which can be easily transformed to the Single programming problem, as shown in Appendix I.

The variables and coefficient matrixes in the Single level problem corresponding to Model (10) are shown in Appendix I.

The Single programming problem can be solved by branch and bound algorithms mentioned before. Denote by $\left(X_H^{1*}, X_H^{2*}, X_H^{D1*}, X_H^{D2*}, Y_H^{1*}, Y_H^{2*}, Z_H^{1*}, Z_H^{2*}, \lambda_H^{*}, \pi_H^{*}\right)$ the optimal solution to Model (10), and it is an optimal production decision for the individual DMU using the average input bundle before merger.

Step 4: Define the total (slack-adjusted) input and output bundles of N systems

$$\begin{aligned}
X_{Total}^{1} &= N \cdot \bar{X}^{1}, \\
X_{Total}^{2} &= N \cdot \bar{X}^{2}, \\
X_{Total}^{D1} &= N \cdot \bar{X}^{D1}, \\
X_{Total}^{D2} &= N \cdot \bar{X}^{D2}, \\
Y_{Total}^{1} &= N \cdot \bar{Y}^{1}, \\
Y_{Total}^{2} &= N \cdot \bar{Y}^{2}, \\
Z_{Total}^{1} &= N \cdot \bar{Z}^{1}, \\
Z_{Total}^{2} &= N \cdot \bar{Z}^{2}.
\end{aligned}$$

Step 5: Solve the Bilevel programming merger DEA problem

Model (11)

$$\begin{aligned}
(P1)\ & \max_{X_M^1, X_M^{D1}, Z_M^1, Y_M^1, \lambda} (Q^{1^T} Z_M^1 + Q^{2^T} Y_M^1) - (P^{1^T} X_M^1 + P^{2^T} X_M^{D1}) \\
& s.t \quad X_M^1 + X_M^2 \geq \sum_{j=1}^{n} X_j^1 \lambda_j + \sum_{j=1}^{n} X_j^2 \pi_j, \\
& \qquad X_M^{D1} \geq \sum_{j=1}^{n} X_j^{D1} \lambda_j, \\
& \qquad Z_M^1 \leq \sum_{j=1}^{n} Z_j^1 \lambda_j, \\
& \qquad Y_M^1 \leq \sum_{j=1}^{n} Y_j \lambda_j, \\
& \qquad X_M^1 + X_M^2 \leq N \cdot E(const.), \\
& \qquad X_M^{D1} \leq X_{Total}^{D1}, \\
(P2)\ & \max_{X_M^2, X_M^{D2}, Z_M^2, Y_M^2, \pi} Q^{3^T} Z_M^2 - (P^{1^T} X_M^2 + P^{3^T} X_M^{D2} + Q^{2^T} Y_M^2) \qquad (11) \\
& s.t \quad X_M^{D2} \geq \sum_{j=1}^{n} X_j^{D2} \pi_j, \\
& \qquad Y_M^2 \geq \sum_{j=1}^{n} Y_j \pi_j, \\
& \qquad Z_M^2 \leq \sum_{j=1}^{n} Z_j^2 \pi_j,
\end{aligned}$$

$$X_M^{D2} \le X_{Total}^{D2},$$
$$Y_M^2 \le Y_M^1,$$
$$X_M^1, X_M^2, X_M^{D1}, X_M^{D2}, Y_M^1, Y_M^2, Z_M^1, Z_M^2, \lambda_M, \pi_M \ge 0$$

Model (11) can be also reformulated in a standard Bilevel programming form which can be easily transformed to the Single programming problem, shown in Appendix I.

The variables and coefficient matrixes in the Single level problem corresponding to Model (11) are shown in Appendix I.

Then the Single programming problem can be solved. Denote by $\left(X_M^{1*}, X_M^{2*}, X_M^{D1*}, X_M^{D2*}, Y_M^{1*}, Y_M^{2*}, Z_M^{1*}, Z_M^{2*}, \lambda_M^*, \pi_M^*\right)$ the optimal solution to Model (11), and it is an optimal production decision for the merger involved DMUs after merger.

Proposition 4

Under the constant returns to scale (CRS) assumption, the optimal solution to Model (11) $\left(X_M^{1*}, X_M^{2*}, X_M^{D1*}, X_M^{D2*}, Y_M^{1*}, Y_M^{2*}, Z_M^{1*}, Z_M^{2*}, \lambda_M^*, \pi_M^*\right)$ is N multiple of the optimal solution to Model (10) $\left(X_H^{1*}, X_H^{2*}, X_H^{D1*}, X_H^{D2*}, Y_H^{1*}, Y_H^{2*}, Z_H^{1*}, Z_H^{2*}, \lambda_H^*, \pi_H^*\right)$.

Proof

Multiply both of the object functions and both sides of all the constraints in Model (10) by N, and define this new model by Model (12).

Model (12)

$$(P1) \max_{X_H^1, X_H^{D1}, Z_H^1, Y_H^1, \lambda} N \cdot \left[(Q^{1^T} Z_H^1 + Q^{2^T} Y_H^1) - (P^{1^T} X_H^1 + P^{2^T} X_H^{D1}) \right]$$
$$s.t \quad \mathrm{N} \cdot \left(X_H^1 + X_H^2 \right) \ge N \cdot \left(\sum_{j=1}^n X_j^1 \lambda_j + \sum_{j=1}^n X_j^2 \pi_j \right),$$
$$\mathrm{N} \cdot X_H^{D1} \ge \mathrm{N} \cdot \sum_{j=1}^n X_j^{D1} \lambda_j,$$
$$\mathrm{N} \cdot Z_H^1 \le \mathrm{N} \cdot \sum_{j=1}^n Z_j^1 \lambda_j,$$
$$\mathrm{N} \cdot Y_H^1 \le \mathrm{N} \cdot \sum_{j=1}^n Y_j \lambda_j,$$
$$\mathrm{N} \cdot \left(X_H^1 + X_H^2 \right) \le \mathrm{N} \cdot E,$$
$$\mathrm{N} \cdot X_H^{D1} \le \mathrm{N} \cdot \bar{X}^{D1},$$

$$(P2)\ \max_{X_H^2, X_H^{D2}, Y_H^2, Z_H^2, \pi} \mathrm{N}\cdot\left[Q^{3^T} Z_H^2 - (P^{1^T} X_H^2 + P^{3^T} X_H^{D2} + Q^{2^T} Y_H^2)\right] \tag{12}$$

$$
\begin{aligned}
s.t\quad & \mathrm{N}\cdot X_H^{D2} \ge \mathrm{N}\cdot \sum_{j=1}^{n} X_j^{D2}\pi_j,\\
& \mathrm{N}\cdot Y_H^2 \ge \mathrm{N}\cdot \sum_{j=1}^{n} Y_j\pi_j,\\
& \mathrm{N}\cdot Z_H^2 \le \mathrm{N}\cdot \sum_{j=1}^{n} Z_j^2\pi_j,\\
& \mathrm{N}\cdot\ X_H^{D2} \le \mathrm{N}\cdot \bar{X}^{D2},\\
& \mathrm{N}\cdot Y_H^2 \le \mathrm{N}\cdot Y_H^1,\\
& X_H^1, X_H^2, X_H^{D1}, X_H^{D2}, Y_H^1, Y_H^2, Z_H^1, Z_H^2, \lambda_H, \pi_H \ge 0
\end{aligned}
$$

Define new variables that are equal to N times of those in Model (12).

$$
\begin{aligned}
\tilde{X}_H^1 &= N\cdot X_H^1,\\
\tilde{X}_H^2 &= N\cdot X_H^2,\\
\tilde{X}_H^{D1} &= N\cdot X_H^{D1},\\
\tilde{X}_H^{D2} &= N\cdot X_H^{D2},\\
\tilde{Y}_H^1 &= N\cdot Y_H^1,\\
\tilde{Y}_H^2 &= N\cdot Y_H^2,\\
\tilde{Z}_H^1 &= N\cdot Z_H^1,\\
\tilde{Z}_H^2 &= N\cdot Z_H^2,\\
\tilde{\lambda}_H &= N\cdot \lambda_H,\\
\tilde{\pi}_H &= N\cdot \pi_H
\end{aligned}
$$

Then the adjusted Model (12) is shown as follows.

$$(P1)\ \max_{\tilde{X}_H^1, \tilde{X}_H^{D1}, \tilde{Z}_H^1, \tilde{Y}_H^1, \tilde{\lambda}} (Q^{1^T} \tilde{Z}_H^1 + Q^{2^T} \tilde{Y}_H^1) - (P^{1^T} \tilde{X}_H^1 + P^{2^T} \tilde{X}_H^{D1})$$

$$
\begin{aligned}
s.t\quad & \tilde{X}_H^1 + \tilde{X}_H^2 \ge \sum_{j=1}^{n} X_j^1\tilde{\lambda}_j + \sum_{j=1}^{n} X_j^2\tilde{\pi}_j,\\
& \tilde{X}_H^{D1} \ge \sum_{j=1}^{n} X_j^{D1}\tilde{\lambda}_j,\\
& \tilde{X}_H^1 \le \sum_{j=1}^{n} Z_j^1\tilde{\lambda}_j,\\
& \tilde{Y}_H^1 \le \sum_{j=1}^{n} Y_j\tilde{\lambda}_j,\\
& \tilde{X}_H^1 + \tilde{X}_H^2 \le N\cdot E,\\
& \tilde{X}_H^{D1} \le N\cdot \bar{X}^{D1},
\end{aligned}
$$

$$(P2)\max_{\tilde{X}_H^2,\tilde{X}_H^{D2},\tilde{Y}_H^2,\tilde{Z}_H^2,\tilde{\pi}} Q^{3^T}\tilde{Z}_H^2-(P^{1^T}\tilde{X}_H^2+P^{3^T}\tilde{X}_H^{D2}+Q^{2^T}\tilde{Y}_H^2) \tag{13}$$

$$\begin{aligned} s.t \quad & \tilde{X}_H^{D2}\geq\sum_{j=1}^{n}X_j^{D2}\tilde{\pi}_j,\\ & \tilde{Y}_H^2\geq\sum_{j=1}^{n}Y_j\tilde{\pi}_j,\\ & \tilde{Z}_H^2\leq\sum_{j=1}^{n}Z_j^2\tilde{\pi}_j,\\ & \tilde{X}_H^{D2}\leq N\cdot\bar{X}^{D2},\\ & \tilde{Y}_H^2\leq\tilde{Y}_H^1,\\ & \tilde{X}_H^1,\tilde{X}_H^2,\tilde{X}_H^{D1},\tilde{X}_H^{D2},\tilde{Y}_H^1,\tilde{Y}_H^2,\tilde{Z}_H^1,\tilde{Z}_H^2,\tilde{\lambda}_H,\tilde{\pi}_H\geq 0\end{aligned}$$

Given that $X_{Total}^{D1}=N\bar{X}^{D1}$ and $X_{Total}^{D2}=N\bar{X}^{D2}$, it is easy to see Model (13), which is the adjusted Model (12), using the new variables is exactly the same with Model (11). Therefore, the optimal solution to Model (11) is N times of that to Model (10). Thus, the proposition holds. □

Based on the average input bundle, intermediate output/input bundle and output bundle from Step 2, setting

$$\bar{P}^S=(Q^{1^T}\bar{Z}^1+Q^{3^T}\bar{Z}^2+Q^{2^T}\bar{Y}^1)-[(P^{1^T}\bar{X}^1+P^{2^T}\bar{X}^{D1})+(P^{1^T}\bar{X}^2+P^{3^T}\bar{X}^{D2}+Q^{2^T}\bar{Y}^2)],$$

$$\bar{P}^L=(Q^{1^T}\bar{Z}^1+Q^{2^T}\bar{Y}^1)-(P^{1^T}\bar{X}^1+P^{2^T}\bar{X}^{D1}),,$$

$$\bar{P}^F=Q^{3^T}\bar{Z}^2-(P^{1^T}\bar{X}^2+P^{3^T}\bar{X}^{D2}+Q^{2^T}\bar{Y}^2).$$

Based on the optimal solution from Step 3, denote the total profit of the Bilevel system, the profit of the Leader and the profit of the Follower under the average input assumption by P_H^S, P_H^L and P_H^F, respectively. Then

$$P_H^S=(Q^{1^T}Z_H^{1*}+Q^{3^T}Z_H^{2*}+Q^{2^T}Y_H^{1*})-[(P^{1^T}X_H^{1*}+P^{2^T}X_H^{D1*})+(P^{1^T}X_H^{2*}+P^{3^T}X_H^{D2*}+Q^{2^T}Y_H^{2*})],$$

$$P_H^L=(Q^{1^T}Z_H^{1*}+Q^{2^T}Y_H^{1*})-(P^{1^T}X_H^{1*}+P^{2^T}X_H^{D1*}),$$

$$P_H^F=Q^{3^T}Z_H^{2*}-(P^{1^T}X_H^{2*}+P^{3^T}X_H^{D2*}+Q^{2^T}Y_H^{2*}).$$

Based on the optimal solution from Step 5, denote the total profit of the Bilevel system, the profit of the Leader and the profit of the Follower under merger assumption by P_M^S, P_M^L and P_M^F, respectively. Then

$$P_M^S = (Q^{1^T} Z_M^1 + Q^{3^T} Z_M^2 + Q^{2^T} Y_M^1) - [(P^{1^T} X_M^1 + P^{2^T} X_M^{D1}) + (P^{1^T} X_M^2 + P^{3^T} X_M^{D2} + Q^{2^T} Y_M^2)]$$

$$P_M^L = (Q^{1^T} Z_M^1 + Q^{2^T} Y_M^1) - (P^{1^T} X_M^1 + P^{2^T} X_M^{D1})$$

$$P_M^F = Q^{3^T} Z_M^2 - (P^{1^T} X_M^2 + P^{3^T} X_M^{D2} + Q^{2^T} Y_M^2)$$

The merger efficiency of the whole Bilevel system is measured in Wu and Birge (2011) as

$$E_m^S = \frac{P_M^S}{N \cdot \bar{P}^S} = \frac{P_H^S}{\bar{P}^S} \cdot \frac{P_M^S}{N \cdot P_H^S} = H^S \cdot S^S \tag{14}$$

where $H^S = \frac{P_H^S}{\bar{P}^S}$ represents the harmony effect and $S^S = \frac{P_M^S}{NP_H^S}$ represents the scale effect.

Similarly, the merger efficiency, harmony effect and scale effect of the Leader and the Follower can be defined. Denote the merger efficiency, harmony effect and scale effect of the Leader by E_m^L, H^L and S^L. Denote the merger efficiency, harmony effect and scale effect of the Follower by E_m^F, H^F and S^F.

If E_m is greater than 1, the N merger members will benefit from the potential profit generated by the merger, otherwise, it would be more efficient to keep them separate. The harmony effect measures the ratio between the profit gained under the average input assumption and the average profit of N individual merger members prior to the merger.

From Proposition 4, we can get that $P_M^S = NP_H^S$, $P_M^L = NP_H^L$ and $P_M^S = NP_H^S$ which indicate the scale effect S should equal to 1 under the CRS assumption. Then we can further get that the merger efficiency E_m equals to the harmony effect H under the CRS assumption.

The scale effect may be less than, equal to or greater than 1. If the score of the scale effect is greater than 1, the potential profit from the merger will be more than the sum of the profit produced by each member using the average input bundle.

Chapter 5

Two numerical studies

5.1 The first numerical example

To illustrate the theoretic findings, the following Bilevel decision example is solved by use of the model and algorithm proposed in this thesis. In this example, there are 8 branches in all, and each one can be seen as a DMU facing a hierarchical optimization problem consisting of two levels, the Leader's level and the Follower's level, and the framework of this hierarchical optimization problem is shown below in Fig. 3.

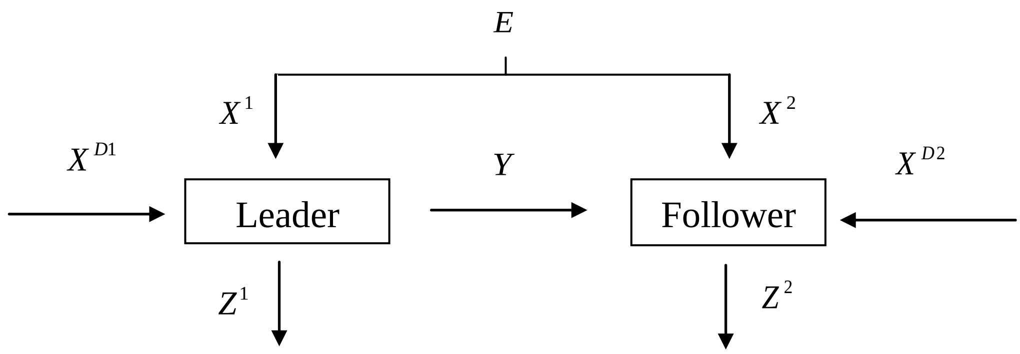

Fig. 3 The framework of Bilevel programming problem in the 1st example

In order to test our proposed approach better, the data is created at random and is made more complex. For the Leader, we employ three inputs (two direct inputs X^{D1} and one shared input X^1) and three outputs (two direct outputs Z^1 and one intermediate output Y). For the follower, we utilize four inputs (two direct inputs X^{D2}, one shared input X^2 and one intermediate input Y) and two direct outputs Z^2. The input and output data is exhibited in Table 1. Denote the profit of the Leader and the Follower by P^L and P^F respectively. And all of the unit price vectors to the inputs and outputs are set to be unit.

Table 1 The input and output data for the 8 branches in the 1st example

Branch	X^{D1}		X^1	Z^1		Y	X^{D2}		X^2	Z^2		P^L	P^F
DMU1	2.5	13	4	35	60	30	1.5	12	16	55	65	105.5	60.5
DMU2	7	12	13.4	76	53	55	5.6	13	6.6	87	45	151.6	51.8
DMU3	3	7	9.8	52	42	40	4	15.4	10.2	65	56	114.2	51.4
DMU4	9	18	4.6	63	71	70	8.8	11.2	15.4	78	89	172.4	61.6
DMU5	2.3	12.5	5	33	62	35	1.6	12.3	15	52	65	110.2	53.1
DMU6	7.4	11.7	14	73	50	53	5.8	13	6	85	42	142.9	49.2
DMU7	3.5	7.5	10	57	45	38	4	15.6	10	69	56	119	57.4
DMU8	8.8	17.9	5	60	70	72	9	11.5	15	75	90	170.3	57.5

5.1.1 Pre-merger

Considering the two situations we discussed above in this thesis.

Under the first situation, we investigate the profit efficiency of the system, the Leader and the Follower using Model (5), under the constant returns-to-scale (CRS) and variable returns-to-scale (VRS) assumptions respectively. We write the programs using Matlab language, first we set the observed input and output data and the unit price vectors to the inputs and outputs, second define the objective functions of the Leader and the Follower, third set the coefficient matrixes based on the constrains, last the extended branch and bound algorithm proposed by Shi et al. (2006) is implemented to solve the optimization problem, finally, the profit efficiency values of the Leader, the Follower and the system are computed. But the extended branch and bound algorithm is sometimes found to be ineffective when there are large numbers of variables involved in the model. In this case, instead, we use the Matlab optimization toolbox function *fmincon* to solve constrained nonlinear optimization problem.

Table 2 lists the CRS and VRS DEA profit efficiency values of the whole system and the two sublevels. It is worth remarking that the DEA profit efficiency values under CRS assumption are greater than those under VRS assumption, which is consistent with current existing DEA literature (Cooper et al., 2000). None of the systems are efficient under CRS assumption but 4 are efficient under VRS assumption. We also find that the whole system is efficient only when both of the sublevels are efficient, which is consistent with our Proposition 3. It can be seen that the profit efficiency of every Follower is 1 under both assumptions, but the profit efficiency of the whole system is not always 1, which implies that during the profit optimizing process, most of the potential improvement profit is obtained by the Leader, which is the dominant level.

Table 2 Profit efficiency scores under both CRS and VRS models in the 1st situation

Branch	CRS			VRS		
	PE^L	PE^F	PE^S	PE^L	PE^F	PE^S
DMU1	0.936	1	0.958	1	1	1
DMU2	0.86	1	0.892	0.883	1	0.908
DMU3	0.947	1	0.963	0.95	1	0.965
DMU4	0.881	1	0.91	1	1	1
DMU5	0.958	1	0.971	1	1	1
DMU6	0.808	1	0.85	0.939	1	0.954
DMU7	0.988	1	0.992	1	1	1
DMU8	0.821	1	0.86	0.975	1	0.981

The optimized solutions for each DMU to be efficient under CRS and VRS assumptions are reported in Table 21 and Table 22 respectively in Appendix II.

Under the second situation, where the positions of the Leader and the Follower are swapped, we calculate the profit efficiency again. Table 3 documents the profit efficiency scores under both the CRS and VRS assumptions. And we can also find that the profit efficiency of every new Follower is 1 but one exception under both assumptions, meanwhile the profit efficiency of the system is not always 1. It reflects that during the profit optimizing process, most of the potential improvement profit is still obtained by the dominant level, the new Leader.

Table 3 Profit efficiency scores under both CRS and VRS models in the 2nd situation

Branch	CRS			VRS		
	PE^L	PE^F	PE^S	PE^L	PE^F	PE^S
DMU1	0.894	1	0.958	1	1	1
DMU2	0.800	1	0.940	0.998	0.963	0.972
DMU3	0.890	1	0.963	0.894	1	0.965
DMU4	0.869	1	0.962	1	1	1
DMU5	0.916	1	0.971	1	1	1
DMU6	0.732	1	0.914	0.842	1	0.954
DMU7	0.976	1	0.992	1	1	1
DMU8	0.796	1	0.939	0.93	1	0.981

The corresponding optimized solutions for each DMU under this condition to be efficient under CRS and VRS assumptions are reported in Table 23 and Table 24 respectively in Appendix II.

5.1.2 α -Strategy

Obviously, the reciprocal of the efficiency shows the promotion from the observed profit. For example, if the efficiency is 0.8, the reciprocal of the efficiency is 1.25. There is 25% of potential improvement from the observed profit. And it is clear to see that dominant level (the Leader) gains much more potential improvement profit than what the lower level (the Follower) gains. Thus the hierarchical structure is not steady.

To encourage the Follower to participate, the Leader promises to share α of his profit to the Follower which is called α -Strategy. Therefore, the total profit that the Follower would get is his actual profit plus α of the Leader's profit. The total profit of the system remains unchanged under this α -Strategy.

Under the first situation with CRS assumption which we proposed above, in order to find a suitable α for the Leader under this strategy, we calculate the efficiency ratio between the Leader's adjusted optimized profit and the observed profit in Fig. 4, and the efficiency ratio between the Follower's adjusted optimized profit and the observed profit in Fig. 5 when α changes from 0 to 0.1 with 0.01 increment.

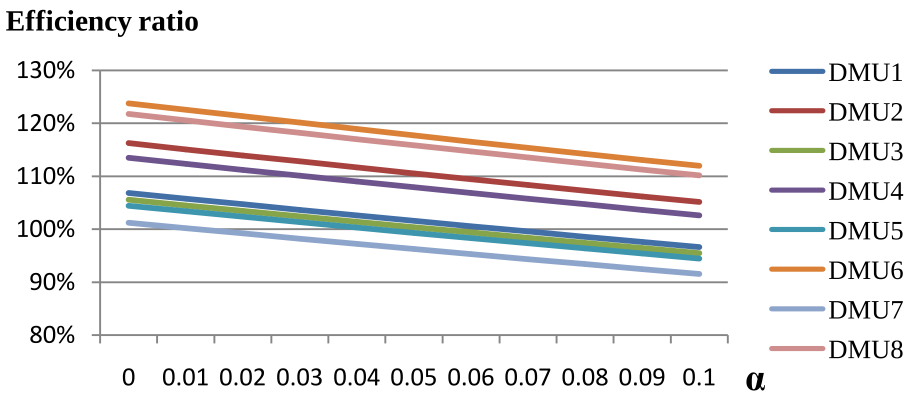

Fig. 4 The efficiency ratio of the Leader under α -strategy

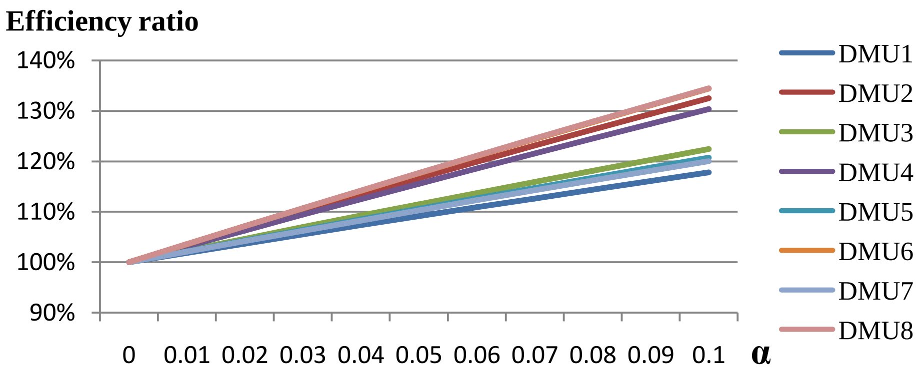

Fig. 5 The efficiency ratio of the Follower under α -strategy

From these two figures as well as the conception of α -Strategy, it is obvious that the efficiency ratio of the Leader decreases with the growth of α, while the efficiency ratio of the Follower increases with the growth of α. To ensure the benefit of the Leader, the efficiency ratio of the Leader must be greater than or equal to 1. Thus the value of α is set to be 0.01 based on Fig. 4 in this case. Under α -strategy with $\alpha = 0.01$, the optimized profit of the system, the Leader and the Follower and the observed profit of them are listed in Table 4. In this table, the optimized profits of the system, the Leader and the Follower are all improved comparing to the observed profits for each branch.

Table 4 The optimized profit and the observed profit with α =0.01

Branch	Observed Leader profit	Optimized profit of the Leader	Observed Follower profit	Optimized profit of the Follower	Observed system profit	Optimized profit of the system
DMU1	105.5	111.569	60.5	61.627	166	173.196
DMU2	151.6	174.449	51.8	53.562	203.4	228.011
DMU3	114.2	119.337	51.4	52.605	165.6	171.942
DMU4	172.4	193.646	61.6	63.556	234	257.202
DMU5	110.2	113.907	53.1	54.251	163.3	168.158
DMU6	142.9	175.099	49.2	50.969	192.1	226.068
DMU7	119	119.223	57.4	58.604	176.4	177.827
DMU8	170.3	205.327	57.5	59.574	227.8	264.901

5.1.3 Post merger

For the purpose of considering potential mergers, we examine what potentially profit could be gained by merging each pair of branches. This leads to a total of 28 possible mergers. Therefore, the relative profit efficiency of these 28 possible mergers is computed with reference to the original DMU by our Bilevel programming DEA model.

According to the two situations we proposed above, the merger, the harmony effect and the scale effect are evaluated respectively under both CRS and VRS assumptions. The numbers of the effective mergers are listed in Table 5.

Table 5 indicates that the number of the effective mergers for the dominate level is always greater than that for the other level. Thus it implies that the dominate level tends to favor merger much more than the other does in this case.

Table 5 The numbers of the effective mergers under both CRS and VRS assumptions

	Merger efficiency measures(>100%)	number under CRS	number under VRS
1st situation	effective mergers for the Leader	20	3
	effective mergers for the Follower	8	2
	effective mergers for the whole system	18	3
2nd situation	effective mergers for the new Leader	24	12
	effective mergers for the new Follower	9	0
	effective mergers for the whole system	20	12

In the first situation, 20 mergers are found to be efficient from the Leader's perspective among the total 28 mergers under CRS assumption. And we have proved that merger efficiency E_m equals to harmony effect H in this case. Table 6 lists the top 10 most promising mergers from the Leader's perspective.

Table 6 The top 10 promising mergers in the 1st situation under CRS

Merger	$E_m^L = H^L$	$E_m^F = H^F$	$E_m^S = H^S$
4,5	1.125	0.999	1.092
3,4	1.098	0.999	1.072
5,8	1.09	1.001	1.067
4,7	1.081	1.001	1.059
3,8	1.06	1.001	1.045
1,4	1.059	0.999	1.042
1,8	1.057	0.999	1.041
7,8	1.047	0.999	1.035
5,6	1.042	0.999	1.031
4,6	1.035	1.001	1.027

The merger efficiencies in the first row indicate that the profit of the Leader will increase by 12.5%, the total profit of the system will increase by 9.2%, and the profit of the Follower will decrease by 0.1%, if the merger of DMU 4 and DMU 5 induces best practice. We can find that among these 10 mergers, none of the Followers benefit much from the merger. And a merger is regarded to be coordinated effective only when merger efficiency scores of the Leader, the Follower and the whole Bilevel system are all greater than 1. Based on this definition, we find that there are 6 coordinated effective mergers. And these promising coordinated mergers are listed in Table 7.

Table 7 The promising coordinated mergers in the 1st situation under CRS

Merger	$E_m^L = H^L$	$E_m^F = H^F$	$E_m^S = H^S$
5,8	1.09	1.001	1.067
4,7	1.081	1.001	1.059
3,8	1.06	1.001	1.045
4,6	1.035	1.001	1.027
6,8	1.023	1.001	1.018
2,8	1.014	1.001	1.011

Under the VRS assumption, only 3 mergers are efficient from the Leader's perspective. The merger scores of them are shown in Table 8. For example, the merger of branch 2 and branch 6 has a merger efficiency value of 1.084 from the Leader's perspective, which implies that merger of these two separate branches would gain 4.6% more profit than the combined profits of these two branches using their individual input bundles respectively, from the Leader's perspective. The harmony effect of the Leader shows that these two branches, each using the average input bundles, will together gain 15.5% more profit than what they would gain collectively using their individual input bundles. The scale effect of the system, which is 0.921, indicates that a single branch using twice the average input bundle would gain 7.9% lower profit than the combined profits of these two branches if each uses the average input bundles. However, in this case the positive harmony effect dominates the negative scale effect from the whole system's perspective. And it is easy to see that from the Follower's perspective, the negative scale effect dominates the positive harmony effect, which leads to an ineffective merger for the Follower. In conclusion, we find that none of these 3 mergers is coordinated effective.

Table 8 The effective mergers in the 1st situation under VRS

Merger	E_m^L	H^L	S^L	E_m^F	H^F	S^F	E_m^S	H^S	S^S
2,6	1.118	1.155	0.921	0.996	1.004	0.97	1.084	1.112	0.933
2,7	1.027	1.189	0.868	0.996	1.01	0.987	1.017	1.13	0.903
2,3	1.012	1.156	0.884	0.996	0.996	0.995	1.007	1.106	0.916

Furthermore, we depict the scores of both harmony and scale effect of the Leader under VRS assumption in Fig. 6. From the figure, it is easy to see that the scores of harmony effect are greater than 1, however scale effect is ineffective from merger.

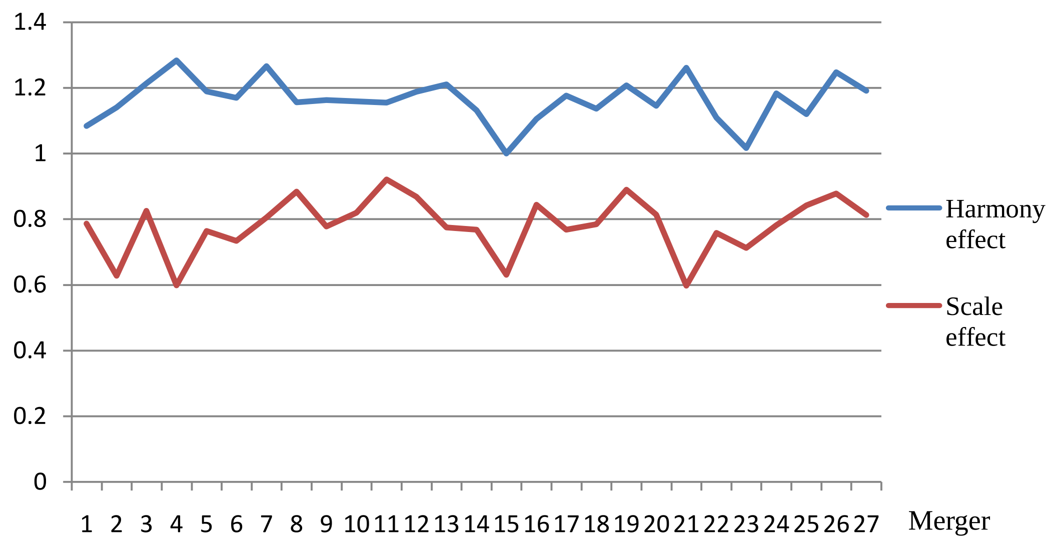

Fig. 6 The VRS scores of both harmony and scale effects of the Leader

In the second situation, where the positions of the Leader and the Follower are swapped, under CRS assumption, 24 mergers are found to be efficient from the new Leader's perspective and 6 of them are coordinated effective. Table 9 lists the top 10 most promising mergers from the new Leader's perspective. From the table, we can find that only 2 of these 10 mergers are coordinated effective.

Table 9 The top 10 promising mergers in the 2nd situation under CRS

Merger	$E_m^L = H^L$	$E_m^F = H^F$	$E_m^S = H^S$
3,4	1.094	1.001	1.029
3,8	1.093	0.999	1.029
2,5	1.083	0.999	1.027
4,5	1.079	1.001	1.025
5,8	1.078	0.999	1.025
4,7	1.074	0.999	1.023
7,8	1.073	0.999	1.023
3,6	1.071	0.999	1.023
2,3	1.063	0.999	1.021
6,7	1.058	0.999	1.019

Under the VRS assumption, 12 mergers are effective from the new Leader's perspective, but none is effective from the new Follower's perspective, which means the Leader benefits much more from the merger than the Follower in this case. The merger scores of the top 10 most promising mergers from the new Leader's perspective are shown in Table 10.

Table 10 The top 10 promising mergers in the 2nd situation under VRS

Merger	E_m^L	H^L	S^L	E_m^F	H^F	S^F	E_m^S	H^S	S^S
2,5	2.566	2.792	0.919	0.999	1	0.999	1.172	1.197	0.979
2,7	2.326	2.783	0.836	0.999	1	0.999	1.158	1.212	0.955
2,6	2.204	2.954	0.746	0.999	1	0.999	1.136	1.222	0.93
2,3	2.11	2.613	0.808	0.999	1	0.999	1.134	1.196	0.949
1,5	2.083	1.89	1.102	0.999	1	0.999	1.236	1.194	1.035
1,4	1.738	1.77	0.982	0.999	1	0.999	1.146	1.152	0.994
1,8	1.736	1.75	0.992	0.999	1	0.999	1.147	1.15	0.997
1,3	1.669	1.713	0.974	0.999	1.007	0.993	1.151	1.167	0.987
1,2	1.639	1.739	0.942	0.999	1	0.999	1.117	1.135	0.984
1,7	1.571	1.831	0.858	0.999	1	0.999	1.127	1.185	0.951

5.2 The second numerical example

In the second example, we use the data from Wu and Birge (2011), regarding 30 branches from a large Canadian Mortgage bank, and the input-output framework is demonstrated in Fig. 7. And obviously it is also a hierarchical system with two levels.

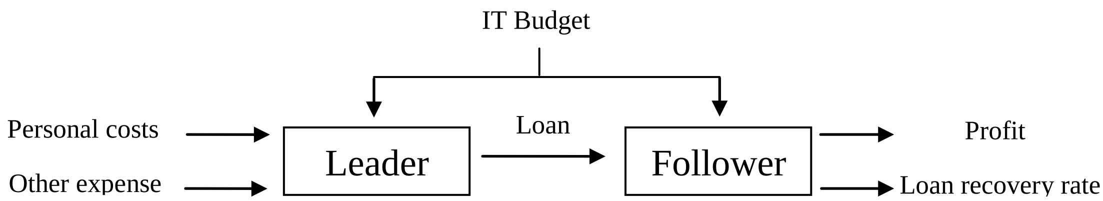

Fig. 7. The framework of Bilevel programming problem in the 2nd numerical example

The data includes two direct inputs: Personal Costs and Other Expenses, one shared input: IT Budget for both the Leader and Follower, one intermediate output: loans and two outputs: profit and Loan recovery. Assuming the variables equally important by the decision makers, all unit price vectors of them are set to be unit. Raw data for the inputs and outputs are shown in Table 11. And the data in this table is unified into one magnitude: 10^5.

Table 11 Raw data of 30 branches in the 2nd example

Branch	X^{D1} Other Expense (×10^5)	X^{D1} Personal Cost (×10^5)	X^1 IT Budget (×10^5)	Y Loan (×10^5)	X^2 IT Budget (×10^5)	Z^2 Profit (×10^5)	Z^2 Loan Recovery (×10^5)	P^L (×10^5)	P^F (×10^5)
DMU1	71.3	1.5	0.133	1447.8	2.5	523.2	1427.7	1374.9	500.6
DMU2	107.1	1.7	0.169	1950.2	2.3	534	1923.3	1841.2	504.8
DMU3	122.4	2.35	0.24	2095.2	1.65	536.3	2066	1970.2	505.4
DMU4	41	1.1	0.159	1364.4	2.9	495.4	1324.8	1322.1	452.9
DMU5	36.3	2.11	0.156	1390.2	1.89	521.1	1365.2	1351.6	494.2
DMU6	40.9	1.33	0.18485	1520.6	2.67	523.7	1496.3	1478.2	496.7
DMU7	91.8	0.6	0.5642	8118.6	3.4	610.3	8005.2	8025.6	493.5
DMU8	123.5	0.71	0.12	1144.1	3.29	519.9	1126.9	1019.8	499.4
DMU9	182.1	1.2	0.198	1742.5	2.8	527.4	1712.9	1559	495
DMU10	191.5	1.2	0.198	1742.5	2.8	527.4	1712.9	1549.6	495
DMU11	302.8	2	0.137	3153.7	2	442.9	2980.6	2848.8	267.8
DMU12	544	3.8	0.297	4517.7	0.2	386	4300.9	3969.6	169
DMU13	87.4	0.5	0.131	1434.2	3.5	517.7	1412.7	1346.2	492.7
DMU14	691.8	3.7	0.125	3249.1	0.3	564.8	3070.4	2553.5	385.8
DMU15	458	4	0.138	2622	0	402	2283.8	2159.9	63.84
DMU16	124.1	1.1	0.144	1749.3	2.9	524.3	1728.3	1624	500.4
DMU17	45	0.53	0.076	951.2	3.47	506.7	932.18	905.59	484.2
DMU18	589.2	3.45	0.155	4246.9	0.55	600.2	4026.1	3654.1	378.8
DMU19	713.8	3.82	0.14	3915.8	0.18	372.5	3559.5	3198	15.98
DMU20	97.3	1.28	0.126	1898.7	2.72	524.3	1870.4	1800	493.3
DMU21	229.4	1.36	0.12843	1876.5	2.64	487.1	1805.2	1645.6	413.2
DMU22	44.4	0.55	0.059	754.6	3.45	515.3	744.79	709.59	502
DMU23	50.8	0.57	0.057	759.5	3.43	512.3	749.78	708.07	499.1
DMU24	37	0.98	0.141	1690.6	3.02	523.3	1658.5	1652.5	488.2
DMU25	39.5	1.04	0.146	1726.4	2.96	526.3	1697.1	1685.7	494
DMU26	268	2.06	0.196	3643	1.94	560.1	3577.4	3372.7	492.6
DMU27	78.1	0.67	0.105	1158.1	3.33	512	1143	1079.2	493.6
DMU28	87.2	1	0.121	2220.7	3	524.8	2158.5	2132.4	459.6
DMU29	175.7	0.106	0.127	2067	3.894	525.3	2042.2	1891.1	496.6
DMU30	193.9	1.72	0.165	2132.5	2.28	493.5	2030.1	1936.7	388.9

5.2.1 Pre-merger

Considering the two situations we discussed above in this thesis.

Under the first situation, we investigate the profit efficiency of the system, the Leader and the Follower of these 30 branches using Model (5), under the CRS and VRS assumptions respectively. The results of the profit efficiency scores are listed in Table 12. And we figure the result in Fig. 8.

Table 12 Profit efficiency scores under both CRS and VRS models in the 1st situation

Branch	CRS			VRS		
	PE^L	PE^F	PE^S	PE^L	PE^F	PE^S
DMU1	0.628	0.609	0.623	0.667	0.992	0.731
DMU2	0.705	0.533	0.673	0.71	0.996	0.717
DMU3	0.502	0.555	0.512	0.52	0.997	0.576
DMU4	0.562	0.51	0.548	0.679	0.896	0.724
DMU5	0.591	0.564	0.584	1	1	1
DMU6	0.552	0.512	0.542	0.703	0.98	0.756
DMU7	1	0.338	0.898	1	0.976	0.999
DMU8	0.483	0.634	0.524	0.496	0.99	0.567
DMU9	0.436	0.625	0.471	0.462	0.982	0.529
DMU10	0.432	0.627	0.467	0.456	0.982	0.524
DMU11	1	0.386	0.88	0.999	0.532	0.929
DMU12	0.689	0.225	0.636	0.768	0.34	0.73
DMU13	0.609	0.6	0.606	0.625	0.972	0.664
DMU14	0.866	0.674	0.835	0.927	0.803	0.909
DMU15	0.666	0.12	0.589	0.699	0.136	0.625
DMU16	0.691	0.569	0.658	0.741	0.989	0.799
DMU17	0.707	0.763	0.726	0.967	0.957	0.964
DMU18	0.999	0.714	0.963	1	0.936	0.994
DMU19	0.952	0.029	0.823	1	0.036	0.883
DMU20	0.806	0.63	0.76	0.902	0.973	0.916
DMU21	0.645	0.57	0.628	0.649	0.815	0.677
DMU22	0.914	1	0.948	1	1	1
DMU23	0.903	1	0.941	1	1	1
DMU24	0.79	0.587	0.732	1	0.967	0.992
DMU25	0.776	0.583	0.722	0.96	0.981	0.964
DMU26	0.907	0.656	0.865	0.951	0.971	0.953

DMU27	0.581	0.672	0.607	0.608	0.974	0.69
DMU28	1	0.586	0.889	1	0.911	0.983
DMU29	0.812	0.593	0.794	0.814	0.979	0.818
DMU30	0.631	0.509	0.607	0.653	0.772	0.67

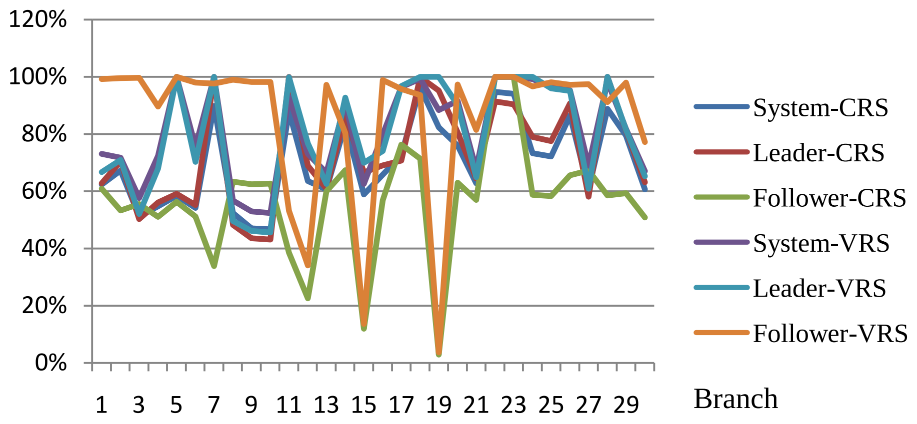

Fig. 8. The profit efficiency scores in the 1st situation

From Fig. 8, the DEA profit efficiency values under CRS assumption are still found to be greater than those under VRS assumption. And we also find that profit efficiency scores of the system are close to those of the Leader. But profit efficiency scores of the Follower are found to be distinctive from those of the system. In short, the profit efficiency of the system is mainly affected by the profit efficiency of the Leader under this situation.

In the second situation, the profit efficiency of the system, the new Leader and the new Follower is evaluated using Model (6), under both CRS and VRS assumptions. The results are depicted in Fig. 9, and the profit efficiency scores are shown in Table 13.

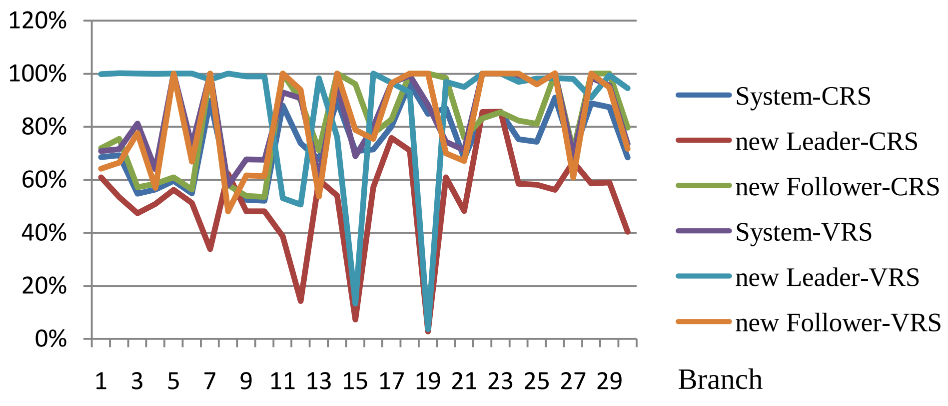

Fig. 9. The profit efficiency scores in the 2nd situation

Table 13 Profit efficiency scores under both CRS and VRS models in the 2nd situation

Branch	CRS			VRS		
	PE^L	PE^F	PE^S	PE^L	PE^F	PE^S
DMU1	0.718	0.608	0.685	0.64	0.997	0.708
DMU2	0.754	0.533	0.692	0.663	1.001	0.715
DMU3	0.57	0.472	0.547	0.775	0.999	0.812
DMU4	0.583	0.508	0.562	0.569	0.999	0.639
DMU5	0.609	0.561	0.595	1	1	1
DMU6	0.562	0.511	0.548	0.667	1	0.728
DMU7	1	0.338	0.898	1	0.976	0.999
DMU8	0.579	0.622	0.593	0.48	0.999	0.579
DMU9	0.539	0.481	0.523	0.615	0.989	0.677
DMU10	0.534	0.48	0.52	0.613	0.989	0.675
DMU11	1	0.386	0.88	1	0.53	0.929
DMU12	0.898	0.141	0.737	0.938	0.506	0.906
DMU13	0.71	0.598	0.676	0.536	0.982	0.61
DMU14	1	0.54	0.899	1	0.761	0.96
DMU15	0.96	0.072	0.708	0.787	0.132	0.689
DMU16	0.774	0.571	0.714	0.753	1	0.799
DMU17	0.827	0.758	0.801	0.963	0.964	0.964
DMU18	1	0.71	0.963	1	0.929	0.993
DMU19	1	0.027	0.848	1	0.036	0.881
DMU20	0.983	0.608	0.868	0.699	0.968	0.744
DMU21	0.758	0.482	0.68	0.67	0.949	0.712
DMU22	0.831	0.855	0.841	1	1	1
DMU23	0.854	0.856	0.855	1	1	1
DMU24	0.822	0.584	0.752	1	0.968	0.992
DMU25	0.81	0.58	0.743	0.958	0.981	0.963
DMU26	1	0.561	0.909	1	0.982	0.998
DMU27	0.709	0.667	0.695	0.607	0.98	0.69
DMU28	1	0.585	0.888	1	0.909	0.983
DMU29	1	0.589	0.873	0.946	0.994	0.956
DMU30	0.796	0.402	0.684	0.716	0.945	0.735

To further investigate the profit efficiency, we figure the profit efficiency scores of the Leaders in these two situations together in Fig. 10.

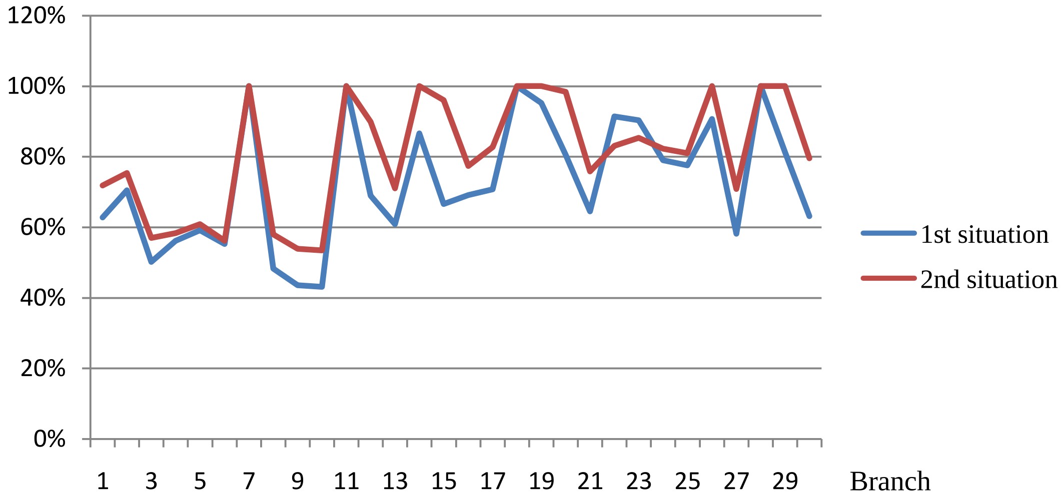

Fig. 10 The profit efficiency scores of the Leaders in the two situations

From Fig. 10, it can be seen that most of the profit efficiency scores of the Leaders in the second situation are slight higher, which means the potential improvement from the observed profit under the second situation is a little less than that under the first situation.

The profit efficiency scores of the Followers in these two situations are shown together in Fig. 11.

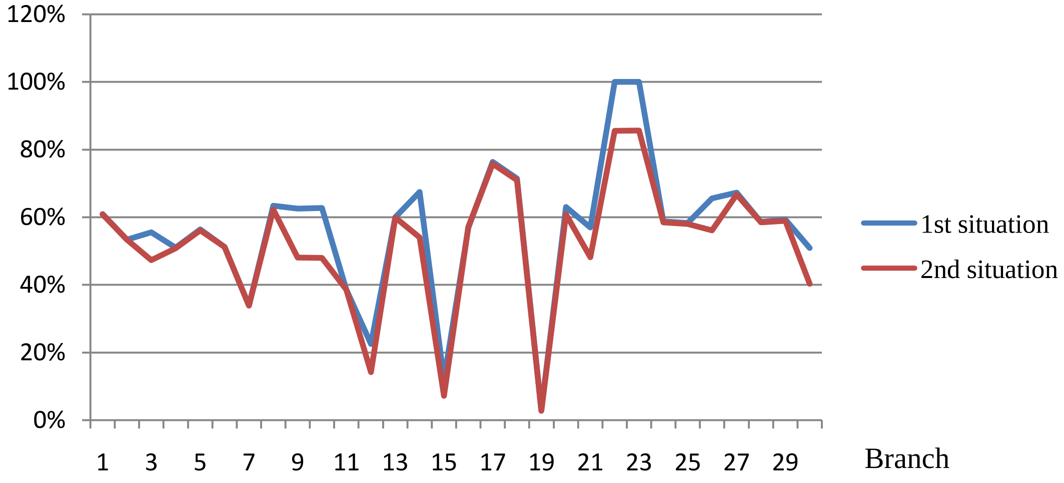

Fig. 11 The profit efficiency scores of the Followers in the two situations

It is clear to see that the Followers get lower profit efficiency scores in the second situation comparing with the other one. It implies that the Followers can benefit more potential profit from the DEA optimization process in the second situation.
Thus we infer that the both the Leader and Follower must favor the situation, where they are at the dominant level.

5.2.2 Post merger

For the purpose of considering potential mergers, we examine what potentially profit could be gained by merging each two branches. And this leads to totally 435 possible mergers. Therefore, the relative profit efficiency of these 435 possible mergers is computed with reference to the original DMU by our Bilevel programming DEA model.

According to the two situations we proposed above, the merger, the harmony effect and the scale effect are evaluated respectively under both CRS and VRS assumptions. The numbers of the effective mergers and the average merger efficiency scores $\bar{E}_m$ are listed in Table 14.

Table 14 The numbers of the effective mergers under both CRS and VRS assumptions

	Merger efficiency measures (>100%)	CRS		VRS	
		number	$\bar{E}_m$	number	$\bar{E}_m$
1st situation	effective mergers for the Leader	218	1.023	213	1.779
	effective mergers for the Follower	191	1.02	13	1
	effective mergers for the whole system	225	1.022	213	1.61
2nd situation	effective mergers for the new Leader	299	1	70	1
	effective mergers for the new Follower	249	1.035	113	1.263
	effective mergers for the whole system	335	1.007	112	1.194

Table 14 indicates that there are numerous mergers effective under CRS assumption, and relatively fewer under VRS assumption. And the results of the average merger efficiency scores $\bar{E}_m$ show significant gains from mergers and relatively higher under VRS assumption.

Based on the concept of the coordinated effective merger, which we have defined in the first numerical example, the numbers of coordinated effective mergers in two situations under both CRS and VRS assumptions are exhibited in Table 15.

Table 15 The number of the coordinated effective mergers

	number under CRS	number under VRS
1st situation	163	7
2nd situation	193	3

From Table 15, it can be seen that the number of the coordinated effective mergers under CRS assumption are much more than those under VRS assumption in all these two situations. Especially, 44.3% of the total 435 mergers, which are 193 mergers, are coordinated effective under CRS in the second situation.

In the first situation, 218 mergers are found to be efficient from the Leader's perspective and 74.8% of them, which are 163 mergers, are coordinated effective. Table 16 lists the top 10 most promising mergers for the Leader. It is easy to see that all of these 10 mergers are coordinated effective, so it is recommended to achieve them since both of the members obtain potential gains from the merger. And it is worth to mention that the Leaders benefit much more from the merger than the Followers among these top 10 promising coordinated mergers.

Table 16 The top 10 promising mergers in the 1st situation under CRS

Merger	$E_m^L = H^L$	$E_m^F = H^F$	$E_m^S = H^S$
22,23	1.327	1.138	1.253
2,22	1.188	1.051	1.146
2,23	1.186	1.056	1.147
22,29	1.173	1.061	1.138
16,22	1.155	1.005	1.109
16,23	1.147	1.055	1.119
4,22	1.137	1.001	1.095
17,23	1.132	1.076	1.112
17,22	1.129	1.076	1.111
13,23	1.106	1.019	1.081

There are 213 merger efficiency scores of the Leader are larger than 1 under the VRS assumption in the first situation, however only 7 of these mergers are found coordinated effective. The top 10 promising mergers from the Leader's perspective in the first situation under VRS assumption are listed in Table 17.

Table 17 The top 10 promising mergers in the 1st situation under VRS

Merger	E_m^L	H^L	S^L	E_m^F	H^F	S^F	E_m^S	H^S	S^S
5,24	4.111	1	4.111	0.996	1	0.996	3.334	1	3.333
5,27	4.084	1.237	3.3	0.997	1	0.997	3.335	1.18	2.827
20,22	3.867	1.295	2.985	0.997	1	0.997	3.087	1.215	2.54
1,5	3.715	1.212	3.064	0.997	1	0.996	3.1	1.164	2.662
22,30	3.48	1.261	2.76	0.998	1	0.998	2.947	1.205	2.446
24,27	3.398	1.11	3.063	0.996	1	0.996	2.851	1.085	2.628
17,30	3.311	1.116	2.966	0.998	1	0.998	2.836	1.092	2.596
1,17	3.213	1.002	3.208	0.999	1	0.999	2.655	1.001	2.652
23,29	3.168	1.137	2.786	0.997	1.001	0.997	2.639	1.104	2.39
16,22	3.136	1.088	2.882	0.997	1	0.997	2.629	1.068	2.463

Furthermore, under VRS assumption in the first situation, to examine the harmony effect and the scale effect, we depict the scores of them of the Leader in Fig. 12. From the figure, it is easy to see that the scores of harmony effect are greater than 1, however some of scale effect scores are greater than 1 and some of them are less than 1. And it means the scale effect is ineffective from merger when the scale effect score is less than 1.

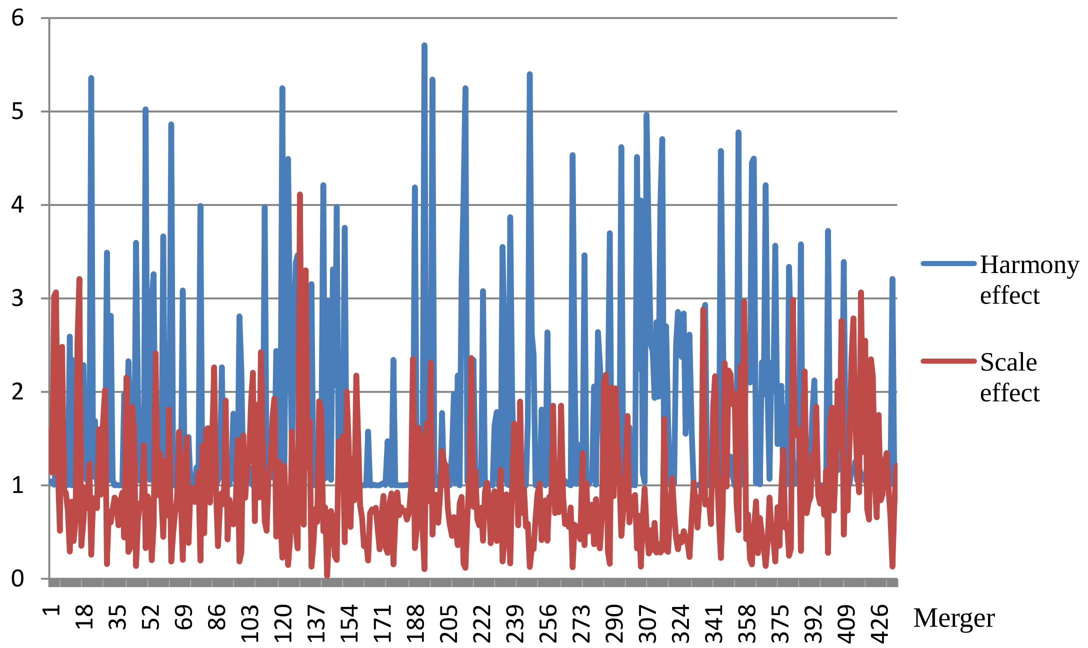

Fig. 12 The VRS scores of both harmony and scale effects of the Leader in the 1st situation

In the second situation, where the positions of the Leader and the Follower are swapped, under CRS assumption, 299 mergers are found to be efficient from the new

Leader's perspective and 193 coordinated effective mergers are found. Table 18 lists the top 10 most promising mergers from the new Leader's perspective. And it is easy to show that all of these 10 mergers are coordinated effective.

Table 18 The top 10 promising mergers in the 2nd situation under CRS

Merger	$E_m^L = H^L$	$E_m^F = H^F$	$E_m^S = H^S$
15,29	1.047	1.033	1.001
21,29	1.027	1.019	1.001
15,23	1.002	1.002	1.004
14,23	1.001	1.031	1.111
15,17	1.001	1.002	1.005
28,29	1.001	1.001	1.003
15,24	1.001	1.002	1.006
21,25	1.001	1.001	1.005
13,21	1.001	1.001	1.004
12,16	1.001	1.009	1.036

There are 70 merger efficiency scores of the new Leader larger than 1 under the VRS assumption in the second situation, but only 3 of these mergers are found coordinated effective. The top 10 promising mergers from the new Leader's perspective in the first situation under VRS assumption are listed in Table 19. And the 3 coordinated effective mergers are shown in Table 20.

Table 19 The top 10 promising mergers in the 2nd situation under VRS

Merger	E_m^L	H^L	S^L	E_m^F	H^F	S^F	E_m^S	H^S	S^S
21,27	1.008	1.708	0.59	0.961	0.872	1.101	0.969	1.024	0.946
4,21	1.008	1.977	0.51	0.818	0.928	0.881	0.847	1.092	0.776
28,30	1.007	1.819	0.554	0.858	0.944	0.909	0.883	1.089	0.81
3,29	1.007	1.999	0.504	1.004	1.177	0.853	1.005	1.326	0.758
21,28	1.007	1.755	0.574	0.887	0.938	0.946	0.908	1.077	0.843
13,21	1.007	1.802	0.559	0.682	0.818	0.833	0.733	0.974	0.752
7,21	1.006	2.481	0.406	0.384	0.973	0.395	0.435	1.097	0.397
20,21	1.006	1.773	0.567	0.804	0.795	1.01	0.836	0.95	0.88
15,19	1.005	1.603	0.627	0.78	0.916	0.851	0.811	1.01	0.803
4,30	1.005	2.033	0.494	0.627	0.934	0.671	0.685	1.103	0.621

Table 20 The coordinated effective mergers in the 2nd situation under VRS

Merger	E_m^L	H^L	S^L	E_m^F	H^F	S^F	E_m^S	H^S	S^S
3,29	1.007	1.999	0.504	1.004	1.177	0.853	1.005	1.326	0.758
16,29	1.001	1.719	0.582	1.181	0.961	1.231	1.146	1.107	1.035
23,26	1.001	1.664	0.601	1.202	1.031	1.167	1.162	1.155	1.006

Chapter 6

Conclusions, limitations and future research

Within the recent two decades, many real operation problems are modeled into a hierarchical structure, where more than one player is involved. Bilevel programming is a very useful tool to solve the special optimization problems which consist of two objectives, one is the Leader's, and the other is Follower's. The Leader is at the dominant level and the Follower is at the submissive level. As we already know, Bilevel programming is applied in many fields. In this thesis, we develop a Bilevel programming DEA model to evaluate the profit efficiency of the systematic and sub-unit levels under both CRS and VRS assumptions. The resources are limited which often happens in reality. The results demonstrate that the sub-level benefits more potential profit from the Bilevel programming DEA optimization process when sub-level is in the dominant level, but not at the submissive level.

The system is found to be efficient only if sub-levels are both efficient. In order to stabilize the hierarchical structure, α -Strategy is proposed here to stimulate the follower to actively participate.

Expecting potential gains from merger, we build new models to evaluate the merger efficiency under both CRS and VRS assumptions, and decompose the merger efficiency into harmony effect and scale effect. The applicability of the proposed approach is further illustrated in two case studies: a numerical example and a practical example about the branches in a big Canadian bank are conducted. The results indicate considerable potential gains from the promising mergers and relative higher potential gains under the VRS assumption. The concept of coordinated effective merger is also discussed, and it is very important since every member in the system benefits from the merger.

The limitations of this study are that internal relationship of the two situations and the differences of the total profit of the system under these two situations are not deeply investigated. During the computation process, the extended branch and bound algorithm is sometimes found to be ineffective when there are large numbers of variables involved in the model.

Nevertheless, future research about Bilevel programming DEA with bounded output is suggested, taking into account the maximum capacity of the output. Future extension of our merger study could examine the effects of the type of the merger, and the method of financing, and the stock-price returns of the merger-involved firms, both pre-merger and post-merger. Especially, Markov-regime-switching GARCH models are recommended to characterize the takeover or merger process.

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Appendix

Appendix I: The standard linear Bilevel programming form of each Bilevel programming model, and the variables and coefficient matrixes in the corresponding Single level problem

The standard linear Bilevel programming form of Model (5)

$$(P1)\ \min_{\tilde{Z}_J^1,\tilde{Y}_J^1,\tilde{X}_J^1,\tilde{X}_J^{D1},\lambda} \begin{pmatrix} -Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix}$$

$$s.t \quad \begin{pmatrix} 0 & 0 & -1 & 0 & X_1^1 \cdots X_n^1 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 & 0 & X_1^2 \cdots X_n^2 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & 0 & -1 & X_1^{D1} \cdots X_n^{D1} \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -Z_1^1 \cdots -Z_n^1 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \le 0,$$

$$\begin{pmatrix}0 & 1 & 0 & 0 & -Y_1 \cdots -Y_n\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda\end{pmatrix} \le 0,$$

$$\begin{pmatrix}0 & 0 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda\end{pmatrix} + \begin{pmatrix}0 & 1 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J \\ \pi\end{pmatrix} \le E,$$

$$\begin{pmatrix}0 & 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda\end{pmatrix} \le X_J^{D1},$$

$$\begin{pmatrix}-Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda\end{pmatrix} \le (P^{1^T} X_J^1 + P^{2^T} X_J^{D1}) - (Q^{1^T} Z_J^1 + Q^{2^T} Y_J^1)$$

$$(P2)\ \min_{\tilde{X}_J^2, \tilde{X}_J^{D2}, \tilde{Y}_J^2, \tilde{Z}_J^2, \pi} \begin{pmatrix}-Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi\end{pmatrix}$$

$$s.t \quad \begin{pmatrix}0 & 0 & -1 & 0 & X_1^{D2} \cdots X_n^{D2}\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi\end{pmatrix} \le 0,$$

$$\begin{pmatrix}0 & 0 & 0 & -1 & Y_1 \cdots Y_n\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi\end{pmatrix} \le 0,$$

$$\begin{pmatrix}1 & 0 & 0 & 0 & -Z_1^2 \cdots -Z_n^2\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi\end{pmatrix} \le 0,$$

$$\begin{pmatrix}0 & 0 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi\end{pmatrix} \le X_J^{D2},$$

$$\begin{pmatrix}0 & -1 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda\end{pmatrix} + \begin{pmatrix}0 & 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi\end{pmatrix} \le 0,$$

$$\begin{pmatrix}-Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0\end{pmatrix}\begin{pmatrix}\tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi\end{pmatrix} \le -Q^{3^T} Z_J^2 + (P^{1^T} X_J^2 + P^{3^T} X_J^{D2} + Q^{2^T} Y_J^2)$$

$$\tilde{X}_J^1, \tilde{X}_J^2, \tilde{X}_J^{D1}, \tilde{X}_J^{D2}, \tilde{Y}_J^1, \tilde{Y}_J^2, \tilde{Z}_J^1, \tilde{Z}_J^2, \lambda, \pi \ge 0$$

The variables and coefficient matrixes in the Single level problem corresponding to Model (5)

$$x = \begin{pmatrix}\tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda\end{pmatrix}, y = \begin{pmatrix}\tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi\end{pmatrix},$$

$$c_1 = \begin{pmatrix}-Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0\end{pmatrix}, d_1 = 0,$$

$$A_1 = \begin{pmatrix} 0 & 0 & -1 & 0 & X_1^1 \cdots X_n^1 \\ 0 & 0 & 0 & -1 & X_1^{D1} \cdots X_n^{D1} \\ 1 & 0 & 0 & 0 & -Z_1^1 \cdots -Z_n^1 \\ 0 & 1 & 0 & 0 & -Y_1 \cdots -Y_n \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 0 & -1 & 0 & 0 & X_1^2 \cdots X_n^2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$b_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ E \\ X_J^{D1} \\ (P^{1^T} X_J^1 + P^{2^T} X_J^{D1}) - (Q^{1^T} Z_J^1 + Q^{2^T} Y_J^1) \end{pmatrix},$$

$$c_2 = (0 \quad 0 \quad 0 \quad 0 \quad 0), d_2 = \begin{pmatrix} -Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, B_2 = \begin{pmatrix} 0 & 0 & -1 & 0 & X_1^{D2} \cdots X_n^{D2} \\ 0 & 0 & 0 & -1 & Y_1 \cdots Y_n \\ 1 & 0 & 0 & 0 & -Z_1^2 \cdots -Z_n^2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ -Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0 \end{pmatrix},$$

$$b_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ X_J^{D2} \\ 0 \\ -Z_J^2 \\ -Q^{3^T} Z_J^2 + (P^{1^T} X_J^2 + P^{3^T} X_J^{D2} + Q^{2^T} Y_J^2) \end{pmatrix},$$

The standard linear Bilevel programming form of Model (6)

$$(P1) \min_{\tilde{Z}_J^2, \tilde{Y}_J^2, \tilde{X}_J^2, \tilde{X}_J^{D2}, \pi} \begin{pmatrix} -Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix}$$

$$s.t \quad \begin{pmatrix} 0 & -1 & 0 & 0 & X_1^2 \cdots X_n^2 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix} + \begin{pmatrix} 0 & 0 & -1 & 0 & X_1^1 \cdots X_n^1 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & -1 & 0 & X_1^{D2} \cdots X_n^{D2} \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & 0 & -1 & Y_1 \cdots Y_n \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -Z_1^2 \cdots -Z_n^2 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix} \le X_J^{D2},$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix} + \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \le E,$$

$$\begin{pmatrix} -Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix} \le -Q^{3^T} Z_J^2 + (P^{1^T} X_J^2 + P^{3^T} X_J^{D2} + Q^{2^T} Y_J^2),$$

$$(P2) \qquad \min_{\tilde{X}_J^1, \tilde{X}_J^{D1}, \tilde{Y}_J^1, \tilde{Z}_J^1, \pi} \begin{pmatrix} -Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix}$$

$$s.t \qquad \begin{pmatrix} 0 & 0 & 0 & -1 & X_1^{D1} \cdots X_n^{D1} \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \leq 0,$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -Z_1^1 \cdots -Z_n^1 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \leq 0,$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & -Y_1 \cdots -Y_n \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \leq 0,$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \leq X_J^{D1},$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^2 \\ \tilde{X}_J^2 \\ \tilde{X}_J^{D2} \\ \tilde{Y}_J^2 \\ \pi \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \leq 0,$$

$$\begin{pmatrix} -Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0 \end{pmatrix} \begin{pmatrix} \tilde{Z}_J^1 \\ \tilde{Y}_J^1 \\ \tilde{X}_J^1 \\ \tilde{X}_J^{D1} \\ \lambda \end{pmatrix} \leq -(Q^{1^T} Z_J^1 + Q^{2^T} Y_J^1) + (P^{1^T} X_J^1 + P^{2^T} X_J^{D1}),$$

$$\tilde{X}_J^1, \tilde{X}_J^2, \tilde{X}_J^{D1}, \tilde{X}_J^{D2}, \tilde{Y}_J^1, \tilde{Y}_J^2, \tilde{Z}_J^1, \tilde{Z}_J^2, \lambda, \pi \geq 0$$

The variables and coefficient matrixes in the Single level problem corresponding to Model (6)

$$x=\begin{pmatrix}\tilde{Z}_J^2\\ \tilde{X}_J^2\\ \tilde{X}_J^{D2}\\ \tilde{Y}_J^2\\ \pi\end{pmatrix}, y=\begin{pmatrix}\tilde{Z}_J^1\\ \tilde{Y}_J^1\\ \tilde{X}_J^1\\ \tilde{X}_J^{D1}\\ \lambda\end{pmatrix},$$

$$c_1=\begin{pmatrix}-Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0\end{pmatrix}, d_1=\begin{pmatrix}-Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0\end{pmatrix},$$

$$A_1=\begin{pmatrix}0 & -1 & 0 & 0 & X_1^2\cdots X_n^2\\ 0 & 0 & -1 & 0 & X_1^{D2}\cdots X_n^{D2}\\ 1 & 0 & 0 & 0 & -Z_1^2\cdots -Z_n^2\\ 0 & 0 & 0 & -1 & Y_1\cdots Y_n\\ 0 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ -Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0\end{pmatrix}, B_1=\begin{pmatrix}0 & 0 & -1 & 0 & X_1^1\cdots X_n^1\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{pmatrix},$$

$$b_1=\begin{pmatrix}0\\ 0\\ 0\\ 0\\ X_J^{D2}\\ E\\ -Q^{3^T}Z_J^2+(P^{1^T}X_J^2+P^{3^T}X_J^{D2}+Q^{2^T}Y_J^2)\end{pmatrix},$$

$$c_2=(0\quad 0\quad 0\quad 0\quad 0), d_2=\begin{pmatrix}-Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0\end{pmatrix},$$

$$A_2=\begin{pmatrix}0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0\end{pmatrix}, B_2=\begin{pmatrix}0 & 0 & 0 & -1 & X_1^{D1}\cdots X_n^{D1}\\ 0 & 1 & 0 & 0 & -Y_1\cdots -Y_n\\ 1 & 0 & 0 & 0 & -Z_1^1\cdots -Z_n^1\\ 0 & 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0 & 0\\ -Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0\end{pmatrix},$$

$$b_2=\begin{pmatrix}0\\ 0\\ 0\\ X_J^{D1}\\ 0\\ -(Q^{1^T}Z_J^1+Q^{2^T}Y_J^1)+(P^{1^T}X_J^1+P^{2^T}X_J^{D1})\end{pmatrix},$$

The standard linear Bilevel programming form of Model (10)

$$(P1)\quad \min_{Z_H^1, Y_H^1, X_H^1, X_H^{D1}, \lambda} \begin{pmatrix} -Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0 \end{pmatrix} \begin{pmatrix} Z_H^1 \\ Y_H^1 \\ X_H^1 \\ X_H^{D1} \\ \lambda \end{pmatrix}$$

$$s.t \quad \begin{pmatrix} 0 & 0 & -1 & 0 & X_1^1 \cdots X_n^1 \end{pmatrix} \begin{pmatrix} Z_H^1 \\ Y_H^1 \\ X_H^1 \\ X_H^{D1} \\ \lambda \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 & 0 & X_1^2 \cdots X_n^2 \end{pmatrix} \begin{pmatrix} Z_H^2 \\ X_H^2 \\ X_H^{D2} \\ Y_H^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & 0 & -1 & X_1^{D1} \cdots X_n^{D1} \end{pmatrix} \begin{pmatrix} Z_H^1 \\ Y_H^1 \\ X_H^1 \\ X_H^{D1} \\ \lambda \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -Z_1^1 \cdots -Z_n^1 \end{pmatrix} \begin{pmatrix} Z_H^1 \\ Y_H^1 \\ X_H^1 \\ X_H^{D1} \\ \lambda \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & -Y_1 \cdots -Y_n \end{pmatrix} \begin{pmatrix} Z_H^1 \\ Y_H^1 \\ X_H^1 \\ X_H^{D1} \\ \lambda \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} Z_H^1 \\ Y_H^1 \\ X_H^1 \\ X_H^{D1} \\ \lambda \end{pmatrix} \le \bar{X}^{D1},$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} Z_H^1 \\ Y_H^1 \\ X_H^1 \\ X_H^{D1} \\ \lambda \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Z_H^2 \\ X_H^2 \\ X_H^{D2} \\ Y_H^2 \\ \pi \end{pmatrix} \le E,$$

$$(P2)\ \min_{X_H^2, X_H^{D2}, Y_H^2, Z_H^2, \pi} \begin{pmatrix} -Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0 \end{pmatrix} \begin{pmatrix} Z_H^2 \\ X_H^2 \\ X_H^{D2} \\ Y_H^2 \\ \pi \end{pmatrix}$$

$$s.t \quad \begin{pmatrix} 0 & 0 & -1 & 0 & X_1^{D2} \cdots X_n^{D2} \end{pmatrix} \begin{pmatrix} Z_H^2 \\ X_H^2 \\ X_H^{D2} \\ Y_H^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & 0 & -1 & Y_1 \cdots Y_n \end{pmatrix} \begin{pmatrix} Z_H^2 \\ X_H^2 \\ X_H^{D2} \\ Y_H^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -Z_1^2 \cdots -Z_n^2 \end{pmatrix} \begin{pmatrix} Z_H^2 \\ X_H^2 \\ X_H^{D2} \\ Y_H^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} Z_H^2 \\ X_H^2 \\ X_H^{D2} \\ Y_H^2 \\ \pi \end{pmatrix} \le \overline{X}^{D2},$$

$$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Z_H^1 \\ Y_H^1 \\ X_H^1 \\ X_H^{D1} \\ \lambda \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} Z_H^2 \\ X_H^2 \\ X_H^{D2} \\ Y_H^2 \\ \pi \end{pmatrix} \le 0,$$

$$X_H^1, X_H^2, X_H^{D1}, X_H^{D2}, Y_H^1, Y_H^2, Z_H^1, Z_H^2, \lambda_H, \pi_H \ge 0$$

The variables and coefficient matrixes in the Single level problem corresponding to Model (10)

$$x=\begin{pmatrix} Z_H^1 \\ Y_H^1 \\ X_H^1 \\ X_H^{D1} \\ \lambda \end{pmatrix}, y=\begin{pmatrix} Z_H^2 \\ X_H^2 \\ X_H^{D2} \\ Y_H^2 \\ \pi \end{pmatrix},$$

$$c_1=\begin{pmatrix} -Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0 \end{pmatrix}, d_1=(0 \quad 0 \quad 0 \quad 0 \quad 0),$$

$$A_1=\begin{pmatrix} 0 & 0 & -1 & 0 & X_1^1\cdots X_n^1 \\ 0 & 0 & 0 & -1 & X_1^{D1}\cdots X_n^{D1} \\ 1 & 0 & 0 & 0 & -Z_1^1\cdots -Z_n^1 \\ 0 & 1 & 0 & 0 & -Y_1\cdots -Y_n \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, B_1=\begin{pmatrix} 0 & -1 & 0 & 0 & X_1^2\cdots X_n^2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, b_1=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ E \\ \bar{X}^{D1} \end{pmatrix},$$

$$c_2=(0 \quad 0 \quad 0 \quad 0 \quad 0), d_2=\begin{pmatrix} -Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0 \end{pmatrix},$$

$$A_2=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{pmatrix}, B_2=\begin{pmatrix} 0 & -1 & 0 & 0 & X_1^2\cdots X_n^2 \\ 0 & 0 & -1 & 0 & X_1^{D2}\cdots X_n^{D2} \\ 0 & 0 & 0 & -1 & Y_1\cdots Y_n \\ 1 & 0 & 0 & 0 & -Z_1^2\cdots -Z_n^2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, b_2=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \bar{X}^{D2} \\ 0 \end{pmatrix}$$

The standard linear Bilevel programming form of Model (11)

$$(P1) \min_{Z_M^1, Y_M^1, X_M^1, X_M^{D1}, \lambda} \begin{pmatrix} -Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0 \end{pmatrix} \begin{pmatrix} Z_M^1 \\ Y_M^1 \\ X_M^1 \\ X_M^{D1} \\ \lambda \end{pmatrix}$$

$$s.t \quad \begin{pmatrix} 0 & 0 & -1 & 0 & X_1^1\cdots X_n^1 \end{pmatrix} \begin{pmatrix} Z_M^1 \\ Y_M^1 \\ X_M^1 \\ X_M^{D1} \\ \lambda \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 & 0 & X_1^2\cdots X_n^2 \end{pmatrix} \begin{pmatrix} Z_M^2 \\ X_M^2 \\ X_M^{D2} \\ Y_M^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & 0 & -1 & X_1^{D1} \cdots X_n^{D1} \end{pmatrix} \begin{pmatrix} Z_M^1 \\ Y_M^1 \\ X_M^1 \\ X_M^{D1} \\ \lambda \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & -Z_1^1 \cdots -Z_n^1 \end{pmatrix} \begin{pmatrix} Z_M^1 \\ Y_M^1 \\ X_M^1 \\ X_M^{D1} \\ \lambda \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & -Y_1 \cdots -Y_n \end{pmatrix} \begin{pmatrix} Z_M^1 \\ Y_M^1 \\ X_M^1 \\ X_M^{D1} \\ \lambda \end{pmatrix} \le 0,$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} Z_M^1 \\ Y_M^1 \\ X_M^1 \\ X_M^{D1} \\ \lambda \end{pmatrix} \le X_{Total}^{D1},$$

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} Z_M^1 \\ Y_M^1 \\ X_M^1 \\ X_M^{D1} \\ \lambda \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} Z_M^2 \\ X_M^2 \\ X_M^{D2} \\ Y_M^2 \\ \pi \end{pmatrix} \le N \cdot E,$$

$$(P2) \min_{X_M^2, X_M^{D2}, Z_M^2, Y_M^2, \pi} \begin{pmatrix} -Q^{3^T} & P^{1^T} & P^{3^T} & Q^{2^T} & 0 \end{pmatrix} \begin{pmatrix} Z_M^2 \\ X_M^2 \\ X_M^{D2} \\ Y_M^2 \\ \pi \end{pmatrix}$$

$$s.t \quad \begin{pmatrix} 0 & 0 & -1 & 0 & X_1^{D2} \cdots X_n^{D2} \end{pmatrix} \begin{pmatrix} Z_M^2 \\ X_M^2 \\ X_M^{D2} \\ Y_M^2 \\ \pi \end{pmatrix} \le 0,$$

$$\begin{pmatrix}0 & 0 & 0 & -1 & Y_1 \cdots Y_n\end{pmatrix}\begin{pmatrix}Z_M^2 \\ X_M^2 \\ X_M^{D2} \\ Y_M^2 \\ \pi\end{pmatrix} \le 0,$$

$$\begin{pmatrix}1 & 0 & 0 & 0 & -Z_1^2 \cdots -Z_n^2\end{pmatrix}\begin{pmatrix}Z_M^2 \\ X_M^2 \\ X_M^{D2} \\ Y_M^2 \\ \pi\end{pmatrix} \le 0,$$

$$\begin{pmatrix}0 & 0 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}Z_M^2 \\ X_M^2 \\ X_M^{D2} \\ Y_M^2 \\ \pi\end{pmatrix} \le X_{Total}^{D2},$$

$$\begin{pmatrix}0 & -1 & 0 & 0 & 0\end{pmatrix}\begin{pmatrix}Z_M^1 \\ Y_M^1 \\ X_M^1 \\ X_M^{D1} \\ \lambda\end{pmatrix} + \begin{pmatrix}0 & 0 & 0 & 1 & 0\end{pmatrix}\begin{pmatrix}Z_M^2 \\ X_M^2 \\ X_M^{D2} \\ Y_M^2 \\ \pi\end{pmatrix} \le 0,$$

$$X_M^1, X_M^2, X_M^{D1}, X_M^{D2}, Y_M^1, Y_M^2, Z_M^1, Z_M^2, \lambda_M, \pi_M \ge 0$$

The variables and coefficient matrixes in the Single level problem corresponding to Model (11)

$$x = \begin{pmatrix}Z_M^1 \\ Y_M^1 \\ X_M^1 \\ X_M^{D1} \\ \lambda\end{pmatrix}, y = \begin{pmatrix}Z_M^2 \\ X_M^2 \\ X_M^{D2} \\ Y_M^2 \\ \pi\end{pmatrix},$$

$$c_1 = \begin{pmatrix}-Q^{1^T} & -Q^{2^T} & P^{1^T} & P^{2^T} & 0\end{pmatrix}, d_1 = (0 \quad 0 \quad 0 \quad 0 \quad 0),$$

$$A_1 = \begin{pmatrix}0 & 0 & -1 & 0 & X_1^1 \cdots X_n^1 \\ 0 & 0 & 0 & -1 & X_1^{D1} \cdots X_n^{D1} \\ 1 & 0 & 0 & 0 & -Z_1^1 \cdots -Z_n^1 \\ 0 & 1 & 0 & 0 & -Y_1 \cdots -Y_n \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0\end{pmatrix}, B_1 = \begin{pmatrix}0 & -1 & 0 & 0 & X_1^2 \cdots X_n^2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\end{pmatrix}, b_1 = \begin{pmatrix}0 \\ 0 \\ 0 \\ 0 \\ N \cdot E \\ X_{Total}^{D1}\end{pmatrix},$$

$$c_2 = (0 \quad 0 \quad 0 \quad 0 \quad 0), d_2 = \left(-Q^{3^T} \quad P^{1^T} \quad P^{3^T} \quad Q^{2^T} \quad 0 \right),$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{pmatrix}, B_2 = \begin{pmatrix} 0 & -1 & 0 & 0 & X_1^2 \cdots X_n^2 \\ 0 & 0 & -1 & 0 & X_1^{D2} \cdots X_n^{D2} \\ 0 & 0 & 0 & -1 & Y_1 \cdots Y_n \\ 1 & 0 & 0 & 0 & -Z_1^2 \cdots -Z_n^2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, b_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ X_{Total}^{D2} \\ 0 \end{pmatrix}$$

Appendix II: The CRS and VRS optimized solutions for each DMU to be efficient in two situations

Table 21 The CRS optimized solutions in the 1st situation

Data	DMU 1	DMU 2	DMU 3	DMU 4	DMU 5	DMU 6	DMU 7	DMU 8
Z^1	36.435	80.241	52	82.24	33	78.74	55.71	88.26
	64.939	67.342	42	81.07	62	65.93	45	84.01
Y^1	37.687	64.435	40	78.94	35	63.06	42.86	81.51
X^1	10.865	18.503	3.458	20	0.142	14	12.43	20
X^{D1}	2.5	5.3047	3	8.65	2.3	5.163	3.214	8.484
	13	12	7	18	12.5	11.7	7.5	17.9
λ_1	4E-12	2E-13	6E-12	3E-16	5E-13	3E-14	3E-12	4E-14
λ_2	4E-13	2E-13	4E-12	6E-16	5E-14	3E-14	4E-12	3E-14
λ_3	0.0631	1.3906	1	0.7	4E-13	1.374	1.071	0.931
λ_4	3E-13	0.1259	2E-12	0.728	3E-14	0.116	4E-12	0.632
λ_5	1.0047	2E-13	8E-12	3E-16	1	3E-14	3E-12	5E-14
λ_6	4E-13	1E-13	3E-12	5E-16	4E-14	2E-14	3E-12	2E-14
λ_7	4E-12	1E-11	4E-11	5E-14	5E-14	7E-12	1E-10	2E-11
λ_8	3E-13	1E-12	3E-12	3E-15	3E-14	4E-13	3E-12	2E-13
Z^2	51.097	82.069	69.94	76.24	53.75	85	68.35	77.63
	59.104	40.552	56.32	60.01	61.91	42	53.74	50.8
X^2	9.1355	1.4966	16.54	7E-15	19.86	6	7.57	5E-13
X^{D2}	1.5	5.6	4	6.506	1.6	5.8	4	5.824
	11.176	12.552	15.4	11.2	11.76	13	15	11.5
Y^2	27.889	51.172	38.92	56.95	29.34	53	38.12	53.6
π_1	0.866	1E-13	0.052	9E-16	0.898	2E-14	5E-12	6E-14
π_2	1E-10	8E-13	0.059	0.497	5E-13	5E-13	0.068	0.696
π_3	0	2E-13	3E-11	4E-16	4E-13	4E-14	2E-11	2E-14
π_4	2E-13	8E-14	9E-12	0.423	6E-14	2E-14	3E-12	0.219
π_5	6E-12	1E-13	4E-11	4E-16	6E-13	2E-14	5E-12	3E-14
π_6	5E-12	0.9655	1E-10	3E-15	3E-13	1	0	2E-13
π_7	0.0503	3E-13	0.899	6E-16	0.063	6E-14	0.905	3E-14
π_8	9E-14	8E-14	7E-12	1E-15	5E-14	2E-14	3E-12	7E-14
u_1	1001.1	611.87	971.5	672.7	734.5	517.3	587.7	638.2
u_2	0	0	0	0	0	0	0	0
u_3	0	0	0	0	0	0	0	0
u_4	0	0	0	0	0	0	0	0

u_5	0	0	0	0	0	0	0	0
u_6	0	0	0	0	0	0	0	0
u_7	761.67	522.4	708.2	427.8	558.8	430.8	466.7	1253
u_8	0	0	0	0	0	0	0	0
u_9	0	0	0	0	0	0	0	0
u_{10}	0	0	0	0	0	0	0	0
u_{11}	1770.6	308.03	1266	244.9	1299	167.8	689.5	232.3
v_1	239.4	89.472	520.9	1003	175.6	178.8	120.9	951.9
v_2	239.4	89.472	263.3	244.9	175.6	86.5	120.9	232.3
v_3	239.4	89.472	263.3	244.9	175.6	86.5	120.9	232.3
v_4	239.4	89.472	263.3	244.9	175.6	86.5	120.9	232.3
v_5	1531.2	218.55	1002	0	1123	81.26	568.5	0
v_6	0	0	257.7	758.5	0	92.32	0	719.6
v_7	0	0	0	0	0	0	0	0
v_8	238.4	88.472	262.3	243.9	174.6	85.5	119.9	231.3
w_1	0	0	0	0	0	0	0	0
w_2	0	0	0	0	0	0	0	0
w_3	0	0	0	0	0	0	0	847.1
w_4	0	0	0	0	0	0	0	0
w_5	0	0	0	0	0	0	0	0
w_6	0	0	0	0	0	0	0	0
w_7	0	3273.2	0	1133	0	2888	1004	1075
w_8	1200.8	37.103	0	0	881	17.29	0	0
w_9	1588.7	1603.9	1670	3458	1166	1694	818.9	3281
w_{10}	10457	4456.8	6396	0	7672	3054	4741	0
w_{11}	1163	3434.8	1417	2744	853.3	3133	1489	2604
w_{12}	1672.5	0	460	379.9	1227	0	148.1	360.5
w_{13}	0	962.54	0	2055	0	1107	0	1950
w_{14}	11440	4658.4	7470	1060	8394	3280	5164	1006

Table 22 The VRS optimized solutions in the 1st situation

Data	DMU 1	DMU 2	DMU 3	DMU 4	DMU 5	DMU 6	DMU 7	DMU 8
Z^1	35	82.46	52	63	33	74.17	57	62.1
	60	58.85	42	71	62	52.57	45	70.73
Y^1	30	66.18	40	70	35	53.75	38	68.96
X^1	4	16.24	3.736	4.6	5	9.948	10	0.556

X^{D1}	2.5	7.347	3	9	2.3	6.727	3.5	8.8
	13	12.14	7	18	12.5	11.7	7.5	17.84
λ_1	1	0.112	1E-12	7E-14	9E-13	8E-12	3E-14	3E-12
λ_2	2E-14	0.624	5E-13	8E-14	5E-14	0.899	6E-14	5E-12
λ_3	1E-13	0.769	1	4E-13	7E-13	3E-09	2E-12	7E-12
λ_4	2E-13	0.6	5E-13	1	4E-15	0.015	2E-13	0.97
λ_5	6E-12	0.119	1E-12	1E-13	1	1E-11	4E-14	0.03
λ_6	2E-14	0.116	6E-13	5E-14	5E-14	4E-11	3E-14	3E-12
λ_7	3E-14	1.054	3E-12	3E-13	4E-13	0.086	1	6E-12
λ_8	1E-13	0.485	3E-13	2E-14	3E-14	6E-10	1E-13	1E-10
Z^2	55	90.08	68.93	78	52	85.46	69	77.44
	65	33.08	56.5	89	65	45.96	56	88.31
X^2	16	3.896	16.26	15.4	15	10.05	10	19.44
X^{D2}	1.5	5.098	4	8.8	1.6	5.414	4	8.614
	12	13.13	15.4	11.2	12.3	13	15.6	11.24
Y^2	30	48.9	38.36	70	35	53.75	38	68.96
π_1	1	0.835	0.031	6E-13	4E-12	0.04	4E-13	0.022
π_2	7E-14	1.404	0.014	4E-16	1E-13	0.945	4E-13	1E-12
π_3	1E-13	0.106	1E-11	3E-14	1E-13	4E-11	1E-13	2E-12
π_4	2E-14	0.219	0.011	1	2E-14	2E-11	3E-14	0.973
π_5	2E-13	0.158	1E-11	8E-14	1	4E-11	8E-14	4E-12
π_6	6E-14	0.621	3E-11	3E-14	1E-13	8E-10	3E-13	9E-12
π_7	3E-13	0.189	0.943	4E-14	8E-13	0.015	1	0.005
π_8	2E-14	0.123	8E-12	9E-14	3E-14	1E-11	3E-14	7E-12
u_1	830.82	3.488	337.2	493.1	197.6	698	772.2	255.4
u_2	0	0	0	0	0	0	0	0
u_3	0	0	0	0	0	0	0	0
u_4	0	0	0	0	0	0	0	0
u_5	0	0	0	0	0	0	0	0
u_6	0	0	0	0	0	0	0	0
u_7	518.63	2.819	181.3	290.3	186.8	454.1	522.1	92.17
u_8	0	0	0	0	0	0	0	0
u_9	0	0	0	0	0	0	0	0
u_{10}	0	0	0	0	0	0	0	0
u_{11}	0	0	0	0	0	0	0	0
u_{12}	0	0	0	0	0	0	0	0

u_{13}	514.72	1.612	208.7	234.6	447.7	244	387.4	163.2
v_1	1039.8	2.282	287.1	800.3	94.55	634.7	566.9	163.2
v_2	357.72	0.669	155.9	215.5	15.04	314.2	326.9	169.1
v_3	312.19	0	155.9	202.8	10.79	244	250.1	163.2
v_4	312.19	0.997	155.9	202.8	10.79	244	250.1	163.2
v_5	202.53	0.943	52.77	31.76	436.9	0	137.3	0
v_6	727.59	1.613	131.2	597.4	83.77	390.7	316.8	0
v_7	45.532	1E-04	0	12.69	4.25	70.19	76.77	5.871
v_8	311.19	7E-10	154.9	201.8	9.785	243	249.1	162.2
v_9	1156.1	2092	5219	651	403.8	1315	1441	8798
v_{10}	967.58	2134	341.3	484.6	4511	613.6	714.9	573.2
w_1	0	0	0	0	0	0	0	0
w_2	0	0	0	0	0	0	0	0
w_3	0	0	0	0	0	0	0	0
w_4	0	0	0	0	0	0	0	0
w_5	0	0	0	0	0	0	0	0
w_6	0	0	0	0	0	0	0	0
w_7	0	0.911	0	136	16.92	0	259.5	0
w_8	537.2	0	0	217.1	336	0	326.5	700.6
w_9	3268.3	1.866	945.4	2536	451.1	1617	1695	1010
w_{10}	2063.1	4.415	0	0	3185	1954	3491	0
w_{11}	2257.8	0.171	1017	1593	0	1820	2081	1145
w_{12}	987.15	0.013	307.1	551.3	330.9	221.5	537.4	1058
w_{13}	1345.9	0.942	0	1355	357.3	0	0	0
w_{14}	3485.5	4.008	616.6	926.5	3276	3031	4583	644.1

Table 23 The CRS optimized solutions in the 2nd situation

Data	DMU 1	DMU 2	DMU 3	DMU 4	DMU 5	DMU 6	DMU 7	DMU 8
Z^2	51.097	83.961	69.94	77.3	53.75	83.61	68.35	79.31
	59.104	54.433	56.32	77.44	61.91	58.52	53.74	78.58
X^2	1.9393	4E-11	10.2	1E-10	15	2E-10	6.143	4E-10
X^{D2}	1.5	5.6	4	7.885	1.6	5.8	4	8.022
	11.176	13	15.4	11.2	11.76	13	15	11.5
Y^2	27.889	55.014	38.92	64.79	29.34	56.15	38.12	66.11
π_1	0.866	0.135	0.052	2E-11	0.898	0.158	5E-14	6E-11
π_2	4E-14	0.768	0.059	0.198	6E-15	0.686	0.068	0.22

π_3	3E-14	2E-12	2E-11	6E-12	5E-15	8E-12	8E-14	2E-11
π_4	1E-14	0.125	1E-12	0.77	6E-16	0.196	1E-14	0.772
π_5	7E-14	4E-12	2E-11	8E-12	7E-15	1E-11	4E-14	2E-11
π_6	3E-14	1E-11	1E-10	6E-11	2E-15	5E-11	3E-13	2E-10
π_7	0.0503	4E-12	0.899	1E-11	0.063	2E-11	0.905	3E-11
π_8	9E-15	6E-12	9E-13	2E-11	6E-16	2E-11	1E-14	6E-11
Z^1	36.435	68.719	52	70.68	33	64.8	55.71	69.73
	64.939	61.314	42	75.02	62	58.63	45	74.41
Y^1	37.687	59.082	40	73.57	35	56.58	42.86	72.86
X^1	18.061	20	9.8	20	5	20	13.86	20
X^{D1}	2.5	5.5142	3	8.86	2.3	5.416	3.214	8.8
	13	12	7	18	12.5	11.7	7.5	17.9
λ_1	4E-14	3E-12	6E-12	1E-11	4E-15	1E-11	1E-14	4E-10
λ_2	3E-15	3E-12	3E-12	9E-12	6E-16	1E-11	2E-14	2E-11
λ_3	0.063	0.972	1	0.279	4E-15	0.867	1.071	0.26
λ_4	2E-15	0.289	1E-12	0.891	7E-16	0.313	1E-14	0.89
λ_5	1.0047	4E-12	7E-12	1E-11	1	1E-11	1E-14	0.005
λ_6	2E-15	2E-12	3E-12	6E-12	5E-16	8E-12	1E-14	2E-11
λ_7	3E-14	2E-11	4E-11	2E-10	6E-16	5E-10	4E-13	4E-10
λ_8	1E-15	2E-11	2E-12	6E-11	8E-16	7E-11	1E-14	2E-10
u_1	472.95	313.38	323.3	240.9	656.4	410.7	340.2	302.5
u_2	0	0	0	0	0	0	0	0
u_3	0	0	0	0	0	0	0	0
u_4	0	0	0	0	0	0	0	0
u_5	0	0	0	0	0	0	0	0
u_6	0	0	0	0	0	0	0	0
u_7	0	0	0	0	0	0	0	0
u_8	0	0	0	0	0	0	0	0
u_9	403.05	267.87	275.6	205.9	567	351	288.3	260.5
u_{10}	0	0	0	0	0	0	0	0
u_{11}	564.71	45.508	246.6	34.98	681.2	59.64	51.95	262.1
v_1	433.8	412.92	353.7	317.4	542.2	541.1	495.9	266.8
v_2	69.893	45.508	47.64	34.98	89.43	59.64	51.95	41.93
v_3	69.893	45.508	47.64	34.98	89.43	59.64	51.95	41.93
v_4	69.893	45.508	47.64	34.98	89.43	59.64	51.95	41.93
v_5	494.82	0	198.9	0	591.8	0	0	220.2

v_6	363.9	367.41	306.1	282.4	452.7	481.5	443.9	224.9
v_7	0	0	0	0	0	0	0	0
v_8	68.893	44.508	46.64	33.98	88.43	58.64	50.95	40.93
w_1	0	0	0	0	0	0	0	0
w_2	0	0	0	0	0	0	0	0
w_3	0	0	0	0	0	0	0	0
w_4	0	0	0	0	0	0	0	0
w_5	0	0	0	0	0	0	0	0
w_6	0	0	0	0	0	0	0	0
w_7	206.36	1046.7	552.8	804.5	198.1	1372	1444	93.01
w_8	2635.8	1099.4	1537	845	3615	1441	1315	1375
w_9	0	0	0	0	287.9	0	0	0
w_{10}	808.24	0	354.4	0	665.6	0	360.9	0
w_{11}	0	917	411.7	704.8	0	1202	1266	0
w_{12}	3574.5	1545.8	2104	1188	4834	2026	1806	1917
w_{13}	174.49	18.841	78.95	14.48	494.3	24.69	30.27	73.39
w_{14}	980.88	165.98	494.3	127.6	916.6	217.5	540.9	125.7

Table 24 The VRS optimized solutions in the 2^{nd} situation

Data	DMU 1	DMU 2	DMU 3	DMU 4	DMU 5	DMU 6	DMU 7	DMU 8
Z^2	55	76.45	68.93	78	52	85.46	69	77.44
	65	38.13	56.5	89	65	45.96	56	88.31
X^2	16	3.397	10.2	15.4	15	0.83	10	15.14
X^{D2}	1.5	4.535	4	8.8	1.6	5.414	4	8.614
	12	12.85	15.4	11.2	12.3	13	15.6	11.24
Y^2	30	41.88	38.36	70	35	53.75	38	68.96
π_1	1	1.296	0.031	8E-11	3E-14	0.04	6E-12	0.022
π_2	1E-11	1.162	0.014	5E-11	2E-15	0.945	6E-12	4E-10
π_3	6E-12	0.224	3E-11	6E-12	2E-15	3E-14	8E-13	6E-11
π_4	6E-12	0.465	0.011	1	4E-16	8E-14	3E-13	0.973
π_5	2E-11	0.309	2E-11	9E-12	1	3E-14	8E-13	3E-11
π_6	1E-11	0.852	7E-11	3E-11	2E-15	4E-13	3E-12	4E-10
π_7	5E-12	0.43	0.943	1E-11	5E-16	0.015	1	0.005
π_8	2E-12	0.25	2E-11	2E-11	1E-16	1E-14	4E-13	6E-11
Z^1	35	68.65	52	63	33	74.17	57	62.1
	60	58.86	42	71	62	52.57	45	70.73

Y^1	30	65.59	40	70	35	53.75	38	68.96
X^1	4	16.73	9.8	4.6	5	19.17	10	4.855
X^{D1}	2.5	6.869	3	9	2.3	6.727	3.5	8.8
	13	12.05	7	18	12.5	11.7	7.5	17.84
λ_1	1	0.246	3E-12	6E-12	1E-13	5E-15	4E-13	3E-11
λ_2	1E-12	0.175	2E-12	9E-12	3E-16	0.899	8E-13	1E-10
λ_3	2E-12	1.361	1	6E-11	5E-15	1E-12	3E-11	1E-10
λ_4	1E-11	1.076	1E-12	1	3E-16	0.015	3E-12	0.97
λ_5	3E-10	0.338	4E-12	2E-14	1	8E-15	5E-13	0.03
λ_6	1E-12	0.169	3E-12	6E-12	2E-16	3E-14	4E-13	2E-11
λ_7	9E-12	1.408	2E-11	5E-11	2E-15	0.086	1	1E-10
λ_8	2E-11	0.55	1E-12	4E-11	2E-16	2E-13	2E-12	2E-09
u_1	383.06	0.514	587.7	268.7	566.7	117.2	128.6	290.1
u_2	0	0	0	0	0	0	0	0
u_3	0	0	0	0	0	0	0	0
u_4	0	0	0	0	0	0	0	0
u_5	0	0	0	0	0	0	0	0
u_6	0	0	0	0	0	0	0	0
u_7	0	0	0	0	0	0	0	0
u_8	0	0	0	0	0	0	0	0
u_9	351.74	0.016	480.1	233.7	492.6	75.26	94.8	238.7
u_{10}	0	0	0	0	0	0	0	0
u_{11}	0	0	0	0	0	0	0	0
u_{12}	0	0	0	0	0	0	0	0
v_1	268.21	0.93	590.8	208.8	909.3	41.91	106.4	663.4
v_2	217.55	2.229	748.5	269	449.8	315	232.9	51.42
v_3	31.325	0.995	107.6	34.97	74.13	41.91	33.76	51.42
v_4	31.325	0.995	107.6	34.97	74.13	41.91	33.76	51.42
v_5	37.981	0	107.6	51.56	136.4	48.89	35.18	74.47
v_6	236.88	0.433	483.2	173.8	835.1	0	72.66	612
v_7	186.22	1.731	640.9	234	375.7	273.1	199.1	0
v_8	6.6563	2E-06	0	16.59	62.24	6.978	1.422	23.05
v_9	30.325	2E-04	106.6	33.97	73.13	40.91	32.76	50.42
v_{10}	527.12	11796	2586	568.4	1927	2671	1537	4375
v_{11}	1442.6	11756	941.1	230.5	659.2	219.3	162.1	502.7
w_1	0	0	0	0	0	0	0	0

w_2	0	0	0	0	0	0	0	0
w_3	0	0	0	0	0	0	0	0
w_4	0	0	0	0	0	0	0	0
w_5	0	0	0	0	0	0	0	0
w_6	0	0	0	0	0	0	0	0
w_7	0	17.05	1756	562.7	521.9	1672	920.9	240.6
w_8	2575.6	0	2842	1281	3561	0	348.1	2292
w_9	702.1	7.47	0	130.7	275	35.6	4.326	1253
w_{10}	319.97	9.121	1192	0	675.3	0	130.6	0
w_{11}	30.74	16.11	1313	397.3	0	1379	735.8	0
w_{12}	3111.3	3.147	4067	1758	4847	341.8	670.8	3173
w_{13}	846.95	4.574	141.7	246.7	747.6	0	0	1406
w_{14}	447.14	10.86	1449	75.58	698.9	76.85	202.1	34.97