Load Bearing Capacity of Super-Light Decks

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30 ECTS thesis submitted in partial fulfillment of a
Magister Scientiarum degree in Civil Engineering

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Abstract

Super-Light Decks (SLD) are an invention of prof. Kristian D. Hertz at DTU. They are one branch of his patented invention of Super-Light structures. The SLDs are intended to be an alternative to Hollow Core Slabs. Instead of the holes in hollow core slabs, around half of the slabs volume is made up of light aggregate concrete blocks. That increases both sound insulation and fire resistance.

In this masters project the behavior of the SLDs has been calculated and tests made to confirm the calculations. The aim is to show that the methods used to calculate the load bearing capacity of SLDs can safely be used in the design of buildings.

The characteristics investigated were: Moment bearing capacity, Stiffness, Shear capacity, Anchorage, Fire resistance, Eigenfrequency and Damping.

The method used to calculate the Shear capacity turned out to be very much on the safe side and the Eigenfrequency did not fit completely with what was expected. Apart from that the calculations seem to be quite reasonable.

Some suggestions are also made to how the Super-Light Decks can be used, in ways not possible with Hollow Core Slabs. The two main things looked at are cantilevered and fixed end Decks.

Útdráttur

Plötueiningarnar (SLD) sem prófaðar voru í þessu verkefni eru uppfinning prof. Kristian D. Hertz við DTU. Þær eru hluti af hugmyndum hans um léttari steyptar byggingar.

SLD einingunum er ætlað að keppa við holplötur. Í staðinn fyrir loftrýmin í holplötum fylla kubbar úr léttsteypu um helming rúmmáls SLD eininganna. Það eykur bæði hljóðeinangrun og brunaþol.

Í þessu Meistaraverkefni hefur hegðun SLD eininganna verið reiknuð út og tilraunir gerðar til að sannreyna niðurstöðurnar. Markmiðið er að sýna fram á að með þeim aðferðum sem notaðar eru í verkefninu sé hægt að hanna byggingar á öruggan hátt.

Eiginleikarnir sem kannaðir voru eru: Vægisburðarþol, stífni, skúfburðarþol, festing járna, brunaþol, eigintíðni og dempun.

Aðferðin sem notuð var til að reikna skúfburðarþol reyndist vanmeta burðarþolið mikið og eigintíðnin sem mæld var passaði ekki fullkomlega við útreikningana. Að öðru leiti voru niðurstöðurnar í góðu samræmi við það sem búist var við.

Einnig eru gerðar tillögur um hvernig er hægt að nota SLD einingarnar til að gera hluti sem eru illmögulegir með holplötum. Skoðað var hvernig mætti gera innspenntar plötur og útkragandi plötur.

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Nomenclature

A_b: Brutto area

A_c: Cross sectional area of normal concrete

A_i: Ideal area

A_l: Cross-sectional area of light concrete

A_p: Cross sectional area of prestress wires

a_p: Height to center of gravity of the reinforcement from bottom of cross section

A_{sl}: Area of tensile reinforcement with sufficient anchorage (EC2)

bf: Bond factor

$b_{w.c}$: Width of normal concrete at the level of prestress wires

$b_{w.l}$: Width of light concrete at the level of prestress wires

b_w: Smallest width of the cross section in the tensile area

C_p: Circumference of prestress wires

d: Height from center of reinforcement to top of the cross section

E: Elastic Modulus

f: Eigenfrequency

F_b: Bond strength of prestressing wires

f_{ck}: Characteristic compressive strength of normal concrete

f_e: Expected frequency

f_m: Measured frequency

f_{pk}: Tensile strength of prestress wires

F_y: Total force needed to reach the tensile strength of all the prestressing wires

g: Linear own weight of the SLD

I_b: Brutto moment of inertia

I_i: Ideal moment of inertia

I_l: Moment of inertia of the light concrete

L: Length of span

l_b: Length of anchorage

M_b: Moment resistance of prestress wire bond

M_g: Maximum moment from own weight

$M_{p.bed}$: Moment around the ideal center of gravity caused by prestressing

$M_{Q.0}$: Moment needed in middle of the span to unload the compression

$M_{R.Q}$: Moment resistance to applied load

M_R: Moment resistance of the cross section

n_l: Ratio between elastic modulus of light concrete and strong one

n_p: Ratio between elastic modulus of prestressing wires and normal concrete

P_{bed}: Total force in all prestress wires while still in the mould

Q_0: Load needed to make the bottom fiber reach zero stress

Q_{max}: Maximum point load that can be applied in the middle of the span

T_c: Temperature of concrete

u_n: Amplitude of a wave

$V_{R.c}$: Shear resistance of the cross section

$V_{Rmin,c}$: Minimum value of Shear resistance

W: Width of the cross section

W_{ib}: Ideal section modulus for the bottom fiber of the cross section

y: Height of the compression zone

y_b: Height of compression zone for bond calculation

z_{bb}: Height to brutto center of gravity from bottom of cross section

z_{ib}: Height to ideal center of gravity from bottom of cross section

z_{lb}: Height to center of gravity of the light concrete from bottom of cross section

ζ: Damping

μ: Linear mass of beam

ξ: Internal damping

ρ_l: Reinforcement ratio

$\sigma_{cb.g}$: Stresses in the bottom fibers of the concrete from own weight

$\sigma_{cb.p}$: Stresses in the bottom fibers of the concrete from prestressing

$\sigma_{cb.Q.0}$: Stresses needed from applied load to unload all the compression

$\sigma_{cp.c}$: Stresses in normal concrete due to prestressing

$\sigma_{cp.l}$: Stresses in light concrete due to prestressing

$\sigma_{p.bed}$: Stresses in prestressing wires while still in the mould

ω: Angular frequency

Acknowledgements

I want to thank my wife Sjöfn Yngvadóttir for her support. Without her encouragement I would not be finishing my MSc degree. Gunnlaugur Róbertsson also helped a lot with his constructive criticism. My advisors Kristian Dahl Hertz and Bjarni Bessason also need mentioning. And finally I want to thank DTU for providing me with the necessary facilities and the laboratory staff for their help.

1 Introduction

1.1 Super-Light Structures

Super-Light Structures are an invention of professor Kristian Hertz at DTU (Technical University of Denmark). The idea is to optimize the use of normal concrete and then support it with light concrete. The name, Super Light Structures is drawn from the fact that steel structures are often referred to as light structures since they are commonly lighter than structures casted in concrete. Using the methods developed by Hertz it is possible to build structures lighter than when using steel, hence Super-Light Structures. (Hertz, 2009)

The best form to take up compression forces is usually an arch and using common modern construction methods arches are very expensive. To be able to optimize the usage of heavy concrete the pearl chain method was developed. A pearl chain beam is made up of precast elements with holes through them. Putting these elements together, one can make beams of almost any shape. A wire is drawn through the holes in the elements and tensioned to give the beam stiffness (*Figure 1-1*).

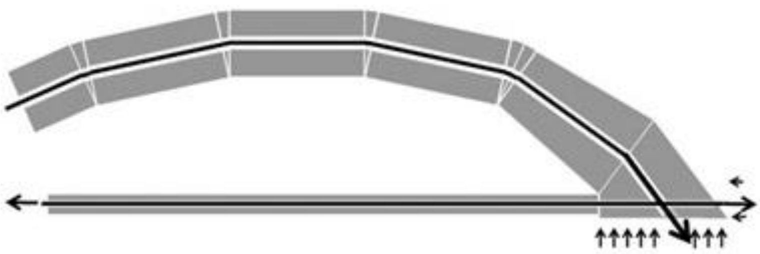

Figure 1-1: A pearl chain arch.

1.2 Light concrete

There are three main types of light concrete. Autoclaved aerated concrete (AAC), foam concrete and light aggregate concrete. What they all have in common is very high porosity which means that a large part of the volume is just air, making them much lighter than normal concrete. That of course reduces their strength and stiffness but it also increases insulation and fire resistance.

AAC concrete is made by mixing aluminum powder with cement, sand and water. The chemical reaction between the aluminum and calcium hydroxide of the cement and water forms hydrogen. The hydrogen forms bubbles in the concrete mix which increses its volume and therefor gives it a lower density. (Hebel, 2010)

When making foam concrete, foam similar to shaving foam is mixed with the concrete mix which gives it the desired lightness.

To make light aggregate concrete a very pourous aggregate is used. It can either be a natural one (Volcanic pumice) or factory made expanded clay. The latter was used to make the light concrete blocks used in the experiments.

1.3 Super-Light Decks

A Superlight Deck (SLD) is made by arranging blocks of light aggregate concrete in a mould and casting over them with normal concrete, after prestress wires have been placed. The blocks have a shape resembling a half cylinder standing on a low box shape (*Figure 1-2*). The dimensions can be seen in Appendix A. These blocks are made of concrete with a unit weight of 600 kg/m³ (Christensen, Hertz, & Brunskog, 2011) using LECA™ (Light Expanded Clay Aggregates).

Figure 1-2: Light aggregate block.

The blocks are placed in parallel rows in the slab casting mould. To make the decks for this project, three rows of 400 mm wide blocks were used making the decks 1200 mm wide. Between the rows of blocks, two 12.5 mm prestressing wires are placed and one wire on the outside of each of the outermost rows, a total of six wires (*Figure 1-3*). Finally the mould is filled with normal concrete. *Figure 1-4* shows the SLD both before and after the normal concrete has been cast.

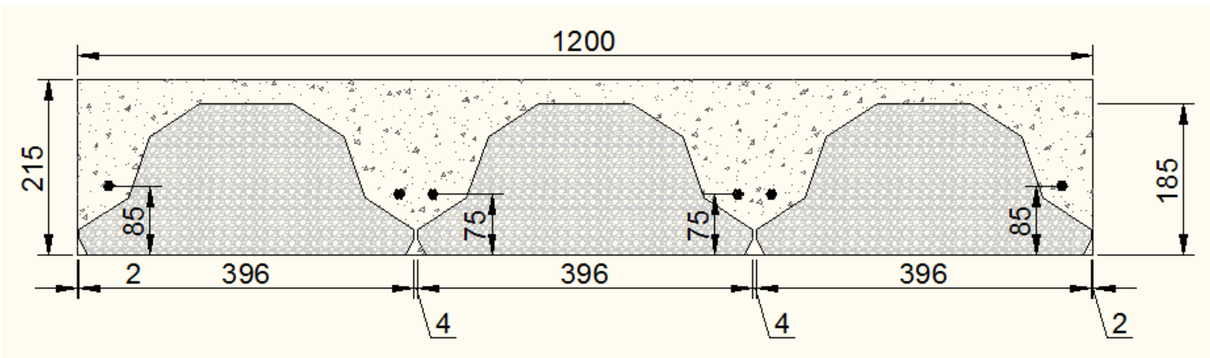

Figure 1-3: Cross-Section through the center of light aggregate concrete blocks.

The idea is to make the compression zone at the top of the element out of normal concrete and cast the prestressing wires in it as well. Then the remaining spaces that add little to the strength of the deck are fitted with light concrete blocks. This decreases the self-weight of the deck. The last 10 cm at either end of each deck are solid normal concrete to give better anchorage to the prestress wires.

From underneath only the light aggregate concrete is visible and the steel wires are never closer to the bottom of the slab than 60 mm (if only one wire is used between the blocks, see Appendix A). The light concrete acts as insulation which gives the slabs very good fire resistance.

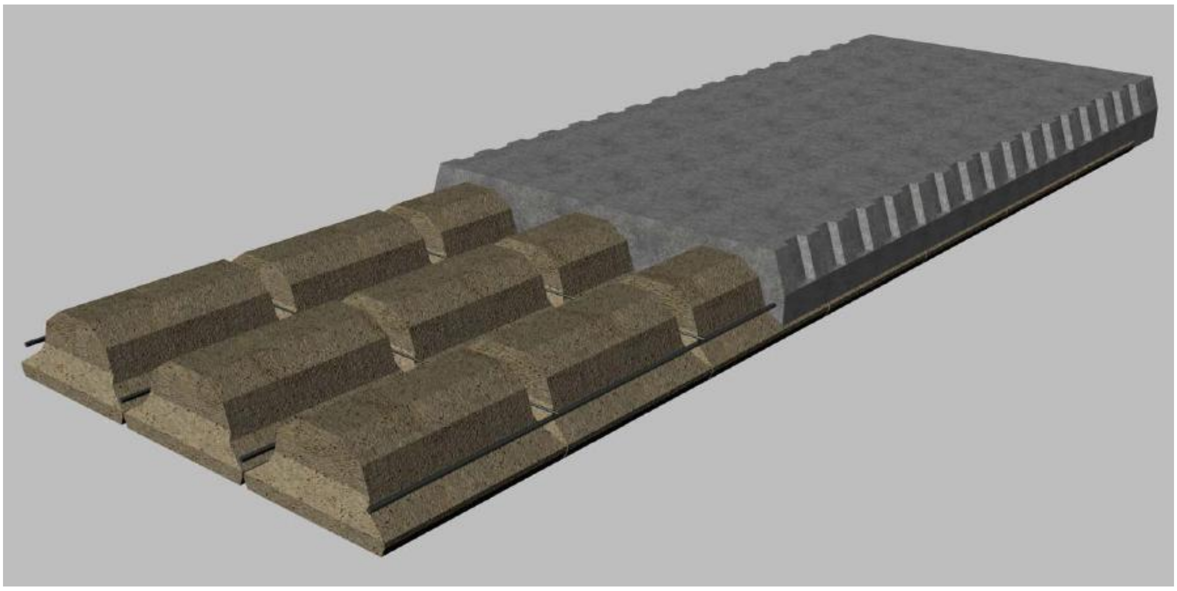

Figure 1-4: An SLD, showing the deck both before and after the normal concrete has been cast.

The reason for the arc shape of the block is sound insulation (*Figure 1-2*). This shape makes the decks less penetrable for sound as shown by studies of Jacob Christiansen (Christensen et al., 2011). He has found the decks to give a sound insulation of 55 dB for airborne sounds which fulfills the demands of the new stricter Danish building regulations. The sound penetration from a standard knocking machine is 79 dB. That is 26 dB higher than the demands of the building regulations so the flooring must add to the insulation against impact sounds as for any other concrete slabs. But sound insulation is not the topic of this project so it will not be addressed further.

1.4 This Project

In this project the goal is to predict with calculations some short term characteristics of the SLD and verify them with tests. The characteristics that will be looked at are: Stiffness, moment bearing capacity, shear resistance, anchorage, eigenfrequency, damping and fire resistance. In some of the tests more than one characteristic will be checked at the same time.

In Chapter 2 the calculations for each characteristic will be discussed. Each test has its own subchapter where calculations relevant to that test are discussed. Most of the calculations are variations of already established methods of designing concrete structures. What complicates them somewhat is that the SLD contains two types of concrete with different elastic modulus and strength.

Chapter 3 is about the tests. It has the same subchapters as chapter 2. That is, one for each test. In each subchapter first the setup of the test is described and then the results are presented.

The conclusions are drawn in Chapter 4, and Chapter 5 has some examples of the possibilities the SLD has to offer.

Figure 1-5: The light concrete blocks in the mould before the normal concrete is cast.

2 Model

In this chapter the methods used to calculate different characteristics of the SLD will be discussed.

2.1 Stiffness and Ultimate Moment Bearing Capacity

2.1.1 Moment of Inertia and Bending Stiffness

Since two types of concrete are used to make the SLD, the usual methods for calculating the bending stiffness of concrete slabs is not valid. It therefore needs to be altered somewhat.

When calculating the stiffness of a normal cracked concrete cross-section the steel is transformed into concrete using the ratio between the Elastic Modulus of the two materials. This method is taken one step further to calculate the stiffness of the SLD cross section. That is, the light concrete is also transformed into normal concrete.

To do this, one needs to find a moment of inertia and center of gravity of a cross-section of only normal concrete, equivalent to the original composite cross-section.

Here the transformed cross-section will be called „ideal“ and the original one „brutto“ denoted with a lowercase *i* and *b* respectively when using their properties in formulas (Fischer, 2011). Normally this method is used to transform steel into concrete but in this case both the light concrete and the steel are transformed into normal concrete.

Table 2-1: Material constants for the steel and the two types of concrete for a 1,2 m wide deck. The cross-section is taken through the middle of the light aggregate blocks. Height of the Center of Gravity is measured from the lowest point of the normal concrete.

	Cross Sectional Area (A)	**Height of Center of Gravity (z)**	**Moment of Inertia (I)**	**Elastic Modulus (E)**	**n coefficient**
	[mm²]	[mm]	[mm^4]	[GPa]	[-]
Normal Concrete	1,03E+05	121,0	2,31E+08	37	1
Light Concrete	1,17E+05	65,4	2,13E+08	3	0,081
Steel	558	46,3	-	190	5,68
Brutto Cross-Section	2,20E+05	91,5	6,13E+08	-	-
Ideal Cross-Section	1,15E+05	114,9	2,86E+08	37	-

First the moment of inertia and center of gravity are found for each section seen in *Figure 2-1*. For the light concrete the lowest part (*labeled I*) is excluded since there is no connection between the blocks in that area and therefore it doesn‘t contribute to the bending stiffness. The values for each material can be seen in *Table 2-1* along with the values for the brutto cross-section and the ideal cross-section. These values are then used to find the ideal cross-section properties for simpler calculations. The full calculations can be seen in Appendix B.

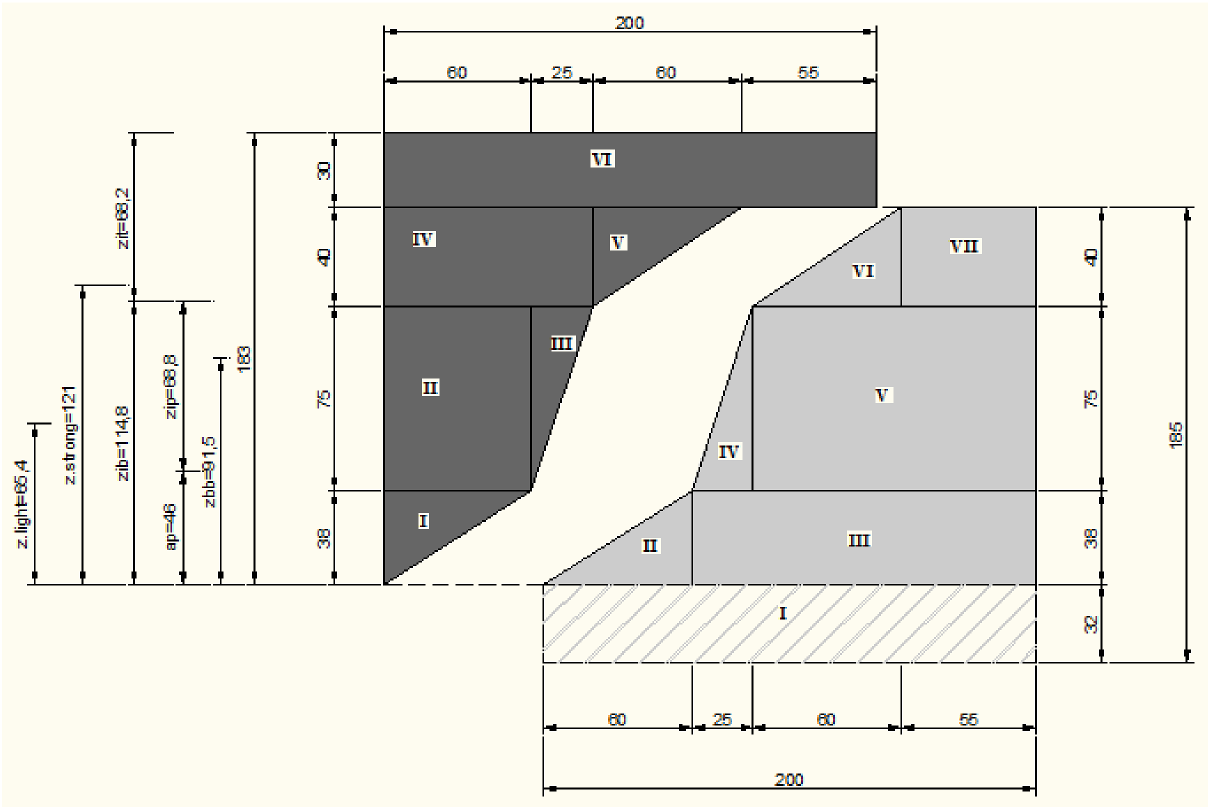

Figure 2-1: One sixth of the cross-section of the slab. The normal concrete is shown in dark grey and the light concrete in a light grey color.

First the area of the ideal cross-section is calculated:

$$A_i = A_b + (n_l - 1) \cdot A_l + (n_p - 1) \cdot A_p = 1{,}151 \cdot 10^5\ mm^2 \qquad \text{(eq. 2-1)}$$

Then the height of the center of gravity:

$$z_{ib} = \frac{A_b \cdot z_{bb} + (n_l - 1) \cdot A_l \cdot z_{lb} + (n_p - 1) \cdot A_p \cdot a_p}{A_i} = 114.8\ mm \qquad \text{(eq. 2-2)}$$

Where: A_x: Area of the material denoted.

z_x: Height to the center of gravity of the material denoted.

n_x: The ratio between the elastic modulus of normal concrete and the material denoted.

X_l: Refers to a property of the light concrete.

X_p: Refers to a property of the prestress cables.

X_I: Refers to a property of the ideal cross-section.

X_l: Refers to a property of the brutto cross-section.

Using these results the moment of inertia of the ideal cross-section can be calculated. Note that when using this method for normal concrete slabs the moment of inertia of the steel around its center of gravity is neglected due to its low value. But that is not the case with the light concrete since its area is much greater than the steels, hence the I_l term:

$$I_i = I_b + A_b \cdot (z_{bb} - z_{ib})^2 + (n_p - 1) \cdot A_p \cdot (z_{ib} - a_p)^2 + (n_l - 1)\left[I_l + [A_l(z_{ib} - z_{lb})^2]\right] = 2{,}872 \cdot 10^8\ mm^4 \quad \text{(eq. 2-3)}$$

Where: I_x: Moment of inertia of the material denoted.

a_p: Average height to the center of prestress cables.

Using this ideal moment of inertia it is possible to calculate the deflections of the slab using just the elastic modulus of the normal concrete (*37 GPa*). That gives a short term bending stiffness of:

$$EI = 10.6\ MNm^2 \quad \text{(eq. 2-4)}$$

2.1.2 Elastic Range

To decide how big the elastic range is, the stresses in the bottom fiber of the normal concrete need to be determined. The concrete is assumed to have no tensile strength so when the stresses in the bottom fiber of the concrete reaches zero it will crack and the bending stiffness calculated before will not apply anymore. Only the main steps are shown here but the full calculations can be seen in Appendix B.

First the stresses in the bottom fiber of the concrete from the prestressing ($\sigma_{cb.p}$) must be determined:

$$W_{ib} = \frac{I_i}{z_{ib}} = \frac{2.87 \cdot 10^8\ mm^4}{114.8\ mm} = 2.5 \cdot 10^6\ mm^3 \quad \text{(eq. 2-5)}$$

$$\sigma_{p.bed} = 0.75 \cdot f_{pk} = 0.75 \cdot 1860\ MPa = 1395\ MPa \quad \text{(eq. 2-6)}$$

$$P_{bed} = A_p \cdot n_p \cdot \sigma_{p.bed} = 93\ mm^2 \cdot 6 \cdot 1395\ MPa = 778.4\ kN \tag{eq. 2-7}$$

$$M_{p.bed} = -P_{bed} \cdot z_{ip} = -53.3\ kNm \tag{eq. 2-8}$$

$$\sigma_{cb.p} = \frac{-P_{bed}}{A_i} + \frac{M_{p.bed}}{W_{ib}} = -28.1\ MPa \tag{eq. 2-9}$$

The calculations can be seen in Appendix B.

$$g = 3.49\ \frac{kN}{m} \tag{eq. 2-10}$$

Using a span of 4 m, which is the intended span for the moment test, gives a maximum moment from dead load of:

$$M_g = \frac{g \cdot L^2}{8} = \frac{3.49 \frac{kN}{m} \cdot (4\ m)^2}{8} = 6.97\ kNm \tag{eq. 2-11}$$

That means the stresses in the bottom fiber from the dead load are:

$$\sigma_{cb.g} = \frac{M_g}{W_{ib}} = \frac{6.97\ kNm}{2.5 \cdot 10^6\ mm^3} = 2.8\ MPa \tag{eq. 2-12}$$

So the stresses from the applied load need to be:

$$\sigma_{cb.Q.0} = -\left(\sigma_{cb.p} + \sigma_{cb.g}\right) = -(-28.1\ MPa + 2.8\ MPa) = 25.3\ MPa \tag{eq. 2-13}$$

$$M_{Q.0} = \sigma_{cb.Q.0} \cdot W_{ib} = 25.3\ MPa \cdot 2.5 \cdot 10^6\ mm^4 = 63.2\ kNm \tag{eq. 2-14}$$

The applied load needed for the bottom fiber of the normal concrete to reach zero:

$$Q_0 = \frac{4 \cdot M_{Q.0}}{L} = \frac{4 \cdot 63.2\ kNm}{4\ m} = 63.2\ kN \tag{eq. 2-15}$$

2.1.3 Ultimate Moment Bearing Capacity

To determine the ultimate moment bearing capacity of the cross-section first one needs to determine if the cross-section is overreinforced. That is whether the cables will yield before the deck brakes or not. To do that one must decide if the strain in the cables when the concrete reaches braking point is larger then the yield strain. If it is, the cables will yield and their yield strength therefore decide the moment capacity of the deck. If the concrete reaches its braking point before the cables yield then the cross section is overreinforced and the compressive strength of the concrete is the deciding factor.

The cable strain is divided into three parts, strain in the tendon after release (ε_p), compressive strain in the concrete due to prestress (ε_{cp}) and total strain in the concrete at level of the tendon from external moment ($\Delta\varepsilon_p$). These three parts are calculated as follows:

$$\varepsilon_p = \frac{P}{E_p \cdot A_p} = \frac{748.5\ kN}{190\ GPa \cdot 558\ mm^2} = 7.1 \cdot 10^{-3} \qquad \text{(eq. 2-16)}$$

$$\begin{aligned}\varepsilon_{cp} &= \frac{P}{E_c(A_i - A_p \cdot n)} + \frac{P(z_{ib} - a_p)^2}{E_c \cdot I_i} \\ &= \frac{748.5\ kN}{37\ GPa\left(1.148 \cdot 10^5 - 558\frac{190}{37}\right)} \\ &+ \frac{748.5\ kN(114.9\ mm - 78.3\ mm)^2}{37\ GPa \cdot 2.858 \cdot 10^8\ mm^4} = 2.75 \cdot 10^{-4}\end{aligned} \qquad \text{(eq. 2-17)}$$

$$\Delta\varepsilon_{\text{p}} = \frac{0.0035(\text{d} - \text{x})}{\text{x}} \qquad \text{(eq. 2-18)}$$

Where: *P*: The force in the cables after elastic shortening.

x: Height from the top of the cross-section to the zero strain line.

Setting the tensile force at ULS in the cross-section equal to the compressive force one gets eq. 2-19. Where the compressive force is on the left.

$$0.8 \cdot x \cdot b \cdot f_{ck} = A_p \cdot E_p\left(\varepsilon_p + \varepsilon_{cp} + \Delta\varepsilon_p\right) \qquad \text{(eq. 2-19)}$$

Finding x from eq. 2-19 gives x = 37 mm which in turn gives $\Delta\varepsilon_p = 9.7 \cdot 10^{-3}$. The yield strain in the cables is:

$$\varepsilon_{yield} = \frac{f_{p0.1k}}{E_p} = \frac{1625\ MPa}{190\ GPa} = 8.55 \cdot 10^{-3} \qquad \text{(eq. 2-20)}$$

And since:

$$\left(\varepsilon_p + \varepsilon_{cp} + \Delta\varepsilon_p\right) = 0.017 > \varepsilon_{yield} = 8.55 \cdot 10^{-3} \quad \text{(eq. 2-21)}$$

Then the cross-section is not overreinforced and the moment bearing capacity can be calculated as follows.

First the force needed to reach the tensile strength of the steel (F_y) is determined and from that the height of the compression zone at the top of the cross-section (y). That is how much of the height of the concrete is needed to withstand the opposite force. Finally the force is multiplied with the height from the center of the reinforcement to the center of the compression zone to get the total moment resistance (M_R).

$$F_y = f_{pk} * A_p = 1860\ MPa * 6 * 93\ mm^2 = 1038\ kN \quad \text{(eq. 2-22)}$$

$$y = \frac{F_y}{f_{ck} * W} = \frac{1038\ kN}{50\ MPa * 1200\ mm} = 17.3\ mm \quad \text{(eq. 2-23)}$$

$$M_R = F_y\left(d - \frac{y}{2}\right) = 1038\ kN\left(136.7\ mm - \frac{17.3\ mm}{2}\right) = 132.9\ kNm \quad \text{(eq. 2-24)}$$

Subtracting the moment from the dead load from the moment resistance gives the resistance to moment from applied load:

$$M_{R.Q} = M_R - M_g = 125.9\ kNm \quad \text{(eq. 2-25)}$$

Since the load applied in the tests is a point load in the center of the span the load the deck should withstand is:

$$Q_{max} = \frac{4 \cdot M_{R.Q}}{L} = 125.9\ kN \quad \text{(eq. 2-26)}$$

2.2 Shear Resistance and Anchorage

2.2.1 Shear Resistance

To calculate the shear strength of the slab a method from Eurocode 2 was used. It is from chapter *6.2.2 Members not requiring design shear reinforcement*. (CEN, 2004a) It uses empirical formulas. Since the SLD is made of two types of concrete some modifications are needed in order to apply the method. The original formula:

$$V_{R,c} = \left[0.18k\sqrt[3]{100\rho_l f_{cm}} + 0.15\sigma_{cp}\right]b_w d \qquad \text{(eq. 2-27)}$$

With a minimum of:

$$V_{Rmin,c} = \left(0.035k^{3/2} \cdot \sqrt{f_{cm}} + 0.15\sigma_{cp}\right)b_w d \qquad \text{(eq. 2-28)}$$

Where: *k*: Constant depending on d (*eq.2-24*).

ρ_l: Reinforcement Ratio.

σ_{cp}: Stress in concrete from prestress.

b_w: Smallest width of the cross section in the tensile area.

d: Height from center of reinforcement to the top of the concrete.

To calculate the shear capacity of the SLDs cross section it is suggested to compute the shear capacity separately for each type of concrete and then add them together:

$$V_{R,c} = V_{R,c.strong} + V_{R,c.light} \qquad \text{(eq. 2-29)}$$

Some of the constants are the same for both parts:

$$k = 1 + \sqrt{\frac{200}{d}} = 1 + \sqrt{\frac{200}{137}} = 2.21 \quad if \ \geq 2.0 \ => k = 2.0 \qquad \text{(eq. 2-30)}$$

$$\rho_l = \frac{A_{sl}}{b_w \cdot d} = \frac{93\ mm^2 \cdot 6}{1200\ mm \cdot 137\ mm} = 0.00339 \qquad \text{(eq. 2-31)}$$

The stresses in each type of concrete from the prestressing depend on the ratio between their elastic modulus:

$$\sigma_{cp.c} = \frac{\sigma_{p.p} \cdot A_{sl}}{A_c + A_l \cdot n_l} = \frac{1294\ MPa \cdot 6 \cdot 93\ mm^2}{103100\ mm^2 + 116500\ mm^2 \cdot \frac{3\ GPa}{37\ GPa}} = 6.42\ MPa \qquad \text{(eq. 2-32)}$$

$$\sigma_{cp.l} = \frac{\sigma_{p.p} \cdot A_{sl}}{\frac{A_c}{n_l} + A_l} = 0.52\ MPa \qquad \text{(eq. 2-33)}$$

Where: $\sigma_{p.p}$: Tension in cables after release. Calculated in Appendix B p. 55.

Since the light concrete has a much lower elastic modulus, the normal concrete will take up most of the stresses.

At the height of the reinforcement the width of the normal concrete is approximately 120 mm in each rib. That gives:

$$b_{w.c} = 3 \cdot 120\ mm = 360\ mm \qquad \text{(eq. 2-34)}$$

and

$$b_{w.l} = W - b_{w.c} = 1200\ mm - 360\ mm = 840\ mm \qquad \text{(eq. 2-35)}$$

From that the shear resistance of both types of concrete is calculated:

$$V_{R,c.strong} = \left[0.18 \cdot 2.0\sqrt[3]{100 \cdot 0.00339 \cdot 58\ MPa} + 0.15 \cdot 6.42\ MPa\right] \cdot 360\ mm \cdot 137\ mm = 95.4\ kN \qquad \text{(eq. 2-36)}$$

$$V_{Rmin,c.strong} = \left(0.035 \cdot 2.0^{3/2} \cdot \sqrt{58\ MPa} + 0.15 \cdot 6.42\ MPa\right) \cdot 360\ mm \cdot 137\ mm = 84.7\ kN \qquad \text{(eq. 2-37)}$$

$$V_{R,c.light} = \left[0.18 \cdot 2.0\sqrt[3]{100 \cdot 0.00339 \cdot 3\ MPa} + 0.15 \cdot 0.52\ MPa\right] \cdot 840\ mm \cdot 137\ mm = 50.6\ kN \qquad \text{(eq. 2-38)}$$

$$V_{Rmin,c.light} = \left(0.035 \cdot 2.0^{3/2} \cdot \sqrt{3\ MPa} + 0.15 \cdot 0.52\ MPa\right) \cdot 840\ mm \cdot 137\ mm = 28.7\ kN \qquad \text{(eq. 2-39)}$$

Since both results of *eq. 2-21* are higher than the minimum value the total shear resistance will be:

$$V_{R,c} = 95.4\ kN + 50.6kN = 146.0\ kN \qquad \text{(eq. 2-40)}$$

2.2.2 Anchorage

The result of *eq. 2-23* is only valid if the prestressing wires are sufficiently anchored. In real constructions slabs often only have a few centimeters of support at either end. Therefor it is important to check if the anchorage of the wires is sufficient when the support is very close to the end of the slab. Therefore, in one of the tests, the SLD had an anchorage length of only 40 mm. The span was 1,995 mm and a load distance of 500 mm. See Figure 3-1 for a picture of the setup.

Bond strength (F_b) of a prestressing wire was described by (Hertz, 2005) with:

$$F_b = C_p \cdot l_b \cdot bf \cdot f_{ck} \qquad \text{(eq. 2-41)}$$

Where: *C_p: Circumference of prestress wires*

l_b: Length of anchorage

bf: Bond factor (0.25-0.41). 0.25 is used to be on the safe side

For prestress wires of 12.5 mm the bond strength is:

$$F_b = \pi \cdot 12.5\ mm \cdot 0.25 \cdot 50\ MPa = 491\ \frac{N}{(mm\ of\ wire)} \qquad \text{(eq. 2-42)}$$

In the case of the SLDs used for the purpose of this project the force needed to pull the prestress wires out of the 100 mm solid concrete at the ends of the decks is:

$$F_b = 6 \cdot \pi \cdot 12.5\ mm \cdot 100\ mm \cdot 0.25 \cdot 50\ MPa = 295\ kN \qquad \text{(eq. 2-43)}$$

To calculate the moment needed to pull the wires out, the height of the compression zone is found:

$$y_b = \frac{F_b}{W \cdot f_{ck}} = \frac{295\ kN}{1200\ mm \cdot 50\ MPa} = 4.9\ mm \qquad \text{(eq. 2-44)}$$

And from there the moment is found:

$$M_b = F_b \cdot (d - y_b) = 295\ kN \cdot (137\ mm - 4.9\ mm) = 39.0\ kNm \qquad \text{(eq. 2-45)}$$

2.3 Eigenfrequency and Damping

2.3.1 Eigenfrequency

According to Teknisk Ståbi p. 68 (Jensen, 2003) the first eigenfrequency of a simply supported beam can be described with the following equations:

$$\omega = \frac{\pi^2}{l^2}\sqrt{\frac{EI}{\mu}} \qquad \text{(eq. 2-46)}$$

and

$$f = \frac{\omega}{2\pi} \qquad \text{(eq. 2-47)}$$

Where: *ω: Angular frequency*

μ: Linear mass of beam

f: Eigenfrequency

Inserting the values for the SLD calculated so far:

$$\omega = \frac{\pi^2}{l^2}\sqrt{\frac{1.06 \cdot 10^7 Nm^2}{355\ \frac{kg}{m}}} = \frac{1705}{l^2}\frac{m^2}{s} \qquad \text{(eq. 2-48)}$$

Which gives an eigenfrequency of:

$$f = \frac{1705/_{l^2}}{2\pi} = \frac{271}{l^2}\frac{m^2}{s} \qquad \text{(eq. 2-49)}$$

2.3.2 Damping

To find the damping of the deck, a load is hung from the deck then cut loose giving the deck acceleration. By looking at how this movement dies out the damping can be calculated from this equation: (Chopra, 1995)

$$\frac{1}{j} ln \frac{u_1}{u_{j+1}} \cong 2\pi\zeta \qquad \text{(eq. 2-50)}$$

Where: j: number of waves used

u_1: Amplitude of the first wave

u_{1+j}: Amplitude of the last wave

ζ: Damping of the system

No attempt will be made to estimate the damping. It will only be measured in the test chapter.

2.4 Fire Resistance

The ConFire program was used to calculate the temperature of the prestress steel under fire load. The program calculates the temperature distribution of a concrete cross section using a finite difference method. (Hertz, 2007b)

The program uses 1.1 kJ/(kg·K) as the specific heat of concrete except for in the range between *20°C* and *120°C* then 0.51 kJ/(kg·K) is added due to the heating and evaporation of water contained in the concrete.

For conductivity of concrete the lower limit curve from Eurocode is used (CEN, 2004b):

$$\lambda_e = \left[1.36 - 0.136\frac{T_c}{100} + 0.0057\left(\frac{T_c}{100}\right)^2\right]\frac{W}{m \cdot K} \qquad \text{(eq. 2-51)}$$

The input values for the program are shown in *Table 2-2*.

Table 2-2: Input values for the Confire Program.

W [m]	0.183	The thickness of the slab. For the calculations the light aggregate concrete was considered as insulation. Therefore this is only the thickness of the normal concrete.
x [m]	0.045	The depth of the point where the temperature is calculated. This is the placement of the lowest prestress strands in the decks. (This is only the normal concrete, since the light concrete is considered to be insulation).
d [m]	0.025	Thickness of insulation. The light aggregate concrete is considered to be insulation.
Conductivity [W/(m*K)]	0.3	Conductivity of the insulation.
Concrete type	Main group concrete	Which type of concrete is to be used.
Fire type	Standard Fire	Opening factor fire or Standard fire.
Steel type	Cold Drawn Prestress wires	Which type of steel is used.

According to (Hertz, 2007a) the loss of strength of reinforcement steel as a function of temperature can be described with the following equation:

$$\xi(T) = k + \frac{1-k}{1 + \frac{T}{T_1} + \left(\frac{T}{T_2}\right)^2 + \left(\frac{T}{T_8}\right)^8 + \left(\frac{T}{T_{64}}\right)^{64}} \quad \text{(eq. 2-52)}$$

Where: *$\xi(T)$: The ratio of full strength at temperature T*

k, T_1, T_2, T_8, T_{64}: Constants depending on the steel type in question

For cold worked prestressing steel as used in the SLDs the constants are (k; T_1; T_2; T_8; T_{64}) = (0; 2,000; 360; 430; 100,000).

Based on results of fire resistance analysis using the ConFire program the strength of the steel as a function of fire time could be computed and then the reduced steel strength used to find the moment resistance of the cross section. The results for different times of the fire are shown in *Table 2-3*.

Table 2-3: Development of steel temperature and strength during a standard fire.

Time [min]	**Steel temperature [°C]**	**0.2% Strength [% of full strength]**	**Maximum Moment [kNm]**
0	20	100	133
30	36	97	129
60	76	92	123
90	122	85	114
120	**165**	**77**	**104**
150	204	70	95
180	240	63	86
210	272	58	79
240	302	52	71

3 Tests

To determine the characteristics of the SLD a series of tests where made on them in 2011. These tests were conducted by the author and a fellow MSc student Andrea Tasello who was working on a parallel Master project. The mechanical tests were conducted at DTUs laboratory hall in building 119. The shear tests on April 12th, the moment, stiffness and dynamic tests on April 13th, and the Anchorage test on June 21st. On April 20th the fire resistance was tested at the Danish Institute of Fire and Security Technology in Copenhagen (DBI).

In the stiffness and ultimate moment bearing capacity tests, loads were applied on the middle of the slabs using two hydraulic jacks. Deflections were measured in the elastic range of the slabs in order to compute their stiffness, then the load was increased until the slabs broke to find the moment capacity.

The setups of the shear strength and anchorage tests were similar to the ones before. Shorter spans and load distances were used to increase the likelihood of getting a shear failure rather than a moment failure. In the first two tests longer anchorage lengths were used as the focus was on the shear strength. On the other hand in the last test the anchorage length was very short, to find the force needed to pull the wires out of the concrete.

The eigenfrequency and damping of the slab were measured using an accelerometer. First the eigenfrequency was measured using only background noice. Then a load of around 100 kg was hung from the slab and released suddenly to produce a free vibration so the damping of the slab could be measured.

Finally two slabs were tested for resistance against standard fire. The six meter slabs were placed above a gas furnace which was heated to follow the standard fire curve. The fire was kept burning for two hours while the slabs were kept under a design load (Catagory A) (CEN, 2001). After the two hours the load was increased while the oven was kept burning to see how much they could take after the fire.

All the slabs that were tested had the same cross-section, shown in *Figure 1-3*. A height of 215 mm and width of 1200 mm, three rows of light aggregate concrete blocks and six 12.5 mm (93 mm²) presstress steel cables with a tensile strength of 1860 MPa.

3.1 Stiffness and Moment Bearing Capacity

3.1.1 Setup

The stiffness and moment tests were setup as shown in *Figure 3-1*. This same setup was also used in the shear and anchorage tests only with different lengths of test slabs and different spans, anchorage lengths and load distances depending on which characteristic was being measured. These different values can be seen in *Table 3-1*.

Table 3-1: Different values used in the hydraulic jack setup depending on which characteristic being measured.

Type of Test	Sample Number	Total Length [mm]	Span [mm]	Load distance [mm]	Anchorage Length [mm]
Moment and Stiffness Tests	1	4200	4000	2000	100
	2	4200	4000	2000	100
Shear Tests	3	2700	2500	500	100
	4	2700	1985	500	350
Anchorage Test	5	2700	1995	500	40

As *Table 3-1* shows, decks of 4.2 m were used to measure the moment capacity and stiffness. Three lines of eight blocks (a total of 24 blocks) were used in these slabs. As each block is 50 cm, they take up 4 m which gives 10 cm of solid normal concrete at each end of the slab. For these tests the anchorage length was 10 cm, the span 4 m and the load was applied in the middle of the span.

The load applied was measured using both a digital pressure measurement device and a manual one. The deflections of the decks were also measured using both manual and digital measuring devices. There were some problems with the digital measurements of the load. The load measured by the digital equipment was much lower than from the manual measurements. After the tests a load cell that measured directly the load from the jacks was used to find the actual load and it confirmed that the manual measurements were correct.

Even so digital load-deflections had the expected shape. Therefore it was scaled up so that the maximum load was the same as for the manual measurements.

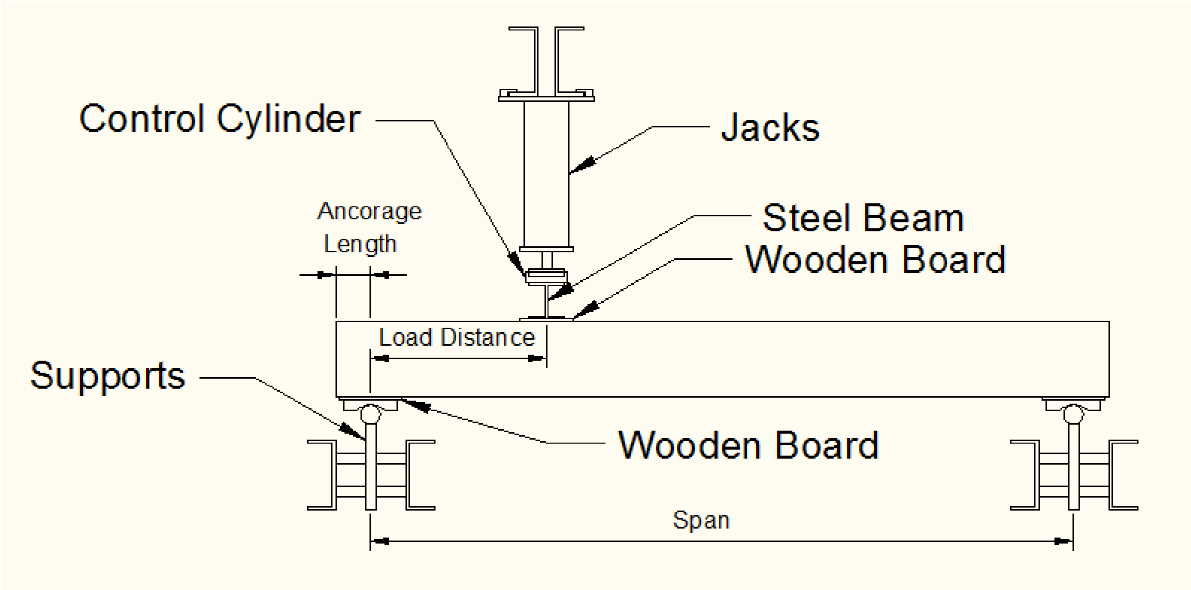

Figure 3-1: Setup of mechanical tests.

Figure 3-2: A roller support for the slabs.

Figure 3-3: The beam supporting the displacement indicators.

The slabs were supported at each end by a rolling support (*Figure 3-2*). Two jacks, each with 98 mm diameter and capacity of 250 kN were used to impose a load on an I-Beam placed on top of the slabs (*Figure 3-4*). Beneath the support, between the support and the slab and between the slab and the I-Beam a 10 mm thick soft wood was placed, to prevent any irregularities in the concrete to cause uneven distribution of forces. Two control discs were welded to the top of the I-Beam to keep the jacks from slipping on the surface of the beam and ensure the load was placed in the center of the beams.

Figure 3-4: The hydraulic jacks.

A steel SHS beam was placed on top of the SLD. It was supported by two U-beams standing on the SLD directly above the roller supports. Two digital and one manual displacement indicators were attached to the SHS beam close to the jacks to measure the deflections of the deck. Supporting them on the deck itself and not on the floor made sure that any vertical movement of the supports did not affect the deflection measurements.

The load on the deck was found measuring the pressure of the oil pumped into the hydraulic jacks. Both manual and digital measurements were taken. With the oil pump being manually controlled it was not possible to keep a constant load rate but the load was applied slowly going from zero to the ultimate load in about 20 minutes.

3.1.2 Results

As discussed in chapter 2 the SLDs were expected to withstand 125.9 kNm moment and the results of the tests confirm this as can be seen in *Table 3-1* Deck 1 broke when the moment reached 139.2 kNm and deck 2 at 140.7 kNm which is around 10% over the calculated value. It is not surprising that the values measured are somewhat higher than the calculated one. Even though no safety factors were used, the values used for the strength of the concrete and the steel (f_{ck} and f_{pk}) are characteristic 5% values which means that 95% of the samples should have larger strength.

Figure 3-5 shows the results of the bending test of deck 1. The blue line shows the results from the digital measurements. There were some problems with the digital measurements of the pressure. They showed much lower values than the manual measurements. After the tests the manual measurements were confirmed using a load cell which directly measured the force from the jacks.

Table 3-2: Ultimate moment load of the decks in the bending test.

	Calculated	**Deck 1**	**Deck 2**
Ultimate Moment Load	125.9 kNm	139.2 kNm	140.7 kNm
Percentage of Calculated value	-	110.5%	111.8%

Since the shape of the digital force-deflections curve seemed to be correct the load measurements were scaled up so that the curve had the same maximum load as was measured manually. To do that a factor of 2.63 was applied. The red curve in the graph shows the scaled results.

The green line shows the manual measurements. The manual displacement indicator only went up to 40 mm therefore the measurements stopped at that point. The purple line shows calculated linear elastic load-deflection curve using the calculated bending stiffness. And finally the black line shows the calculated ultimate load.

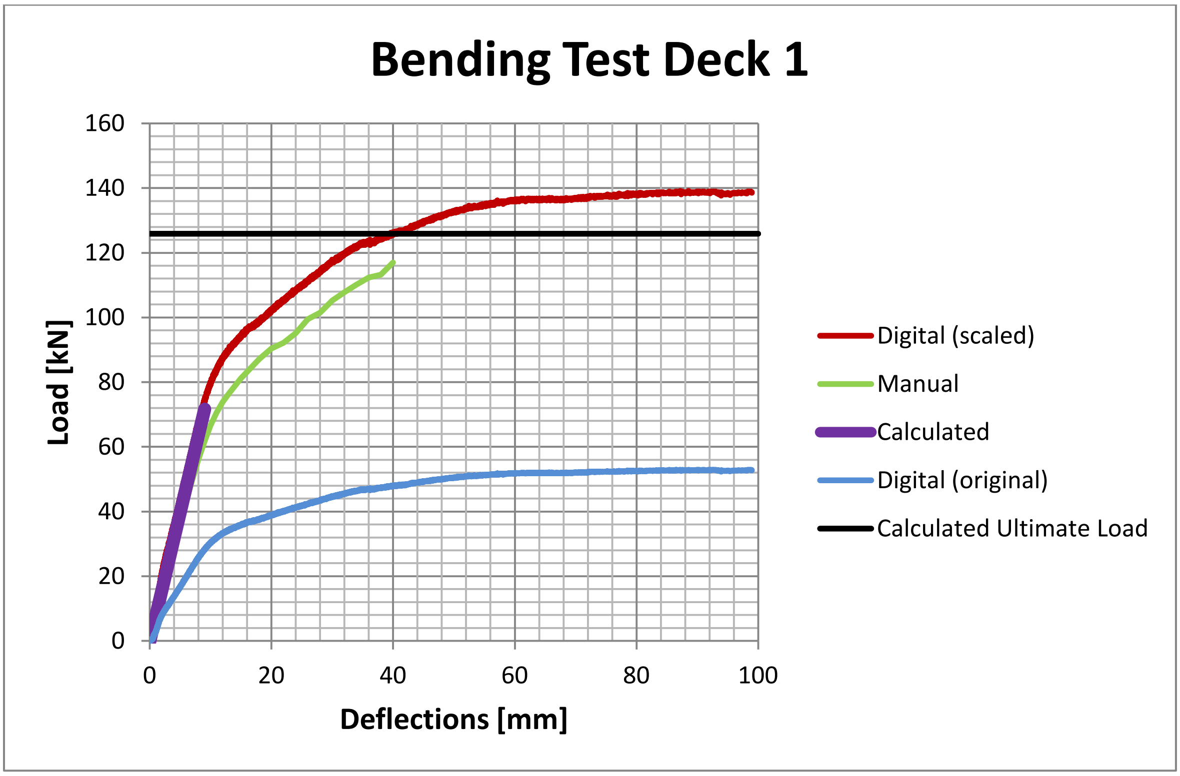

Figure 3-5: Results of bending test 1.

Figure 3-6 shows only the elastic range of the test. As before the purple line shows the calculated curve. The digital measurements show an unexpected behaviour at the beginning which is clearer in the test of deck 2 (*Figure 3-9*). It seems to show a low stiffness at the beginning and then it rises quite fast before going down again. (The stiffness is the slope of the curve). After this beginning phase the slope of the curve is very close to the calculated one. The slope of the manually measured curve is somewhat lower then of the other two.

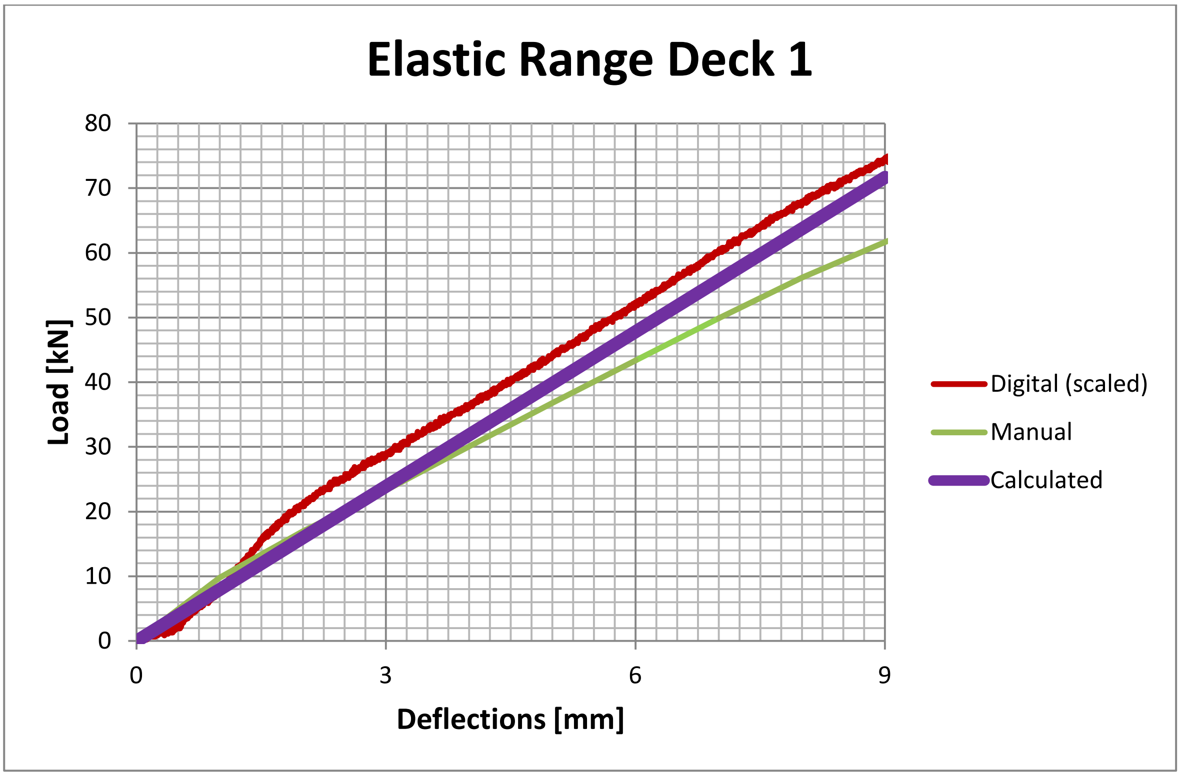

Figure 3-6: Elastic part of bending test 1.

Figure 3-7 shows the secant stiffness calculated from each measuring point of the bending test. That is, drawing a line from the point to the zero point and calculating the slope of that line. The digital curve shows the stiffness as very small right in the beginning and then grows fast before moving slowly down again. No reason was found for this low stiffness at the beginning of the test so it probably has something to do with the measuring instruments rather than being real phenomena.

The manual measurements show somewhat lower stiffness then the digital ones. Still after around 2 mm the curves have almost the same shape which indicates some systematic error in the measurements rather than one of them being totally wrong.

Figure 3-8, *Figure 3-9* and *Figure 3-10* show the results for the test of deck 2 comparable to the results already discussed for deck 1. The scaling factor for the digital load sensor was *2.49*.

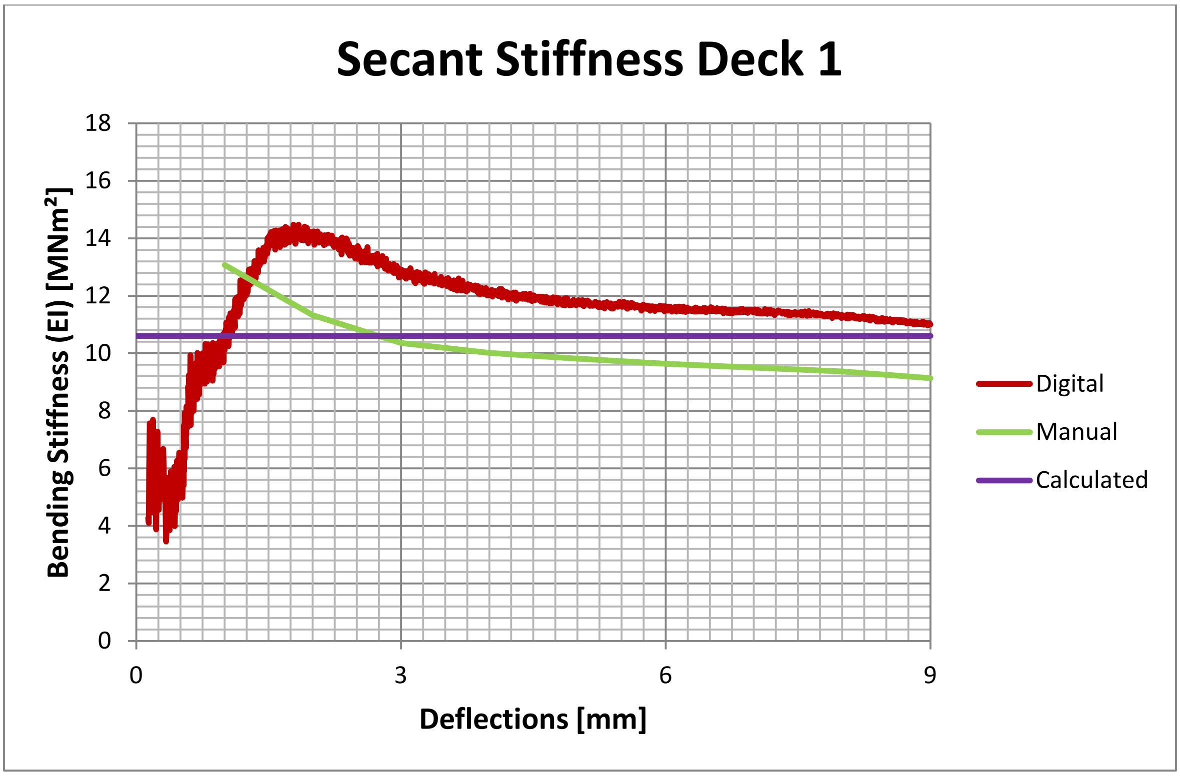

Figure 3-7: Stiffness versus deflection of the first deck.

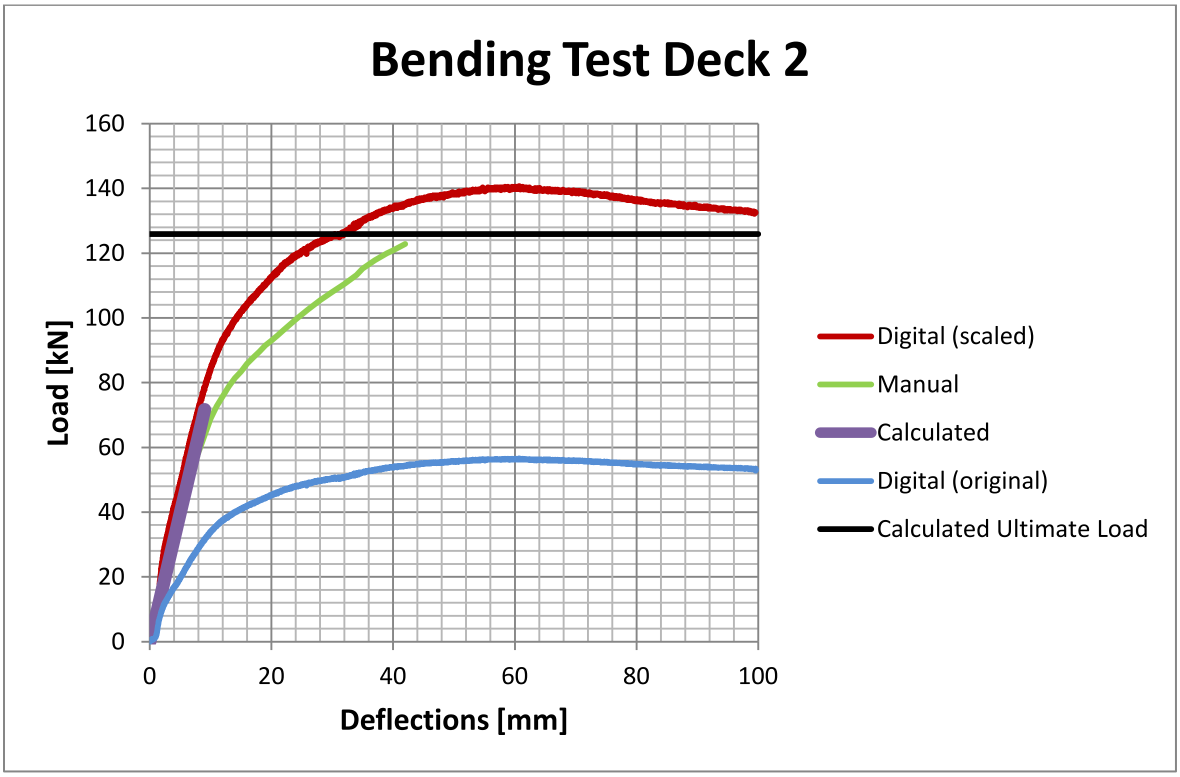

Figure 3-8: Results of bending test 2.

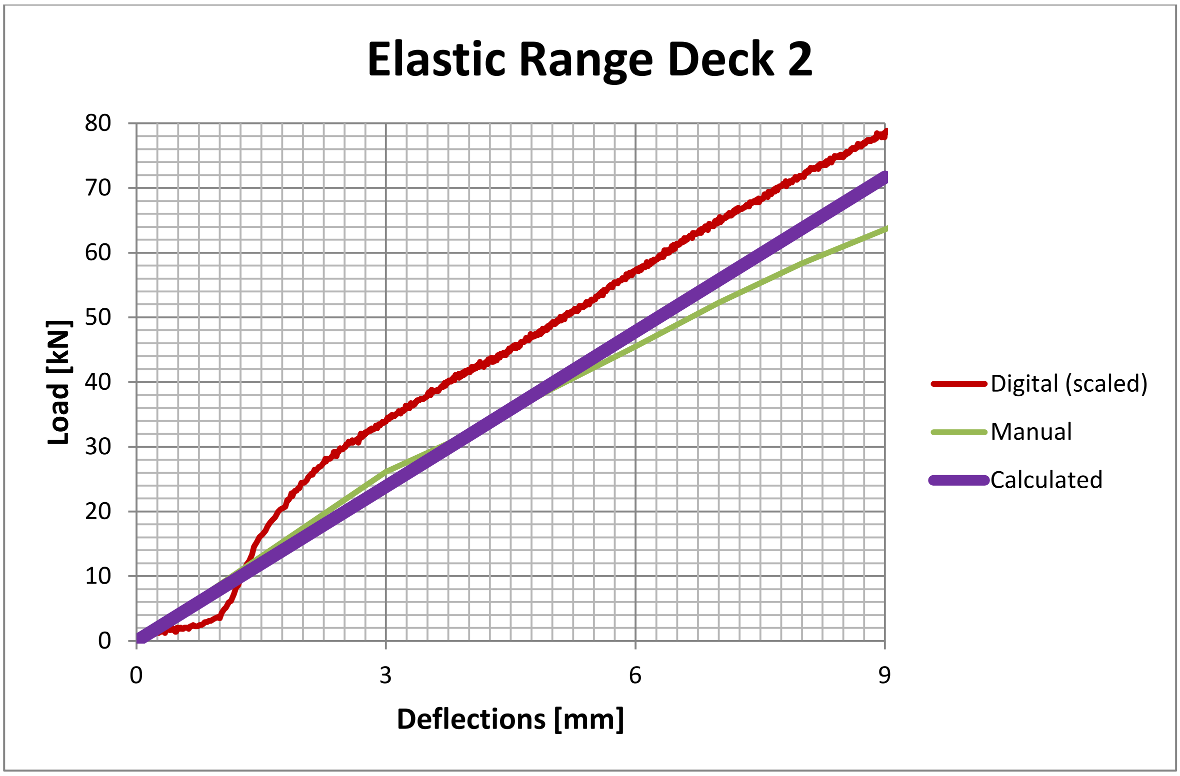

Figure 3-9: Elastic part of bending test 2.

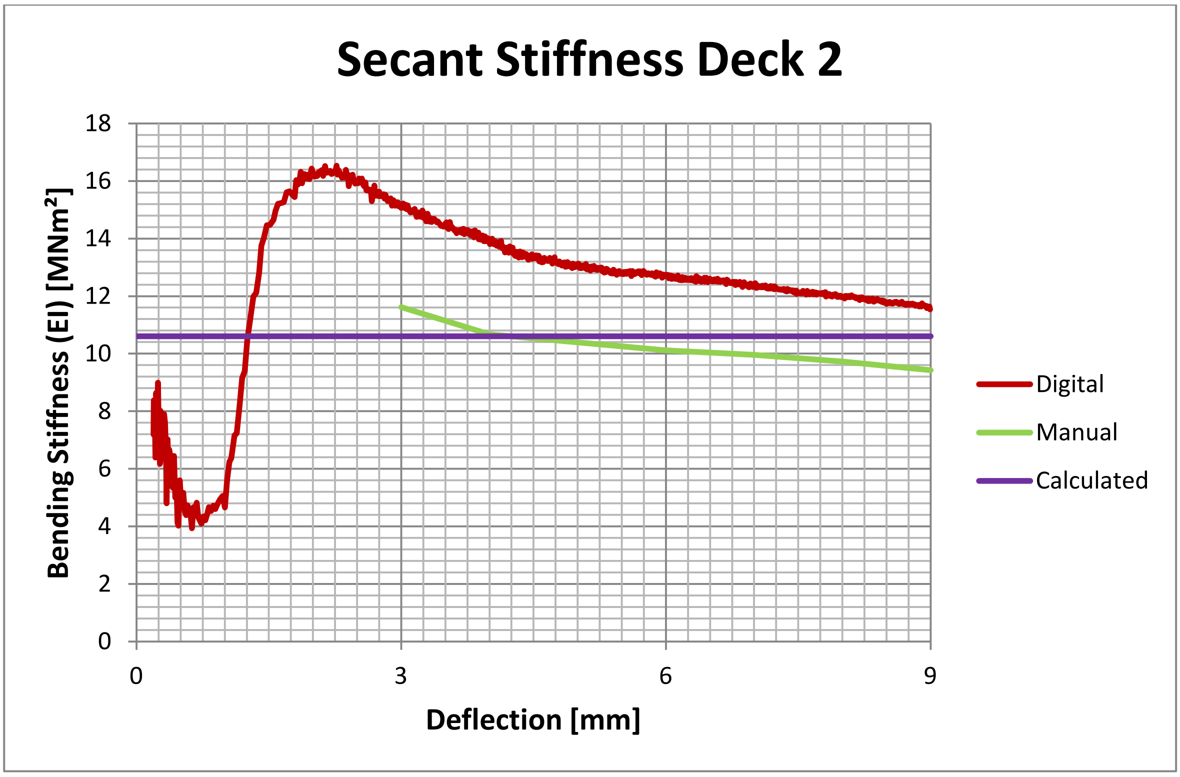

Figure 3-10: Stiffness versus deflection of the second deck.

3.2 Shear Resistance and Anchorage

3.2.1 Setup

The setup for the shear and anchorage tests was the same as for the bending test (*Figure 3-1*). Still different values for spans, anchorage lengths and load distances were used and the deflections were not measured since they are irrelevant to shear resistance.

A short span was used and the load applied close to one of the supports, to increase the ratio of shear load action to moment load, reducing the chances of the failure being a moment failure instead of a shear failure. To increase the shear force in the slab it is best to use a short load distance but due to the arch effect that causes the shear resistance to rise dramatically when getting close to the supports a minimum distance must be kept. According to Eurocode 2 rule 6.2.2(6) (CEN, 2004a) it is safe to count on this effect closer than 2d to the support (measured from the edge of the supporting plate to the edge of the steel beam distributing the load). That is of course on the safe side so a distance of 380 mm (the 500 mm load distance minus half of the supporting plate and half of the steel beam) was used, which is close to 2.8d.

In the first test a span of 2500 mm was used and an anchorage length of 100 mm. The measured shear resistance was lower than expected. Later it was found that the test had been stopped too early as is discussed in the results chapter.

At first the reason was thought to be that it had not been a proper shear failure so for the second test we shortened the span to 1,985 mm to avoid any moment failure and increased the anchorage length to 350 mm to avoid anchorage failure.

In the third and final test which was conducted two months later we used a very short anchorage length (40 mm).

3.2.2 Results

The results of the shear and anchorage tests can be seen in *Table 3-3*. The resistance of the first SLD is probably too low. When the pressure in the pump dropped somewhat, the test was stopped, since it was considered to be the maximum load the specimen could take. Which was shown to be incorrect in the next test. The shear load at that point was 147.3 kN.

In the second test the pressure also dropped at a similar load (133.6 kN) but afterwards pressure could be applied to a much higher level. This drop in pressure was due to the SLD starting to crack. The first test therefore does not show the full shear resistance of the deck.

The second test, shows the shear resistance of the cross-section as 263 kN. At that point the moment had reached the calculated moment resistance (see bending test) so the shear resistance might be a little higher.

In the last test the anchorage length was only 40 mm. The failure was therefore not a shear failure but a bond failure of the prestressing wires. The concrete cracked where the solid normal concrete met the part with the light concrete blocks and the prestressing wires had slid 20 mm in at the end which is the same as the width of the crack. The calculated moment needed to pull out the wires was 39.0 kNm but it took 66.0 kNm. This could mean that it is safe to use a higher bond factor than 0.25 to calculate the bond strength (*eq. 2-35*).

According to this test the bond strength was 856 N/(mm of wire). A bond factor of 0.436 would have given the right result.

Figure 3-11: Anchorage failure.

Table 3-3: Input values and results of Shear and anchorage tests.

	Deck 3	**Deck 4**	**Deck 5**
Span [mm]	2,500	1,985	1995
Anchorage length [mm]	100	350	40
Load Distance [mm]	500	485	500
Load from 100% Pressure [kN]*	431.2	431.2	572.0
Maximum pressure [%]*	42.7	80.7	30.8
Maximum Load from both Jacks [kN]	184.1	348.0	176.2
Max Shear Load [kN]	147.3	263.0	132.0
Max moment [kNm]	73.6	127.5	66.0

** The pump showed the pressure in a range from 0-100.*

3.3 Eigenfrequency and Damping

3.3.1 Setup

The SLD was placed on top of the same supports as in previous tests with a 3920 mm span. On the center of the slab an accelerometer (*Figure 3-12*) was placed. It was connected to a computer that recorded vertical acceleration at center of the span.

The accelerometer was attached to a magnet which was then placed on top of a small steel plate. The steel plate had three pointy screws going through it. These screws were the only parts that touched the concrete. That made sure the unit did not wiggle because of an uneven surface of the SLD.

Figure 3-12: The accelerometer on top of the SLD.

First the background noice was recorded to find the eigenfrequency. That is the acceleration from sounds and movement of the environment was measured.

Then a load of around 100 kg was hung in a rope from beneath the SLD and when the rope was cut the vibrations of the SLD were recorded. By studying how the vibrations die out it is possible to estimate the damping of the deck. This was repeated twice.

3.3.2 Results

Figure 3-13 shows the power spectral density of the vibrations measured. The highest peak shows the eigenfrequency of the subject. In this case it is 14.8 Hz.

The results do not fit with the expected value which was 17.6 Hz. Reversing the eigenfrequency equation to use the measured eigenfrequency to find the bending stiffness gives:

$$EI = \mu \left(\frac{f \cdot 2\pi \cdot l^2}{\pi^2} \right)^2 = 355 \frac{kg}{m} \left(\frac{14.8\,Hz \cdot 2\pi \cdot (3.92\,m)^2}{\pi^2} \right)^2 = 7.44 \cdot 10^7\,kNm^2 \qquad \text{(eq. 3-1)}$$

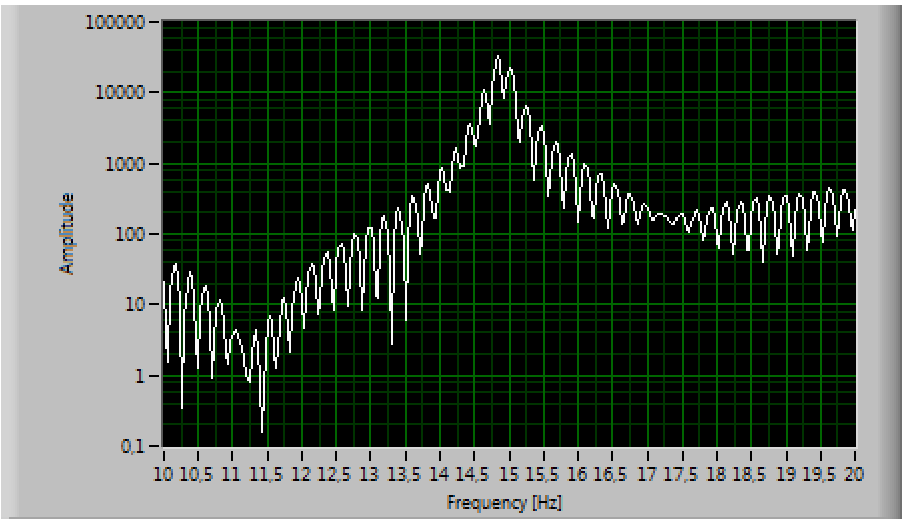

Figure 3-13: Power spectral density of the SLDs vibration.

But that does not fit with the calculated stiffness which was 10.6 MNm². The calculated value was confirmed in the stiffness test. A possible explanation for this is that due to the prestress the deck was cracked in the top. That would reduce the bending stiffness measured in the Eigenfrequency test but when the load was applied in the bending test the cracks closed and stayed in compression and therefore did not affect the stiffness in that test. If this explanation is correct it could be verified by repeating the test using a longer span so that the tensile stresses in the top of the deck would be overcome.

Table 3-4: Calculated and measured eigenfrequency of the SLD.

Calculated eigenfrequency	**Measured eigenfrequency**
17.6 Hz	14.8 Hz

Figure 3-14 show the acceleration time history of the deck after the rope has been cut to release the load. At first it is very irregular but after around half a second it reduces quite regularly.

Enlarging the regular part of the time history one gives a graph like the one in *Figure 3-15*. It shows a reduction in amplitude from 680 to 85 over 32 waves. putting these values into equation 2-50 gives a damping of:

$$\zeta = \frac{\frac{1}{j}\, ln \frac{u_1}{u_{j+1}}}{2\pi} = \frac{\frac{1}{32}\, ln \frac{680}{85}}{2\pi} = 0.010 = 1.0\% \qquad \text{(eq. 3-2)}$$

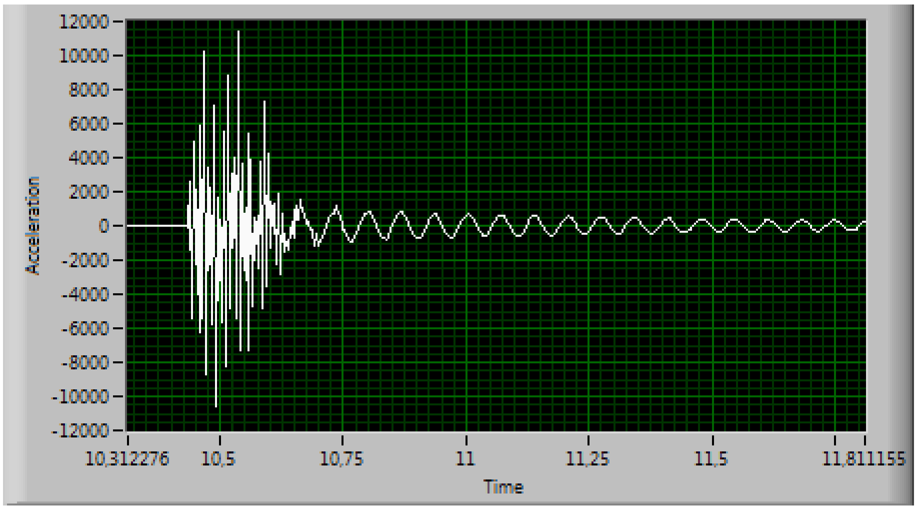

Figure 3-14: Time history of the first damping test.

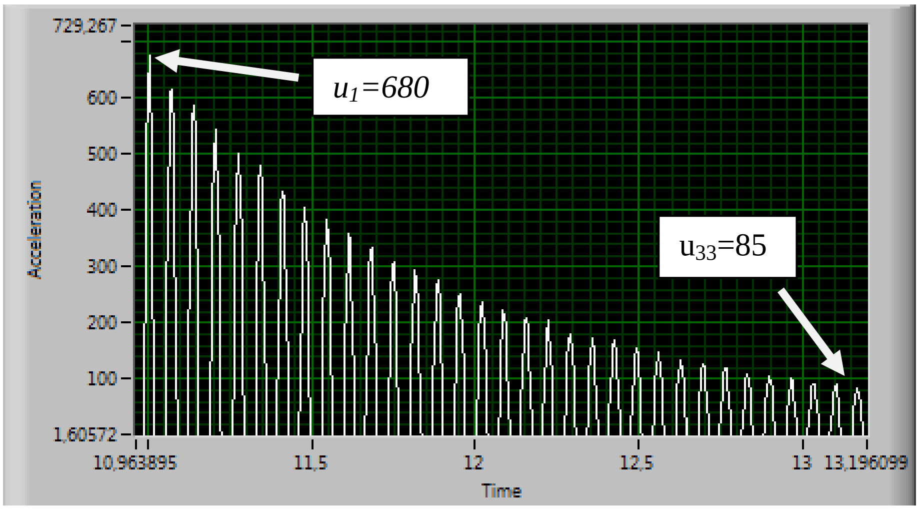

Figure 3-15: Amplitude of the acceleration of the deck.

3-5: Results of damping tests.

Test nr.	number of waves (j)	Amplitude of first wave (u_1)	Amplitude of last wave (u_{j+1})	Damping (ζ) [%]
1	32	680	85	1.0
2	24	258	60	1.0
3	26	590	100	1.1

3.4 Fire Resistance

3.4.1 Setup

Two decks of length 6,200 mm were used for the fire test. The SLDs were placed side by side in a special fire safe frame (*Figure 3-16*) with 100 mm at each end sitting on the frame meaning the span was 6,000 mm. Aerated concrete blocks were used to close the gaps that formed on either side of the decks (Figure *3-17*). Mineral wool was placed between all concrete surfaces so that hot air could not escape through and to make sure movements of the SLDs were not hindered. When the SLDs were cast thermometers were placed on one prestress strand in either deck. After the SLDs had been placed in the frame seven thermometers were placed on top of them and two underneath to follow the heat development of the decks.

Figure 3-16: Second slab placed in the fire safe frame.

Two steel beams were placed across the decks at the quarter points. Across these beams came a large beam and a hydraulic jack on top of that one (*Figure 3-20*). In the same picture the seven displacement indicators can be seen. The first four measured at the quarter points of each deck and the two at the center of the decks. The final one was place on the edge of one of the decks at midspan.

The frame was then lifted (*Figure 3-19*) onto the large gas furnace (*Figure 3-18*) and acted as a lid on top of it. The white things sticking out of the walls of the furnace are thermometers used to control the gas flow to the furnace so the desired temperature curve is followed.

Load was applied to the slabs that gave the same maximum moment as an equally distributed load of 2.5 kN/m². Which is the 1.5 kN/m² imposed load for domestic buildings suggested by the Eurocodes plus a 1.0 kN/m² for flooring and other dead loads apart from the weight of the SLD itself. That gives a maximum moment of:

$$M_{max} = \frac{q \cdot l^2}{8} = \frac{2.5 \; {}^{kN}/_{m^2} \cdot (6\;m)^2}{8} = \frac{11.25\;kNm}{m} \qquad \text{(eq. 3-3)}$$

Figure 3-17: Aerated concrete placed next to slabs to close the gap.

Figure 3-18: The gas furnace that heats up to over 1000 °C.

Figure 3-19: The frame being lifted onto the furnace.

Figure 3-20: Steel beams, jack and displacement indicators.

To get an equal maximum moment for the setup of this experiment the total applied load (Q) needs to be 15.0 kN/m:

$$M_{max} = \frac{Q}{2}\left(\frac{\frac{l}{4}\cdot\frac{3l}{4}}{l} + \frac{\frac{l}{4}\cdot\frac{l}{4}}{l}\right) = \frac{Ql}{8} = \frac{11.25\ kNm}{m} \quad \text{(eq. 3-4)}$$

The total width of the two slabs is 2.4 m which gives a total load of:

$$Q_{tot} = Q \cdot W = 15\frac{kN}{m} \cdot 2.4\ m = 36.0\ kN \quad \text{(eq. 3-5)}$$

The steel beams placed on top of the decks have a weight of 4.2 kN so the hydraulic jack provided the rest or 31.8 kN.

This load was kept constant for two hours while the furnace was heated to follow the standard fire curve:

$$T_g = 20 + 150 \cdot ln(8t + 1) \quad \text{(eq. 3-6)}$$

Where: T_g: Temperature at time t

t: time in minutes

After the furnace had been heated for two hours the load on the SLDs was increased, while the furnice was kept burning, with the rate of approximately 18 kN/min until the maximum capacity of the hydraulic jack was reached. Giving a total load of 249.2 kN (including the load of the steel beams).

3.4.2 Results

The steel temperature (*Figure 3-21*) rose faster for the first 40 minutes and slower after that than the ConFire program had predicted.

The two decks were made at the same time using the same concrete. The thermometers were placed identically on the steel in the decks. Still they record a temperature difference of close to 30 °C after two hours. It is therefore clear that the uncertainty in these calculations is quite big due to some unpredicted differences in the production. Still it seems to be a pretty good approximation especially since it is on the safe side as the fire progresses.

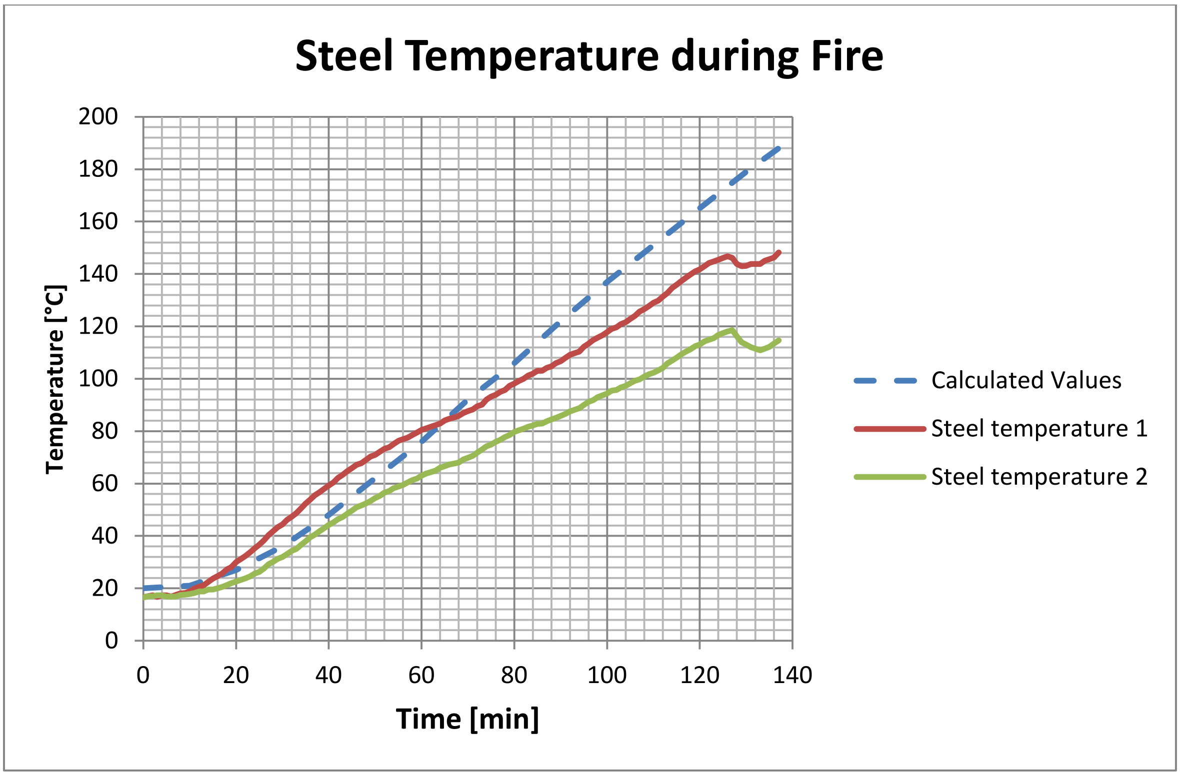

Figure 3-21: Steel temperature during fire.

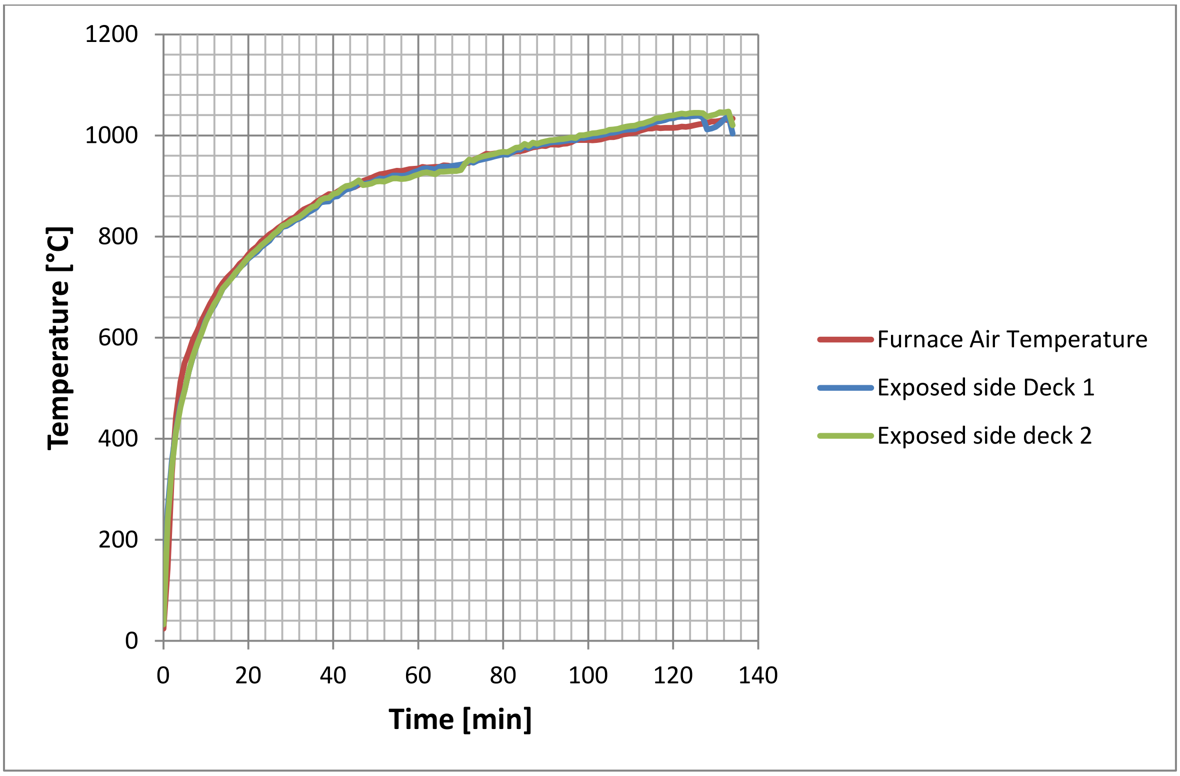

Figure 3-22: Temperature of furnace air and exposed side of decks.

Figure 3-22 shows the furnace temperature and the recordings of the two thermometers placed underneath the SLDs. It is no surprise that these three curves are almost identical. The temperature of the furnace follows the standard fire curve.

The deflections can be seen in *Figure 3-23*. There were seven displacement indicators placed on top of the decks. Four of them were placed at the quarterpoints and three in the center of the span. For clarity only two of these measurements are shown in the figure (one quarterpoint and one in the middle of the span) since the other ones were very similar. In appendix C the graph with all the results can be seen.

During the first 120 minutes the load was kept constant at 36.0 kN as described before. At first the deflections increase as the decks heats up but slow down almost to a halt in the last 30 minutes.

After 120 minutes the load was increased with a rate of 18 kN/min up to 249.2 kN with the furnace still burning. The deflections in the center went up to 200 mm but when the load was released returned to around 50 mm. *Figure 3-25* shows the deck under full load. No visible damage could be seen on the decks, neither from above nor below (the sides of the decks could not be seen).

The 249.2 kN load gives a maximum moment of 93.5 kNm in each deck. Adding that to the 15.7 kNm caused by the self weight of the deck gives 109.2 kNm which is higher then the expected 104 kNm.

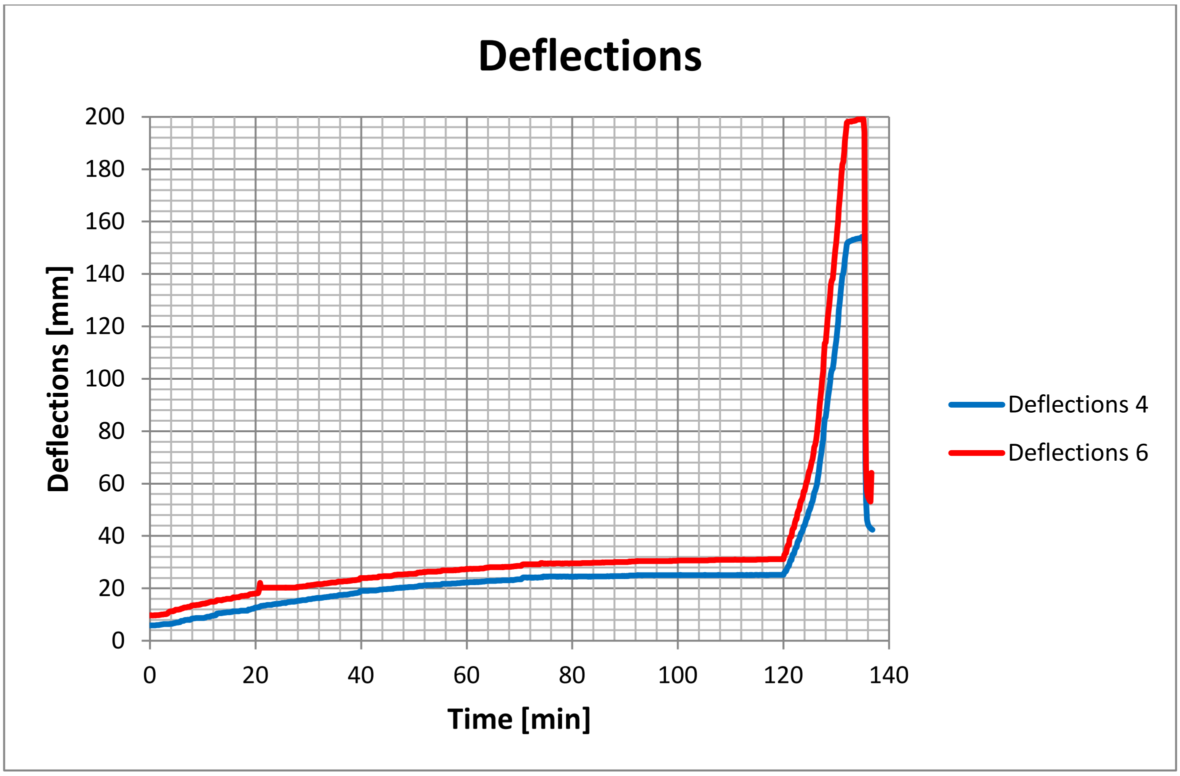

Figure 3-23: Deflections of the decks. Line 4 is at a quarterpoint but lines 6 in the center.

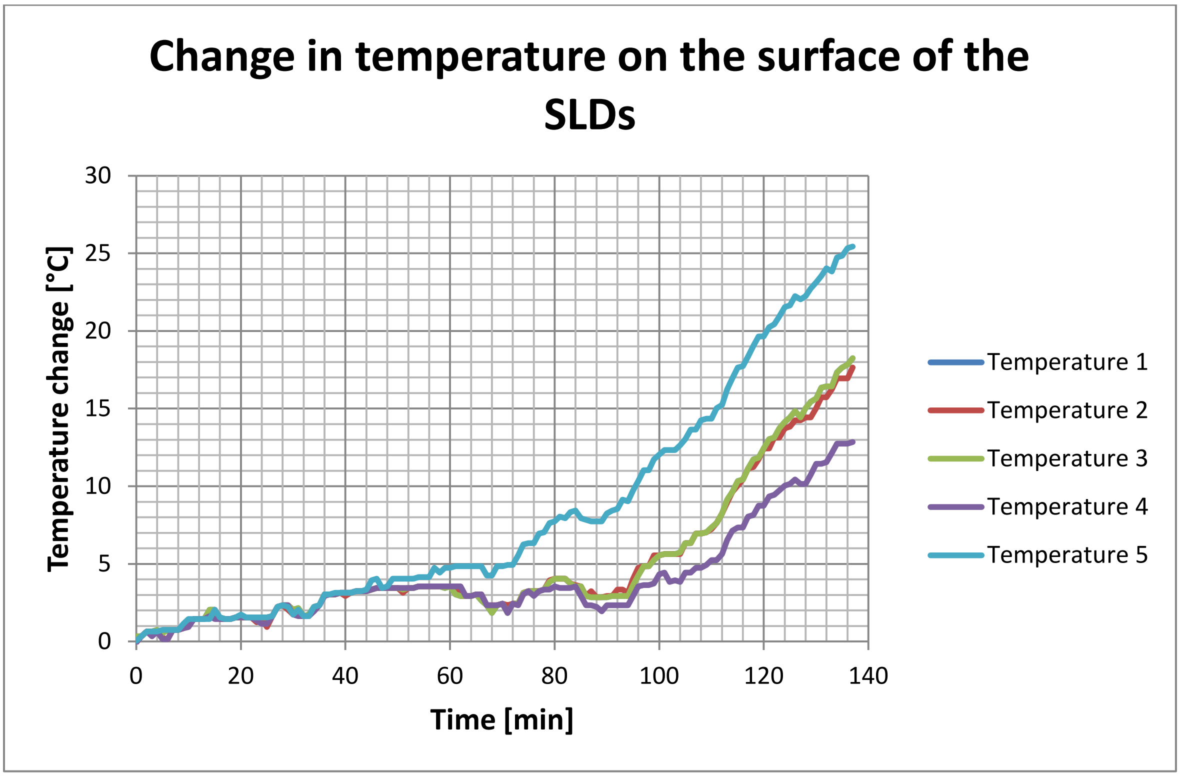

Figure 3-24: Change in temperature on the surface of the SLDs.

Only one of the thermometers on top of the decks showed more than 25°C rise in temperature (*Figure 3-24*) but the other ones show a maximum rise of less than 19°C. One could easily place ones hand on top of the decks without discomfort.

Figure 3-25: The decks under full load. Looking closely the large deflections can be seen.

4 Conclusions

The tests made on the SLD show that most of the calculations made were reasonable (See *Table 4-1*). All the calculations for characteristics regarding the safety of a building were on the safe side. Still the shear and anchorage were probably too far on the safe side as will be discussed further below.

The calculations of the eigenfrequency did not give the same results as the tests. A possible explanation for that is cracking in the top of the cross-section due to the prestress. Further testing using longer span would be required to confirm that.

Table 4-1: Expected Values and Test Results for all the Characteristics Measured.

Characteristic	**Expected value**	**Test Result**
Stiffness	10.6 MNm²	9.3-11.6 MNm²
Moment Bearing Capacity	125.9 kNm	139.2 kNm
Shear Resistance	146.0 kN	263.0 kN
Anchorage	491 N/(mm of wire)	856 N/(mm of wire)
Eigenfrequency	17.6 Hz	14.8 Hz
Damping	-	1.0-1.1%
Moment Resistance after 120 min Standard Fire	104 kNm	≥109.2 kNm

The bending stiffness was estimated to be 10.6 MNm². *Figure 3-7* and *Figure 3-10* show that this is a reasonable estimate. In both of the tests the Secant Stiffness measured digitally is somewhat higher than when measured manually. But the difference stays almost constant, as the stiffness reduces slowly with increased load, which suggest that there is some systematic error in the measurements. The Elastic Modulus of concrete is not constant as is assumed in the calculations but decreases with stress. That explains the reduction of the bending stiffness with increased load.

The Calculated Ultimate Moment Bearing Capacity is 125.9 kNm. The measured values were around 10% higher or 139.2 and 140.7 kNm. The difference is on the safe side and can probably be explained by the material strength (concrete and steel) being somewhat higher than the values used.

Using the method from Eurocode gives 146.0 kN Shear Resistance. The resistance measured was on the other hand 263.0 kN or 77% higher. The method is therefore very much on the safe side. The shear resistance is rarely the limiting factor when it comes to the design of concrete slabs so this is unlikely to cause an overdesign. Still it would have been better to calculate a value closer to the reality.

The anchorage was also a lot higher than the calculations suggested or 856 N/(mm of wire) instead of 491 N/(mm of wire). But that is due to a cautious choice of bond factor in *eq. 2-35*. (Hertz, 2005) suggests a bond factor of 0.25-0.41 and for caution 0.25 was chosen. The test results suggest that the bond factor should be 0.436. But since only one test was made one should be careful when deciding to use a higher factor but 0.3 or 0.35 would most likely be safe since in structure design safety factors add further to the safety of the structure.

The measured eigenfrequency was 14.8 Hz or considerably lower than the expected value of 17.6 Hz. As discussed before cracking in the top of the cross-section due to the prestress could explain this lower value from the tests. Further testing using a longer span could confirm this. No attempt was made to evaluate the damping of the deck but the measurements showed it was in the region of 1.0-1.1% of critical damping.

After being exposed to a Standard Fire for 120 minutes the deck could withstand a moment of at least 109.2 kNm. At that point the hydraulic jack applying the load had reached its maximum capacity but the deck was not visibly damaged. The expected value was 104 kNm. The expected value after four hours of fire is 71 kNm but that was not tested. After the two hours of fire the temperature at the surface of the SLDs had risen less than 25°C so the deck could be touched without getting burned.

The results of the tests performed suggest that it is safe to calculate the short term behavior of the SLD using the methods in this project. It has also been shown that the SLD has very good resistance to fire. Still the long term behavior of the decks might need further studying. For example what effects shrinkage of the light concrete might have.

5 Possible Uses

The SLD can be used in ways not possible with Hollow Core Slabs. The flexible fabrication of the SLD makes alterations to the design easy. For example one can leave a space between the rows of light concrete blocks and place a transverse beam anywhere in the deck. It would also be possible to leave out some of the blocks to fit installation such as lamps within the deck.

In this chapter two main design alterations will be discussed, fixed end decks and cantilevered decks.

5.1 Fixed end Deck

With fixed ends the maximum moment is reduced by one third compared to simply supported slabs. And instead of it being a positive moment in the center of the span it becomes a negative moment next to the supports. But the main advantage is that the maximum deflections go down to one fifth of what it would be for a similar simply supported deck. That makes longer spans possible.

The production of the SLDs does not limit the width of the decks. The limiting factor is the transportation. It has some complications to transport decks wider than 3 m. Since the light concrete blocks are 40 cm wide a deck of 280 cm width would be a good option.

Figure 5-1 shows one of the possibilities. A deck connected to the walls with steel bolts that are tightened when the deck has been placed (*Figure 5-2*). In the center where the two decks meet a vertical steel plate is cast into each deck and a steel bolt placed between them to connect the two decks so that they move together (*Figure 5-3*).

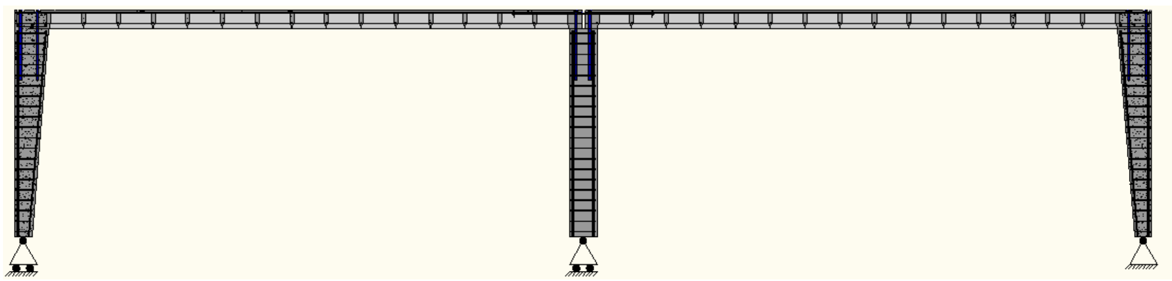

Figure 5-1: Fixed end deck.

Figure 5-2 shows how the fixed end connection is made. The precast walls come with steel bolts sticking out of the top. The deck has holes in it that these bolts fit into. Once the deck has been placed on top of the wall the bolts are tightened so that the decks is securely fastened to the wall. That way the deck and the wall move together and the moment from the deck is transferred down into the wall.

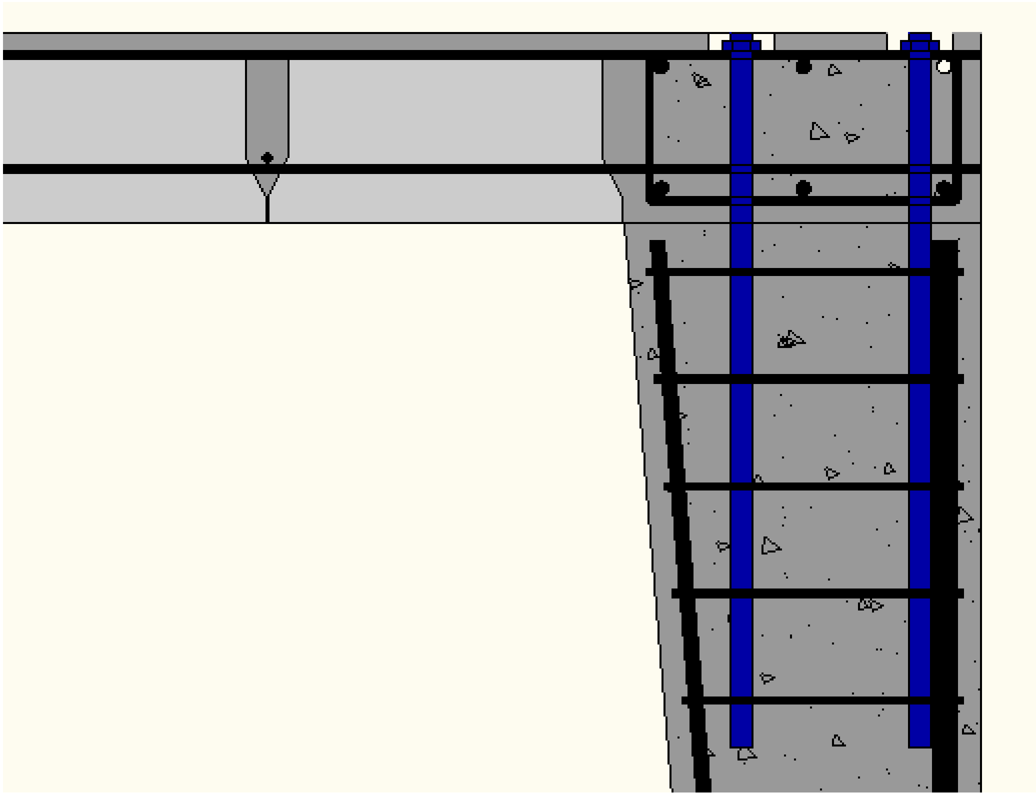

Figure 5-2: The fixed end connection.

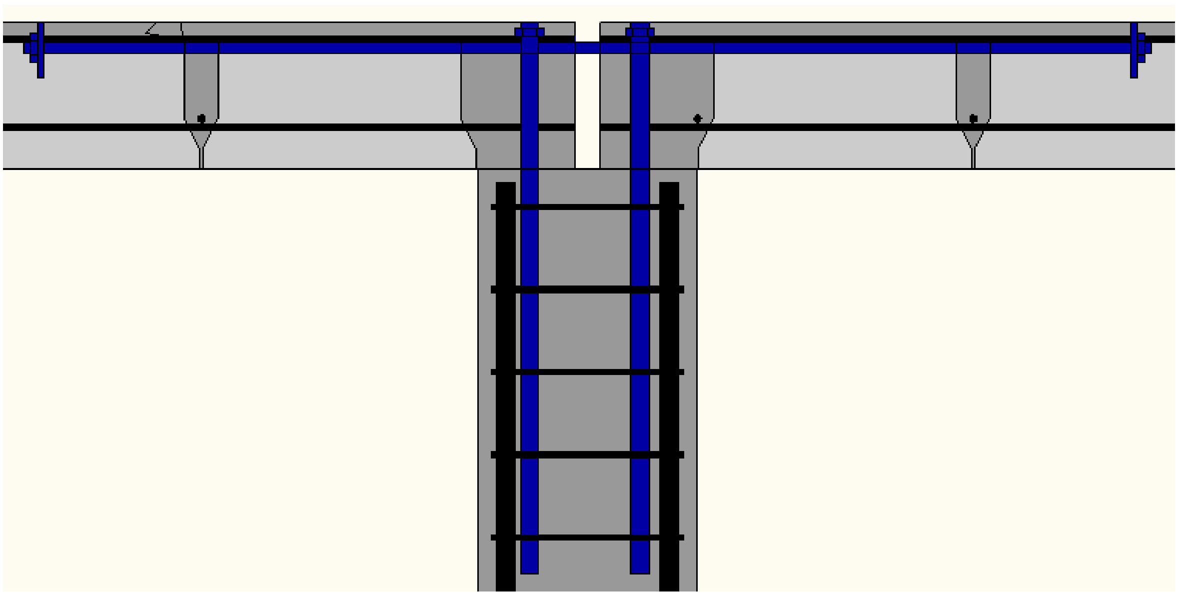

Figure 5-3: Two SLDs connected together over a wall or a column.

5.2 Cantilevered Deck

Cantilevered decks are another thing made possible with the SLD. Since it is possible to place prestressing wires in the top of the deck, the deck can take negative moments.

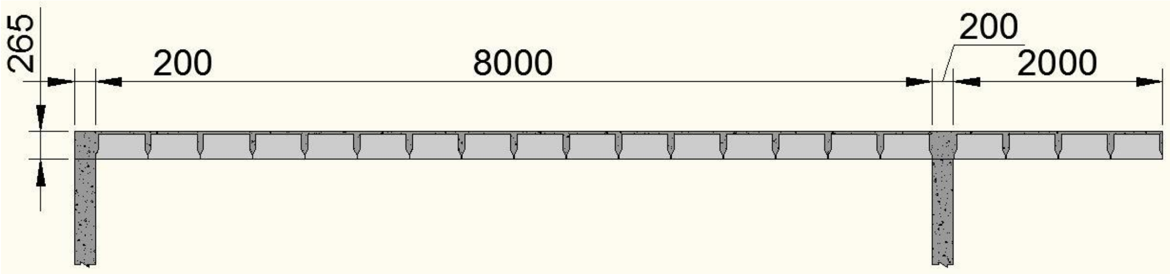

Figure 5-4: Cantilevered deck, side view.

The example taken here could be for a domestic building with balcony. Precast balconies are often quite troublesome to set up. But by using the cantilevered SLD both the balcony and the floor of the house are one unit and therefore placed at the same time. In *Figure 5-4* the deck spans 8 m of floor inside the house and has a 2 m balcony attached to it. It is possible to have longer spans and a bigger balcony but this could be a typical house.

In the design of this structure the following parameters were used:

- Own weight of the deck, 3.63 kN/m²
- Flooring and other own weight, 1 kN/m²
- Live load for domestic building according to EC, 2.0 kN/m²
- Live load for balconies according to EC, 2.5 kN/m²

The maximum moment of 87.2 kNm is 3.8 m from the left support and the maximum negative moment over the right support is -23.0 kNm. These moments are countered with 4 12.5 mm prestressing wires in the top of the deck and 6 in the bottom.

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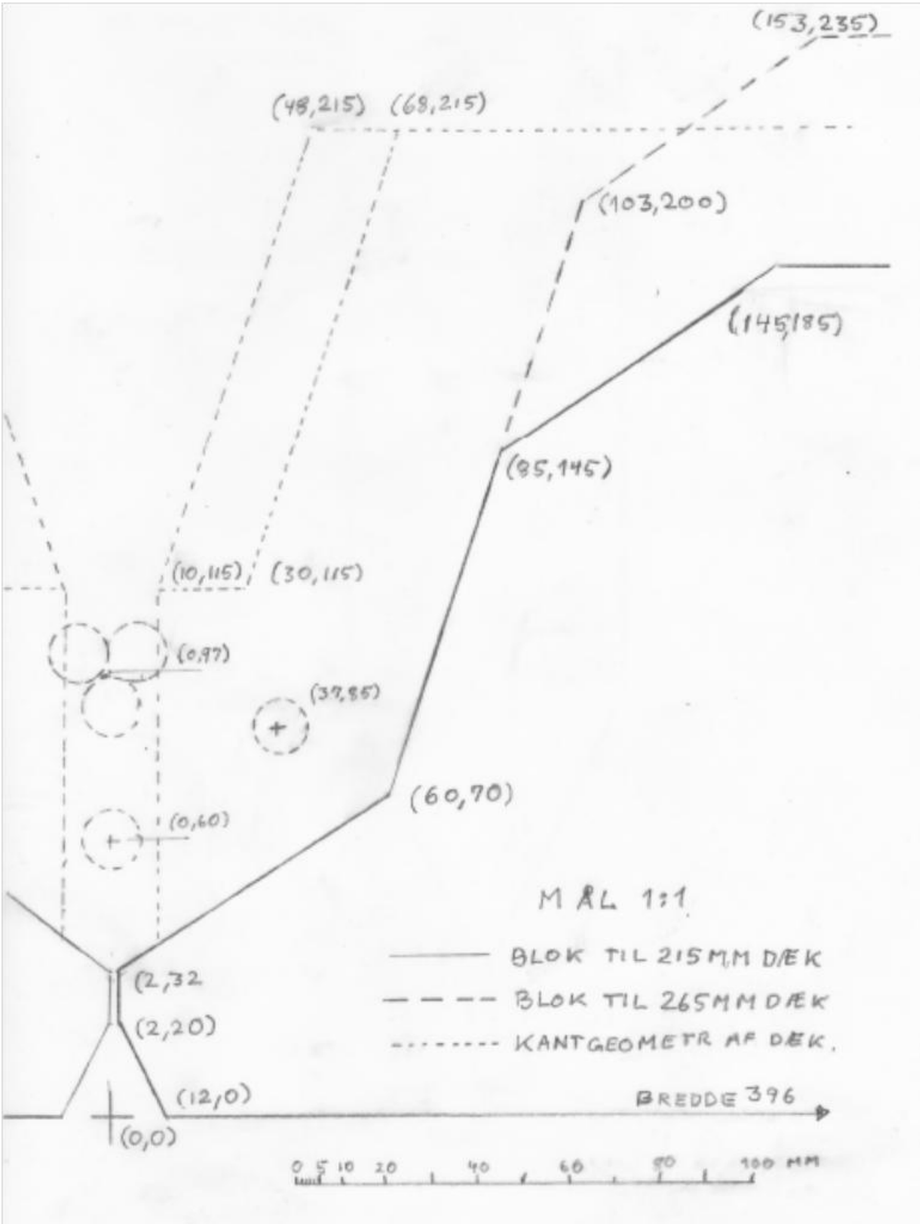
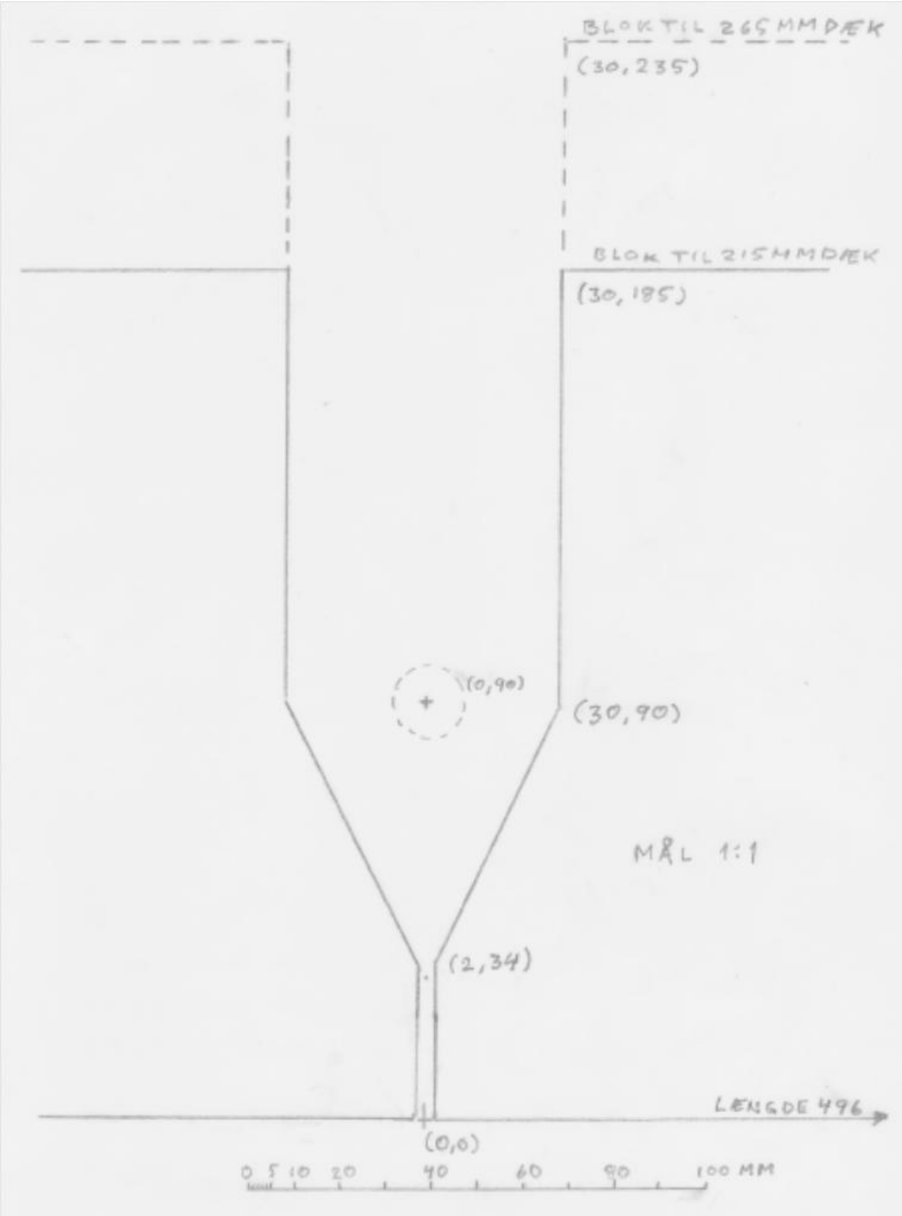
Appendix A – Dimensions of Blocks




BLOK TIL 265MM DÆK
(30,235)
BLOK TIL 215MM DÆK
(30,185)
(0,90)
(30,90)
MÅL 1:1
(2,34)
LÆNGDE 496
(0,0)
0 5 10 20 40 60 80 100 MM

Appendix B – Calculations

Material Properties:

Strong Concrete

Compressive Strength: $f_{ck} := 50 \cdot MPa$

Tensile Strength: $f_{ctk} := 2.9 \cdot MPa$

Safety factor: $\gamma_c := 1.0$

Design compressive strength: $f_{cd} := \frac{f_{ck}}{\gamma_c}$ $\quad f_{cd} = 50MPa$

Unit mass: $\rho_c := 2300\frac{kg}{m^3}$

Light Aggregate Concrete

Compressive Strength: $f_{ck1} := 3 \cdot MPa$

Tensile Strength $f_{ctk1} := 0.44 \cdot MPa$

Unit Mass: $\rho_{c.1} := 600\frac{kg}{m^3}$

Volume per block: $V_b := 23813cm^3$

Mass per block: $m_b := V_b \cdot \rho_{c.1}$ $\quad m_b = 14.29kg$

Prestressing Steel

Strength of Prestress Steel: $f_p := 1860 \cdot MPa$

Safety factor for Prestress: $\gamma_p := 1.0$

Design Strength of Prestress Steel: $f_{pd} := \frac{f_p}{\gamma_p}$ $\quad f_{pd} = 1860MPa$

Area of Prestress Strands: $A_p := 93mm^2$

Capacity of one Strand: $F_{pd} := f_{pd} \cdot A_p$ $\quad F_{pd} = 172.98kN$

Unit mass of steel: $\rho_s := 7850\frac{kg}{m^3}$

Strand linear mass density: $\alpha_p := A_p \cdot \rho_s$ $\quad \alpha_p = 0.73\frac{kg}{m}$

Transverse rebars (Y6)

Strength: $f_{yk} := 550 \cdot 10^6 Pa$

Safety factor: $\gamma_s := 1.0$

Design Strength: $f_{yd} := \frac{f_{yk}}{\gamma_s}$ $f_{yd} = 550MPa$

Area of bars: $A_s := (3mm)^2 \cdot \pi$ $A_s = 28.3\,mm^2$

Linear Mass Density: $\alpha_s := A_s \cdot \rho_s$

Dimensions of Slab $\alpha_s = 0.222 \frac{kg}{m}$

Length of Slab: $L_s := 4196mm$

Span of Slab: $L := 4000mm$

Height of Slab: $H := 215mm$

Width of Slab: $W := 1200mm$

Number of prestress strands $n_p := 6$

Height of Inner Strands: $h_i := 75mm$

Height of Outer Strands: $h_o := 85mm$

Average Height of Strands: $h_{av} := \frac{4 \cdot h_i + 2 \cdot h_o}{6}$ $h_{av} = 78.3\,mm$

Number of light aggregate blocks per m in slab: $n_b := 6$

Number of transverse bars per m in slab: $n_s := 2$

Cross Sectional Area of Slab: $A_{slab} := H \cdot W$

Volume of Strong Concrete per m: $V_c := \frac{\left[A_{slab} \cdot 1 \cdot m - \left(n_b \cdot V_b\right)\right]}{1m}$ $V_c = 0.115m^2$

Mass per m: $\alpha_{slab} := \frac{\left[V_c \cdot 1 \cdot m \cdot \rho_c + n_b \cdot V_b \cdot \rho_{c.1} + n_p \cdot 1 \cdot m \cdot \alpha_p + n_s \cdot \alpha_s \cdot (W - 60 \cdot mm)\right]}{1m}$

$\alpha_{slab} = 355.394 \frac{kg}{m}$

Moment Bearing Capacity

Dead Load: $G := \alpha_{slab} \cdot g$ $G = 3.485 \frac{kN}{m}$

Maximum Moment from Dead Load:

$$M_{max.dead} := \frac{G \cdot L^2}{8} \qquad M_{max.dead} = 6.97 kN \cdot m$$

Total Capacity of Strands: $F_{pdt} := F_{pd} \cdot n_p \qquad F_{pdt} = 1.038 \times 10^3 kN$

Height of Compression zone: $y := \dfrac{F_{pdt}}{f_{cd} \cdot W} \qquad y = 0.017 m$

Moment Bearing Capacity: $M_R := \left(H - \dfrac{y}{2} - h_{av}\right) \cdot F_{pdt}$

$M_R = 132.867 kN \cdot m$

Moment Capacity - Own Weight:

$$M_{R.Q} := M_R - M_{max.dead} \qquad M_{R.Q} = 125.9 kN \cdot m$$

Maximum Load Capacity:

$$Q_{max} := \frac{4M_{R.Q}}{L} \qquad Q_{max} = 125.9 kN$$

Shear Capacity

$d := H - h_{av}$

$d = 0.137 m$

According to EC

$$k := 1 + \sqrt{\frac{200}{d \cdot \frac{1000}{\mathrm{UnitsOf}(d)}}} \qquad k := \mathrm{if}(k > 2, 2, k)$$

$k = 2$

$$\rho_1 := A_p \cdot \frac{n_p}{0.36 m \cdot d} \qquad \rho_1 := \mathrm{if}(\rho_1 > 0.02, 0.02, \rho_1)$$

$\rho_1 = 0.011$

$N_E := F_{pdt} \cdot 0.75$

$$\sigma_{cp} := \frac{N_E}{W \cdot H} \qquad \sigma_{cp} = 3.017 \times 10^6 Pa$$

$$V_{R.EC} := \left[0.18 \cdot k \cdot \left(100 \cdot \rho_1 \cdot \frac{f_{ck}}{10^6 \cdot \mathrm{UnitsOf}(f_{ck})}\right)^{\frac{1}{3}} \cdot Pa + 0.15 \cdot \frac{\sigma_{cp}}{10^6}\right] \cdot 0.36 m \cdot d \cdot 10^6$$

$V_{R.EC}$ 90.314kN

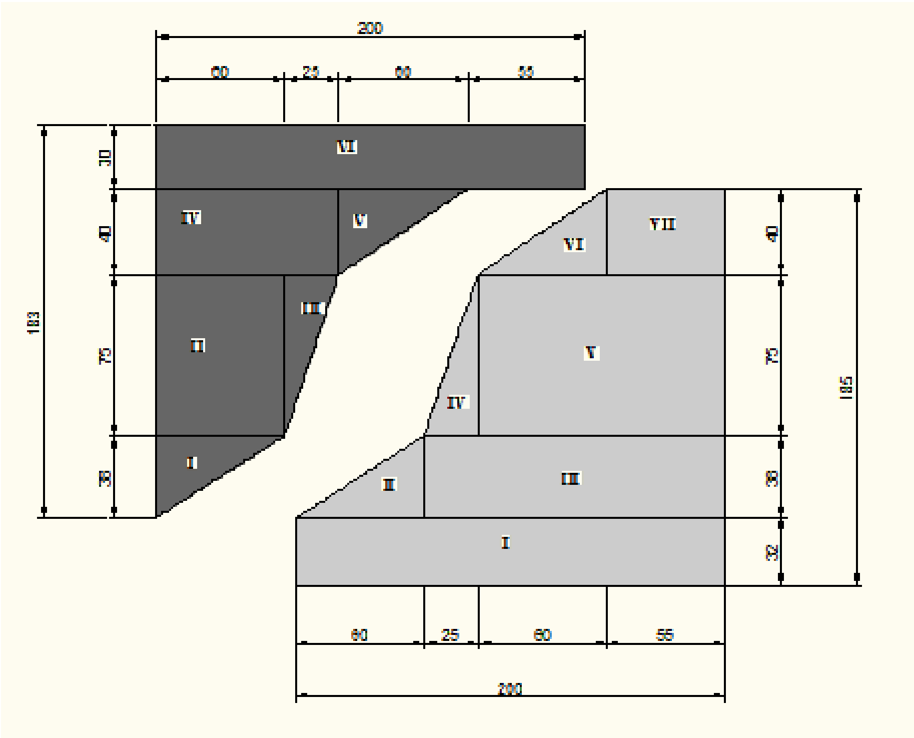
$A_{z.strong} := 118728mm^2$

$A_{z.light} := 70872mm^2$

$V_{R.Kristian} := \left(A_{z.strong} \cdot f_{ctk} + A_{z.light} \cdot f_{ctk1}\right) \cdot \frac{2}{3}$ $\qquad V_{R.Kristian} = 250.33kN$

$A_{z.strong} + A_{z.light} = 0.19m^2$

Moment of inertia



Vertical lengths:

$l_{V1} := 0mm \quad l_{V2} := 38mm \quad l_{V3} := 75mm \quad l_{V4} := 40mm \quad l_{V5} := 30mm$

$l_{V.tot} := l_{V1} + l_{V2} + l_{V3} + l_{V4} + l_{V5} \quad l_{V.tot} = 0.183m$

Horizontal lengths:

$l_{H1} := 60mm \quad l_{H2} := 25mm \quad l_{H3} := 60mm \quad l_{H4} := 55mm$

Normal Concrete:

Area: I

$$A_{S1} := \frac{l_{V2} \cdot l_{H1}}{2} \qquad A_{S1} = 1.14 \times 10^3\,mm^2$$

$$I_{S1} := \frac{l_{H1} \cdot l_{V2}^3}{36} \qquad I_{S1} = 9.145 \times 10^4\,mm^4$$

$$z_{S1} := l_{V1} + l_{V2} \cdot \frac{2}{3} \qquad z_{S1} = 25.333mm$$

II

$$A_{S2} := l_{V3} \cdot l_{H1} \qquad A_{S2} = 4.5 \times 10^3\,mm^2$$

$$I_{S2} := \frac{l_{H1} \cdot l_{V3}^3}{12} \qquad I_{S2} = 2.109 \times 10^6\,mm^4$$

$$z_{S2} := l_{V1} + l_{V2} + \frac{l_{V3}}{2} \qquad z_{S2} = 75.5mm$$

III

$$A_{S3} := \frac{l_{H2} \cdot l_{V3}}{2} \qquad A_{S3} = 937.5mm^2$$

$$I_{S3} := \frac{l_{H2} \cdot l_{V3}^3}{36} \qquad I_{S3} = 2.93 \times 10^5\,mm^4$$

$$z_{S3} := l_{V1} + l_{V2} + l_{V3} \cdot \frac{2}{3} \qquad z_{S3} = 88mm$$

IV

$$A_{S4} := (l_{H1} + l_{H2}) \cdot l_{V4} \qquad A_{S4} = 3.4 \times 10^3\,mm^2$$

$$I_{S4} := \frac{(l_{H1} + l_{H2}) \cdot l_{V4}^3}{12} \qquad I_{S4} = 4.533 \times 10^5\,mm^4$$

$$z_{S4} := l_{V1} + l_{V2} + l_{V3} + \frac{l_{V4}}{2} \qquad z_{S4} = 133mm$$

V

$$A_{S5} := \frac{l_{H3} \cdot l_{V4}}{2} \qquad A_{S5} = 1.2 \times 10^3\,mm^2$$

$$I_{S5} := \frac{l_{H3} \cdot l_{V4}^3}{36} \qquad I_{S5} = 1.067 \times 10^5\,mm^4$$

$$z_{S5} := l_{V1} + l_{V2} + l_{V3} + l_{V4} \cdot \frac{2}{3} \qquad z_{S5} = 139.667mm$$

VI $A_{S6} := (l_{H1} + l_{H2} + l_{H3} + l_{H4}) \cdot l_{V5}$ $A_{S6} = 6 \times 10^3\,mm^2$

$$I_{S6} := \frac{(l_{H1} + l_{H2} + l_{H3} + l_{H4}) \cdot l_{V5}^3}{12} \qquad I_{S6} = 4.5 \times 10^5\,mm^4$$

$$z_{S6} := l_{V1} + l_{V2} + l_{V3} + l_{V4} + \frac{l_{V5}}{2} \qquad z_{S6} = 168\,mm$$

$$A_{c.strong} := 6 \cdot (A_{S1} + A_{S2} + A_{S3} + A_{S4} + A_{S5} + A_{S6})$$

$$A_{c.strong} = 1.031 \times 10^5\,mm^2$$

$$z_{c.strong} := \frac{6 \cdot (A_{S1} \cdot z_{S1} + A_{S2} \cdot z_{S2} + A_{S3} \cdot z_{S3} + A_{S4} \cdot z_{S4} + A_{S5} \cdot z_{S5} + A_{S6} \cdot z_{S6})}{A_{c.strong}}$$

$z_{c.strong} = 121.026\,mm$ $z_{c.strong.top} := l_{V.tot} - z_{c.strong}$ $z_{c.strong.top} = 0.062\,m$

$$I_{c.strong} := 6 \left[I_{S1} + A_{S1} \cdot (z_{c.strong} - z_{S1})^2 + I_{S2} + A_{S2} \cdot (z_{c.strong} - z_{S2})^2 + I_{S3} + A_{S3} \cdot (z_{c.st} \right.$$

$$I_{c.strong} = 2.306 \times 10^8\,mm^4$$

$$I_{c.strong.1} := \frac{I_{c.strong}}{3} \qquad I_{c.strong.1} = 7.687 \times 10^{-5}\,m^4$$

Light Aggregate Concrete

Area: I $A_{L1} := (l_{H1} + l_{H2} + l_{H3} + l_{H4}) \cdot l_{V1}$ $A_{L1} = 0\,mm^2$

$$I_{L1} := \frac{(l_{H1} + l_{H2} + l_{H3} + l_{H4}) \cdot l_{V1}^3}{12} \qquad I_{L1} = 0\,mm^4$$

$$z_{L1} := \frac{l_{V1}}{2} \qquad z_{L1} = 0\,mm$$

II $A_{L2} := \frac{l_{H1} \cdot l_{V2}}{2}$ $A_{L2} = 1.14 \times 10^3\,mm^2$

$$I_{L2} := \frac{l_{H1} \cdot l_{V2}^3}{36} \qquad I_{L2} = 9.145 \times 10^4\,mm^4$$

$$z_{L2} := l_{V1} + \frac{l_{V2}}{3} \qquad z_{L2} = 12.667\,mm$$

III $A_{L3} := (l_{H2} + l_{H3} + l_{H4}) \cdot l_{V2}$ $\quad A_{L3} = 5.32 \times 10^3\,mm^2$

$$I_{L3} := \frac{(l_{H2} + l_{H3} + l_{H4}) \cdot l_{V2}^3}{12} \qquad I_{L3} = 6.402 \times 10^5\,mm^4$$

$$z_{L3} := l_{V1} + \frac{l_{V2}}{2} \qquad z_{L3} = 19\,mm$$

IV $A_{L4} := \frac{l_{H2} \cdot l_{V3}}{2}$ $\quad A_{L4} = 937.5\,mm^2$

$$I_{L4} := \frac{l_{H2} \cdot l_{V3}^3}{36} \qquad I_{L4} = 2.93 \times 10^5\,mm^4$$

$$z_{L4} := l_{V1} + l_{V2} + \frac{l_{V3}}{3} \qquad z_{L4} = 63\,mm$$

V $A_{L5} := (l_{H3} + l_{H4}) \cdot l_{V3}$ $\quad A_{L5} = 8.625 \times 10^3\,mm^2$

$$I_{L5} := \frac{(l_{H3} + l_{H4}) \cdot l_{V3}^3}{12} \qquad I_{L5} = 4.043 \times 10^6\,mm^4$$

$$z_{L5} := l_{V1} + l_{V2} + \frac{l_{V3}}{2} \qquad z_{L5} = 75.5\,mm$$

VI $A_{L6} := \frac{l_{H3} \cdot l_{V4}}{2}$ $\quad A_{L6} = 1.2 \times 10^3\,mm^2$

$$I_{L6} := \frac{l_{H3} \cdot l_{V4}^3}{36} \qquad I_{L6} = 1.067 \times 10^5\,mm^4$$

$$z_{L6} := l_{V1} + l_{V2} + l_{V3} + \frac{l_{V4}}{3} \qquad z_{L6} = 126.333\,mm$$

VII $A_{L7} := l_{H4} \cdot l_{V4}$ $\quad A_{L7} = 2.2 \times 10^3\,mm^2$

$$I_{L7} := \frac{l_{H4} \cdot l_{V4}^3}{12} \qquad I_{L7} = 2.933 \times 10^5\,mm^4$$

$$z_{L7} := l_{V1} + l_{V2} + l_{V3} + \frac{l_{V4}}{2} \qquad z_{L7} = 133\,mm$$

$$A_{c.light} := 6 \cdot (A_{L1} + A_{L2} + A_{L3} + A_{L4} + A_{L5} + A_{L6} + A_{L7})$$

$$A_{c.light} = 1.165 \times 10^5\,mm^2 \qquad A_{c.light.1} := \frac{A_{c.light}}{3} \qquad A_{c.light.1} = 0.03884\,m^2$$

$$z_{c.light} := \frac{6 \cdot (A_{L1} \cdot z_{L1} + A_{L2} \cdot z_{L2} + A_{L3} \cdot z_{L3} + A_{L4} \cdot z_{L4} + A_{L5} \cdot z_{L5} + A_{L6} \cdot z_{L6} + A_{L7} \cdot z_{L7})}{A_{c.light}}$$

$z_{c.light} = 65.387mm$ $\qquad z_{c.light.top} := t_{V.tot} - z_{c.light}$ $\qquad z_{c.light.top} = 0.1176m$

$I_{c.light} := 6\left[I_{L1} + A_{L1}\left(z_{c.light} - z_{L1}\right)^2 + I_{L2} + A_{L2}\left(z_{c.light} - z_{L2}\right)^2 + I_{L3} + A_{L3}\left(z_{c.light} - z_{L}\right.\right.$

$I_{c.light} = 2.129 \times 10^8\, mm^4$

Transformation of material

$E_{cm.strong} := 37GPa$ $\qquad E_{cm.light} := 3GPa$ $\qquad E_p := 190GPa$

$$n_{light} := \frac{E_{cm.light}}{E_{cm.strong}} \qquad n_{light} = 0.081$$

$$n_{prestress} := \frac{E_p}{E_{cm.strong}} \qquad n_{prestress} = 5.135$$

$$I_b := \frac{1200\, mm \left(t_{V.tot}\right)^3}{12} \qquad I_b = 6.128 \times 10^8\, mm^4$$

$A_b := 1200mm \cdot t_{V.tot}$ $\qquad A_b = 2.196 \times 10^5\, mm^2$

$$z_{bb} := \frac{t_{V.tot}}{2} \qquad z_{bb} = 91.5\, mm$$

$h_{av.32} := h_{av} - 32mm$ $\qquad h_{av.32} = 46.333mm$

Ideal Area

$A_i := A_b + \left(n_{prestress} - 1\right)\cdot A_p \cdot n_p + \left(n_{light} - 1\right)\cdot A_{c.light}$ $\qquad A_i = 1.148 \times 10^5\, mm^2$

Ideal Center of Gravity

$$z_{ib} := \frac{A_b \cdot z_{bb} + \left(n_{prestress} - 1\right)\cdot A_p \cdot n_p \cdot h_{av.32} + \left(n_{light} - 1\right)\cdot A_{c.light} \cdot z_{c.light}}{A_i} \qquad z_{ib} = 114.947mm$$

Ideal Moment of Inertia

$I_i := I_b + A_b\left(z_{bb} - z_{ib}\right)^2 + \left(n_{prestress} - 1\right)\cdot A_p \cdot n_p\left(z_{ib} - h_{av.32}\right)^2 + \left(n_{light} - 1\right)\cdot\left[I_{c.light} + \left[A_{c.ligh}\right.\right.$

$I_i = 2.858 \times 10^8\, mm^4$

$z_{it} := t_{V.tot} - z_{ib}$ $\qquad z_{it} = 68.053mm$

$z_{ip} := z_{ib} - h_{av.32}$ $\qquad z_{ip} = 68.613mm$

Ideal section modulus

$W_{it} := \frac{I_i}{z_{it}}$ $\quad W_{it} = 4.199 \times 10^6\, mm^3$

$W_{ib} := \frac{I_i}{z_{ib}}$ $\quad W_{ib} = 2.486 \times 10^6\, mm^3$

$W_{ip} := \frac{I_i}{z_{ip}}$ $\quad W_{ip} = 4.165 \times 10^6\, mm^3$

Load cases

Prestressing

$\sigma_{p.bed} := 0.75 \cdot f_p$ $\quad \sigma_{p.bed} = 1.395 \times 10^3\, MPa$

$P_{bed} := A_p \cdot n_p \cdot \sigma_{p.bed}$ $\quad P_{bed} = 778.41\, kN$

$M_{p.bed} := -P_{bed} \cdot z_{ip}$ $\quad M_{p.bed} = -33.409\, kN \cdot m$

Stresses in the concrete from prestressing

$\sigma_{ct.p} := \frac{-P_{bed}}{A_i} - \frac{M_{p.bed}}{W_{it}}$ $\quad \sigma_{ct.p} = 5.94\, MPa$ Top Fiber

$\sigma_{cb.p} := \frac{-P_{bed}}{A_i} + \frac{M_{p.bed}}{W_{ib}}$ $\quad \sigma_{cb.p} = -28.263\, MPa$ Bottom Fiber

$\sigma_{cp.p} := \frac{-P_{bed}}{A_i} + \frac{M_{p.bed}}{W_{ip}}$ $\quad \sigma_{cp.p} = -19.603\, MPa$ Level of strands

Stresses in strands from prestressing

$\sigma_{p.p} := \sigma_{p.bed} + n_{prestress} \cdot \sigma_{cp.p}$ $\quad \sigma_{p.p} = 1.294 \times 10^3\, MPa$

Dead Load (g)

$M_g := \frac{G \cdot L^2}{8}$ $\quad M_g = 6.97\, kN \cdot m$

Stresses in the concrete from dead load

$\sigma_{ct.g} := \frac{-M_g}{W_{it}}$ $\quad \sigma_{ct.g} = -1.66\, MPa$ Top Fiber

$\sigma_{cb.g} := \frac{M_g}{W_{ib}}$ $\quad \sigma_{cb.g} = 2.804\, MPa$ Bottom Fiber

$\sigma_{cp.g} := \frac{M_g}{W_{ip}}$ $\sigma_{cp.g} = 1.674\,MPa$ Level of strands

Stresses in the strands from dead load

$\sigma_{p.g} := \alpha_{prestress} \cdot \sigma_{cp.g}$ $\sigma_{p.g} = 8.594\,MPa$

Live Load (q)

$Q := 17.28kN$

$M_Q := \frac{Q \cdot L}{4}$ $M_Q = 17.28kN \cdot m$

Stresses in Concrete from live load

$\sigma_{ct.Q} := \frac{-M_Q}{W_{it}}$ $\sigma_{ct.Q} = -4.115MPa$ Top Fiber

$\sigma_{cb.Q} := \frac{M_Q}{W_{ib}}$ $\sigma_{cb.Q} = 6.951MPa$ Bottom Fiber

$\sigma_{cp.Q} := \frac{M_Q}{W_{ip}}$ $\sigma_{cp.Q} = 4.149MPa$ Level of strands

Stresses in the strands from live load

$\sigma_{p.Q} := \alpha_{prestress} \cdot \sigma_{cp.Q}$ $\sigma_{p.Q} = 21.306MPa$

Total stresses

Total Stresses in Concrete

$\sigma_{ct.total} := \sigma_{ct.p} + \sigma_{ct.g} + \sigma_{ct.Q}$ $\sigma_{ct.total} = 0.165MPa$ Top Fiber

$\sigma_{cb.total} := \sigma_{cb.p} + \sigma_{cb.g} + \sigma_{cb.Q}$ $\sigma_{cb.total} = -18.508MPa$ Bottom Fiber

$\sigma_{cp.total} := \sigma_{cp.p} + \sigma_{cp.g} + \sigma_{cp.Q}$ $\sigma_{cp.total} = -13.781MPa$ Level of strands

Total Stresses in Strands

$\sigma_{p.total} := \sigma_{p.p} + \sigma_{p.g} + \sigma_{p.Q}$ $\sigma_{p.total} = 1.324 \times 10^3\,MPa$

Zero Points

For what value of Q is σ.cb.total=0?

$\sigma_{cb.Q0} := -(\sigma_{cb.p} + \sigma_{cb.g})$ $\quad \sigma_{cb.Q0} = 25.459\,MPa$

$M_{Q0} := \sigma_{cb.Q0} \cdot W_{ib}$ $\quad M_{Q0} = 63.292\,kN \cdot m$

$Q_0 := 4 \cdot \dfrac{M_{Q0}}{L}$ $\quad Q_0 = 63.292\,kN$

For what value of Q is σ.cb.total=2.9 (fctk 0.05)

$\sigma_{cb.Q2.9} := \sigma_{cb.Q0} + 2.9\,MPa$ $\quad \sigma_{cb.Q2.9} = 28.359\,MPa$

$M_{Q2.9} := \sigma_{cb.Q2.9} \cdot W_{ib}$ $\quad M_{Q2.9} = 70.502\,kN \cdot m$

$Q_{2.9} := 4 \cdot \dfrac{M_{Q2.9}}{L}$ $\quad Q_{2.9} = 70.502\,kN$

How long can the span be?

$q := 3.6 \dfrac{kN}{m}$

$L_{max} := \left[8 \cdot \dfrac{M_{Q0}}{(q + G)} \right]^{0.5}$ $\quad L_{max} = 8.454\,m$

$u_c := \dfrac{5 \cdot (q) \cdot L_{max}{}^4}{384 \cdot E_{cmstrong} \cdot I_i}$ $\quad u_c = 22.642\,mm$

$OK := u_c \le \dfrac{L_{max}}{250}$ $\quad OK = 1$

Appendix C – Deflection Curves