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Vulnerability of pension fund balances[1]

Ólafur Ísleifsson, MSc, lektor við Háskólann í Reykjavík

Abstract

Although the Icelandic general labour market pension funds are built on the proviso that pension schemes are fully funded these funds are still grappling with the devastating financial effects of the 2008 economic collapse that rendered most of them in a significant actuarial deficit. The public sector pension funds are based on an employer guarantee that makes up for any lack of funding that historically has been quite significant. We identify the relatively high actuarial discount rate and increasing longevity as two factors that add to the vulnerability of the Icelandic pension system. We present a stochastic model in order to obtain reasonably sound estimates of the effect of revising such key parameters of the actuarial assessments of the pension funds and thus obtain a view of the viability of the Icelandic pension system when confronted with the potential necessity of such parameter shifts. We present results of stochastic simulations of this models made to assess effects of changes in these major financial and demographic assumptions in actuarial evaluations of pension fund balances. Our results suggest that the Icelandic pension funds may be significantly less well funded than is generally perceived.

Keywords: Pension funds, actuarial deficit, discount rate, longevity expectations, mortality rates, stochastic model, policy implications.

Introduction

The long-standing actuarial deficit of the Icelandic public pension funds and the adverse effects of the 2008 economic collapse on the finances of Icelandic pension funds in general have been the subject of much discussion. The object of this paper is to probe these actuarial imbalances and address the question of the financial soundness of the Icelandic pension system. We identify the relatively high discount rate applied in actuarial assessment of pension funds and increased longevity among the factors that significantly effect the actuarial balance of pension funds. We present a stochastic model developed with a view to being capable of providing reasonably sound estimates of the effect of revising such key parameters of the actuarial assessments of the pension funds and thus obtain a view of the vulnerability of the Icelandic pension system when confronted with the potential necessity of such parameter shifts. We present results of stochastic simulations of this model made to assess effects of changes in these major financial and demographic assumptions in actuarial evaluations of pension fund balances. Our results suggest that the Icelandic pension funds may be significantly less well funded than is generally perceived.

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The structure of this paper is as follows. Section 1 gives an overview of the actuarial deficits of the Icelandic pension funds. Section 2 reviews the key parameters employed in assessing the actuarial balance of a pension funds. Section 3 provides a stochastic simulation model designed by the author to assess the sensitivity of actuarial assessments to changes in key demographic and financial variables. In Section 4 we offer a discussion of the main problems faced by the Icelandic pension system and draw some conclusions.

1. Actuarial deficits of the Icelandic pension funds

1.1 Defined benefits vs. defined contributions pension funds

Pension funds are categorised depending on whether fund members enjoy a guarantee of pension benefits on the basis of a given formula or whether pension benefits depend upon the accumulated member contributions of premiums and the investment income of these balances in the fund. The first case is referred to as *defined benefit* (DB) pension funds and the second as *defined contribution* (DC) pension funds. In the first case the employer or a third party provides a guarantee for the pension benefits, whereas in the second case no such guarantee applies. A pension fund without a third party guarantee provides its members with pension benefits solely on the basis of accumulated contributions to the fund and the returns of the funds accumulated in this manner. In this case the pensioner alone shoulders the risk of the amount of the pension benefits, whereas this risk is carried by the employer or third party in case of a defined benefit fund. In the Icelandic context the pension funds of the general labour market fall under the second category.[2]

In the following table we list the Icelandic pension funds enjoying a guarantee and rank them according to the size of their net assets. In addition, we state each fund's actuarial position as well as the total position of all Icelandic pension funds.

Table 1. Actuarial deficit of pension funds with employer guarantee 2011, millions of krónur[3]

Data as of 31.12.2011	Net assets	Actuarial position	Actuarial position %
Pension Fund for State Employees			
A-division	179,342	-57,397	-13.1%
B-division	190,874	-373,149	-63.0%
City of Reykjavík pension fund	58,203	-17,965	-22.9%
Pension fund for nurses	22,309	-44,318	-64.3%
Akureyrarbær pension fund	8,076	-4,921	-35.3%
Kópavogsbær pension fund	3,108	-5,223	-57.9%
Reykjanesbær pension fund	3,081	-3,324	-49.5%
Hafnarfjardarkaupstadur pension fund	1,785	-9,222	-79.8%
Akraneskaupstaður pension fund	901	-4,816	-83.1%
Húsavíkurkaupstadur pension fund	602	-2,003	-76.4%
Neskaupstadur pension fund	503	-1,420	-73.9%
Vestmannaeyjabær pension fund	94	-3,822	-94.4%
Retirement fund of the Fisheries Bank	21	-4,116	-99.4%
Funds with guarantee, total	**468,899**	**-531,695**	**-43.0%**
All pension funds, total	**1,888,718**	**-668,105**	**-17.2%**

Thus, the combined actuarial deficit of employer guaranteed funds amounts to close to ISK 532 billion if the LSR A division is included in the figure. Given the combined actuarial deficit of all pension funds of roughly ISK 668 billion we deduce that the combined actuarial deficit of the general labour market pension funds amounts to about ISK 136 billion. According to the figures in the table, the combined actuarial deficit of the Icelandic pension funds amounts to 41% of 2010 GDP. These statistics can act as a reminder of the fact that, taken in isolation, the internationally high Icelandic pension assets in relation to GDP only have a limited value as a measure of the financial soundness of the pension system. For the LSR and the LH, the pension scheme for nurses,[4] taken together this ratio amounts to 25.6-29.1% of GDP depending on whether the LSR A division is included or not. The Financial Supervision Authority, commonly referred to as FME, classifies the LSR A division along with non-guaranteed pension funds. The board of the fund is by the Act on the LSR required each year to decide on an appropriate employer contribution for the scheme to be able to meet its obligations as assessed by an annual actuarial survey.

These figures suggest that the structure and finances of the LSR, in terms of assets the largest pension fund in Iceland, are the single most important source of the overall actuarial deficit in the pension system. In addition there exists a close link between the LSR and the LH that can be viewed as sister funds. Indeed, about 90% of the combined actuarial deficit of guaranteed funds is accounted for by the LSR and the LH. Most of the other defined benefit pension schemes with an employer guarantee are quite small in size.[5]

1.2 A closer look at the actuarial deficits of the LSR A and B divisions

Looking at the LSR A division we see that the current state of the scheme‘s actuarial deficit derives from the effects of the 2008 economic collapse. Before 2008 the A division deficit was within the 5% limits prescribed by the FME. As the Act on the LSR does not include cuts in benefits as an option to counter an actuarial deficit, the FME in 2011 has, as we will look at more closely, demanded that premiums to the scheme be raised from 15.5% to 19.5%. A 13% deficit falls outside of the range that could possibly be covered by investment returns, not least in the light of rather unfavourable investment prospects domestically in the years ahead and investments abroad being precluded by currency controls. Apart from raising premiums the remaining policy options would be Treasury injections of funds and a cut in pension rights or a combination of these. It is a legal issue, however, to what extent accrued pension rights are protected by the constitutional rights of ownership.

Throughout its history the LSR B division has been largely unfunded. Although ad hoc Treasury payments into the B division have had the effect of significantly improving the actuarial position, the effects of the 2008 economic collapse brought the actuarial position back by some 10%, from a 53% deficit to a 63% deficit.

Viewing the LSR as a whole the fund’s combined total actuarial deficit in macro-economic terms amounted in 2011 to roughly ISK 430,000 million, which measures as just over one quarter of 2011 GDP. The B division 2011 ISK 373,000 million deficit measures as roughly 75% of 2011 total Treasury revenues.

For a number of years the actuarial assessment section of the LSR and LH Annual Report has stated an estimate of what year the LSR B division and the LH will become empty of funds. These points in time have varied, in particular depending on investment performance and ad hoc Treasury injections into the funds. Clearly, the effects of the 2008 economic collapse have moved these points up in time. The 2011 Annual Report states that based on an actuarial assessment of the state of assets and obligations at the end of 2011 and assuming no further Treasury injections the LSR B division and the LH will become empty no later than in 2026.[6]

It is natural to ask about the cost dimensions of the problem at hand in terms of added payments into the B division of the LSR to balance the scheme's assets and liabilities. Based on a cash flow analysis and assuming a 3.5% real return over the remaining lifetime of the B division, the LSR has concluded that an annual injection of ISK 7.8 billion would be needed for the period 2012-2051.[7] This amounts to 1.3% of 2011 total Treasury expenditures.

We note that the assumption of the relatively high annual real return of 3.5% for the period indicated suggests that the figure for the balancing injection may be an underestimate. To estimate the sensitivity of this result to changes in the discount rate we have on the basis of the LSR cash flow analysis calculated the annual injection needed over the lifetime of the B division for the period 2012-2051 assuming a discount rate of 3.0% and 2.5%. It turns out that the result presented above is not overly sensitive to changes in the discount rate assumption as presented in the table below.

Table 2. Sensitivity of required annual injections 2012-2051 to changes in the discount rate, figures in billions of krónur

Discount rate	Required injection
3.5%	7.8
3.0%	8.1
2.5%	8.4

Source: Author's calculations based on the LSR cash flow analysis as presented in LSR (October 3, 2011).

We note that whereas the private sector pension funds have responded to actuarial imbalances by cutting rights to benefits, in some cases by iterated actions, such measures have not been an option concerning the public funds. Thus, in a significant way the A division deviates from the private funds in that it lacks modalities for effectively countering the effects of a shortcoming in or a shock to investment performance, as happened as a consequence of the 2008 economic collapse. In particular, the Act on the LSR does not allow for A division pension rights to be cut, as is the normal recourse for the private funds. It should be kept in mind, however, that several of the general labour market pension funds had in the upswing prior to the 2008 economic collapse increased the rights of their members whereas such increases of rights did not apply in the public sector pension funds.

2. Key parameters in assessing pension fund balances

In the Icelandic pension system a sharp distinction exists by way of the fact that the general labour market pension funds are largely characterised by being of the defined contribution type whereas the funds of the central government and the municipalities can broadly be classified as defined benefits pension funds. The public sector pension funds are based on an employer guarantee that makes up for any lack of funding that historically has been quite significant while in case of an actuarial deficit the general labour market pension funds have to exercise cuts in their benefits to meet the solvency requirements of the Pension Act no. 129/1997.

2.1 Actuarial assessments of pension funds revisited

In a defined benefits pension plan rights are provided with benefits defined in terms of a member's final salary or career average salary and the length of membership in the plan.[8]

By the *accrued obligation* of a pension fund we mean an assessment of the fund outlays due to the pension rights that the fund's members on the assessment date have earned by making contributions to the fund. By *future obligation* we mean an assessment of the fund outlays due to the pension rights that the fund's members will earn by their future contributions to the fund. The sum of these two constitutes the pension fund's *total obligation*. By actuarial custom in assessments of this kind only the pension fund members on the assessment date are taken into account.[9] Thus, the assessment is based on the assumption of a constant number of pension fund members, even in the case where there is mandatory participation in the pension fund in its industry or geographical area.

In the Icelandic context, by Regulation no. 391/1998 on the operation of pension funds and mandatory pension insurance, cf. Article 19, actuarial assessments for the defined contribution general market pension funds are based on a discount rate of 3.5% on top of the CPI.[10] The discount factor for pension funds that link pension benefits to wages is a 2% real rate. An assumption of a 1.5% real wage growth makes these two rates equivalent. The primary pension funds that link pension benefits to wages are the B division of the LSR, the LH and municipal pension funds. These pension schemes, however, no longer accept new members.

For an international comparison of the discount rate we first note that no universal standard exists for the selection of the discount rate.[11] In the European context Thornton, Collinson and Lucas conclude that there are

- Different actuarial methods
- Different approaches to setting actuarial assumptions
- Different assumptions used
- Different approaches to valuation of assets
- Different approaches to setting demographic assumptions, and
- Different approaches to valuing options and choices.[12]

2.2 Considerations for a choice of a discount rate

For selected countries we present in the following table corporate pension discount rates.

Table 3. Corporate pension discount rate assumptions for selected countries

	Discount Rate	
	2006	2010
Japan	2.0	1.5
Unted Kingdom	5.1	5.4
Netherlands	4.6	5.1
United States	5.8	5.4
Ireland	4.7	5.2
Canada	5.1	5.2
Switzerland	2.7	2.6

Source: IMF 2012 Global Financial Stability Report.

We note that while the assets of Dutch pension funds have traditionally been marked-tomarket, the liabilities have since 2007 been discounted at the riskfree nominal swap rate.

For a closer look of liability valuation rules in select countries we present the following information based on a table compiled from an International Monetary Fund working paper.

Table 4. Liability valuation rules in selected OECD countries

Belgium	Belgian prudential legislation: the discount rate for the calculation of the technical provisions has to be chosen in a prudent manner and taking into account (i) the return on future returns and/or (ii) the return on bonds of a Member State or on other high-quality bonds.
Finland	3.5-3.8% depending on the plan.
Germany	The maximum discount rate for *Pensionskassen* and *Pensionsfonde* (if the latter offer insurance-like guarantees) is [in July 2009] 2.25% for new schemes. *Pensionsfonde* can use market interest rates on a best estimate basis if they offer no insurance-like guarantees.
Ireland	(a) a pre-retirement discount rate of 7.25%; (b) a long-term post-retirement rate of 4.50%; (c) a pre-retirement price inflation rate of 2.25%; and (d) a post-retirement long-term rate of price inflation of 2.00%.
Netherlands	Swap rate.
Portugal	4.50%
Spain	4% discount rate. Inflation assumption of 1.5-2.0%.
Norway	4% discount rate until 1993. For contributions due after 1 January 2004 and pension funds established after 1993 the maximum rate is 3%. 2.75% for new contracts after 2006.
United States	Simplified yield curve based on a two-year average of high-grade corporate bonds of appropriate duration.
Source: Impavido and Tower (2009).	

Noting that U.S. public plans typically use a fixed discount rate, set on average at 8%,[13] for two major U.S. pension funds we present a table of discount rates.

Table 5. United States: Discount rates of two major pension funds

Pension fund	Discount rate	Inflation assumption
CalPers*	7.5%	3%
CalSTRS**	7.75%	3%
* The California Public Employees' Retirement System .		
** The California State Teachers Retirement System.		
Sources: Funds' websites.		

These relatively high rates have been the subject of criticism in the United States. Given that the higher the rate, the smaller a fund's obligations appear indicating lesser amounts that states need to contribute to their pension funds, critics blame these relatively high rates for contributing to state pension shortfalls, estimated nationwide to total more than $1 trillion.[14]

Novy-Marx and Rauh (2009) offer a sharp criticism of government accounting standards requiring pension schemes to discount their liabilities at the expected return on their assets suggesting that this approach is analytically misguided as the magnitude of pension liabilities and how a pension's funds are invested are two separate issues that should be considered independently. We note that this consideration is not least directed towards the current U.S. rules that contain incentives for states to invest their pension funds in risky assets with higher expected rates of return, as higher expected rates of return allow them to discount liabilities at higher rates.[15] Novy-Marx and Rauh argue that in practice, the current accounting standard sets up a false equivalence between pension payments, which are extremely likely to be made, and the much less certain outcome of a risky investment portfolio.[16]

The question naturally arises how sensitive a fund's actuarial balance is to the discounting rate. For a stylized pension fund we have shown that relatively small changes in the rate can cause significant changes in the fund's balance. Our stochastic simulation model in Section 3 indicates that by lowering the discount rate by 0.5% from 3.5% to 3.0% a fund's balance deteriorates by approximately 11%. This result is broadly comparable to the result derived by the author by a different method. [17]

The choice of the discount rate has been the subject of increasing disagreement in recent years. Blake (2006) puts forward the question: should the discount rate reflect the liabilities to be paid, or should it reflect the pension fund's asset allocation? In other words, should the discount rate reflect the growth rate of liabilities or should it reflect the weighted-average expected return on the assets in the pension fund?[18]

To approach this issue from a somewhat more fundamental vantage point we share the view put forward by Novy-Marx and Rauh that standard financial theory suggests that financial streams of payment should be discounted at a rate that reflects their risk, and in particular their covariance with priced risks.[19] Further, if pension payments were not subject to uncertainty as to whether and when payments will need to be made, in order to arrive at a measure of the amount of pension liabilities they should be discounted using the risk-free interest rates, like the interest rate on Treasury bills and bonds. Given that actual pension payments are uncertain due to a variety of economic and demographic factors the question remains how to select a discount rate that reflects the risks involved.

In a series of papers Martin L. Weitzman has probed the question how the distant future should be discounted when discount rates are uncertain.[20] Weitzman begins by pointing out that the concept of a "discount rate" is central to economic analysis as it allows effects occurring at different times to be compared by converting each future dollar amount into equivalent present dollars. Indeed, Gollier and Weitzman go on to state that because of the centrality of the concept of discounting to economics the choice of an appropriate discount rate is one of the most critical issues in economics.[21] Weitzman considers events that happen in the "distant future" – a term purposely left vague, but meaning, loosely, generations and even centuries from the present time.[22] Weitzman's well known main result is that the interest rate for discounting among events within the far distant future should be its lowest possible limiting value.[23]

Given that pension payments stretch out over decades into the future we believe that Weitzman's result, although being discussed on the basis of examples from global climate change and other such long-term phenomena, has a bearing on the question of the choice of a discount rate for future pension payments. At the least, Weitzman's statement in the title of his 1998 paper to the effect that the far-distant future should be discounted at is lowest possible rate suggests the direction to which a prudent decision would be made in making this choice.

In sum, we believe that in seeking an answer to the delicate question of which discount rate to apply in assessing the actuarial balance of Icelandic pension funds the following considerations should be taken into account: Following Novy-Marx and Rauh the magnitude of pension liabilities and how a pension's funds are invested are two separate issues that should be considered independently. Following standard financial theory financial streams should be discounted at a rate that reflects their risk, and in particular their covariance with priced risks. Following Weitzman and general prudency considerations the choice of a discount rate should be directed towards the lower end of possible range being considered. Further, in the Icelandic context, given the limitations of the domestic bond market the proposition that selected market yields might be used to value the liabilities of pension funds does not appear viable. These considerations in effect limit the available options of methods for selecting a discount rate to value the liabilities of the Icelandic pension funds. Thus, on the basis of the considerations above and for prudential reasons there seems to be a case for adopting in actuarial assessments a lower discount rate than is currently applied.

2.3 Effects of increased longevity

A pension fund's financial position is heavily influenced by demographic factors such as life expectancies and disability frequencies.[24] Life expectancy has increased considerably in the past 100 years. This applies both to life expectancy at birth, which has increased greatly and seen a linear increase since about 1900, and to life expectancy at older ages, a concept even more important for pension analysis than life expectancy at birth, which has also improved significantly over the past 100 years. Life expectancy at age 60 in advanced economies in Europe, for example, rose from 15 years in 1910 to 24 years in 2010, and is expected to rise further.[25]

In the Icelandic context actuaries Bjarni Gudmundsson and Helgi Bjarnason (2011) have made the case that, given that life expectancy is generally believed to be increasing and therefore implying increases in future assessments of pension obligations, actuarial assessments of pension funds should be based on life expectancy predictions rather than historical data. In this regard we note that regulation no. 391/1998 on the operation of pension funds and mandatory pension insurance provides general assumptions for the actuarial appraisal of pension funds. The regulation stipulates, cf. Article 14, the use of the most recent data on death frequencies as published by the Icelandic Actuarial Society.

By applying Statistics Sweden predictions of the yearly reduction of mortality rates for men and women by age for different periods for the period 2009-2060[26], indicating, for example an increase in life expectancy for males aged 20-40 years of 3.6-4.7 years compared with the historical Icelandic data currently applied, on a fully financed "typical pension fund", Bjarni Gudmundsson and Helgi Bjarnason conclude that these new demographic assumptions would assess the fund as having close to a 10% deficit.[27] As presented in Section 2 of this chapter by applying the Statistics Sweden population projection for 2009-2060 instead of mortality rates based on historical experience our own stochastic simulation model predicts that a fund's balance would deteriorate by no less than 10% and possibly even 11-12%.

We note that in the European context there are different approaches to setting demographic assumptions. The practice ranges from a full complete set of demographic assumptions to considering mortality and retirement only. Also, practice varies from using standard tables specified in regulations, as in Iceland and Denmark, to complete freedom of choice for the actuary.[28]

Given that the size of the deficit of the typical pension fund presented by Bjarni Gudmundsson and Helgi Bjarnason amounts to roughly one third of the present value of its future premiums, given unchanged benefits, the 12% premium rate would need to increase by one third to about 16% to balance the fund by a premium increase alone. Given that the deficit amounts to about one tenth of total liabilities, given an unchanged contribution rate, pension benefits would need to be cut by up to 10%.[29] The high increase in premiums necessitated by the change is explained by the fact that current fund members, having paid premiums that have been too low in view of increased life expectancy, not only for the current accumulation of pension rights but also for the pension rights already accumulated in the fund, will be paying premiums only for the remaining part of their working lives, and pensioners, of course, will not be paying any additional premiums at all.[30]

The natural response to increased life expectancy is to extend the retirement age. Bjarni Thórdarson has estimated that extending the retirement age by one year lowers pension obligations by 5-6%.[31] As presented in Section 2 of this chapter by increasing the retirement age by one year from 67 to 68 years of age our own stochastic simulations suggest that a fund's balance improves by approximately 6%. These considerations suggest that a gradual extension of the retirement age by close to two years would be in order, if such a course were to be taken. Further, we note the observation made in the Statistic Sweden report that the overall picture is that mortality for the foreseeable future should drop in varying degrees for the different age groups.[32] This observation suggests that the benefit age might be indexed as is being frequent around the world.[33]

3. A stochastic simulation model for pension schemes

In this section we present the results of a stochastic simulation model designed by the author.

We modelled a pension fund established by a group of 10,000 individuals with no previously accrued pension rights. We ran the fund over time during which members pay contributions, thus acquiring pension rights as defined by fund rules as specified below, while the fund pays out pension benefits based on whether any given individual reaches pension age or becomes disabled. When the last member of the group has died contributions and pension payments are discounted to the start of the simulation using a fixed discount rate. Averaging over the simulation results thus gives an approximation of the actuarial or expected present value of premiums and pension benefits and the present value referring to the start of the fund, or, equivalently, the simulation. The purpose of the simulation exercise was to establish numerically the sensitivity of the results of actuarial valuations to assumptions made on major financial and demographic parameters.

The assumptions that are varied are the discount rate used for present value calculations and the rates of mortality and invalidity. Also, the effects of different gender mixes are studied, keeping in mind that the accrual rules used by the Icelandic pension funds apply equally to both genders.

At the outset the group of 10,000 individuals is aged 25 to 71 years, the number in each age group is chosen so that it accurately reflects the Icelandic population in 2011 by using Statistics Iceland demographic data on the number of persons for each given sex and age.

This group is given a wage structure by applying Statistics Iceland data on income from employment for different age groups. To this wage structure we added an assumption of increase in productivity as an annual fixed 1.5% rise in the pay scale. Thus, as a given individual matures she/he assumes the wages indicated by the corresponding age group. In any given year an individual may die or another year added to her/his life. In the same manner an individual of working age may continue to make contributions to the scheme or begin to receive benefits due to disability. After the last fund member has died we calculated the balance of the fund as measured by the present value of contributions less the present value of pension payments in relation to the latter. The present values are calculated at the beginning of the fund's lifetime.

We applied mortality rates published by the Icelandic Association of Actuaries based on population data from the years 2004 to 2008. In order to assess the effects of changes in life expectancy these mortality rates were modified by a projection published by Statistics Sweden for the period 2009-2060. The Statistics Sweden projection is given as a yearly lowering of the mortality rate, with a differing amount by gender, age and calendar year. The version used was the projection based on the main or baseline assumption given in the report.

We note that to calculate the future change in mortality rates Statistics Sweden uses the method advocated by Lee and Carter (1992), cf. Statistics Sweden (2009) p. 148. In this model the logarithm of the mortality rate is a linear function of trends over time in the mortality rate. In a recent paper, Alho, Bravo and Palmer (2012), it is demonstrated that the tendency towards accelerating mortality improvements in older ages, depending on the development phase of a population, poses a challenge for statistical modeling of mortality. Linear models like the Lee-Carter model do not take into account the second derivative that otherwise would capture the acceleration factor. For this reason such models risk underestimating life expectancy.[34]

We applied disability figures published by the Iceland Association of Actuaries, which are based on national historical data.

For pension rights created by a given amount paid by a fund member as contribution to the fund we applied tables of rights accruals as determined by the Pension Fund of Commerce and by the Gildi Pension Fund, the two largest pension funds in the Icelandic general labour market. We noted that for all Icelandic pension funds pension rights accruals are gender neutral, this feature being a part of the coinsurance nature of the funds as determined by laws and labour agreements. In the Icelandic general labour market pension rights accruals are age-dependent and this applies to a certain extent also in the public sector pension schemes. In the case of increasing the retirement age we applied figures for rights created for ages 67 and 68 determined by our own simple extrapolation from the existing tables.

As a given individual matures she/he assumes the wage structure indicated by the Statistics Iceland data. At any given year for any individual the simulation procedure draws a random number from the uniform distribution in the interval [0, 1] and compares it with the relevant mortality probability to check whether the individual adds another year to her/his life. In the same manner, by comparison with the relevant disability likelihood, the procedure checks whether an individual of working age continues to make contributions to the scheme or begins to receive disability benefits.

Following Pitacco (1986) and Bacinello (1988) the model is focused on a given individual in regard to two events, death or becoming disabled. There are two ways to simulate an individual's lifespan in a simulation: method A is to randomize whether an individual dies or becomes disabled every year until death; or method B which is to randomize the age at death and the age at disability. We used method A, meaning that in the simulation, for each year for every individual still alive, a random number r is drawn from the range [0, 1]. This range is split into three sub-ranges as follows:

$$[0,x],\ [x,x+y] \text{ and } [x+y,1],$$

where x is the probability depending on sex and age that the individual dies and y is the probability, also depending on sex and age, that the individual becomes disabled. The probability that r is drawn from the first range is x, i.e., the same as the mortality probability. The probability that r is drawn from the second range is equal to the length of the range which is $(x+y) - x = y$, i.e., the same as the disability probability.

If r is in the first range, the individual is considered to have died; if r is in the second range the individual is considered to have become disabled. On the other hand, if r is in the last range the individual is considered to remain healthy and to continue to contribute to the fund.

- The following simplifying assumptions are made for the simulation:
- Women and men are assumed to have the same salary.
- We assume that the fund pays no spousal and child benefits.
- We ignore the cost of running the fund.

We consider four main changes in underlying assumptions:

- A change in the discount rate applied for the actuarial assessment.
- An increase in the retirement age.
- Changes in life expectancy as presented by changes in expected mortality odds.
- Variations in incidence of disability.

We simulated each change 100 times and present the average outcome along with its standard deviation. We applied the population statistics described above scaled down to make a member group of 10,000 individuals. For each of the two pension funds we considered two sets of gender distributions, the first one broadly reflecting the real distribution in each of the funds, the second as an alternative to test for the sensitivity of this factor. Thus, we considered the following alternatives to the gender structure by looking at male/female ratios of 50:50 and 70:30 in the case of the Pension Fund of Commerce (LV) and 70:30 and 90:10 in the case of the Gildi Pension Fund (Gildi). In regard to disability incidence, for LV the base case assumed the standard disability rates, whereas for Gildi the base case assumed 130% of the standard disability rates. Our code is written in the *Python* programming language.

Our results of averages for each of the 100 simulation exercises are presented in the following tables. We stress that the results have to be considered in the light of the simplifying assumptions listed above. In no case did the standard deviation exceed 1.5%.

Table 6. LV 50:50 Results of a stochastic sumulation

Demographic or financial factor	Retirement age	Discount rate	Disability variation	Swedish mortality rates	Fund balance	Compared to base case
Base case	67	3.5%	1	No	7.5%	0.0%
2.5% discount rate	67	2.5%	1	No	-13.2%	-20.7%
3% discount rate	67	3.0%	1	No	-3.3%	-10.8%
4% discount rate	67	4.0%	1	No	19.1%	11.7%
68	68	3.5%	1	No	13.3%	5.8%
69	69	3.5%	1	No	19.1%	11.6%
Swedish mortality rates	67	3.5%	1	Yes	-3.6%	-11.1%
0.9 disability	67	3.5%	0.9	No	10.1%	2.6%
1.1 disability	67	3.5%	1.1	No	5.0%	-2.5%
3% discount rate and Swedish mortality rates	67	3.5%	1	No	-13.8%	-21.3%

Note. Statistics Iceland population data amended to reflect a 50:50 male/female ratio applied to LV pension rights by age. The term Swedish mortality rates refers to the Statistics Sweden projection for the period 2009-2060 of lowered mortality applied to the current Icelandic mortality rates. The numbers 68 and 69 refer to an increase in the retirement age from 67 to 68 or 69 years of age.
Source. Author's calculations based on a Python programming language simulation code.

Figure 1. Graphical representations of simulation results for LV 50:50

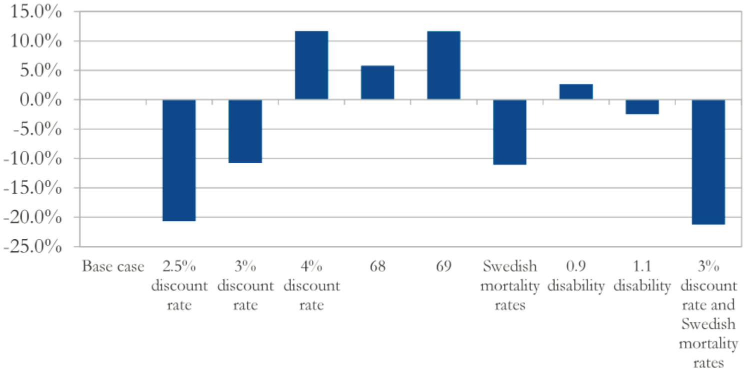


Table 7. LV 70:30 Results of a stochastic sumulation

Demographic or financial factor	Retirement age	Discount rate	Disability varation	Swedish mortality rates	Fund balance	Compared to base case
Base case	67	3.5%	1	No	12.4%	0.0%
2.5% discount rate	67	2.5%	1	No	-9.2%	-21.6%
3% discount rate	67	3.0%	1	No	1.2%	-11.2%
4% discount rate	67	4.0%	1	No	24.6%	12.2%
68	68	3.5%	1	No	19.0%	6.6%
69	69	3.5%	1	No	25.5%	13.1%
Swedish mortality rates	67	3.5%	1	Yes	-0.4%	-12.8%
0.9 disability	67	3.5%	0.9	No	15.0%	2.6%
1.1 disability	67	3.5%	1.1	No	10.0%	-2.5%
3% discount rate and Swedish mortality rates	67	3.5%	1	No	-11.0%	-23.4%

Note. Statistics Iceland population data amended to reflect a 70:30 male/female ratio applied to LV pension rights by age. The term Swedish mortality rates refers to the Statistics Sweden projection for the period 2009-2060 of lowered mortality applied to the current Icelandic mortality rates. The numbers 68 and 69 refer to an increase in the retirement age from 67 to 68 or 69 years of age.
Source. Author's calculations based on a Python programming language simulation code.

Figure 2. Graphical representations of simulation results for LV 70:30

Table 8. Gildi 70:30 Results of a stochastic sumulation

Demographic or financial factor	Retirement age	Discount rate	Disability varation	Swedish mortality rates	Fund balance	Compared to base case
Base case	67	3.5%	1	No	10.9%	0.0%
2.5% discount rate	67	2.5%	1	No	-9.9%	-20.8%
3% discount rate	67	3.0%	1	No	0.1%	-10.8%
4% discount rate	67	4.0%	1	No	22.5%	11.7%
68	68	3.5%	1	No	16.5%	5.6%
69	69	3.5%	1	No	22.2%	11.4%
Swedish mortality rates	67	3.5%	1	Yes	-1.4%	-12.2%
0.9 disability	67	3.5%	0.9	No	13.8%	2.9%
1.1 disability	67	3.5%	1.1	No	8.0%	-2.8%
3% discount rate and Swedish mortality rates	67	3.5%	1	No	-11.5%	-22.4%

Note. Statistics Iceland population data amended to reflect a 70:30 male/female ratio applied to Gildi pension rights by age. The term Swedish mortality rates refers to the Statistics Sweden projection for the period 2009-2060 of lowered mortality applied to the current Icelandic mortality rates. The numbers 68 and 69 refer to an increase in the retirement age from 67 to 68 or 69 years of age.
Source. Author's calculations based on a Python programming language simulation code.

Figure 3. Graphical representations of simulation results for Gildi 50:50

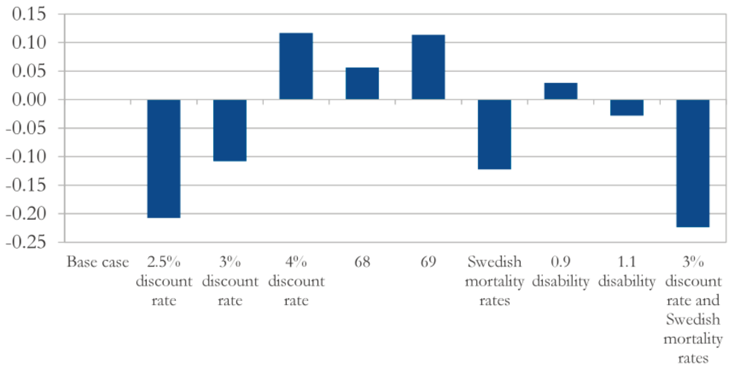
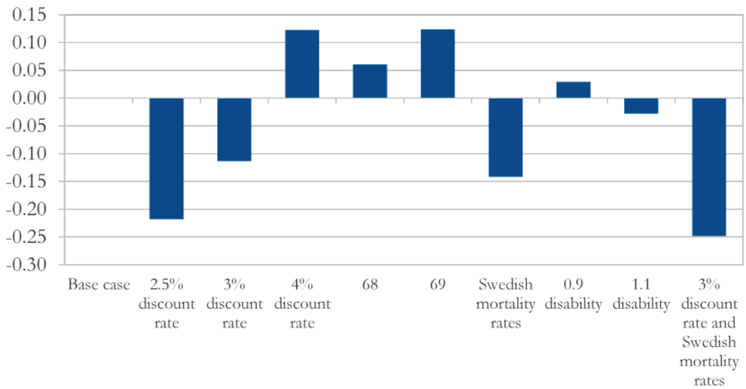


Table 9. Gildi 90:10 Results of a stochastic sumulation

Demographic or financial factor	Retirement age	Discount rate	Disability varation	Swedish mortality rates	Fund balance	Compared to base case
Base case	67	3.5%	1	No	16.8%	0.0%
2.5% discount rate	67	2.5%	1	No	-5.0%	-21.8%
3% discount rate	67	3.0%	1	No	5.4%	-11.4%
4% discount rate	67	4.0%	1	No	29.0%	12.3%
68	68	3.5%	1	No	22.8%	6.1%
69	69	3.5%	1	No	29.2%	12.4%
Swedish mortality rates	67	3.5%	1	Yes	2.6%	-14.2%
0.9 disability	67	3.5%	0.9	No	19.7%	2.9%
1.1 disability	67	3.5%	1.1	No	14.0%	-2.8%
3% discount rate and Swedish mortality rates	67	3.5%	1	No	-8.0%	-24.8%

Note. Statistics Iceland population data amended to reflect a 90:10 male/female ratio applied to Gildi pension rights by age. The term Swedish mortality rates refers to the Statistics Sweden projection for the period 2009-2060 of lowered mortality applied to the current Icelandic mortality rates. The numbers 68 and 69 refer to an increase in the retirement age from 67 to 68 or 69 years of age.
Source. Author's calculations based on a Python programming language simulation code.

Figure 4. Graphical representations of simulation results for Gildi 90:10



We note that our results, as presented in the tables above and derived as deviations from a base scenario, are relatively robust, given the variations in the tables in gender composition and rights accrual between the two pension funds considered. The results can be roughly summarised as follows:

By lowering the discount rate by 0.5% from 3.5% to 3.0% a fund's balance deteriorates by approximately 11%. This result is broadly comparable to the result derived by the author by a totally different method based on a model of a pension fund characterised only by the duration of its assets and liabilities and presented in Chapter 2, Section 3.2, of this dissertation.

By increasing the retirement age by one year from 67 to 68 years of age a fund's balance improves by approximately 6%. By increasing the retirement age by two years from 67 to 69 years of age the fund's balance improves by close to 12%. For comparison Bjarni Thórdarson (2010) estimated that extending the retirement age by one year lowers pension obligations by 5-6%.

By applying the Statistics Sweden population projection for 2009-2060 a fund's balance deteriorates by no less than 10% and possibly even 11-12%. This result is similar to the result of close to 10% presented by Bjarni Gudmundsson and Helgi Bjarnason (2011).

By changing the disability odds either way by 10% a fund's balance improves or deteriorates by 2.5-3.0% depending on the direction of the changed odds.

In addition we note that taken together the assumption of a 3.0% discount rate and applying the Statistics Sweden population forecasts for 2009-2060 leads to a deterioration of a fund's balance by around 23%. This simulation result does not come as a surprise given the simulated effects of each of these two important factors taken separately and in light of the general plausibility of the simulation outcomes.

4. Conclusion

We believe our analysis has significant policy implications, the most important of which is the need to formulate effective policies on how to avert or at least ameliorate the heavy foreseeable burden on taxpayers through the effects of the Treasury guarantee becoming operative when the public schemes discussed above become empty of funds. Delays in implementing measures are certain to exacerbate the problems faced by the Treasury in this regard. Early measures, none of which could be termed easy by any standard, would, however, serve to lessen the severity of the choices that ultimately have to be made, including Treasury injections into the schemes and possibly a downward revision of pension rights, at least for rights accrued in the future.

For the Icelandic pension system as a whole our analysis suggests that decisions have to be made on aspects fundamental to its financial sustainability. First, consideration needs to be given towards gradually reducing the actuarial 3.5% discount rate down to a level that would be sustainable in light of reasonable expectations of the funds' investment performance in the period ahead. Lowering the discount rate includes adjusting pension rights downward to reflect the funds' ability to meet their commitment to their members. Lowering the discount rate includes adjusting pension rights downward to reflect the funds' ability to meet their commitment to their members. A prudent choice of a discount rate serves to prevent unwarranted expectations on the level of pension benefits that a discount rate misaligned from market reality is bound to induce. Should fund performance exceed the assumed returns, pension benefits might be increased, as otherwise current pensioners would be deprived of their fair share of the funds' returns. It is therefore a delicate balancing act not to grant rights to one generation at the cost of another. The stability of the pension system as such would, however, have to enjoy the benefit of any doubt in this regard as another approach would undermine the credibility of the system. Second, given the high likelihood of increased life expectancy, extending the retirement age should be considered as the most natural response to increased life expectancy rather than meeting this challenge by increased premiums or cuts in pension rights.

Taken together the two fundamental factors discussed here, a possibly unwarranted discount rate and outdated mortality assumptions in actuarial assessments of pension funds, indicate that as a whole the Icelandic pension system is underfunded to a larger extent than shown by the prevailing actuarial assessments. Our stochastic simulation model presented in Section 3 above suggests that that when taken together the assumption of a 3.0% discount rate and applying the Statistics Sweden population forecasts for 2009-2060 leads to a deterioration of a fund's balance by at least 20%. This figure sugges the dimensions of the challenge ahead in formulating and executing measures to ensure a satisfactory level of funding for the Icelandic pension system.

Further, given the uncertainty surrounding the future of price indexation as a prominent feature of the Icelandic bond market the refinancing risk of indexed bonds also needs to be taken into account when assessing the funding level of the Icelandic pension system. This issue seems to warrant further study.

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Icelandic names here appear in their correct Icelandic spelling and are alphabetized accordingly. To assist the reader, these names have been transliterated in the main text.

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Margrét Indriðadóttir, Head of unit, Statistics Iceland, May 8, 2012.
Vigfús Ásgeirsson, Actuary, October 3, 2011, and various other occasions.
Þorkell Sigurgeirsson, LSR CFO, October 26, 2011 and other occasions.

Notes

1 This paper is based on research presented in the author's PhD dissertation presented in August 2012 to the Economics Department of the University of Iceland. The author would like to thank doctoral committee members, professors Þorvaldur Gylfason, Lans Bovenberg and Sverrir Ólafsson, and appointed opponents, professors Casper van Ewijk and Edward Palmer,for useful commens and Bjarni Guðmundsson and Þórarinn V. Þórarinsson for helpful advice. Thanks go also to two anonymous referees for useful comments. I am grateful to Sigurður Guðjón Gíslason for effective research assistance. Thanks go also to Hjörtur Bjarnason and Trausti Sæmundsson for writing the code in the Python programming language for the stochastic simulation model. Any errors are mine.

2 On the structure of the system of the general labour market pension funds and their finances see Ólafur Ísleifsson (2007) and (2009).

3 Since 1997 Pension Fund for State Employees (commonly referred to as LSR) consists of two separate divisions. The B division, the traditional old LSR before the Act on LSR in 1997, is closed to new members. New fund members enter A-division created in 1997 which offers pension rights more in line with what prevails in the general labour market. We apply the FME classification of pension funds as presented in the 2011 Annual Pension Report with the exception that we add the LSR A division to the table. See Benedikt Jóhannesson (2011) for a list of actuarial deficits applied to a broader definition of public pension funds.

4 In Icelandic Lífeyrissjóður hjúkrunarfræðinga, formerly Lífeyrissjóður hjúkrunarkvenna, commonly referred to as the LH.

5 For an extensive analysis of the Icelandic public pension funds, in particular the LSR and the LH, see Ólafur Ísleifsson (2011).
6 LSR and LH 2011 Annual Report, p. 42.
7 LSR (October 3, 2011).
8 On pension fund solvency requirements and international actuarial practice in regard to actuarial assessments of pension funds see Ólafur Ísleifsson (2009).
9 Bjarni Gudmundsson (2006). This rule is also part of Icelandic law, cf. Article. 39 of Act no. 129/1997 on mandatory pension insurance and the operations of pension funds.
10 Bjarni Gudmundsson (2006), p. 104.
11 The authorization for the regulation of the discount rate is in Article 24 of Act no. 129/1997 on the operation of pension funds and mandatory pension insurance. The 3.5% discount rate had previously been generally used in actuarial appraisals and had also been agreed upon by the parties to the labour market; cf. Ólafur Ísleifsson (2007), p. 164. Gudmundur Gudmundsson (1998), p. 25, rejects the 3.5% real rate as being too low, given the prevailing interest rates, as the expected pension benefits of members currently receiving benefits would be lower than for members receiving benefits at a later point in time who would be likely to enjoy the effects of higher than the assumed returns.
12 I thank Gylfi Arnbjörnsson for pointing out sources on international pension discount rates.
13 Thornton, Collinson and Lucas (2002).
14 Impavido and Tower (2009), p. 32.
15 Novy-Marx and Rauh (2009). For a popular discussion see e.g. http://www.publicsectorinc.com/forum/2011/03/bill-gates-to-criticize-public-pension-discount-rates-at-ted-conference.html.
16 Novy-Marx and Rauh (2009), p. 202.
17 Op. cit., p. 192.
18 Ólafur Ísleifsson (2009), pp. 130-131.
19 Blake (2006), p. 258.
20 Novy-Marx and Rauh (2009), p. 195.
21 Cf. Weitzman (1998 and 2001) and Gollier and Weitzman (2009).
22 Gollier and Weitzman (2009), p. 1.
23 Weitzman (1998), p. 201.
24 Op.cit., p. 205.
25 Chapter 4 of the 2012 IMF Global Financial Stability Report deals extensively with the financial impact of longevity risk, retrieved June 10, 2012, at http://www.imf.org/External/Pubs/FT/GFSR/2012/01/pdf/text.pdf.
26 International Monetary Fund (2012), p. 3.
27 Statistics Sweden (2009).
28 Bjarni Gudmundsson and Helgi Bjarnason (2011).
29 Thornton, Collinson and Lucas (2002).
30 Calculations to this effect are explained in Chapter 2 by reference to the basic pension fund model presented in section 2 of that chapter.
31 I thank Bjarni Gudmundsson for pointing this out.
32 Bjarni Thórdarson (2010).
33 Statistics Sweden (2009), p. 109.
34 I thank professor Edward Palmer for suggesting this point.
35 Cf. Statistics Iceland data retrieved June 9, 2012, at http://hagstofa.is/?PageID=2593&src=/temp/Dialog/varval.asp?ma=MAN00101%26ti=Mannfj%F6ldi+eftir+kyni+og+aldri+1841%2D2012+++++%26path=../Database/mannfjoldi/Yfirlit/%26lang=3%26units=Fj%F6ldi.
36 Cf. the Statistics Iceland publication *Landshagir 2011*, retrieved June 9, 2012, at http://hagstofa.is/lisalib/getfile.aspx?itemid=13167.

37 Statistics Sweden (2009).

38 I thank Professor Edward Palmer for pointing out this reference.

39 Cf. the Icelandic Association of Actuaries website http://actuaries.is/birtingar.php. The data was provided to the author by Bjarni Gudmundsson in an e-mail dated May 9, 2012.

40 The Pension Fund of Commerce rules, retrieved June 18, 2012, at www.live.is/media/utgefid-efni/vidauki2011.pdf and the Gildi Pension Fund rules, retrieved June 18, 2012, at www.gildi.is/media/files/1309880584/Samthykktir_ Gildis_1.9.2011.pdf. The members of the Gildi Pension Fund are blue-collar workers and seamen. The Pension Fund of Commerce rules explicitly state that they assume a discount rate of 3.5%, mortality rates based on the experience during 2004-2008, disability rates based on the fund‘s experience in 1998-2006. Further, the male/female ratio is assumed to be 50:50. Vigfús Ásgeirsson, Gildi‘s actuary, has informed the author that in the Gildi actuarial assessments the disability rates applied are equivalent to 130% of the rates in the standard disability table. Hence, in the stochastic simulations below when considering a 10% variation in disability rates in the case of LV we applied 90% and 110% of the standard table and in the case of Gildi we applied 117% and 143% of the standard table as a 10% variation from the value of 130% on which the Gildi rights accruals table is based.

41 At the simulation technical level we note that the random seed in the simulation is set to the value 12 in the beginning of the program. Any number, however, could be used, but we chose 12 for the simple reason that the pension contribution rate in Iceland is 12%. The seed initializes the random generator so that it gives the same sequence of random numbers in every simulation. In other words, the program gives the same results for each type of input, so if the program is run again with the same parameters it gives the same averages and the same standard deviations. If the random seed were not set, the computer would use the system clock to initialize the random generator and thus different results would appear every time the same parameters were given to the program.

42 Cf. the Python official website www.python.org. I thank Bjarni Gudmundsson for providing a simple pension fund simulation written in the Python language that served as a prototype for the actuarial principles applied in the code written for the model presented here.

43 In each of the following four tables the figures presented in the rightmost columns should be read in the context of the variable J* presented in the simple model of Section 3.3 of Chapter 2.