Lateral Dike Propagation Forecasting Model

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LATERAL DIKE PROPAGATION FORECASTING MODEL

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I want to dedicate this thesis to all the fantastic people I’ve had the great fortune of working with these last two years. I especially want to thank Freyasteinn Sigmundsson, Páll Einarsson and Andy Hooper which all have taught me so much. I also want to dedicate this to my family who have always been supportive.
Abstract

An important aspect of eruption forecasting is predicting the path of a propagating dike. Where dikes propagate to the surface eruptions start, which can cause great threat to life and property. Dikes have also been known to trigger explosive eruptions when hot basaltic magma comes in contact with more evolved volatile saturated magma. In Iceland dikes can erupt under glaciers causing explosive eruptions and jökulhlaups. Understanding how and where dikes propagate is thus of great importance in mitigation of volcanic hazards. Most literature suggests that dikes propagate perpendicular to the least compressive principal stress ($\sigma_3$). It is true that orientation of the principal axis of stress is very important in determining the orientation of a dike. However, recent observations of dike propagation indicate that this is not always the case. In this thesis the role of topography and gravity on dike propagation is explored. A model is developed to forecast the most probable path of laterally propagating dikes. This model considers minimizing the potential energy of the system, thus accounting for possible influences which may be difficult to take into account otherwise. Model predictions are compared with observed propagation path of the 2014 Bárðarbunga dike, as well as the inferred path of an 1996 Bárðarbunga dike. The results show that both accumulated plate boundary strain prior to the event and topography influence the path of dikes and a model to forecast lateral dike propagation is presented that fits observations.
Mikilvægur þáttur í eldgosaspám er mat á myndun og þróun kvikuganga. Þar sem kvikugangar ná til yfirborðs myndast hraun sem geta valdið miklu tjóni á mannvirkjum. Gangar geta einnig valdið sprengigosum þegar basaltkvika kenst í tæri við þróaðri og gasríkari kviku. Á Íslandi geta gangar valdið gosum undir jökli með sprengivirkni og jökulhu laupum, sem ogna bæði mannsliðum og mannvirkjum. Það er því mjög mikulvægt að skilja hvar og hvernig gangar myndast til þess að megi draga úr þeirri vá sem stafar af eldvirkni. Flestar rannsóknir á myndum kvikuganga hafa komist að þeirri niðurstöðu að þeir myndist hornrétt á lægstu höfuðspennuna ($\sigma_3$).

Þó vissulega sé stefna $\sigma_3$ mjög mikilvæg þá hafa nýlegar mælingar bent til þess að þetta sé ekki allttaf raunin. Í þessari ritgerð eru áhrif landslags og þyngdarkrafts á myndun kvikuganga skoðuð. Spáðikan fyrir myndun kvikuganga sem ferðast samþéða yfirborði járðar er sett fram. Þetta líkan bygdir á því að skoða hvætti kerfisins nær lágmarki. Pannin má taka til greina aðra áhrifaþætti sem annars getur verið erfitt að meta. Lýkan er borið saman við leiðir ganganna sem mynduðust í Bárðarbungu árið 2014 og 1996. Niðurstöður sína að áhrif vegna landslags og uppsafnaðar spenu í jarðskorpu vegna plötuhreyfinga hafa áhrif á leið ganga og hægt er að setja fram spáðikan um myndun þeirra sem fellur að mælingum.
Preface

This thesis has two main chapters. Firstly Chapter 2 which contains a description and results of a dike model I calculated based on the combined strain and gravity potential changes in relation to the 2014 dike in Bárðarbunga in consultation with Andrew Hooper and Freysteinn Sigmundsson. This work has already been published as a part of the paper by Sigmundsson et al. (2015). The aim of that model was to explain the variations in strike of the different segments of the 2014 Bárðarbunga dike. Secondly, Chapter 3 contains a draft manuscript by E.R. Heimisson, A. Hooper and F. Sigmundsson. This is work in preparation for a submission to an international journal. These two chapters are independent of each other, as a result there will be some repetition. When the content of Chapter 2 will be referenced it will be done as Sigmundsson et al. (2015) or (Sigmundsson et al., 2015). In the end there is a summary chapter which will compare and discuss the results from Chapter 2 and 3.

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1 Introduction

On 16 August 2014 a major unrest began at the Bárðarbunga volcanic system in the central part of Iceland. Initial seismic activity was sporadic and occurred in several clusters but quickly it became evident that one of these clusters showed lateral propagation of seismicity. Seismic swarms which exhibit such behaviour in volcanic settings are usually associated with dike intrusions (Brandsdottir and Einarsson, 1979; Einarsson and Brandsdottir, 1978; Wright et al., 2012). Modelling of ground displacements measured with GPS verified this interpretation of the seismic observations. On 31 August an eruptive fissure opened in the Holuhraun area north of Vatnajökull. New crust had indeed been formed in Iceland via laterally propagating dike.

From Figure 1.1 it is clear that the seismicity associated with the 2014 Bárðarbunga dike showed intervals where the epicenters fit very nicely to a line. We refer to these linear parts as segments. The segments of the dike have been accurately determined by Kristín Vogfjörð and the staff of the Icelandic Meteorological Office, and presented in the paper Sigmundsson et al. (2015). These segments are in some cases separated by abrupt changes in strike. At two locations these strike changes are close to 90°. The most prominent locations are where segment 1 (see Figure 1.2) is directed towards south-east whereas segment 2 is directed towards north-east. Another large turn is between segments 4 and 5. The dike propagation was not only segmented in terms of geometry but also in the temporal sense. The dike halted at several locations that corresponded to the end of the segments. The longest halt was around 4 days which occurred at the end of segment 4. The dike did however not halt due to decrease in magma supply. The geodetic measurements showed very clearly that the dike inflated, mostly at the tip, during these halts.

In 1996 Bárðarbunga went through another period of unrest. It started with an intense seismic swarm concentrated in the vicinity of the caldera faults of Bárðarbunga. The seismicity then propagated to the south of the caldera. This resulted in the Gjálp subglacial fissure eruption midway between Bárðarbunga and Grímsvötn, which has been called the Gjálp eruption (Einarsson et al., 1997). The chemistry of tephra from the Gjálp eruption showed similarities with the products from the Grímsvötn volcano (Sigmarsson et al., 2000). A possible explanation for these observations is that a dike from Bárðarbunga intersected magma from Grímsvötn and triggered the Gjálp eruption (Pagli et al., 2007). Similarities between the event of
1 Introduction

Figure 1.1: Relative locations of earthquakes from 16 August to 6 September 2014 (dots) and horizontal ground displacements. Epicentres and horizontal displacements associated with the dike are colored according to time (colorbar in the top left corner), other earthquake locations during the diking event are in grey. The red to blue colorscale at Bárðarbunga caldera shows subsidence up to 16 m inferred from radar profiling on 5 September. The star shows the location of the magma source inferred from inversion of geodetic data. Ice cauldrons formed are shown by circles. At the northern tip of the dike are the outlines of a lava flow mapped from a radar image on 6 September. Thin black lines are segments which are fitted through the lineaments in the dike’s seismicity. Figure from Sigmundsson et al. (2015)

1996 and 2014 strongly suggest that a dike did indeed travel laterally to the south of Bárðarbunga in 1996. In this thesis it is assumed that the epicentral locations from 1996 and the mapped Gjálp eruptive fissure delineates the path of that dike.

The segmented nature of the 2014 Bárðarbunga dike is a very interesting aspect of its formation. It raises many fundamental questions concerning the physics of dike propagation. The questions I try to answer in this thesis are mainly: What is controlling the changes in strike and why does the dike halt at certain locations. These questions are addressed in Chapter 2 and 3. Both chapters present a forecasting model for dike propagation. The models are tested by hindcasting, which is to apply a forecasting model to past events. Hindcasting is commonly used to test models in some fields of earth science such as oceanography and meteorology.

The common goal of Chapter 2 and 3 of this thesis is to look at how and if lateral
basaltic dike propagation is governed by the principle of minimum potential energy (Reddy, 2013). The idea that a dike tip is emplaced in such a way that the potential energy of the dike and the crust is minimized is explored. In Chapter 2 this is done with a very simplified model of crack opening and comparison to actual recorded propagation of the 2014 Bárðarbunga dike (Sigmundsson et al., 2015). Because of how successful that simple model was in anticipating the strike changes we tried a more sophisticated and computationally feasible boundary element crack model in Chapter 3. We test if it can be used for hindcasting the propagation path of both the 1996 and 2014 Bárðarbunga dikes. In Chapter 3 we develop a probability distribution for the dike paths and find that is agrees well with both the 2014 and 1996 dikes. More detailed introduction to each model can be found in Section 2.1 and 3.1.
2 Simple crack model

2.1 Introduction

When a dike started propagating away from the Bárðarbunga caldera in August 2014 one of the most surprising observations was the change in strike of the dike. This is especially clear considering it first starts propagating away from the caldera to the southeast but then suddenly turned towards more northeast orientation. Along its path, there are a few significant changes in strike. Although we do not know the absolute state of stress in the crust, this behavior was not what would be expected from the stress field induced solely by plate motion.

Earlier studies have suggested influence of topography on dikes (e.g., Fialko and Rubin, 1999; Pollard and Muller, 1976). The hypothesis was thus presented that the topography was influencing the strike changes, especially the Bárðarbunga edifice. To test this we came up with the simple crack model which allows us to estimate the strain and gravitational potential energy changes for each segment of the 2014 Bárðarbunga dike.

We found that a model combining both strain and gravitational energy changes could explain the changes in strike. We furthermore found that the dike stopped below a topographic low for four days. This furthermore strengthens our conclusions that dike propagation is influenced by topography.

2.2 The simple crack model

All parameters that go into the simple crack model are estimated from seismic and geodetic data, except the strike of a dike segment. The location and length of each segment was determined from relative earthquake locations described by Sigmundsson et al. (2015). The crack is assumed to be a rectangular tensile dislocation with no strike–slip or dip–slip component. The depth to the top of each dislocation was fixed to 2 km for all segments, the width (height) was fixed to 4 km and opening was fixed to 3 m. The starting point of each segment is considered fixed. Dike
segments of varying hypothetical strike are then tested and the change in strain and gravitational potential energy are estimated. The strike of each segment is varied by rotating it around the starting point which ensures that only energy states which assume continuation of the magma flow are considered. The strike is varied well over 180° in search of the minimum energy for emplacement of the new segment. To implement the approach we performed two integrations, one in three dimensions for the strain potential energy and one in two dimensions for the gravitational potential energy.

2.3 Total potential energy function

The total potential energy function $U_T$ can be written as a sum of the relevant terms

$$U_T = \Theta_s + \Phi_g$$

(2.1)

The total strain energy is given by the sum $\Theta_s = \theta_c + \theta_f$ where $\theta_c$ is the strain energy of the crust and $\theta_f$ the strain energy of the fluid inside the crack respectively. We assume that $\theta_f \approx$ constant. That is because here we consider lateral propagation with small or no variations in depth and making this assumption saves a great deal of computational time. The total gravitational energy is the latter term $\Phi_g = \phi_c + \phi_f$ which is composed of the gravitational energy of the crust and the gravitational energy of the fluid inside the crack respectively.

2.3.1 Strain potential

Assuming that the crust is made from a isotropic, homogeneous and linear elastic material the strain potential energy can be written by:

$$\theta_c = \frac{1}{2} \int \varepsilon_{ij} \sigma_{ij} dV$$

(2.2)

where $\varepsilon_{ij}$ and $\sigma_{ij}$ are the strain and stress tensors respectively. Indexing follows the Einstein summation convention. Several problems come up when applying 2.2 in practice. Not only is calculating the integral computationally expensive and tedious but there are also stress singularities formed at the edges of rectangular dislocations. We used an *ad hoc* method to avoid these: All strain energy densities above three orders of magnitude larger than an estimated average where given a value of 0. These high energy density value are all in the immediate vicinity of the dislocation. They are not caused by interaction of the dislocation strain and tectonic strain fields and thus independent of strike. In Chapter 3 this problem is avoided by integrating over the crack surface instead of the volume surrounding it.
2.3.2 Gravitational potential

In the simple crack model we assume that the dike is propagating at fixed depth with respect to a fixed coordinate system thus rendering $\phi_f = \text{constant and irrelevant}$. The value of $\phi_c$ is however dependent on the strike because of irregular topography. Because we assume a fixed depth of the each segment in the simple model this can be approximated as a two dimensional integral. We account for the topography by assuming it can be represented by distribution of point masses on top of the half space. An area where the topography is higher thus has greater mass.

Let us assume the we are considering a small area $dA$ which lies under $l_t$ meters of rock and $l_i$ meters of ice. The potential energy increase of the system required to lift that $dA$ by $u_z$ is thus:

$$d\phi_c = (l_t \rho_t + l_i \rho_i) g u_z dA$$

where $\rho_t$ and $\rho_i$ are the densities of rock and ice respectively. By integrating 2.3 we get an approximation of the gravitational potential energy change of the crust.

Here we are assuming that $u_z$ is the same at the reference surface below the topography and at the surface between the rock and ice layers. We do this in the simple crack model because it is much more computationally feasible when the topography is complex. A correction could be applied where the depth to the crack is changed depending on the elevation of the rock. This is done in Chapter 3.

2.4 Methods

2.4.1 Stress model

In order to evaluate the integral in equation (2.2) we need to assume tectonic stress. We modelled the stress field with a infinitely long and wide rectangular dislocation with top at 10 km depth. The magnitude of the least compressive stress from this model is shown in Figure 2.1, considering that the axis of the rift zone has been inferred to lie below Askja (Sturkell and Sigmundsson, 2000). The dislocation was placed under the Askja volcano with a strike of N 12° E. This type of models have been used to model plate boundary surface deformation in Iceland (Arnadóttir et al., 2006). The buried dislocation model is only a good approximation of the deformation field close to the surface. We, therefore, assumed the model was two dimensional. We calculated the stresses close to the surface and assumed they where independent of depth. The dislocation was opened 4 m which would roughly correspond to
Figure 2.1: A map view of the model of the tectonic stresses. Colorbar shows magnitude of the least compressive stress, $\sigma_3$, where tensile stress is positive. Dots are epicenters both around the caldera and caused by the dike propagation.

stresses built up by 200 years of plate movement. That is estimated here to be the time since the last dike intrusion in this area, which is has been inferred to have formed the old Holuhraun lava field (Hartley and Thordarson, 2013). It should be noted that there might have been other intrusions in the meantime, but they are not considered.

The stress model that we consider in this thesis does not have the highest tensile stress along the center axis of the rift zone. Although this kind of model has been used successfully to model plate boundary deformation this distribution of tensile stress raises the question if the model accurately represents the strain field. Throughout this thesis we use the buried dislocation model to approximate the stress field due to plate motion. However, we suggest that other models should be tested for this purpose in future work.
2.5 Results

Figure 2.2: Preferred direction of diking for different segments based on a model of combined strain and gravitational potential energy release. Dots are relatively located epicenters colored according to days. Thin dark blue lines which are fitted through the epicenters represent dike segments. Black dots indicate the beginning of each segment and surrounding arc of coloured points represent possible end points for different strikes of propagation. The red to blue colorbar indicates \((E - E_{\text{min}})/(E_{\text{max}} - E_{\text{min}})\), where \(E\) is the energy state for a particular strike, and \(E_{\text{min}}\) and \(E_{\text{max}}\) are the minimum and maximum energy state for that segment. Lower (cold colors) values indicate strikes which are favored for dike emplacement. Background shows bedrock topography with calderas outlined (in black). Grey thick line shows the edge of the Vatnajökull ice cap. This figure is a modified version of a figure in Sigmundsson et al. (2015).

We compare our model predictions to observed propagation path in Figure 2.2. We see that the model does predict the change in strike from segment 1 to 2. Although both segments show two minima there is a shift between the absolute minima which agrees with the strike change. There is, however, some offset for the first 4 segments between the predicted minimum value and the observed strike but for segments 5, 6, 7a and 8a the agreement is very good even though these segments are not quite perpendicular to the least compressive stress due to plate motion. As can be seen by Figure 2.3 the gravitational term is dominating the strike in the first segments but then the strain term becomes dominant in the later segments.
Another prominent feature of the 2014 Bárðarbunga dike was not only the changes in strike but also halts or periods where the dike stopped. One of these halts lasted for 4 days. This occurred at the end of segment 4 (see Figure 2.2). Although the dike was arrested it still increased in volume and inflated mostly at the tip. After 4 days the dike broke through the barrier and continued to propagate. We found that the area where the dike halted correlated with increase in lithostatic pressure caused by uphill bedrock topography in front of the dike (see Figure 2.4). We interpret this as either the higher lithostatic pressure increasing the strength of the rock thus forming a barrier, or if the dike travelled along the level of neutral buoyancy it would have needed to build up pressure to propagate uphill and thus halted.
Figure 2.3: Energy profiles for dike segments 1, 2, 3, 4, 5, 6, 7a and 8a, as described in Figure 2.2. Blue lines indicate the strain energy potential change as a function of the strike, and the red lines the gravitational potential change. Green is the total potential energy change which is scaled and displayed in the colorbar of Figure 2.2. Energy is shown in terajoules ($10^{12}$ J). The lowest point on each energy curve is defined as 0 TJ. Error bar indicate $1\sigma$ error on the numerical integration. Figure from Sigmundsson et al. (2015).
2 Simple crack model

Figure 2.4: a) Bedrock (brown line) and ice topography (light blue line) along the dike path.
b) Depth of earthquake hypocentres (grey dots) below sea level projected on the dike segments and lines (red) of constant lithostatic pressure, assuming constant crustal density of 2,800 kg/m³ and ice density of 920 kg/m³. Line spacing corresponds to change in lithostatic pressure of 25 MPa.
c) Lithostatic pressure at sea level calculated along dike segments 1, 2, 3, 4, 5, 6, 7a, 8a and 8b (blue line). The calculations take into account both the subglacial bedrock topography and ice thickness. Light blue triangles indicate the beginning of a segment and red triangles the end of a segment. It is assumed that between segments the dike propagates along a straight path. Dike propagation was halted for the longest time at the end of segment 4. Figure from Sigmundsson et al. (2015).
2.6 Discussion

In Figure 2.4 panel b we see the earthquake depths plotted along with lines of equal lithostatic pressure. We found the average depth of the earthquakes correlated well with the changes in lithostatic pressure ($R^2 \approx 0.7$). This might suggest level of neutral buoyancy (LNB) propagation. However the depth changes in the earthquakes are on the range of factor 5–6 larger than the changes expected from lithostatic pressure changes. One possible explanation for this is that the depth changes are increased because the dike is inflating more in topographic lows causing fractures to extend both deeper and shallower in correlation with the lithostatic pressure, as will be discussed further in next chapter. This behavior can be expected both for a dike travelling at LNB or dike travelling at fixed depth.

2.7 Conclusions

We found that the model of combined strain and gravitational energy changes could explain well the changes in strike which the Bárðarbunga dike demonstrated. While the orientation of the least compressive stress is an important part of determining the orientation of dikes our results show that other factors are likely to contribute as well.

We observed that the dike halted in a topographic low. This furthermore suggests that topography not only influences the strike but also the propagation speed of the dike.
3 Boundary element crack model

3.1 Introduction

Physically complete and realistic numerical models of dike propagation have, at present, not been developed. This is due to the complexity and computational impracticality of considering all factors that might affect dike propagation (Rivalta et al., 2015). The aim of this chapter is not to present the most physically realistic or sophisticated model of dike propagation to date, but rather to present simple and fast approach for forecasting the path of a laterally propagating dike. In the study of the Earth there will always be a number of assumptions and sources of errors due to its heterogeneous nature. A forecasting model of any sort should account for such errors and propagate errors of the input parameters whenever possible. For complex and computationally expensive numerical models such error propagation might be difficult on the time scale which dike propagates and results are required as soon as possible. Here we present an approach for doing this which is able to hindcast the path of 2014 and 1996 intrusions in the Bárðarbunga volcanic system (Sigmundsson et al., 2015), (Einarsson et al., 1997).

The 2014 dike intrusion in the the Bárðarbunga volcanic system, is well documented, demonstrating segmented propagation and significant changes in strike. The orientation of laterally propagating dikes is usually considered to be governed by the orientation of the least compressive stress ($\sigma_3$). However, measuring deviatoric stresses in the crust is very difficult and thus verifying with observations that volcanic dikes propagate exactly perpendicular to $\sigma_3$ difficult as well. The Bárðarbunga 2014 intrusion showed that the dike emplacement was not completely perpendicular to the least compressive stress. This was seen in modelling of geodetic data as significant strike slip motion accompanied the dike formation (Sigmundsson et al., 2015). Few lateral diking events have been monitored with such high accuracy in hypocentral locations and geodetic measurements as the 2014 Bárðarbunga dike. In some cases deviations, which do not agree with the theory that dike emplacement is purely determined by $\sigma_3$, could perhaps be contributed to measurement error. That is certainly not the case with Bárðarbunga 2014 where relative errors of hypocenters are generally less than 200 m in longitude and latitude. The 2014 Bárðarbunga dike is no exception because modelling of the Upptyppingar 2007-2008 dike showed
3 Boundary element crack model

that the geodetic data couldn’t be explained without a shear displacement along the plane of the dike. This strongly suggests emplacement not perpendicular to \( \sigma_3 \) (Hooper et al., 2011). While it is evident that orientation of the principal stresses play a major role in determining dike propagation orientation, the cases of the 2014 Bárðarbunga dike and Upptyppingar 2007-2008 dike show that other factors should also be considered.

Several studies have found that other factors than deviatoric stresses can influence dike formation. Pollard and Muller (1976) found that gradient in magma pressure or lithostatic pressure was a plausible explanation for the tear drop shape some dikes show in vertical cross section. This indicates that the equilibrium configuration of the crust after a dike intrusion, where the total potential of an elastic body is at a minimum (Reddy, 2013), is influenced by pressure gradients. Dahm (2000) concluded that the tectonic stress gradient, length of fractures and buoyancy all influenced the orientation of vertically propagating magma. The study by Sigmundsson et al. (2015) showed that a dike propagation model which considered strain and gravitational energy changes due to topography could well explain the observed changes in strike well. Here we explore further the role of topography and how the fact that it is easily measurable can allow us to forecast lateral dike propagation with reasonable accuracy.

A propagating dike tip can be expected to be emplaced in such a way that the total potential of the system is at a minimum. The minimum potential energy principle is a far-reaching variational principle (Reddy, 2013). One can equivalently state that the emplacement should be such that it allows for the greatest energy release. This can be interpreted as the path of least resistance. We apply it here to quasi-static dike propagation, which means we do not consider the details of the magma flow within the dike. Similar methods have been used successfully in studying dike propagation (Dahm, 2000; Maccaferri et al., 2011, 2014). Here we introduce several new aspects to this approach, including pressure changes due to realistic topography and improved approach for estimating gravitational potential energy variations.

The total potential energy function \( U_T \) can be written as a sum of the relevant terms \( U_T = \Theta_s + \Phi_g \) where \( \Theta_s = \theta_c + \theta_f \), i.e., the sum of the strain energy of the crust and of the fluid inside the crack, respectively. We assume that \( \theta_f \approx \) constant. That is because here we consider a laterally propagating dike tip which is propagating at level of neutral buoyancy (LNB) and thus not exposed to significant changes in confining pressure. The latter term \( \Phi_g = \phi_c + \phi_f \) is composed of the gravitational energy of the crust and the gravitational energy of the fluid inside the crack, respectively.

In our model, the dike is a vertical, laterally propagating crack where the upper and lower margins are at fixed depth with respect to the elevation of the topography. The crack is systematically lengthening and the crack tip is emplaced in such a way
3.1 Introduction

that it minimizes the potential energy of the crust. In the following we will look at
the change in these potential terms: \( \Delta \phi_f, \Delta \theta_c \) and \( \Delta \phi_c \), rather than their absolute
value when evaluating different directions. We are determining the propagation path
that causes the greatest energy release or, in other words, the greatest lowering of
the total potential. Evaluating the absolute potential energy is not necessary.

The approach we present here allows us to create a probability density function
(PDF) for dike propagation paths in near-real time without requiring great com-
putational power. All computation presented in this paper was carried out on a
regular laptop. Although we apply a similar method as Sigmundsson et al. (2015),
the method presented in that study was very computationally expensive and could
not be used for building a distribution of the most likely path of the dike in near-real
time. We estimate the approach in this paper to be on the order of 1000 times faster
than that of Sigmundsson et al. (2015). In Sigmundsson et al. (2015) the dike was
assumed to propagate at fixed depth with respect to sea level. Here we look at a
dike propagating at level of neutral bouyancy (LNB) and thus at fixed depth below
the topography. The dike is assumed to propagate at 2.5 km below the topographic
elevation, and having a width (height) of 5 km. This is estimate from the geodetic
modelling of the dike by Sigmundsson et al. (2015). LNB propagation has been pro-
posed as a likely explanation for lateral flow of magma in dikes (Fialko and Rubin,
1999; Fiske and Jackson, 1972; Lister and Kerr, 1991). We presented a method to
evaluate the gravitational energy change due to topography in Sigmundsson et al.
(2015) that assumed the topography to be point masses distributed on a surface of
a half-space. Here we introduce a different correction to the topography by shift-
ing the depth to the dislocation in elastic half-space depending on the topographic
elevation. We, furthermore, present a method to evaluate the gravitational energy
change of the crust that does not require us to assume the dike is at fixed depth.
The crack model presented by Sigmundsson et al. (2015) to evaluate the propagation
path assumed the opening to be constant and the crack tip was composed of only
one rectangular dislocation. Assuming constant opening which is independent of
strike requires information about the actual dike opening which is difficult to attain
without first modelling geodetic displacement. Here, the crack tip is a single column
of 10 rectangular dislocations and generally shorter in the along strike direction. In
Sigmundsson et al. (2015) we required ad hoc measures to avoid stress singularities
in a strain energy integral, which may have led to inaccurate integral evaluations.
We build on a previous study (Maccaferri et al., 2011) and take their approach of
using a boundary element method (BEM) coupled with evaluating the strain energy
integral as a surface integral to resolve these issues of volumetric strain integral in
Sigmundsson et al. (2015). BEM allows us to solve for opening of the dike tip.
This resolves the aforementioned problems of assuming constant opening indepen-
dent of strike. Furthermore, we expand on the work by Maccaferri et al. (2011),
which looked at vertical migration of magma with constant mass in two dimensions.
We extend their method into three dimensions, which allows us to study laterally
propagating dikes. We also compare our results to actual observations of dike propa-
gation and show that the topography has a large influence on the propagation path. Our results show that even though a large uncertainty is allowed in the model of the stress field, the influence of the topography is so great that a dike propagation path can be constrained with relatively good accuracy. Our results suggest that near real-time prediction of the propagation path of laterally propagating dikes is possible.

3.2 Propagation model

3.2.1 Strain energy change

To evaluate the strain energy change from a crack opening we need the absolute stress field. From the stress field we can calculate traction on the crack surface. Once the crack is opened and filled with low viscosity fluid, the shear traction will vanish; however, the normal component of the traction vector does not. From these changes in traction we can calculate the displacements of the crack surface using a boundary element method. Once the displacements are estimated, we can calculate the strain energy change.

The strain energy change $\Delta \theta_c$ due to slip on surface $\Sigma$ is given by Aki and Richards (1980). One side of the surface is $\Sigma^-$ and the other one $\Sigma^+$. $\nu$ is the normal to $\Sigma$. The displacement is described by $[u]$, which is the difference between displacements on $\Sigma^+$ and $\Sigma^-$. We have

$$\Delta \theta_c = -\frac{1}{2} \int_{\Sigma} [u_i] (\sigma_{ij}^0 + \sigma_{ij}^1) \nu_j d\Sigma$$  \hspace{1cm} (3.1)

where $\sigma_{ij}^0$ is the stress tensor acting at a point on $\Sigma$ before slip and/or opening, and $\sigma_{ij}^1$ after slip and/or opening. In the following we adapt the methodology presented by Maccaferri et al. (2011) to three dimensions to evaluate this integral.

The crack surface are discretized into rectangular patches each assumed to have constant stress changes on the surface. We can write the stress before slip as the sum of the following terms on one patch:

$$\sigma_{ij}^0 = P_{\text{litho}} \delta_{ij} + \sigma_{ij}^T$$  \hspace{1cm} (3.2)

where $P_{\text{litho}}$ is the lithostatic pressure, $\delta_{ij}$ the Kronecker delta and $\sigma_{ij}^T$ the stress
3.2 Propagation model

contribution of tectonics, loading or other causes that contribute to stresses in the crust other than pressure. Lithostatic pressure is calculated assuming the density model introduced in Section 3.2.2. It is more convenient for the BEM if equation (3.2) is written in terms of components of the traction vector in the basis of the unit normal $\nu_i$, the dip vector $\nu_d^i$ and the strike vector $\nu_s^i$ of each point of the crack:

Normal: $T_n^0 = \sigma_{ij}^0 \nu_j \nu_i$
Dip: $T_d^0 = \sigma_{ij}^0 \nu_j \nu_d^i$
Shear: $T_s^0 = \sigma_{ij}^0 \nu_j \nu_s^i$  

(3.3)

The stress vector in the basis of these orthogonal vectors before crack opening is thus:

$$T^0 = (T_n^0, T_d^0, T_s^0)$$  

(3.4)

The stress vector after crack opening is:

$$T^1 = (T_n^0 - \Delta P, 0, 0)$$  

(3.5)

Once a crack opens and is filled with fluid, all shear terms will become zero. $\Delta P$ is the difference between the fluid pressure and the confining pressure:

$$\Delta P = P_{fluid} - P_{conf}$$  

(3.6)

The fluid pressure can be written as:

$$P_{fluid} = \rho_f g(z - z_T) + P_{over} + P_{litho}^1$$  

(3.7)

where $\rho_f$ is the density of the magma, $z$ is the vertical location of the patch and $z_T$ is the top rim of the dike. The term $\rho_f g(z - z_T)$ thus represents the hydrostatic pressure inside the dike tip. The $z$ axis is considered positive upwards. $P_{over}$ is the fluid overpressure which may be caused by a connection to a pressurized magma reservoir. 

The overpressure should roughly be equal to the tensile strength of the host rock. Here we consider the overpressure to be constant. $P_{litho}^1$ is the lithostatic pressure at the top rim of the dike where it first opens and begins to propagate. Where the dike first opens it must have fluid pressure equal to the effective normal stress on the crack surface plus overpressure to fracture the rock. We use the lithostatic pressure instead of the effective normal stress in equation (3.7) to estimate a realistic fluid pressure in the model because the fluid pressure is not in reality dependent on the effective
normal stress. Otherwise the model would have variable fluid pressure depending on the orientation of the first segment. We thus use the lithostatic pressure which at a constructive plate boundary functions as a upper limit to the effective normal stress. The confining pressure is, however, a function of the strike and we account for that in equation (3.8). We set the overpressure from 5 MPa to 25 MPa which should represent a reasonable range see Section 3.2.5). Because the dike tends to propagate downhill the hydrostatic pressure increases in the tip. This is the reason that the dike tip inflates in as seen Figure 3.1. Even though the lithostatic pressure is constant, the fluid pressure is generally increasing. We assume the fluid to be incompressible because the dike is at constant confining pressure and the changes in magma pressure are too small to cause significant changes in density.

Figure 3.1: Example of how the dike tip opening changes depending on the topography. In this example simulation the dike path was constrained to go along straight line to make the figure simpler. The blue line above shows the bedrock topography above the dike tip. Notice how low topography correlates with large opening. This is because in the topographic lows the magma pressure is increased.
The confining pressure is:

\[ P_{\text{conf}} = P_{\text{litho}}(z) + \sigma_{ij}^T \nu_i \nu_j \]  

(3.8)

where \( \sigma_{ij}^T \nu_i \nu_j \) is the component of the tectonic stress that is normal to \( \Sigma \). Note that the normal traction after opening of the dike is just the magma pressure at that depth. We, however, calculate \( \Delta P \) according to equation (3.6) because \( -\Delta P \) is the change to the normal traction before and after opening and is the quantity we require to solve for opening in the BEM model.

Now equation (3.1) can be replaced by a sum over \( k \) patches:

\[ \Delta \theta_c \approx -\frac{1}{2} \sum_{l=1}^{k} \left[ b_{n}^{l}(T_{n}^{0,l} + T_{n}^{1,l}) + b_{d}^{l}T_{d}^{0,l} + b_{s}^{l}T_{s}^{0,l} \right] \Sigma^{l} \]  

(3.9)

Where \( l \) is the number of a patch and \( \Sigma^{l} \) is the area of patch number \( l \). \( T_{n}^{0,l} + T_{n}^{1,l} \) are the \( i \)-th component of the stress vector acting on patch number \( l \) where 0 indicates the stress vector before opening and 1 indicates the stress vector after opening. The components of the vector, \( b^{l} = (b_{n}^{l}, b_{d}^{l}, b_{s}^{l}) \), represent the opening, the slip in the strike and the dip direction of patch number \( l \), respectively.

### 3.2.2 Gravitational potential change of the crust

A dike travelling laterally will change the gravitational potential of the crust. This potential change is:

\[ \Delta \phi_c = \int g \rho_c u_z dV \]  

(3.10)

where \( u_z \) is the vertical component of the deformation field and the integration volume is the whole medium. In practice equation 3.10 is very difficult to apply using Okada’s Green’s functions because the integral is not convergent. Furthermore, the edges of the dislocation contain singularities. This means that deformation field in the immediate vicinity of the dislocations is very different from a realistic non-singular crack. Although the singular values are removed from equation (3.10) it gives unrealistically large values for the integration. One might argue that it doesn’t matter if the values are very high, in our method, as only the variations with strike matter. This is true. However, these unrealistically large values require very high
relative accuracy of the integral evaluation, which is difficult to attain. For example, a tensile dislocation with opening of 1 m, with top edge at 1 km depth and bottom edge at 6 km and 2 km long in strike direction, will give gravitational energy increase of $71900 \pm 200$ TJ ($10^{12}$ J). This assumes a box extending $\pm 500$ km in latitude and longitude and from depth of 0 to 500 km. In Sigmundsson et al. (2015) we found that the variability with strike was on the order of tens of TJ, so a 1σ error of 200 TJ is around two orders of magnitude too large for a meaningful comparison. We found that reaching a 1σ error of 200 TJ took several hours of computation. It should be noted that integral was evaluated with a Monte Carlo method which gives good constraints on error. Using other more robust numerical integration method would reduce this computation time. We, however, want to be able to evaluate each segment in a fraction of second which is very difficult to reach in a three dimensional integral when such high relative accuracy is needed. We thus conclude that an approximation is needed which will allow us to evaluate the gravitational energy change of the crust in faster and computationally less expensive manner.

Here we estimate the gravitational energy change of the crust from only the surface displacements and the crack opening. Figure 3.2 is a part of the crust before (top) and after (bottom) opening of a dike tip.

B is the part of the crust that contains the extent of the topography before opening. Part C is the altered topography which is modified by the opening displacement field. Parts A and A’ are composed of three layers each with constant density. A1: extending from $z = 0$ to $z = D_t$, where $D_t$ is the $z$ location of the top of the dike tip. A2: Extending from $z = D_t$ to $z = D_b$, where $D_b$ is the $z$ location of the bottom of the dike tip. A3: Extending from $z = D_b$ to $z = D_{\text{max}}$ where $D_{\text{max}}$ is some large depth where no deformation from the dike tip has occurred. We assume that the box is large enough so only the top boundary deforms, i.e. the vertical component of the deformation field has become negligible at the bottom and the vertical boundaries of the box. Note that layers A1 and A3 do not affect the final results this becomes evident in equation (3.21).

We now consider the $z$ coordinates of the centers of mass of each layer in part A: $Z_{A_1}$, $Z_{A_2}$, $Z_{A_3}$. Due to symmetry these are unchanged before and after opening. However, the mass of layer A2 has been changed so the total center of mass of part A: $Z_A$ and of A': $Z'_A$ are not the same. We call $Z_B$ is the $z$-component of the center of mass of B and $Z_C$ the center of mass of C.

The mass of the crust is conserved before and after the crack opening. We can thus estimate how much the center of mass has moved before and after the crack opening and from that find and approximation for equation (3.10).

Let us call the center of mass of the crust before opening $Z_{CM}$ and after $Z'_{CM}$. We
3.2 Propagation model

Figure 3.2: a) A three density layered crustal model before crack opening. b) The crust after crack opening.

\[ Z_{CM} = \frac{M_A Z_A + M_B Z_B}{M_A + M_B} \]  
(3.11)

where the mass of A is \( M_A = M_{A1} + M_{A2} + M_{A3} = V_{A1} \rho_{A1} + V_{A2} \rho_{A2} + V_{A3} \rho_{A3} \). Here \( V_{A1}, V_{A2} \) and \( V_{A3} \) are the volumes of layers A1, A2 and A3 respectively. The center of mass of A is:

\[ Z_A = \frac{M_{A1} Z_{A1} + M_{A2} Z_{A2} + M_{A3} Z_{A3}}{M_A} \]  
(3.12)

The z coordinate of the center of mass of B is simply half the mean elevation of the topography over an area \( S \):

\[ Z_B = 0.5 \bar{h}(x, y) \]  
(3.13)

where \( h(x, y) \) is the function which describes the elevation of the topography and the bar over \( h \) indicates a mean value. The area \( S \) is the integration area of the
3 Boundary element crack model

following integral to evaluate the mass of B:

\[ M_B = \rho A_1 \int \int h(x, y) \, dx \, dy \]  \hspace{1cm} (3.14)

Note the relationship which can save computation time: \( Z_B = M_B/(2\rho A_1 S) \). \( S \) needs to be large enough so the vertical displacement field is close to 0 at the boundaries. As can be seen from equation (3.21), computing \( M_A, M_{A1}, M_{A3}, Z_{A1} \) and \( Z_{A3} \) is not necessary, \( M_{A2} \) and \( Z_{A2} \) are given by:

\[ M_{A2} = (D_t - D_b)\rho A_2 S \]  \hspace{1cm} (3.15)

\[ Z_{A2} = \frac{1}{2}(D_t + D_b) \]  \hspace{1cm} (3.16)

Note that \( D_b < D_t < 0 \). The center of mass of the crust after the crack tip opening is given by:

\[ Z'_{CM} = \frac{M'_{A2}^2 + M_CZ_C}{M'_{A2} + M_C} \]  \hspace{1cm} (3.17)

The z coordinate of the center of mass of part C is:

\[ Z_C = 0.5(h(x, y) + u_z(x, y)) \]  \hspace{1cm} (3.18)

i.e. half the mean of \( h(x, y) \) and the vertical surface displacements \( u_z \) over area \( S \). The mass of C is given by:

\[ M_C = \rho A_1 \int \int (h(x, y) + u_z(x, y)) \, dx \, dy \]  \hspace{1cm} (3.19)

We assume that \( M'_{A1} = M_{A1} \) and \( M_{A3} = M'_{A3} \). However due to the crack opening in layer A2 we need to adjust the mass. The mass of the crust is conserved although the volume is not. Conservation of mass gives: \( M_{A1} + M_{A2} + M_{A3} + M_B = M'_{A1} + M'_{A2} + M'_{A3} + M_C \). This gives:

\[ M'_{A2} = M_{A2} + M_B - M_C \]  \hspace{1cm} (3.20)

The approximation for the gravitational energy change of the crust becomes:

\[ \Delta \phi_c = gM(Z'_{CM} - Z_{CM}) = g(M_CZ_C - M_BZ_B + Z_{A2}(M_B - M_C)) \]  \hspace{1cm} (3.21)

where \( M = M'_{A2} + M_C = M_A + M_B \). Note that all terms related to layers A1 and A3 cancel. This means that the density \( \rho_{A3} \) doesn’t matter.

The volume change of the crust for a Poisson’s ratio of 0.25 is around 75% of the crack’s volume. Layer A2 has the highest volumetric strain due to the crack opening and thus likely to hold the bulk of the compressed volume. It is thus reasonable
3.2 Propagation model

to assume that all the mass stored as compressed volume will be left symmetrically
distributed around the crack opening in layer A2.

In reality the crack opening would deform the layers which we have assumed to
have constant density. However in the brittle crust in Iceland there are generally
no ubiquitous sharp density contrasts (Gudmundsson and Högnadóttir, 2007). The
effects of internal movements of mass on the center of mass will therefore be limited.
We have estimated average values for the densities. Gudmundsson and Högnadóttir
(2007) estimated from the work of Carlson and Herrick (1990) and Christensen and
Wilkens (1982) a plausible density range for the crust in Iceland as a function of
depth. We have taken these ranges and estimated $\rho_{A1} = 2400$ kg/m$^3$ and $\rho_{A2} = 2700$
kg/m$^3$. These constant values should thus be thought of as average values. At
the surface, however, a sharp density contrast occurs between rock and air. That
contrast and the opening of the crack is what we consider in this approximation to
be the main contributions to gravitational energy change of the crust.

Most of the propagation path of the dikes were under the Vatnajökull icecap, so we
need to also consider the ice thickness and density in this model. We take the ice
thickness $l_i$ and multiply with the ratio between the ice density and rock density
$\rho_i/\rho_t$ and add to the bedrock topography’s elevation. Because the ice has around three
times less density then the rock, it is less important than the bedrock topography.

The influence of the topography is taken into account by adding $h(x', y')$ to the
depth of the dike tip model, when the surface displacement is evaluated at point
$(x', y')$. This is a good approximation when the topography is not very steep, which
is the case for the area surrounding the Bárðarbunga dike.

3.2.3 Gravitational potential change of the magma

The gravitational potential of the magma in a dike tip can be written:

$$\phi_f = \rho_f V g Z^1$$  \hspace{1cm} (3.22)

Where $Z^1$ is the z component of the center of mass of the magma in the tip and $V$
is the volume of the tip. To get the change in potential energy we must consider
where the magma is being supplied. Assuming that it being transported laterally
from the starting point of the first segment of the dike we can express the difference
in gravitational energy change of the magma:

$$\Delta \phi_f = V \rho_f g (Z^1 - Z^0)$$  \hspace{1cm} (3.23)

Where $Z^0$ is the center of mass of the first segment. Note that if the dike is always
travelling at a fixed depth with respect to sea level this model predicts no change
in gravitational energy of the magma, as would be expected.
If we consider a LNB propagation laterally away from a cone the dike begins by travelling downhill but once it is outside the cone it will move at fixed depth with respect to the z axis. At first glance one might think that the magma would at that point have no change in gravitational energy. This is not true, the initial downhill emplacement of the dike results in magma flowing downhill when new segments are added to the end. This will favor crack orientation that allows for the greatest opening because that results in most mass downhill mass transfer. This is also the orientation that is perpendicular to $\sigma_3$.

We assume that the magma has the density $\rho_f = 2700 \text{ kg/m}^3$ which is the same as the average density for layer A2 in Section 3.2.2. The criteria for LNB propagation is therefore met.

### 3.2.4 Boundary element crack model

To solve for crack tip opening using the changes in traction on the crack surface we use a boundary element method (BEM). Assuming the crack tip is composed of a finite number of rectangular dislocations, we can use Okada’s Green’s functions ($Okada$, 1992) to form the matrix $G$ presented in (3.24). $G$ has dimensions $3k \times 3k$ where $k$ is the number of dislocations of the crack tip. Elements 1 to $k$ of column $l$ in $G$ are the contributions of traction in the strike direction at the center of all dislocations induced by a unit strike slip displacement of dislocation $l$. Elements $k + 1$ to $2k$ are the traction in the dip direction induced by a unit strike slip in dislocation $l$ and elements $2k + 1$ to $3k$ in the same column are the contributions to the normal traction due to a unit strike slip on $l$. In the same way, column $l + k$ contains the strike, dip and normal tractions induced by a unit dip slip on $l$ and column $l + 2k$ the strike, dip and normal tractions induced by opening on dislocation $l$. This matrix is calculated for all dislocation 1 to $k$:

$$\Delta T = GB \Rightarrow B = G^{-1} \Delta T$$

(3.24)

$\Delta T$ is the vector of change in traction and $B$ the vector displacement of the crack surface. Given $\Delta T$ we use (3.24) to solve for $B$, which is contains the strike slip $b_1^s, \ldots, b_k^s$, the dip slip $b_1^d, \ldots, b_k^d$ and opening $b_1^o, \ldots, b_k^o$ of the dislocations,
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Figure 3.3: Figure shows a patch of area $\Sigma^l$, dipping into the page, on a crack surface approximated by a single dislocation. Left patch is before opening and right is after opening. The traction vector before opening $T^0$ on $\Sigma^l$ indicates shear component. After opening the shear is removed and the stress vector $T^1$ is normal to the patch and equal to the magma pressure at that depth. The vector of opening is $b$, its components are the opening, strike slip and dip slip motion of the dislocation.

respectively. $\Delta T$ is given by:

$$
\Delta T = \begin{pmatrix}
-T^{0,1}_s \\
\vdots \\
-T^{0,k}_s \\
-T^{0,1}_d \\
\vdots \\
-T^{0,k}_d \\
-\Delta P^1 \\
\vdots \\
-\Delta P^k 
\end{pmatrix}
$$

(3.25)

Figure 3.3 shows the stress vectors before and after the crack is filled with magma as well as vector of displacement $b$ for one finite crack surface $\Sigma^l$. 

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3.2.5 Probability distribution and path prediction

To infer where a dike is most likely to propagate we carry out multiple simulations of the preferred path of propagation while varying the stress model input parameters. We calculate the tectonic stress in the same way as Sigmundsson et al. (2015), i.e. using a buried dislocation. We treat it as a two dimensional stress model assuming that stresses close to the surface are independent of depth.

The simulation process goes as follows:

1. Model parameters of a stress model are randomly sampled from plausible ranges.

2. The starting point of the first vertical dyke segment is located in a predetermined x-y coordinates

3. It is rotated around its start point and the strike which minimizes the potential energy is picked.

4. Another segment is added to the end of the previous one and again the most favorable strike is found. This process is repeated for a fixed number of times.

5. Steps 1 to 4 are repeated to generate multiple simulations

Figure 3.4 shows this process in a schematic way.

Figure 3.4: Schematic figure of the dike emplacement process in the each simulations. Segments 1 and 2 have already been emplaced under this simple cone shaped volcano with orientation minimizing the potential energy. Segment 3 is being rotated around end point of segment 2 to determine the most favourable orientation.
3.2 Propagation model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening of buried dislocation</td>
<td>1.0–4.2 m</td>
</tr>
<tr>
<td>Depth to top of buried dislocation</td>
<td>6–8 km</td>
</tr>
<tr>
<td>Strike of buried dislocation</td>
<td>12°–14°</td>
</tr>
<tr>
<td>Horizontal shift of buried dislocation</td>
<td>± 5 km</td>
</tr>
<tr>
<td>Magma overpressure</td>
<td>5–25 MPa</td>
</tr>
</tbody>
</table>

We randomly sample the parameters of the stress model from within a plausible range at the start of each simulation. We assume a uniform probability distribution for each parameter within these ranges. The opening of the buried dislocation is varied within a range from 1.0 m to 4.2 m. This roughly corresponds to the build-up of tectonic stresses in Iceland for 50 to 220 years of plate motion. From seismic monitoring in Iceland we infer that in the last 50 years prior to the 2014 Báðarbunga event no dike has propagated the same way. A fissure eruption fed by the Báðarbunga system is inferred to have occurred between 1794 to 1864 (Hartley and Thordarson, 2013). The 2014 eruption reactivated same craters as the one between 1794 to 1864. It is thus likely that a similar diking event caused that eruption. The center location of the buried dislocation is through the center of the Askja volcanic system, which geodetic measurements have suggested to be the center of the plate boundary in that region of Iceland (Sturkell and Sigmundsson, 2000). We vary this location by ±5 km in longitude to estimate the error in the actual location of the plate boundary. The depth to the top of the dislocation is from 6 km to 8 km. This depth should roughly reflect the depth to the brittle ductile boundary. The brittle-ductile boundary in the Askja area has been suggested to be between 6-8 km (Key et al., 2011; Soosalu et al., 2010). Finally the strike of the dislocation is varied from 12° to 14°. Plate motion models show that the strike of the plate motion vector of the Eurasian plate with respect to the North American plate is at angle of 102° to 104° in Iceland and we assumed the buried dislocation to be perpendicular to that vector. We also vary the dike overpressure from 5 MPa to 25 MPa. These ranges for the parameters are summarized in Table 3.1.

The step size in the simulations is 2 km, looking at smaller step sizes is not reasonable because the bedrock topography has a 0.5 km posting and any interpolation is likely to bias the results.

The starting point of all simulations is fixed where the earthquake swarm exits the caldera. This is the beginning of the first segment identified with relative hypocenter locations presented by Sigmundsson et al. (2015). The first segment in the simulation is forced to have strike between 97° and 157° this is done so the simulation starts in the direction of the propagating swarm which exited the caldera which was at 127°. We allow for a range of about ±30° to test the power of the method. Once
the first segment has been emplaced the next one is added by searching for most the favorable orientation within \( \pm 75^\circ \) of the strike of the previous segment this allows the dike path to have sharp turn as was observed by Sigmundsson et al. (2015). The orientations are evaluated in 50 equally spaced steps. We pick the most favourable strike at random but give weights to the probability of each strike based on how much it deviates from the lowest value. Our weighing discriminates strongly against values that are significantly higher then the lowest value. This is done so that some strike orientations which give values very close to the lowest one can also be picked. More details follow in Section 3.2.6. The stress changes caused by previous segments are considered by calculating the stresses induced by the previous dike tip openings. Because our crack model has stress singularities at the edges while real cracks do not we only calculate the stresses from segments 1 to \( n - 2 \) when the most favorable orientation of segment \( n \) is being determined. We found that including segment \( n - 1 \) gave unrealistic behavior due to stress singularities being too close to the points where the stress was evaluated. One could design a tapered crack that would minimize the influence of these singularities. Such model would be computationally more expensive, however, and the use of dislocation elements that are too small can lead to the \( G \) matrix in equation (3.24) becoming close to singular.

We then evaluate the locations of the center point of each segment of every simulation and how they are distributed in the x-y plane. We form a histogram by binning the frequency of points in a two dimensional grid of 5 km x 5 km. We scale the histogram by taking the frequency of each bin and dividing by the maximum frequency. This means that the bin which had highest frequency gets a value of 1. We do this scaling so that all histograms can be displayed using one colorbar. We assign white color to areas of lower value than 0.05 to reflect that the dike is unlikely to propagate there. In each simulation the total dike length is 50 patches along strike so that each simulation results in 100 km long path prediction. This results in longer paths taken than was observed in the Bárðarbunga dikes. However, for the hindcasting we should assume that we would not know beforehand how long the dike is going to be and thus assume that it might be very long. The extent of the bedrock topographic map is limited, we remove points which fall outside our area of study which contains the 2014 and 1996 Bárðarbunga dikes. The bedrock DEM is displayed in Sigmundsson et al. (2015). The description of the bedrock DEM is given by Björnsson and Einarsson (1990).

The method does not tell us directly where a dike is going to erupt. It does however tell us where it is likely to propagate. It is thus possible to qualitatively estimate where a likely eruption site might be. For example, in the 2014 event the dike erupted in a topographic low facing a steep topographic increase of the Askja volcanic system. The model indicates that it was very unlikely for the dike to continue onwards even though the dike simulations are long enough to do so. When the simulations reach such areas the model becomes trapped; the tip goes back or start showing other erratic behavior. While a real dike would not behave in such a manner, this does
3.3 Results

Constrain our distribution and can be interpreted as areas where the dike is likely to erupt. This behavior happens mostly in topographic lows or where the dike faces uphill topography.

3.2.6 Random sampling of strikes

When the strike that gives greatest energy release is identified there maybe other orientation that give very similar results. Even though one orientation might give slightly lower value than another, this could simply just be an artifact of the resolution of the bedrock topography, or numerical error in the integration of gravitational potential change of the crust. We take this into account by not always picking the strike that gave the minimum but randomly sample with weighted probability to form a probability density function of the dike path. The weight of the i-th element of the strike vector is:

\[ w_i = \exp\left( - \left( a \frac{\Delta U^i_T \Delta U^{\text{min}}_T}{\Delta U^{\text{max}}_T - \Delta U^{\text{min}}_T} \right)^2 \right) \]  

where \( \Delta U^i_T \) is the total energy change for the i-th element of the strike vector, \( \Delta U^{\text{min}}_T \) is the energy change for the strike that minimizes the total potential energy, \( \Delta U^{\text{max}}_T \) is the energy change which maximizes the total potential energy and \( a \) is a constant. The higher the value of \( a \) the more likely it is that the strike with lowest potential is picked. When \( \Delta U^i_T = \Delta U^{\text{min}}_T \) then \( w_i = 1 \) however when \( \Delta U^i_T = \Delta U^{\text{max}}_T \) then \( w_i = e^{-a^2} \). We set \( a = 6 \) this means that when the ratio \( (\Delta U^i_T - \Delta U^{\text{min}}_T)/(\Delta U^{\text{max}}_T - \Delta U^{\text{min}}_T) = 1/4 \) then \( w_i \approx 0.1 \). The quadrant of the strike values that give the lowest potential energy values will thus have significant chances of being picked. While values other strikes have very little chances of being picked.

3.3 Results

3.3.1 Cone shaped topography

We compare our model predictions based on our approach to the classical case of a dike propagating laterally away from a cone shaped volcano.

Figure 3.5 shows the results for a cone with elevation of 1000 m and radius of 10 km which is exposed to 1 MPa of tensional stress along the y-axis.
The model predicts curved dikes which do not align to perpendicular to the regional stress field in the vicinity of the volcanic edifice, without the assumption of local deviatoric stress field. Note that the topography causes local stress field because it changes pressure but not local deviatoric stress field. The deviatoric stress tensor is given by:

$$\sigma'_{ij} = \sigma_{ij} - \delta_{ij}\sigma_{kk}/3$$  \hspace{1cm} (3.27)

where $\sigma_{ij}$ is the absolute stress tensor and $\delta_{ij}\sigma_{kk}/3$ is the pressure component of the absolute stress. It follows that when only pressure contributes to the absolute stress then $\sigma'_{ij} = 0$. In a study by Roman and Jaupart (2014) they found that radial dikes which curve and eventually align to the regional stress field in Spanish Peaks could be explained by stress field changes induced by loading of a volcanic edifice. However, loading stresses, other than pressure, will eventually be, at least partially, released through faulting and fracturing. It is difficult to estimate the remaining deviatoric loading stresses in the brittle crust at time of diking. Our results suggest that topography alone, if all the deviatoric loading stresses have been released, can explain curved dikes in the vicinity of volcanic edifices. The model run presented in Figure 3.5 does not predict dikes that start out radially.
parallel to the least compressive stress of the regional field. However, a steeper volcanic edifice can produce this effect. In Figure 3.5 the dikes travel along almost straight lines under the edifice. This is because the simplified cone topography has constant slope radially away from its center. If we would introduce a gradient in the slope, as is common in shield volcanoes, the dikes would show more curvature.

### 3.3.2 Probability distribution for Bárðarbunga dikes

We compare the probability distribution from our path prediction model to the observed path of the 2014 and 1996 dikes in Figure 3.6. We see that the region of high probability for the dike propagation is in reasonable agreement for the actual dikes in 2014 and 1996, which appear to have exited the caldera in a similar location. It should be noted that the epicenter locations for the 1996 dike are less certain than for the 2014 dike.

We investigate what combination of potential energy terms can best hindcast the parts of the dikes. The only model that can hindcast both dike paths is the one that combines strain energy and gravitational energy change of the crust and the gravitational energy change of the magma ($\theta_c + \phi_c + \phi_f$). To determine if all three energy terms are necessary we consider if other combination of energy terms can hindcast the paths better or equally well as in Figure 3.6. We find (Figure 3.7) that other combinations of these terms cannot, with reasonable accuracy, hindcast the 2014 dike although all predict the 1996 dike.

From Figure 3.7 we conclude that the model best suited for forecasting lateral dike propagation considers all the terms $\Delta \theta_c$, $\Delta \phi_c$ and $\Delta \phi_f$. In panel a we consider only the strain energy term which would generally be considered to be the most important. We find that although it is showing some tendency towards the 2014 dike, it cannot confidently hindcast its path. Including both strain and magma gravitational energy change (panel b) does somewhat increase the tendency for the model to resolve the 2014 path but the observed path does not fall in a high probability area. Considering the strain energy change and the gravitational energy change (panel c) gives rather scattered results and we see that the 2014 dike does propagate outside the area of high probability.

To explore further the importance of different types of stresses we consider the end member case in Figure 3.8 panel a where all stress is in the form of pressure due to topography and the contribution to the stress field from plate motion is none. In panel b we consider the case where the path is purely determined by the plate motion stresses.

In panel a of Figure 3.8 there are no stresses from plate motion considered. Thus the
3 Boundary element crack model

![Figure 3.6: The dike path PDF for the preferred model. Red dots are relative relocations of epicenters associated with the 2014 diking event. The locations are from Sigmundsson et al. (2015). Pink dots are located events from the 1996 unrest and inferred dike propagation to the south, the dashed line is the approximate location of the 1996 Gjálp eruptive fissure. Earthquake locations and fissure for the 1996 event is from Einarsson et al. (1997). The colorbar shows scaled frequency and colored square patches are top view of a 3D histogram. The most probable location gets a value of 1 the least probable 0. All areas with value less than 0.05 are assigned a white color. White squares that are not next to a colored areas are removed. The black triangle is the starting point of all simulations and the starting point of the first segment identified by Sigmundsson et al. (2015). Upper case A, B and G indicate centers of the Askja, Bárðarbunga and Grímsvötn volcanic systems respectively. The four black dots are the corners of the continuous bedrock DEM used, all segments that lie outside this box are removed before binning to form the histogram.]

The propagation model is only governed by topography and pressure changes due to the topography. The path of the 2014 dike is resolved quite well although small portion
3.3 Results

Figure 3.7: The same as Figure 3.6 except hindcasts do not use all three terms ($\Delta\theta_c, \Delta\phi_c, \Delta\phi_f$).

a) Only strain energy change ($\Delta\theta_c$).

b) Strain energy change and gravitational energy change of the magma ($\Delta\theta_c + \Delta\phi_f$).

c) Strain energy change and gravitational energy change of the crust ($\Delta\theta_c + \Delta\phi_c$).

Figure 3.8: The same as Figure 3.6 except the stress model has been changed.

a) No deviatoric stresses assumed in the crust.

b) All influence of topography on the model has been removed and only the plate boundary stress model determines the path.

c) All topography above 500 m.a.s.l. is assumed to be loading the crust, causing both pressure changes and deviatoric stresses.

of it is in an area of low probability. However, the 1996 dike and fissure extend into the a low probability area. Due to the 1996 dike being considerably shorter than
the 2014 we would expect it to be predicted with higher accuracy. From this we can conclude that the 2014 dike was more topography driven. Panel b is the end member where no influence of topography is considered. In this case the model predicts propagation perpendicular to $\sigma_3$. The 1996 dike is well constrained but the 2014 is not hindcasted. Comparison of panels a and b suggests that the 1996 dike released deviotoric stresses in the vicinity of the exit path from the Bárðarbunga caldera and that sent the 2014 dike on a more topography driven route.

We have so far assumed topography to only change pressure in the crust. However in reality loading the crust does also induce deviatoric stresses. These stresses caused by build up of topographic structures decay with time by faulting and fracturing. Panel c considers the end member case where all topography higher than 500 m.a.s.l. is acting as a load on the crust both changing pressure but also inducing deviatoric stresses. We find that this model does not hindcast the 2014 dike. This might suggest the reality, which is somewhere between these two end members, i.e. that all deviatoric loading stresses have been released and no deviatoric loading stresses have been released. Figure 3.6 and Figure 3.8 panel c display these two end member cases. Comparison of the two suggests the end member where all deviatoric loading stresses have been released is a better approximation for the the crust. At least for a very tectonically active area such as the east rift zone of Iceland which we are considering.

3.4 Discussion

3.4.1 The influence of topography

One argument against the influence of gravitational potential energy change on propagation path would be that when the rock in front of a propagating dike tip is fractured, the stresses act first, before any influence from the topography, in determining the fracture orientation.

While it may be true that the rock fractured in front of the propagating dike tip might be largely governed by stresses, as they will be first to act, we cannot claim that this fracture will simply be filled with magma without any alterations. It is also not clear how long this fracture which precedes the magma is or if it even occurs at depth. Let us assume that a significantly long tensile crack is formed in front of the dike before it is filled with magma. As magma starts to fill the crack the fracture surface will be deformed. While a dike is propagating it is unlikely to inflate except at its tip. If the dike has been arrested while still collecting magma. The dike will inflate but mostly at the tip (Irwan et al., 2006; Sigmundsson et al.,
When the dike tip inflates it creates a deformation field. This deformation field requires net transport of material upwards in the gravitational field. This deformation field spreads out with velocity of the same order as seismic waves. Thus, presumably, will act while magma is being transported into the crack. The gravitational energy change can thus alter how the fracture is deformed once it has been created. Furthermore, the equilibrium configuration of the crust will depend on the work of body forces which in this case is the gravitational force. This is because the equilibrium configuration of the crust is indeed where the total potential is at a minimum (Reddy, 2013). How the dike will be accommodated will thus depend on the gravitational potential of the crust. The deformation field spreads out over a large area, not only directly above the dike, this is how variations in topography influence the dike emplacement. The opening of the dike is also dependent on both pressure and deviatoric stresses and larger opening is directly proportional to increase in gravitational energy change.

Furthermore, the topography also influences the stress field by varying the lithostatic pressure. A dike that propagates through high lithostatic pressure requires more energy than one which propagates through less pressure. Even if it were true that the gravitational energy change of the crust could be ignored topography would still have an effect through pressure changes.

One could argue that for a dike that travels along level of neutral buoyancy the variations in lithostatic pressure should not matter. This is not completely true, although the lithostatic pressure remains constant the magma pressure changes depend on the topography. This changes the expansion of the dike. The expansion of the dike will reflect how much strain energy is released.

Hjartardottir and Einarsson (2015) showed that fracture density in rift zones decreases greatly in the vicinity of shield volcanoes in Iceland. This indicates that such structures might act as barriers for dikes or even arrest them. Sigmundsson et al. (2015) found that the dike halted for 4 days in a topographic low where it met steep topographic increase. It still inflated at the tip and until it could break through the barrier. This furthermore suggests dike propagation is influenced by topography. Even though our model does not say anything about the changes in propagation speed it still predicts that the dike will try to bypass such structures, if possible, and the opening of the tip to correlate negatively with the elevation of the topography (see Figure 3.1). This corresponds well with the findings of Pollard and Muller (1976) which suggest the increase in lithostatic pressure or decrease in magma pressure does restrict the opening of the dike. If the opening is restricted this will influence the fluid flow, which can arrest the dike and allow pressure build up. During that process new fractures might form and the dike can find an alternative path, erupt, or eventually break through the barrier. From this we suggest that topographic lows are plausible eruption site for laterally propagating dikes. If the dike is propagating at LNB pressure must build in the tip in order for the dike to
move upwards promoting fractures to propagate closer to the surface. It should be noted that whether or not the dike requires pressure build-up to overcome the low will depend on the overpressure and how or if the overpressure has decayed during the propagation.

### 3.4.2 Propagation depth

Here we assumed a depth range for the dike based on what was inferred from geodetic modelling. The LNB model we have considered here is a simplification. The depth and depth range of laterally propagating dikes is likely to be governed by multiple processes and factors that are very difficult to estimate for a volcano. These include density changes, changes in material properties, previous dike emplacements, stress history and viscoelastic relaxation. To be able to present a model that can predict dike propagation path based on easily observable factor we need to assume that all these processes are secondary to the influence of the path and we can simply assume the dike to be propagating at fixed depth below the topography. Our agreement with observations strengthens this assumption.

### 3.4.3 Modelling dike arrest

As shown by (Dahm, 2000) and (Maccaferri et al., 2011, 2014) determining the arrest where the dike might stop is possible by considering when the energy release does not exceed specific fracture energy threshold. However modelling a dike growing in mass, as we have done, and determining the arrest is more complicated. This is because the energy release will depend on the overpressure and how the overpressure decays with time. To model this process accurately requires the consideration a coupled system of a dike and a magma chamber feeding the dike. If we were to apply the model presented here for forecasting we would not know beforehand how the overpressure decays. It is therefore not reasonable to include such considerations of arrest for this purpose due to the extremely variable possible decay rates. It should be noted that the energy release for the crack tip in our preferred model at most favorable strike was generally on the order of 500 – 1500 TJ.

### 3.5 Conclusions

We have developed a method for forecasting the path of laterally propagating dikes given the starting point of propagation. The method is computationally feasible and
can be used to form a probability distribution for dike location in near-real time. We have compared our model to the propagation of the 2014 and 1996 dikes in the Bárðarbunga volcanic system and found that the agreement between predicted and observed propagation was reasonable.

Our results suggest that topography plays an important role in determining the path of laterally propagating dikes. It influences the pressure changes in the dike and thus the strain energy change, the gravitational energy change of the crust which results from dike opening, and the gravitational energy change of the magma. We found that the all the three terms that we considered combined give the best results. We found that to fully explain both 2014 and 1996 dike the stress model had to take into account deviatoric stress field from plate motion and pressure changes from topography where required. It should be noted that a model which only included pressure changes from topography could almost explain the observations satisfactorily. It appears that the 1996 dike was more dominated by devitoric stresses due to plate motion while the 2014 dike was topography dominated. We suggest that the 1996 dike released deviotoric stresses in the vicinity of the caldera and thus the 2014 dikes was directed on a path which is better explained by topography.

We found that curved dikes around a cone shaped topography can be explained by topography alone without the assumption of local deviatoric stress field. This agrees with the observation that radial dikes are not always connected to a central reservoir. This has been explained by rotation of deviatoric stresses due to the loading of an volcanic edifice (Roman and Jaupart, 2014). We, however, point out that such deviatoric stresses are not permanent as they are eventually, at least partially, released through faulting, while pressure change remains. Nevertheless, it is likely that such dikes are influenced by both local deviatoric stress and topography. We found a model that assumed the topography around Bárðarbunga to be act as a load on the crust inducing both deviatoric and pressure changes showed significantly less agreement with observations than a model that only considers pressure changes from topography.

We suggest that laterally propagating dikes are most likely to erupt in topographic lows. In topographic lows the magma pressure is highest and thus the dike will be more inflated in the lows. To propagate out of a low the dike will need to inflate even more and build up pressure. This furthermore promotes the propagation of fractures towards the surface.
4 Summary

Although the methods in Chapter 2 and 3 are quite different they seem to be in agreement with one another. Both found that gravitational energy change of the crust was necessary to explain the observations and both indicate two main branches of propagation given the starting point. One which agrees with the 1996 dike and another one that agrees with the 2014 dike. No discrepancy is between the two models and both models provide compelling evidence that gravitational energy change of the crust does influence dike propagation. That is a factor that has usually not been considered before even by author applying similar methods (Dahm, 2000; Maccaferri et al., 2011, 2014).

Although deviatoric stresses are important to determine the orientation of a laterally propagating dike our results suggest that dikes are influenced by other factors as well. The most important factor other than deviatoric stresses is quite possibly topography. The evidence that pressure changes caused by topography on the shape of intrusions is quite compelling. We can thus expect it might influence the propagation path. This thesis presents evidence, supported by observation, that the path is indeed influenced by topography.

The method in Chapter 2 was essentially the simplest approach to testing the influence of topography on the strike changes in the 2014 Bárðarbunga dike. This method was then improved and the boundary element crack model was the result of those improvements and was presented in Chapter 3. Not only does it make the model more physically realistic but also computationally less expensive. Due to the fact that it’s computationally much faster allows us to account for the uncertainty in the stress model and approach the problem of forecasting the path in terms of probabilities. We furthermore could easily test what the influence of topography versus strain was and from that conclude that the 1996 Bárðarbunga dike was mostly governed by deviatoric stresses and the 2014 dike, at least during the first kilometers of propagation, by topography.
Bibliography


