Toward a cognitive historiography of mathematics education

Iason Kastanis* and Nikos Kastanis**
* Universidad de Barcelona. Spain
** Aristotle University of Thessaloniki. Greece

Abstract

In recent years, the cognitive approach to mathematics education and the history of mathematics has seen a rapid development. This development favors the emergence of a new interest in the historiography of mathematical teaching and learning. Here we provide a motivation for the introduction of this approach into the history-oriented approach to mathematics education. We shall concentrate on the concepts of "practices of knowledge" and "cultural practice" as employed in relevant mathematical and scientific studies, aiming at showing their meaning and role in the renewed historiography of mathematical teaching and learning.

Introduction

The cognitive approach to Mathematics Education is nowadays at the very core of educational processes and studies in school mathematics. Two indicative manifestations of this realization are found in:

1. Schoenfeld, A. H. (ed.): *Cognitive science and mathematics education* (1987)
2. Greeno, J. G., Collins, A., & Resnick, L. B.: Cognition and learning (1996).

The cognitive viewpoint, according to James Greeno, "emphasizes conceptual structures and representations of information" (Greeno, p. 116), as well as "conceptual changes" and "discourses of practice" (*ibid*, p. 85). And this viewpoint has influenced, in recent decades, several areas of scientific thought. It actually expresses one of the present trends in Epistemology. In other words, the understanding and the analysis of scientific thought have nowadays as one of their theoretical bases a new kind of epistemology, the Cognitive Epistemology.

This new epistemology has not left untouched the historiography of mathematics. And this can be easily seen through the titles of some relatively recent publications, for example

1. Netz, R.: *The shaping of deduction in Greek mathematics: A Study in cognitive History* (1999)
2. Borzacchini, L.: Incommensurability, music and continuum: A cognitive approach, (2007)
3. Chrisomalis S.: The cognitive and cultural foundations of numbers (2008).

These approaches emphasize the conceptual understanding of the emergence, organization, representation, and change of mathematical knowledge, departing from the corresponding historical and cultural contexts. Netz in particular has perceptively focused on the "practices of knowledge", enriching and deepening the cognitive orientation of the historiography of mathematics.

It is clear that practices form a contemporary component of the cognitive approach in mathematics. And this component adds a new dimension to the history-oriented understanding of mathematics and also a more refined historiographical metaknowledge. Further, the application of this modern viewpoint to specific historical studies leads to the development of a new kind of historical awareness of mathematics.

Within this momentum, it is natural for the historiography of mathematics education to be challenged toward an analogous modernization. What impact could such a challenge have?

The *practice* in the history of mathematics

During a conference on New Trends in the History and Philosophy of Mathematics that took place in August 1998 in Roskilde, Denmark, the goal was the promotion of new ideas and approaches to the historiography of mathematics. And the expectations on the richness of new approaches towards the historiography of mathematics were fulfilled in 2004, through the publication of the conference proceedings. Another allusion in this direction is made in the introduction to the Proceedings, pointing toward the thesis of Moritz Epple:

> it is the weave of concrete scientific action rather than an abstract life of mathematical ideas that historians need to analyse in a detailed and realistic way if they wish to adhere to the goal of topology [*or any other mathematical knowledge*, I.K./N.K.] (Kjeldsen, et al, p. 13).

As Epple further clarifies:

> I take the practice of mathematical argumentation to be a complex of actions, such as defining, conjecturing, proving, etc. (These *mathematical actions* are immersed in communicative and *social actions* like publishing, giving talks, applying for positions, organizing meetings, and the like.) (Epple, 1997, p. 183).

Through these remarks, the practices of mathematics emerge as a contemporary focal point in the historiography of mathematics. This is a new approach to the history of mathematics, not limited to methodological and theoretical declarations, but beginning to be applied to various case studies of history-oriented investigations. Some examples of pertinent publications are the following:

1. Epple, M.: Knot Invariants in Vienna and Princeton during the 1920s: Epistemic Configurations of Mathematical Research (2004)
2. Jones, M. L.: *The Good Life in the Scientific Revolution: Descartes, Pascal, Leibniz and the Cultivation of Virtue* (2006)
3. Kjeldsen, T. H.: Egg-Forms and Measure-Bodies: Different Mathematical Practices in the Early History of the Modern Theory of Convexity (2009a).

This historiographical approach aims at the illumination of historical choices, changes, and adoptions (or rejections) of practices of mathematics in the respective cultural, social, and institutional contexts.

Concerning this historical orientation, Tinne Kjeldsen remarks:

> At present the general opinion among historians of mathematics is that … on concrete practices of mathematics, acknowledging that … mathematical

> knowledge is produced by mathematicians who live, interact, and communicate in concrete social settings.
> Through mathematicians' activities, mathematical ideas and knowledge emerge and develop at local places and in specific intellectual contexts and time. Problems, concepts, definitions, and proofs emerge, develop, and change through mathematical activities. To understand how this happens, it is productive on the one hand to investigate how and why mathematicians have introduced certain concepts and definitions, and employed particular strategies of proof; and on the other hand to identify changes in the understanding of mathematical entities, notions, and approaches (Kjeldsen, 2008, p. 756).

And she adds:

> In this line of thinking, mathematics is viewed as a cultural and social phenomenon, despite its universal character [that of a strongly Platonic view, (*which encounters*) mathematics as an autonomous science with unchanging eternal subject-matter to be gradually uncovered over the course of time] (ibid., p. 67).

In their history-oriented investigations, Epple and Kjeldsen analyzed the distinct research practices that led to new theories of modern mathematics – that is, the simultaneous research practices that promoted knot theory, or the theory of convex sets, or nonlinear programming. They noticed the local character of the respective research activities that expresses the local traditions of mathematical practices, that is the distinct mathematical cultures.

On the other hand, Matthew Jones illuminated, in his book (Jones, 2006), the moral and ethical aspects of the different mathematical practices of three key early modern scientists: Descartes, Pascal, and Leibniz. He showed that these three prominent seventeenth century intellectuals developed and articulated their cognitive practices in improving scientific and mathematical techniques and thinking as useful for cultivating virtue and for pursuing a good life. He discerned that their scientific and philosophical innovations related with their understanding of mathematics and science as cognitive and spiritual exercises that could create their authentic mental and spiritual nobility. An aspect of Jones' study is the connection between the new mathematical techniques and the discursive practices, influenced from the Christian spirituality, rhetorical strategies, and pedagogy of the time. It is worth pointing out that this historical connection of practices of mathematics with pedagogical contexts opens a new historiographical prospect for the practices of mathematics education.

Even though these cases have a common historiographical origin, the mathematical practices, they differ with respect to methodological choices. Epple and Kjeldsen focused on the differentiations among research activities and behaviors with respect to mathematical meanings or objects, as well as mathematical tools or techniques. On the contrary, Jones followed a different methodological way. He "places himself firmly within the ranks of historians of science concerned with the myriad practices involved in the social construction of knowledge" (Whitmer, 2007). This means that his historical analysis was sensitive to the connection between science practices and contexts of any given historical period. More specifically, Jones (2006) focused on the discursive practices that were developed within the broader cultural and cognitive dimensions of seventeenth century mathematics.

There is no doubt that these historiographical attempts[1] promote a refreshing perspective for a new understanding of historical changes and tendencies of mathematical culture.

An insight into this history-oriented approach to mathematics would be facilitated by certain observations on its cognitive background.

The *practice* as a term in the historiography of science

During the post-positivist period of epistemology and historiography of science, Thomas Kuhn's ideas played, undoubtedly, a crucial role in the reforming tendencies toward the understanding of scientific knowledge and thought in the Western world. These ideas opened a new horizon in the study of, and reflection upon, the scientific development.

Along these lines, the cognitive and social dimensions of historical evolutions of the scientific trends and communities gained a strong momentum. Directly or indirectly, Kuhn and his followers "reorient the philosophy of science toward an account of scientific practices rather than scientific knowledge" (Rouge, J. 2003, 115-116). And this underlying component emerged, in the early 80's, within the movement of Sociology of Scientific Knowledge, which was and still is akin to Kuhnian approach. The concept of scientific practice acquired thus a central position in the more recent historical and epistemological studies; this did not leave untouched the movement of Cognitive Science or the corresponding educational tendencies (Nersessian, 2003; Lave, 1988; Lave & Wenger, 1991).

Under this perspective, the cognitive history of science is presented as "the investigation of the *thinking practices through which scientists create, change, and communicate their representations of nature* and examines *the cognitive tools scientists employ and the artifacts they construct in theoretical and experimental thinking practices*. An aspect of scientific practice that is at once shown to be relevant for cognitive history is the use of cognitive tools" (Heinz, 2007, p. 336; Nersessian, 1995, p. 194). The practice-oriented investigation of scientific knowledge tended to emphasize the manner in which scientists "do" things, and thus intervention and experimentation were studied more intensely than the production of propositional or theoretical knowledge.

The realization that every scientific practice possesses both individual and contextual features leads to corresponding components in the studies of cognitive history of science. So, on the one hand the scientists are seen as having interests,

[1] Here are some related publications: (Alexander, 1995); (Alexander, 2006); (Bernard, 2003); (Hill, 1998); (Jesseph, 1989); (Johnston, 1994); (Kutz, 1997); (Mazzotti, 1998); (Neal, 2003); (Netz, 1999); (Netz, 2004).

beliefs, motivations, choices, aims and skills, and on the other hand their activities are seen as socially and culturally loaded endeavors. This opens up the possibility toward the development of historico-psychological approaches to the scientific practices of any given era, as well as socioculturally localized studies on them.

The historico-psychological viewpoint, for example, leads to the investigation of the interplay between the case studies of historical scientific practices and the corresponding problem-solving ways of thinking, human reasoning and representations (Nersessian, 1995, p. 195). And the sociocultural viewpoint calls for an analysis of scientific activities within the cultural and social environment of a specific historical period and region. Under the unifying perspective of cognitive historiography "the scientific practices can be investigated at different levels of analysis: at the level of researchers as individual, embodied, social, tool-using agents; at the level of groups of such researchers; at the level of the material and conceptual artifacts comprising the context of activities, such as laboratory research; and as various combinations of these" (Nersessian, 2008, p. 9).

The development of new ways of thinking signifies either the creation of new ideas or the assignment of new meanings to already established concepts. These conceptual changes are closely related to new cognitive tools like a novel scientific instrument or a novel theoretical method. And the promotion or cultivation of these novel scientific practices is impossible to materialize in the absence of new social needs, new motives, new demand, and new collective backings. For example, the development of linear perspective in the fifteenth century emerged as a novel esthetic and geometric representation of space that was required not only by the reforming tendencies and sensibilities of certain social groups, but also by the new representational demands of the scientific needs of the times (like cartography). In this particular case, some older ideas and techniques of stenography and optics were recontextualized and reorganized, both conceptually and methodologically. It appears that this whole change took place neither spontaneously nor arbitrarily: it was cultivated within the abacus culture (Kuhn, 1990, p. 119; Camerota, 2006; Camerota, 2004; Davis, 1977, pp. 16-18) and was perceived and promoted by the Roman Catholic Church (Woods, 2005, pp. 130-132)[2], as well as by networks of interested actors and scientists. Besides contextual motives and backings, this new artistic and geometric practice developed and employed new tools: conceptual tools, like grid frameworks, and material tools, like perspectographs. It is worth mentioning that through this scientific dynamic a new cognitive means emerged: the idea of system of reference and the method of coordinates. That was a new cognitive tool, echoing broader social and cultural tendencies of Early Renaissance, like cartography & navigation or validity and verification of paper money, which subsequently played a most crucial role in the renewal of geometrical thought.

[2] By contrast, the Protestants rejected it for some time, and the Orthodox reject it to this day, (Andersen, 2007, p. 489; Florensky, 1992).

The example of abacus mathematics is a very good historical case that can illuminate the historical dimensions of mathematical practice. A strong boost to this endeavor is provided by the following two passages from the history-oriented activity of Jens Høyrup:

> The norm system which governed the practice of abacus mathematics was not identical with that of Greek-inspired Humanist and university mathematics, and could not be already because the practices they governed were different in spite of similarities" (Høyrup, 2007a, p. 24).
>
> As early as [1900: 166], it is true, Cantor had spoken of the existence throughout the fourteenth century of two coexisting "schools" of mathematics, one "geistlich" ("clerical", that is, universitarian), the other "weltlich oder kaufmännisch" ("secular or commercial", supposedly derived from Leonardo Fibonacci's work) (Høyrup, 2007b, p. 3).

The emphasis on the differentiation between the norm systems associated with these two kinds of mathematical culture during the Later Middle Ages and the Renaissance points to the differentiation between the corresponding institutional and cultural norms. This further suggests that the nature of mathematical knowledge and practice, as well as the standards or the ideals, in the corresponding choices, representations, manipulations (or procedures), and the ways of justification or proof are treated differently. This divergence reflects on different necessities, preferences, and mentalities, as well as on different forms of organization and reproduction of mathematical knowledge.

The social norms and cultural forms affect not only the organizational contexts of the handling and promotion of the scientific conventions of a given historical period, but also the issues of legitimacy of the cognitive and functional structures of the corresponding scientific practice. Gert Schubring has characteristically made the following remark:

> Actually, the emergence of a new discipline entails its institutionalisation, since […] a scientific discipline was understood as a mainly university-based social subsystems, constituted by a stock of theoretical knowledge, a plurality of problematical questions supported by a paradigm, and both a set of research methods on the one hand and a scientific community with specific career patterns and socialization procedures and a proper clientele on the other.
>
> […] 'Acceptance' of a new discipline thus implies that is has to prove its legitimacy, its agreeing with the overall values of the embracing system. These values use not be universal, but culturally specific (Schubring, 2003, p. 1049).

David Rowe claimed something similar, pointing out that:

> [the] mathematical knowledge depends on numerous contextual factors that have dramatically affected the meanings and significance attached to it. Reaching such a contextualized understanding of mathematical knowledge, however, implies taking into account the variety of *activities* that produce it, an approach that necessarily deflects attention from the finished products as such - including the 'great works of the masters' - in order to make sense of boarder realms of 'mathematical experience'.

> […] [A]n actor-oriented, realistic approach that takes mathematical ideas and their concrete contexts seriously offers a way to bridge the gulf that divides these two camps. Such an approach can take many forms and guises, but all share the premise that the type of knowledge mathematicians have produced has depended heavily on cultural, political, and institutional factors that shaped the various environments in which they have worked" (Rowe, 2002, pp. 113-114).

The institutionalizations of scientific practices incorporate the social norms and cultural values into the cognitive, ideological, and professional goals of certain social groups. These norms and values are also diffused into the corresponding institutionalized infrastructures of assessment and attitude, which is into the collective conventions of recognition, validity, and acceptance of certain scientific knowledge and activities or the underestimation, disapproval, and rejection of certain others. This setting leads to the cultivation and establishment of diverse scientific discourses, diverse groups of practitioners or scientific communities, and diverse contexts of professionalization of scientists.

A remark on the professional context of mathematicians in the Later Middle Ages and the Renaissance was made by Mario Biagioli:

> The pattern of mathematical disciplines and professions existing inside and outside the university seems to have been quite stable in Italy during the later Middle Ages. As shown by the organization of the teaching of mathematics at Bologna and in other Italian cities, there were two disciplinary and professional types of mathematical practitioners who […] remained socially distinct. In the first group we find the book-keepers, the land surveyors, and the engineers-masons. Their professional culture was represented by the chairs 'ad arithmeticam et geometriam' and by the public teaching of the abacus. Socially and professionally distinct from these practitioners we find the astrologer-physicians represented by the chairs 'ad astrologiam' (Biagioli, 1989, pp. 42-43).

The formation of diverse structures of scientific occupation echoes respective social standards and may be viewed as a process of professional normalization. And such normative procedures do not only constrain, but they also generate new objects of knowledge and forms of identity. So in the case of abacus practitioners, "the content and social consistency of the 'culture of abacus' was quite homogeneous"(*ibid*, p. 43), that is, they had a professional and scientific identity, that of the abacist. This identity of the abacists expresses their historical and social legitimacy as mathematical practitioners, that is, as craftsmen (*ibid*, 56). As craftsmen they formed a social group distinct from that of university scholars. And, "from the mid-seventeenth century onwards this practical tradition gradually ceased to be separate from the learned one" (Field, 2005, p. 324).

The dominant feature of the world of the abacus and the cohesive identity of the abacists in particular is seen mainly in their special mathematical discourse and their specific means of activities. It is a fact that the abacists' discourse was distinct from the learned discourse of the universities of the same era (O'Halloran, 2005, p. 34). It was a folk mathematical discourse dominated by calculation issues and procedures of commercial or other practical content (Van Egmond, 1980, p. 15; Swetz, 1987, p. 21). On the other hand, the means of their activity differed from the theorizing scholasticism of the mathematical tradition of the universities of the time. They employed practical examples as the basic means of presentation and communication of mathematical knowledge and methods. Abacus mathematics may be characterized as a culturally located practice in mathematics. In other words, it expressed a specific practice of mathematical knowledge that materialized within specific cultural and institutional contexts. And the world of abacus echoed a specific community of mathematical practice, that is, a specific culture of mathematical practice.

More generally, the practices of scientific knowledge and the cultures of scientific practice were, during the recent decades, the focus of the cognitive-historical and cognitive-educational studies (Shapin, 1994; Biagioli, 1994; Golinski, 1990; Hackett et al, 2008). A very characteristic example is the historiographical emphasis on the practices of scientific knowledge given by Steven Shapin, who, in one of his influential studies, stressed from the beginning:

> A *Social History of Truth* is concerned with questions about the grounds of scientific knowledge […] I have followed the questions, and the resources for addressing them, wherever they happened to be in academic culture. My purpose is […] to assist in the reconstitution of what might count as historical practice, and to make a theoretically driven detailed historical narrative the sort of thing which engages the attention of philosophers and sociologists because they recognize it as a significant way of dealing with their problems (Shapin, 1994, p. xv).

And he referred that

> the practice devoted to interpreting historical action in historical actors' terms (*ibid*, p. xvi).

An analogous position was adopted by Nancy Nersessian in her book *Creating Scientific Concepts*, pointing out in her introduction:

> The method used in my analysis is thus "cognitive-historical," drawing on cognitive science research to understand the basis of the scientific practices, and reflecting back into cognitive science many considerations that arise in analyzing scientific problem solving (Nersessian, 2008, p. xi).

It is worth noticing that the idea of *scientific practices* was promoted by the contemporary movement of *Sociology of Scientific Knowledge*. It would nonetheless be very unfair to overlook other historiographical and epistemological movements that developed and cultivated a similar idea. One such case is the *sociohistorical movement of activity theory* (Radford, 1997; Stepin, 2005). An analogous interest also emerged through the hermeneutic considerations (Bernstein 1983; Lynch & Woolgar, 1988), as well as the pragmatistic or neo-pragmatistic approaches (Fujimura, Star, & Gerson, 1987) to the history of science.

The *practice* as means in the historiography of mathematics education

Naturally, this contemporary historiography of science was not overlooked by other cognitive fields of study and research, such as, for example, the history of economy (Weintraub, 2001), of literature (Laden, 2004; Mazzio, 2009), of medicine (Shapin, 2000), of education (Shapin & Barnes, 1976). Among these, the historiography of science and mathematics education seems to be directly related to the historiography of science, as well as the historiography of education.

In relation to this second component, it is noticed that, in contrast to the history of science, the practices of knowledge do not appear to have an analogous development in the historiography of education[3]. Nonetheless, there are some signs of awareness. One such example is Derrick Armstrong's paper "Historical voices: philosophical idealism and the methodology of 'voices' in the history of education" (Armstrong, 2003), where it is pointed out that

> Historical explanation may be understood, from the poststructuralist perspective, as a knowledge practice which reproduces power, legitimized by the 'authority' of academic discourse. […]
>
> [The poststructuralist theorizing] has encouraged a view of educational history as a politicized cultural practice concerned with the making and remaking of power/knowledge discourses (Armstrong, 2003, p. 212, p. 216).

It has actually been noted that, in recent years, this approach began to be of broader appeal to education researchers. And this fact is corroborated by the recent collective monograph "*Educational Research: the Educationalization of Social Problems*", edited by Paul Smeyers and Marc Depaepe, where the concept of practices has permeated all its essays. This climate began to affect, more and more, the modern historical approaches to education. Related studies began to emphasize the interweaving of pedagogical practices with discursive practices and cognitive

---

[3] For an overview of this issue one may point at the positions of the following three education historians:

1. Léon, (1985), where no allusion is made to the practices of knowledge or, more generally, to cognitive approaches.
2. *History of Education in the Postmodern Era*, special issue of the *Paedacogica Historica* [32(2), 1996, pp. 301-450], where some authors occasionally touch upon related methodological issues. For example, H.-E Tenorth stresses the terms Praxis and pädagogischer Praxis relating them to the historiographical ideas of Thomas Kuhn, while S. Cohen points to the new cultural history and the cultural practices connecting them to the epistemology of Michel Foucault.
3. Popkewitz, T.S. et al, (2001), where there are certain vague insights toward new cultural history, with no focus on any solid historiographical direction centered on cultural or cognitive practices.

changes in their social and cultural contexts (Casale, 2004; Ashby & Gordon, 2005; Depaepe, 2007; Milewski, in print 2009).

An analogous tendency has been cultivated, during the recent decades, within the historiography and the history of science and mathematics education through the insights of Gert Schubring, Kathryn Olesko, Lewis Pyenson, Bruno Belhoste, Hélène Gispert, Andrew Warwick, and David Kaiser[4]. It is worth pointing out certain characteristics of this dynamic, focusing our attention to the work of Andrew Warwick, who is inspired by this contemporary tendency of the historiography of science and science education, applied at the investigation of the emergence of mathematical physics in the context of Cambridge University in the nineteenth century.

Warwick's book *Masters of Theory, Cambridge and the Rise of Mathematical Physics*, may be viewed as a milestone in the historiography of mathematics: it makes a turn from the standard perception of "focusing mainly on the history of mathematical innovation[s]" (Warwick, 2003, p. 11) toward a contemporary social and cultural approach centered mainly around mathematical practices and their dependence upon local pedagogical contexts. It is about a pedagogical historiography of mathematics and mathematical physics, promoting the change of the scientific and educational tradition of Cambridge University and the promotion of educational legitimacy and cultivation of mathematical physics and relative theory in the British scientific culture during the period 1820-1920. Through this history-oriented analysis, Warwick shows "how a system of values that holds in the mathematics and science education, the institutional conditions of pedagogical practices, the coordination of secondary and higher education, and the assignation of social prestige to the new scientific careers and to the science teaching led to transform the state of a local scientific culture" (Olesko, 2005, 2p. 79).

The historiographical means employed in Warwick's monograph were drawn mainly from the epistemological ideas developed by Thomas Kuhn, Michel Foucault, and the movement of the Sociology of Scientific Knowledge. Very characteristically, Kuhn's influence lies in his claim that "scientific knowledge consisted in a shared collection of craft-like skill learned through the mastery of canonical problem solutions" (Warwick, 2003, pp. 3-4), as well as his influential concepts of "paradigms" and "normal science" (*ibid*, p. 172). From Foucault Warwick borrowed the view that "regimes of institutionalized training introduced in the decades around 1800 found new productive capacities in those subjected to its rigors, and imposed a new pedagogical order on scientific knowledge" (*ibid*, p. 4). And from the movement of the Sociology of Scientific Knowledge he got the concept of "practice" (*ibid*, p. 12).

Two historiographical remarks in Warwick's monograph are of particular interest for the historiography of mathematics education. The first one refers to

[4] Some rather enlightening papers are the following: (Belhoste, 1998); (Schubring, 2001); (Warwick & Kaiser, 2005); and (Olesko, 2006).

the "pedagogical revolution" by the development and impact of such pedagogical devices as "face-to-face training in problem solving on paper, written examinations, educationally orientated treatises, and end-of-chapter exercises, and by listening to the comments of tutors and students" (*ibid*, p. 38).

The following question arises spontaneously: when did the use of exercises in the textbooks of various countries or cultural communities begin, and how and why did it spread? This is a question on the context of the history of mathematical education and its importance lies in its potential to illuminate a historical change in the pedagogical practices of mathematics. A similar question arises about the history of the appearance and development of pedagogical examples in the teaching and the textbooks of mathematics education. The historical value of these questions becomes even more transparent in view of the fact that Euclid's *Elements*, a model of mathematical teaching and learning for many centuries and for many civilizations, contain neither exercises nor examples.

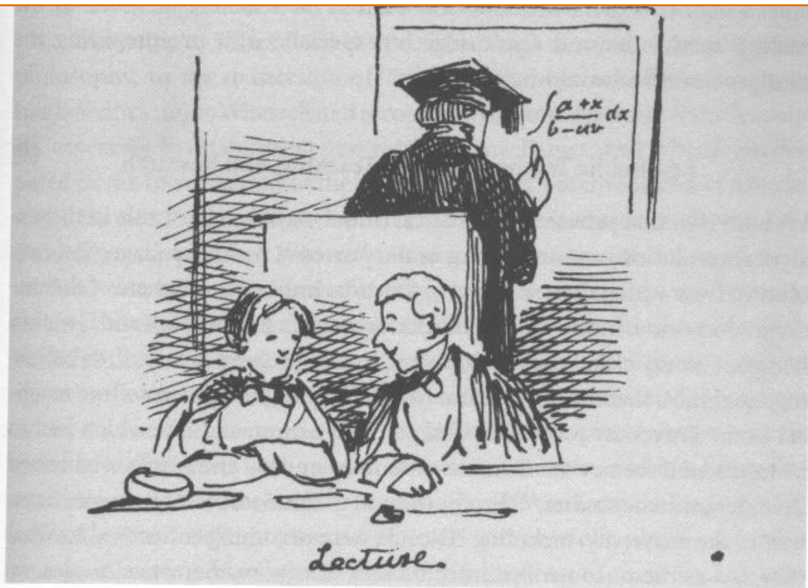

FIGURE 2.5. This sketch by J. L. Roget (2SO 1850) illustrates the relative unimportance of college lectures (as compared with private tuition). The sketch also provides a rare illustration of a blackboard, which was a fairly recent pedagogical innovation at this time (chapter 5). Roget, *Cambridge Customs and Costumes* (1851). (By permission the Syndics of Cambridge University Library.)

Warwick's second remark concerns the appearance of blackboard in teaching (see, also, Kidwell, et al, 2008, ch. 2). He presented a related 1850 cartoon and made the following allusion in his caption:

> This sketch by J. L. Roger (1850) illustrates the relative unimportance of college lecturers (as compared with private tuition). The sketch also provides

> a rare illustration of a blackboard, which was a fairly recent pedagogical innovation at this time (*ibid*, p. 93).

This naturally generates related questions: which pedagogical necessities or pedagogical theories motivated the idea of blackboard? How did this pedagogical tool spread to various countries? Was it first used in the teaching of mathematics? Or did it come from other pedagogical needs, expanding subsequently to mathematics and other subjects?

Our survey of the historiographical approach employed by Warwick in the monograph discussed here makes it clear that the monograph in question is dominated by the spirit of historiography of practices of knowledge, which according to Netz constitutes a cognitive history of mathematics. And since the subject of Warwick's monograph is a case study related, to a degree, to the history of mathematics education, it may be viewed as an example of an application of cognitive historiography to mathematics education.

Very close to this kind of historiography are the case studies in the history of mathematics education by Gert Schubring (Schubring, 1989; Schubring, 2005) and Lewis Pyenson (Pyenson, 1979; Pyenson, 1983).[5] For they do systematically employ the institutional, epistemological, and cultural approaches that are shared with the contemporary cognitive historiography of mathematics. But they are not identical in spirit with it, due to the absence from their toolbox of the practices of knowledge.

It is worth noting that the theoretical background of the cognitive historiography of mathematics and, more generally, science, is in need of a deeper and broader effort in order to interweave its two components, that of Sociology of Scientific Knowledge and that of Cognitive Science. The asynchrony between these two components has recently been mentioned by Nersessian, who points out that "sociologists and anthropologists have shown the fruitfulness of studying research laboratories and groups for thinking about the cultures of science, but scarcely any research has looked at cognitive practices … or with the problem of integration in mind" (Nersessian, 2008, p. 206). Such a development is going to be a result of both the meta-theoretical insights into the historiographical domain of sciences and the expansions and refinements of the case studies of the cognitive history of science, as well as the cognitive-historical analyses of issues of mathematics education.

Conclusion

Through this investigation of the cognitive historiography of mathematics and science we point to certain aspects of a new way of understanding their development through the ages. We also attempt to extend somewhat this historiographical orientation to the domain of history of mathematics education.

[5] See, also, (Miguel, 2005).

According to all indicators, the prospects of this trend, in general and in mathematics education in particular, are fruitful.

In this fashion we promote a new understanding of the ways of development of active mathematical conduct and conventions, as well as of sociopolitical conditions and their influences. This new understanding departs from the older historical perceptions that were usually focused on descriptions of absolutist ideas and statements by eminent scientists or educators, in combination with educational mechanisms of realizing such goals.

This new perspective on history of mathematics and mathematics education is fully compatible with social constructivism, nowadays relevant in the epistemology of mathematics and mathematics education.

References

Alexander, A. R. (1995), The imperialist space of Elizabethan mathematics, *Studies in History and Philosophy of Science, 26*, 559-591.

Alexander, A. R. (2006), Tragic mathematics. Romantic narratives and the refounding of mathematics in the early nineteenth century, *Isis, 97*, 714-726.

Andersen, K. (2007). *The geometry of an art. The history of the mathematical theory of perspective from Alberti to Monge,* New York: Springer.

Armstrong, D. (2003). Historical voices: philosophical idealism and the methodology of 'voices' in the history of education, *History of Education, 32*(2), 201-217.

Ashby, R., & Gordon, P. (2005). *Understanding history. Recent research in history education. International Review of History Education*. Vol. 4. London, New York: Routledge Falmer.

Belhoste, B. (1998). Pour une réévaluation du role de l'enseignement dans l'histoire des mathématiques. *Revue d'Histoire des Mathématiques, 4,* 289-304.

Bernard, A. (2003). Ancient rhetoric and Greek mathematics: A response to a modern historiographical dilemma, *Science in Context, 16*(3), 391-412.

Bernstein, R. J. (1983). *Beyond objectivism and relativism: Science, hermeneutics, and praxis*, Philadelphia: University of Pennsylvania Press.

Biagioli, M. (1989). The social status of Italian mathematicians, 1450-1600, *History of Science*, 27, 41-95.

Biagioli, M. (1994). *Galileo, Courtier: The practice of science in the culture of absolutism*, Chicago, IL - London: University of Chicago Press.

Borzacchini, L. (2997). Incommensurability, music and continuum: A Cognitive Approach*, Archives for the History of Exact Sciences, 61*, 273–302,

Camerota, F. (2004). Renaissance descriptive geometry: The codification of drawing methods. In W. Lefèvre (Ed.), *Picturing machines, 1400-1700* (pp. 175-208). Cambridge, MA.: MIT Pres.

Camerota, F. (2006). Teaching Euclid in a practical context: Linear perspective and practical geometry, *Science & Education, 15*(2-4), 323-334.

Casale, R. (2004). The educational theorists, the teachers, and their history of education, *Studies in Philosophy and Education, 23*, 393–408.

Chrisomalis, S. (2008). The cognitive and cultural foundations of numbers. In E. Robson & J. Stedall (Eds.), *The Oxford handbook of the history of mathematics* (pp. 495-517). Oxford: Oxford Univ. Press.

Depaepe, M. (2007). Philosophy and history of education: Time to bridge the gap? *Educational Philosophy and Theory, 39*(1), 28-43.

Epple, M. (1997). Styles of argumentation in late 19th century geometry and the structure of mathematical modernity. In M. Otte and M. Panza (Eds.), *Analysis and synthesis in mathematics*. Dordrecht, Boston, London: Kluwer Academic Publishers, 177-198.

Epple, M. (2004). Knot invariants in Vienna and Princeton during the 1920s: Epistemic configurations of mathematical research, *Science in Context, 17*(1/2), 131–164.

Field, J. V. (2005). *Piero della Francesca. A mathematician's art*, New Haven, London: Yale University Press.

Florensky, P. (1992). *La perspective inversée suivi de L'Iconostase,* Lausanne: Éditions L'Age d'Homme.

Fujimura, J. H., Star, S.L., & Gerson, E. (1987). Méthodes de recherche en sociologie des sciences: Travail, pragmatisme et interactionnisme symbolique, *Cahiers de Recherche Sociologique, 5*, 65-85.

Golinski, J. (1990). The theory of practice and the practice of theory: Sociological approaches in the history of science, *Isis*, 81, 492-505.

Greeno, J. G. and the Middle-School Mathematics through Applications Project Group (1997). Theories and practices of thinking and learning to think, *American Journal of Education, 106*, 85-126.

Greeno, J. G., Collins, A., & Resnick, L. B. (1996). Cognition and learning. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology* (pp. 15-41). New York: Macmillan.

Hackett, E. J. et al (Eds.) (2008). The handbook of science and technology studies, Cambridge, MA.: MIT Press.

Heinz, Chr. (2007). *Cognition scientifique et évolution culturelle*, Doctorat nouveau régime, École des Hautes Études en Sciences Sociales.

Hill, K. (1998). 'Juglers or Schollers?': Negotiating the role of a mathematical practitioner, *The British Journal for the History of Science, 31*(3), 253-274.

Høyrup, J. (2007a). What did the abacus teachers really do when they (sometimes) ended up doing mathematics?, *Filosofi og Videnskabsteori på Roskilde Universitetscenter.* 3. Række: *Preprints og Reprints*, Nr. 4
http://www.akira.ruc.dk/~jensh/Publications/2007%7Bc%7D_What%20did%20the%20abacus%20teachers%20really%20do.pdf

Høyrup, J. (2007b). Jacopo da Firenze's Tractatus algorismi and early Italian abacus culture. Basel, Boston: Birkhäuser.

Jesseph, D. M. (1989). Philosophical theory and mathematical practice in the seventeenth century, *Studies in History and Philosophy of Science, 20*, 215-244.

Johnston, S. (1994). Making mathematical practice: Gentlemen, practitioners, and artisans in Elizabethan England, Ph.D. diss., Cambridge Univ.

Jones, M. L. (2006). *The Good life in the scientific revolution: Descartes, Pascal, Leibniz and the cultivation of virtue*, Chicago, IL: University of Chicago Press.

Kidwell, P. A., Ackerberg-Hastings, A., & Roberts, D. .L. (2008). *Tools of American mathematical teaching, 1800-2000*, Washington, DC: Smithsonian Institution.

Kjeldsen, T. H. (2008). Abstraction and application: new context, new interpretations in twentieth-century mathematics. In E. Robson & J. Stedall (Eds.), *The Oxford handbook of the history of mathematics* (pp. 755-778). Oxford: Oxford Univ. Press.

Kjeldsen, T. H. (2009). Using history as a means for the learning of mathematics without losing sight of history: the case of differential equations. In F. Furinghetti et al. (Eds.), *Pre-Proceedings of CERME 6*, *Working Group 15-The Role of History of Mathematics in Mathematics Education: Theory and Research*, 66-75.

http://educmath.inrp.fr/Educmath/recherches/actes-en-ligne/wg15.pdf
Kjeldsen, T. H. (2009a). Egg-forms and measure-bodies: Different mathematical practices in the early history of the modern theory of convexity, *Science in Context, 22* (1), 85–113.
Kjeldsen, T. H., et al (Eds.) (2004). *New trends in the history and philosophy of mathematics*, Odense: University Press of Southern Denmark.
Kuhn, J. (1990). Measured appearances: documentation and design in early perspective drawing, *Journal of the Warburg and Courtauld Institutes, 53*, 114-132.
Kutz, E. M. (1997). *Representation practices of differential calculus: A historical-cognitive approach*, Ph.D. Graduate College of Bowling Green State University.
Laden, S. (2004). Recuperating the archive: Anecdotal evidence and questions of "Historical Realism". *Poetics Today*, 25(1), 1-28.
Lave, J. (1988). *Cognition in practice,* Cambridge, New York: Cambridge University Press.
Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation,* Cambridge, New York: Cambridge University Press.
Léon, A. (1985). *The history of education today*, Paris: Unesco.
Lynch, M., & Woolgar, S. (1988). Introduction: Sociological orientations to representational practice in science, *Human Studies, 11*, 99-116.
Mazzio, C. (2009). Shakespeare and science, c. 1600, *South Central Review, 26 (1 & 2)*, 1-23.
Mazzotti, M. (1998). The geometers of God: Mathematics and reaction in the Kingdom of Naples, *Isis, 89*, 674-701.
Miguel, A. (2005). History, philosophy, and sociology of mathematical education in teachers training: a research program. *Educação e Pesquisa, 31*(1), 137-152, http://www.scielo.br/scielo.php?pid=S1517-97022005000100010&script=sci_arttext&tlng=en
Milewski, P. (in print, 2009), Educational reconstruction through the lens of archaeology, *History of Education*.
Neal, K. (2003). Mathematics and empire, navigation and exploration: Henry Briggs and the Northwest, *Isis,* 93(3), 435-453.
Nersessian, N. J. (1995). Opening the black box: Cognitive science and history of science, *Osiris, 10*, 194–211.
Nersessian, N. J. (2003). Kuhn, conceptual change, and cognitive sciente. In T. Nickles (Ed.), *Thomas Kuhn* (pp. 178-211). Cambridge, New York: Cambridge Univ. Press.
Nersessian, N. J. (2008). *Creating scientific concepts.* Cambridge, MA: MIT Press.
Rouge, J. (2003). Kuhn's philosophy of scientific practice. In T. Nickles (Ed.), *Thomas Kuhn* (pp. 101-121). Cambridge, New York: Cambridge Univ. Press.
Netz, R. (1999). *The shaping of deduction in Greek mathematics: A study in cognitive history,* Cambridge, New York: Cambridge Univ. Press.
Netz, R. (2004). *The transformation of mathematics in the early Mediterranean world. From problems to equations,* Cambridge, New York: Cambridge Univ. Press.
O'Halloran, K. L. (2005). *Mathematical discourse: Language, symbolism and visual images*, London, New York: Continuum Publ.
Olesko, K. (2005). Training for the Tripos, *American Scientist, 92*, 276-279.
Olesko, K. M. (2006). Science pedagogy as a category of historical analysis: Past, present, and future, *Science & Education, 15*, 863-880.
Popkewitz, T. S. et al (2001). *Cultural history and education. Critical essays on knowledge and schooling*, London, New York: Routledge Falmer.

Radford, L. (1997). On psychology, historical epistemology and the teaching of mathematics: Towards a socio-Cultural history of mathematics, *For the Learning of Mathematics, 17*(1), 26-33.

Pyenson, L. (1979). Mathematics, education, and the Gottingen approach to physical reality, 1890-1914, *Europa, A Journal of Interdisciplinary Studies, 2*(2), 91-127.

Pyenson, L. (1983). *Neohumanism and the persistence of pure mathematics in Wilhelmian Germany*, Philadelphia, PA: American Philosophical Society.

Rowe, D. E. (2002). Mathematical schools, communities, and networks. In M. J. Nye, (Ed.), *The modern physical and mathematical sciences, vol. 5 of The Cambridge history of science*, (pp. 113-132). Cambridge, New York: Cambridge University Press.

Schoenfeld, A. H. (Ed.). (1987). *Cognitive science and mathematics education.* Mahwah, NJ: L. Erlbaum Associates.

Schubring, G. (1989). Pure and applied mathematics in divergent institutional settings in Germany: The role and impact of Felix Klein. In D. E. Rowe & J. McCleary (Eds.), *The History of modern mathematics*, *Vol. II, Institutions and Applications* (171-220). Boston, San Diego, New York, Berkeley, London, Sydney, Tokyo, Toronto: Academic Press.

Schubring, G. (2001). Production mathématique, enseignement et communication, *Revue d'Histoire des Mathématiques, 7*, 295-305.

Schubring, G. (2003). Recent developments in research on the institutional history of mathematics. *Llull*, 26, 1045-1059.

Schubring, G. (2005). *Conflicts between generalization, rigor, and intuition. number concepts underlying the development of analysis in 17th-19th century France and Germany*, New York: Springer.

Shapin, S. (1994). *A social history of truth: Civility and science in seventeenth-century England*, Chicago, IL: University of Chicago Press.

Shapin, S. (2000). Descartes the doctor: Rationalism and its therapies, *The British Journal for the History of Science, 33*(2), 131-154.

Shapin, S., & Barnes, B. (1976). Head and hand: Rhetorical resources in British pedagogical writing, 1770-1850, *Oxford Review of Education, 2*(3), 231-254.

Stepin, V. (2005). *Theoretical knowledge*, Dordrecht: Springer.

Swetz, F. J. (1987). *Capitalism and arithmetic*, La Salle, IL.: Open Court Publ.

Van Egmond, W. (1980). *Practical mathematics in the Italian Renaissance: A catalog of Italian abbacus manuscripts and printed books to 1600*, Firenze: Instituto e Museo di Storia della Scienza.

Warwick, A. (2003). *Masters of theory, Cambridge and the rise of mathematical physics*, Chicago, IL: University of Chicago Press.

Warwick, A., & Kaiser, D. (2005). Conclusion: Kuhn, Foucault, and the power of pedagogy. In D. Kaiser (Ed.), *Pedagogy and the practice: Historical and contemporary perspectives* (pp. 393-409). MIT Press.

Whitmer, K. J. (2007). Review of Matthew L. Jones. *The good life in the scientific revolution: Descartes, Pascal, Leibniz and the cultivation of virtue.* In *H-Net Reviews in the Humanities & Social Sciences*, February,
http://www.h-net.org/reviews/showrev.php?id=12897

Weintraub, E.,R. (2001), Making economical knowledge: Reflections on Golinski's constructivists history of science, *Journal of the History of Economic Thought, 23*(2), 277-282.

Woods, T. E., Jr (2005). *How the Catholic Church built western civilization*, Washington, DC: Regnery Publ.