Professional debate and social structure in Swedish mathematics education, 1905-1962. The case of geometry instruction at the lower secondary level

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Abstract

In this article a prosopography regarding the people who engaged in the professional debate on geometry instruction at the lower secondary level in Sweden during the period 1905-1962 is presented. The analysis is based on Bourdieu's theory on capital and field. The main focus of the investigation is a debate in the teacher periodical Elementa. Certain aspects of the debaters' backgrounds and their arguments are treated. The primary finding is that the debate has features of a field. This result is used to illuminate further known facts and previous research.

Introduction

During the period 1905-1962 important changes occurred in Swedish secondary schools. The number of students increased dramatically. In 1910, the number of students who completed the lower and upper level was 1502 and 1544, respectively. In 1960, the numbers were 27968 and 9136. This increase in students was naturally followed by an increase in the number of teachers. At the same time, primary and secondary schools became more and more integrated; course plans, grades and admission standards were harmonized. Moreover, there was an ongoing battle between those who advocated humanistic versus realistic ideals in education. Hence, the conditions for mathematics instruction changed.1

In this paper another aspect of the conditions of mathematics instruction is investigated: the professional debate about mathematics instruction. I delineate the status of the debaters, what they spoke about and how they spoke.

The study is restricted to geometry at the lower secondary level. This is because geometry constituted a large part of the mathematics curriculum in the Swedish secondary schools. Moreover, the debate focused on the lower secondary level. The most significant part of the professional debate on geometry instruction at the lower secondary level occurred in the periodical Elementa. The periodical was founded in 1917 and was a main forum for teachers in mathematics, physics and chemistry in the secondary schools. Its original name was Tidskrift för elementär matematik, fysik och kemi and it was changed in 1938. The people who took part in the debate were the following:

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1 See for instance (Prytz, 2007) for a more thorough description of the Swedish school system and mathematics education during the period 1905-1962.

The debate consisted of two parts. The first occurred during the period 1917-1927 and involved Petrini, Meyer, Hedström and Olson. The second occurred during the period 1938-1939 and involved Olson, Sjösted and Nyhlén.

Compared to today’s professional debate about mathematics education, the situation was somewhat different. From 1905 to the late 1950s, there were no handbooks or teaching literature about geometry instruction at the secondary level, see (Prytz, 2007).

The aim of the article is to present a prosopography regarding the people who engaged in the professional debate on geometry instruction at the lower secondary level in Sweden during the period 1905-1962. The analysis is based on Bourdieu’s theory on capital and field. This theory is explained in the section on method and theory. The questions that I have set out to answer are the following:

- What capital connected to education and mathematics did the debaters possess?
- In the debate, what were the main arguments about geometry instruction at the lower secondary level?
- In what respect can we consider the professional debate on geometry instruction a field?

These questions are treated in three separate sections. In a fourth section, I give an example of how the low results on the geometry exercises of the final exams of the lower secondary schools can be explained on the basis of what we know about the professional debate as a field. In the final section, I relate my discoveries to previous research.

**Method and theory**

Following (Broady, 2002b), a prosopography can be considered a collective biography. The collective in this case comprises the people who engaged in geometry instruction at lower secondary level during the period 1905-1962. By ‘engaged’ I mean being a teacher, taking part in debates in teacher periodicals, textbook authoring and taking part in teacher education. The focus of the study is the relations between the people who took part in the professional debate about geometry instruction. By ‘relation’ I mean a pattern or regularity between different properties of the debaters. The investigated properties are education, academic positions, positions in the educational system, book production and standpoints and arguments regarding mathematics education, in particular geometry instruction. Thus, my primary goal is not to describe some kind of social network.
Moreover, my intention is not to delineate the life of the debaters as closely as possible.

The analysis of the relations between the properties mentioned is based on Bourdieu’s (2000) theory on capital and field and Broady’s (1991, 2002b) description of how they can be used.

The parts of Bourdieu’s theory that have been applied in the analysis are the following: A ‘capital’ is something which is valued by a group of people. This can be certain objects, actions, opinions, arguments, skills, educations, professions, cultural habits, material possessions, social manners, social relations or even parents. A person can have various capitals and each capital can be valued differently in different contexts.

A ‘field’ exists when a group of people, often specialists of some kind, fights over something they find important. A group of specialists in geometry instruction is treated in this paper. A field consists of the relations between the combatants’ assets of capital, the positions they occupy and their actions. The type of field that I investigate is sometimes called field of production; the people who are active in this type of field produce standpoints, arguments and values that other people take into account. Authors of textbooks and teaching literature and debaters in teacher periodicals are examples of people who might be active in a field of production. They produce standpoints, arguments and values regarding the content and the teaching methods that teachers take into consideration.

Just because people fight over something they consider important does not mean that they must be active in a field. The important property of a field is autonomy. The autonomous field has the following characteristics: There is a specific kind of capital that can be earned within the field. This means that the people in the field value and award certain types of education, skills, actions, objects, arguments or opinions. This also means that the people act and achieve recognition according to a certain logic which is connected to the capital specific for the field; big assets of the specific capital render a person a high position and vice versa. An important aspect of this logic is that the specific capital is appreciated more than other types of capital; examples of the latter are financial assets, social background or political opinions. Moreover, this logic is manifest in the persons’ actions, to which I include standpoints and argumentation. Persons with great assets of the specific capital build their standpoints on assumptions, arguments and facts that are specific to that field. Moreover, they refrain from using arguments that are valued in other fields, for instance the political or economic field. Let us say that we can consider a field of mathematics instruction to be distinctly different from the political field and the economic field. Then the people active in the field of mathematics instruction do not use strictly party-political or economic arguments to support their viewpoints. For instance, a textbook in mathematics is then not valued on the basis of the amount of exercises with correct party-political references.
Another starting point for my investigation has been consideration of the educational system and its parts as objects for different conflicts and debates that include different groups. I discriminate between two types of groups that can be involved in these conflicts: professionals and non-professionals. The investigated professionals in this study are persons who interpreted and implemented the directives of the authorities by authoring textbooks, taking part in debates in teacher periodicals and being involved in teacher education. Using the field concept, they may have constituted an intermediary field of production between on the one hand the authorities and a public debate on education and on the other hand the teachers. The value of studying this group is that they interpreted the course plans and gave them a more concrete form.

The investigated text material comprises texts produced for and used by teachers in their profession, i.e. articles in teacher periodicals, teaching literature, textbooks, final exams, corrections standards for the final exams and reports about the results of the final exams. Apart from textual sources, statistics about the results of the final exams and book production have been gathered. A more detailed description of the sources is available in (Prytz, 2007). The statistics regarding book production are based on the LIBRIS, the catalogue of the Swedish academic and research libraries plus about twenty public libraries. Various encyclopedias and biographic lexicons have been used to collect information about the debaters.

The debaters and their capital

The starting point for the description of the debaters’ capital is different types of careers that are connected to mathematics and education. By focusing on a person’s career we delineate parts of that person’s interests and competences, i.e. things that were valued by that person. But more importantly, we delineate competences that the person excels in and that are being recognized by others, i.e. things that are valued by others.

One type of career common to all the debaters was a scientific career in mathematics: they all had a PhD in mathematics. However, it was just Petrini who was successful in terms of university positions with connection to mathematics and publications in scientific journals in mathematics. About a year after his PhD exam in 1890 he was given a teaching position at Uppsala University, which he held until 1901, see (Svenskt biografiskt lexikon, 1997). He also published papers in the international journals Acta Mathematica and Journal de Mathématique. He also was reviewed in Jahrbuch über die Fortschritte der Mathematik.2 In 1908, Petrini applied for a professorship in mechanics and mathematical physics at Uppsala University, but he was not appointed.

Apart from Petrini, Sjöstedt also succeeded in a scientific context, however not in mathematics but in philosophy. He received his PhD in mathematics in 1930

2 My sources regarding the debaters’ published works in mathematics are the search engine MathSci.net and (Gårding, 1998).
and the same year he became a PhL in philosophy; the subject of the later dissertation is philosophy of mathematics, and the title is \textit{Zur Erkenntnistheorie der Geometrie}. Sjöstedt did not have a formal position in philosophy at the university level; he did however belong to a group around Adolf Phalen, professor in philosophy at Uppsala University from 1916 to 1931, see (Svenskt biografiskt lexicon, 1997).

Compared to the other debaters, Petrini stands out as the most successful mathematician. None of the other debaters published in prominent international scientific journals specialized in mathematics.

In addition to their scientific careers, both Petrini and Sjöstedt authored a great number of books\footnote{According to LIBRIS, Petrini published 41 individual works and Sjöstedt 33. For the other four the numbers are the following: Meyer 6, Hedström 2, Olson 3 and Nyhlén 1.} that were not textbooks intended for primary, secondary or university level. These books treated various subjects such as philosophy, religion, science, language and education. Petrini focused on science, religion and education; Sjöstedt focused on philosophy, language and education. Hence, both of them had a general education outside mathematics that was recognized.

The careers that the most debaters entered and succeeded in were connected to the school system, in particular at the secondary level. In order to distinguish different types of competences, two types of careers in the school system are considered: a career connected to mathematics education and one connected to school administration.

Regarding the careers in mathematics education, there are several sources\footnote{In biographic lexicons, historical works on individual schools, flyleaves in textbooks state that the authors worked as teachers.} that imply that the debaters worked or had worked as mathematic teachers at the secondary level. The only exception is Nyhlén; I have not found any explicit note indicating that he worked or had worked as a teacher. However, he authored textbooks and he debated about mathematics education in the professional journal \textit{Elementa}. In addition to being teachers, all debaters authored textbooks in mathematics. Hence, being a teacher or textbook author is a common characteristic of the debaters. All of them also authored textbooks in geometry. However, Meyer, Hedström, Olson and Sjöstedt stand out as they reached certain positions in mathematics education.

\textit{Meyer}

Founder and editor of the periodical \textit{Elementa} in 1917, see (Meijer et al., 1904-1926).

\textit{Hedström}

Founder and editor of the teacher periodical \textit{Elementa}, see (Meijer et al., 1904-1926).

Involved in the education of mathematics teachers\footnote{In an article in \textit{Elementa}, Hedström declares that the article is based on a lecture given for teacher students.}
Olson
Involved in the education of mathematics teachers\(^6\)
Became editor of the periodical *Elementa* after Meyer in 1926, see (Meijer et al., 1904-1926).

Sjöstedt
Responsible for mathematics education at the central school board, see (*Svenskt biografiskt lexicon*, 1997)
1940-1962: advisor in education
1952-1962: head of department
1960-1962: deputy director general

Apart from that, Hedström, Olson and Sjöstedt produced a large amount of textbooks in mathematics for the secondary level. Moreover, their textbooks in geometry for lower secondary schools were among the most used in teaching, see (Prytz, 2007). Hence, this trio had an interest in writing textbooks in mathematics and a competence in doing so that was valued by publishing houses and teachers. In comparison to Sjöstedt, who authored several books in other areas and not only textbooks, the authorship of Olson and Hedström was restricted to mathematical textbooks.

The other type of school career was connected to school administration. Once again, it is the trio Hedström, Olson and Sjöstedt that stands out.

Hedström\(^7\)
Director of an all-girls school (secondary level) 1907-1918
From 1918, headmaster of a secondary school in Stockholm

Olson\(^8\)
Director of the local school board in Stockholm, 1921-1929
Advisor at the ministry of education, 1924
Advisor at the central school board, 1934-1935
Headmaster of a secondary school in Stockholm, 1945-1950

Sjöstedt\(^9\)
From 1939, headmaster of a secondary school in Borås
Responsible for mathematics education at the central school board
1940-1962: advisor in education
1952-1962: head of department
1960-1962: deputy director general
Member of the board of an institute specialized in pedagogy and psychology

\(^6\) In an article in *Elementa*, also Olson declares that the article is based on a lecture given for teacher students.

\(^7\) All data is collected in (Meijer et al., 1904-1926)

\(^8\) All data is collected in (Dahl, 1949).

\(^9\) All data is collected in (*Svenskt biografiskt lexicon*, 1997)
These careers indicate that the members of the trio had an interest and skills in school administration. Moreover, I see these appointments as a sign of recognition of these skills.

So far I have not mentioned Nyhlén. He authored a few textbooks in mathematics, but no other books. Moreover, he has left no marks in encyclopaedias or biographic lexicons. His textbooks do not indicate the school where he worked.

If we consider the debaters and the three types of careers, we can discern different patterns. However, we get a better understanding of these patterns if they are related to the arguments of the debaters.

The main arguments of the debaters

Regarding the main goal of geometry instruction, a consensus prevailed more or less among debaters: the subject should facilitate training in reasoning. However, this goal did include more than the ability to master logic; the students were also supposed to develop a critical attitude about reasoning, language and spatial intuition.

Considering the debaters’ textbooks, they also agreed on essential parts of the content. In large parts, they treated the same concepts and theorems as in traditional versions of Euclid’s *Elements*, for instance (Heath, 1956). Moreover, the textbooks had a basic design similar to Euclid’s *Elements*. At the beginning the concepts were defined and the axioms presented. In some textbooks, however, the concepts were not introduced by a strict list of definitions, but in a more experimental fashion, and some axioms were given when needed in the proofs. Definitions and axioms were followed by theorems along with their proofs. Each proof was based on definitions, axioms, and previous theorems only.

The disagreements mainly concerned content and method. By ‘method’ in this case I mean how the content should be communicated with the students. Notice that I used ‘method’ in a broad sense. It denotes not only teachers’ actions in the classroom, but also ways of communicating the content textually and visually as well as orally. However, the discussions about method also included content issues as some concepts were considered more appropriate from a pedagogical point of view; according to the proponents certain concepts made learning easier.

Petrini initiated the first part of the debate 1917–1927 as he criticized contemporary Swedish textbooks. He identified four types of flaws that were linked to alterations in the Euclidean system, see (Prytz, 2007).

- If a theorem is proved by means of the fifth postulate, i.e. the parallel postulate, the theorem becomes less general than if the fifth postulate is not used. If the fifth postulate is not used, the theorem applies for Euclidean geometry as well as non-Euclidean geometries. Thus, proofs in

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10 For a more thorough description of the textbooks, see (Prytz, 2007).
which the fifth postulate is included become less general. Moreover, the teachers lose a good opportunity to discuss theorems that apply to non-Euclidean geometries.

- Theorems proved by Euclid are presented as axioms, as for instance the proposition that the sum of two sides in a triangle is greater than the third. According to Petrini, as few axioms as possible should be used.
- The construction theorems serve as proofs of existence in the Elements. This makes it almost impossible to alter the order of the theorems.
- Translations or movements of figures in the plane should not be used. Here, Petrini suggested that theorem I.4 should be considered an axiom.\textsuperscript{11}

These criticisms do match a textbook of an author named Asperén, but also Olson’s textbook. At the time of Petrini’s articles, Asperén’s geometry textbook was the most popular one in Sweden. However, Olson’s textbook in geometry, which became increasingly popular in the following years, can in many respects be seen as a continuation of Asperén’s way of authoring geometry textbooks, see (Prytz, 2007).

Petrini also maintained that Euclid’s Elements was still the best textbook from a logical point of view. However, a high level of rigor also had a pedagogical advantage, according to Petrini; it brought a clear account which in turn made the content more accessible for the students, especially the less gifted ones.

On the other side of the debate we find Meyer, Hedström and Olson. Taken together, their standpoints can be summarized as follows:\textsuperscript{12} A scientific level of rigor does not suit school geometry and Euclid’s Elements is not suitable as a textbook for young students.

- A scientific level of rigor does not suit school geometry. It must be allowed to move geometrical objects as if they were real objects; there should not be a specific axiom regarding movements of geometrical objects, and movements of geometrical objects should be used more often than in Euclid’s Elements. Otherwise textbooks would be too complicated.
- Theorems that treat the same concept shall not be kept apart as in Euclid’s Elements. They should be treated thematically.
- Some proofs in Euclid’s Elements are complex and awkward, both in terms of their design and their function. For instance, Meyer disputed that 13-year-olds could grasp the idea of proofs of existence in connection to the construction theorems. Moreover, a good proof should not only show the correctness of a theorem from a logical point of view. Meyer argued that a good proof should reveal not only the “nature” of the theorem but also a cause. Olson argued that a proof should appeal to our spatial intuition.

\textsuperscript{11} Most likely, Petrini was here referring to Hilbert’s axiom III:6. If so, the suggestion is a bit surprising since Petrini, seven years earlier, considered this axiom too far fetched for the students at Realiskolan.

\textsuperscript{12} See (Prytz, 2007) for a more thorough description of Meyer’s, Hedström’s and Olson’s arguments.
Such proofs included movements of geometrical objects and the symmetry concepts.

- Definitions and axioms should be introduced when needed. Otherwise the account would confuse the students.

An important feature of the argumentation of Meyer, Hedström and Olson is that they made a distinction between scientific standards and educational standards regarding the content. According to them schoolbooks in geometry could not only be valued on the basis of scientific standards. They also criticized Petrini for being ignorant of the educational standards.

The second issue discussed in the first part of the debate on geometry instruction was about the introduction of the axiomatic method. The position of Meyer, Hedström and Olson was that there should be a so-called “gentle” transmission from a less formal geometry to a more formal one. In the beginning, theorems should be introduced through empirical investigations. In this way, the student should attain a better understanding of the theorems. However, the teacher should also ask questions about the reliability of the investigations; this should make the students discover the advantage of an axiomatic method over strictly empirical and inductive reasoning regarding precision. In contrast, Petrini argued that axiomatic geometry and a high level of rigor should be introduced only briefly. He did not find it useful to give explicit teaching about the function and the value of the axiomatic method.

The second part of the debate (1938-1939) involved Olson, Sjöstedt and Nyhlén and it shared some characteristics with the first part, see (Prytz, 2007). It was initiated by Nyhlén’s attack on the two most popular contemporary textbooks: this time it was Olson’s and Sjöstedt’s. Also this time, the charge was that the level of rigour was too low; the authors were accused of relying on spatial intuition on several occasions. The cause of this reliance on spatial intuition was tracked to a lack of certain explicit axioms. According to Nyhlén, both Olson and Sjöstedt lacked explicit axioms regarding the movement of geometric objects and its shape and size and explicit axioms regarding the geodetic property of a straight line. Olson’s and Sjöstedt’s replies were to some extent similar. Both maintained that an elementary level textbook should employ a more tangible everyday language. Therefore they found it more appropriate to use formulations in which geometrical objects were moved as if they were real objects. Moreover, they did not find it relevant to refer to an axiom about movements of geometrical objects each time a geometrical object was moved; it would have hampered the account.13 Hence, both Olson and Sjöstedt claimed that one had to differentiate between scientific geometry and school geometry and that a pedagogical standard had to be realized in school geometry.

13 Actually, in his textbook Olson did make an explicit assumption about movements of a geometrical object and its shape and size. However, he did not make explicit references to it in the proofs. Sjöstedt, on the other hand, did not include such an axiom in his textbook at all.
However, Sjöstedt took his defence a bit further and attacked Nyhlén’s idea of science, axiomatic systems and spatial intuition. Sjöstedt objected to the idea that it is possible to determine a geometrical object completely by means of an axiomatic system. Instead, our understanding of a geometrical concept, and eventually also theorems and proofs, is based on our spatial intuition. Thus, Sjöstedt differentiated between understanding a theorem and a logically correct proof of a theorem and he considered them both necessary items in a proof. In this way Sjöstedt tried to avoid Nyhlén’s critique since the axioms Nyhlén requested had no logical function in the textbook, according to Sjöstedt.

Actually, Sjöstedt’s reply to Nyhlén was also relevant in relation to the first part of the debate since Petrini did explicitly take the position Sjöstedt criticized.

A difference in Olson’s and Sjöstedt’s argumentations is that Sjöstedt tried to show that there must not be a conflict between scientific and educational standards. Olson made no such attempts.

These debates and their arguments should by no means be considered mere words. The way Meyer, Hedström, Olson and Sjöstedt designed geometry textbooks does correspond with their standpoints regarding content and method. Moreover, the geometry textbooks of both Olson and Sjöstedt eventually became the most popular textbooks. Petrini did however not author a textbook on his own, but he edited a version of Euclid’s *Elements*. Nyhlén did produce a textbook, but his design deviated on several points from the others. For instance, he used algebra and he designed proofs with a more empirical and experimental approach. This textbook was however not a success; only two editions were printed.

Taken together, the debate contained two types of arguments: scientific and educational. Teaching methods and textbooks were valued on the basis of both scientific and educational standards. Some of the debaters, i.e. Meyer, Hedström, Olson and Sjöstedt, stated that scientific geometry and school geometry were two different practices. Moreover, they argued that educational standards should be superior if there was a conflict between scientific and educational standards. However, the debaters did not apply administrational arguments, which are arguments based on how schools work or economical considerations for schools. Actually, all debaters refrained from making explicit references to guidelines and goals in course plans, government documents, official reports or policy documents.

**The professional debate on geometry instruction as a field**

A characteristic of a field, which is autonomous, is the existence of a specific capital. My findings suggest that two types of capitals were valued by the debaters: a scientific capital and an educational capital. The common assets of the debaters were PhD’s in mathematics and experience in teaching and textbook authoring.

\[14\] According to Sjöstedt this spatial intuition was not based on experience.

\[15\] See (Prytz, 2007) for a description of the textbooks and a comparison of arguments and textbooks.
These assets were also reflected in the debate as both scientific and educational arguments were applied, which is typical for a field. A characteristic of the investigated debate is that it was sufficient for a debater to possess greater amounts of only one type of these capitals; it was not necessary to be an authority in both mathematics and mathematics education in order to be heard in the debate. My investigations also suggest that a greater amount of scientific capital was not an asset in reaching leading positions in mathematics instruction. Without having a great amount of scientific capital, Meyer, Hedström, Olson and Sjöstedt reached positions such as editor of a leading professional journal (Elementa), involvement in teacher education and advisor in the school administration regarding mathematics instruction issues.

But what did this specific capital comprise? What did Meyer, Hedström, Olson and Sjöstedt have that Petrini and Nyhlén did not have? I think that one part of this capital was the recognition of two types of standards and the superiority of one of them. Meyer, Hedström, Olson and Sjöstedt underscored the difference between scientific geometry and school geometry. They also made a clear distinction between scientific standards and educational standards. School geometry had to apply to both. But in cases in which a conflict arose, the latter should prevail.

Petrini and Nyhlén, on the other hand, did not make a clear distinction between scientific and educational standards. Petrini even argued that a textbook that met high scientific standards also was the most pedagogically efficient due to its clarity.

According to my research it is not possible to give a broader description of competences related to mathematics education, but my findings suggest that textbook authoring was one such competence. Hedström, Olson and Sjöstedt produced a large number of textbooks. Moreover, the geometry textbooks authored by them were amongst the most popular during the period 1905-1962. Petrini and Nyhlén did not have such success. Moreover, textbook design was a central issue throughout the debate, which indicates that textbook authoring was considered important. The discussions about scientific and educational standards were to a large extent about textbooks.

On the other hand, I have no material from some kind of appointment process where somebody’s competence is explicitly valued on the basis of his or her textbook production. However, an interesting aspect of competence evaluation in connection to instruction and education is that textbooks are a way to document and show pedagogical competence. You then show an actual product of your competence that is possible to evaluate, which is something different from an official certificate.

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16 It may be possible to investigate this since Sjöstedt was employed at the central school board. Hence, there should be some records from the employment process. Moreover, Hedström, Olson and Sjöstedt had positions as headmasters. Also from these employment processes there should be records.
Another characteristic of a field is the logic in which the specific capital is valued more than other types of capital. The investigated debate constitutes an example in that respect. In the section about the debaters, three types of careers are delineated and associated with different capitals: scientific capital, mathematics education capital and school administration capital. However, in the debate only scientific and educational arguments were applied. Hence, only the scientific capital and the mathematics education capital had corresponding arguments in the debate. Furthermore, the debaters with high positions in mathematics education, especially Olson and Sjöstedt, also had greater amounts of school administrative capital, but they did not use that type of argument. None of them acted as school officials in the debate and their argumentation was by no means explicitly based on formulations in government documents. This indicates that the specific capital in this case consists of scientific capital and mathematics education capital, but not school administration capital. These circumstances also indicate that the professional debate was autonomous in relation to the school administration.

An explanation for the low results on the final exams

On the basis of what we know about the professional debate as a field, I think it is possible to give an explanation for the low results on the geometry tasks on the final exams in mathematics of the lower secondary schools. Throughout the period 1905-1962, the solution frequency of the tasks that involved two to four theorems and some kind of formal proof was each year among the lowest, if not the lowest, and very seldom over 20 percent. According to the correction standards and the reports about the correction the students were supposed to perform a formal proof. However, the students’ abilities to master proofs like the ones in the textbooks, i.e. those that were discussed in the professional debate, were not decisive for the result. The reports and the correction standards reveal that it was sufficient to discover certain relations between straight lines or between angles in a given figure and to compute a correct answer in order to receive a pass on the task. Yet, a proper formal proof that supported the answer would probably have given a higher grade in mathematics. My point here is that the crucial competence for success was related to insight and discovery, not formal proofs.

Here we can observe a difference between the practice of the teachers and the professional debate on geometry instruction. The professional debate was in large part focused on textbooks and proofs and how they could be developed, but also treated in the classroom. We can also observe that newer textbooks contained innovations in this respect: new concepts, theorems and proofs. Discovery and insight, on the other hand, were not an important issue in the debate. A confirmation of the low interest in insight and discovery is the late translation of Pólya’s book on problem solving. It was published in English in 1945, but it appeared in Swedish first in 1971.

17 See (Prytz, 2007) for a description of the exercises and statistics of the exam tests.
My point is that the professional debate on geometry instruction seems to have been a system on its own; it functioned as a field. I would therefore say that when the debaters entered the debate, their prime concern focused on issues related to their positions as textbook authors, not the situation in the schools. However, I do not say that the debaters were ignorant of the situation in the schools, but they debated just a part of the situation, i.e. textbooks and proofs. Regarding this part, they discussed different ways of improvement, they developed concepts to address issues in this area and they gave suggestions about how the classroom activity should be organized. Moreover, they did develop new textbooks and new proofs and the teachers did buy them. This meant that other parts, even crucial parts for the teachers and the students, for example issues about insight and discovery, did not receive the same attention. Such issues were not important in relation to the debaters’ status. As a consequence of the debaters’ lack of interest in insight and discovery, the professional debate did not offer specific concepts to treat insight and discovery and there was no explicit suggestion of how such aspects should be treated in the classroom. This may be one reason for the low results on the geometry tasks on the final exams. Notice that there were no specific handbooks or teaching literature regarding geometry instruction at the secondary level until the late 1950’s. Nor did the formal curriculum provide extensive descriptions of goals, content or teaching methods.

Final comments

A benefit of the type of study presented above is that we can achieve a better understanding of how the school system worked. Especially if we want to understand the influence of arguments it is fruitful to understand the debaters’ position in the school system and what types of arguments they valued. If we, for instance, compare the more public debate about science and mathematics education in Swedish secondary schools during the period 1900-1965, investigated by Löwheim (2006), we can see that the professional debate on geometry instruction in lower secondary schools during the same time was narrower. In the professional debate, the main goal, but also the unquestioned goal, for geometry instruction was to train a general ability in reasoning and to foster a critical attitude regarding language, spatial intuition and reasoning. This type of goal regarding mathematics and science education was conveyed also in the more public debate. In the public debate, however, there were more goals, and the goal about training in reasoning was questioned by groups that advocated other types of goals. My point here is that the professional debate about mathematics instruction by no means was a complete reflection of the public debate on mathematics and science education. A more appropriate metaphor is that the professional debate functioned as an amplifying filter that sorted out parts of the public debate and amplified others. By amplification in this case I mean that a goal was approved by people with high professional status, but also that the professional debate provided expressions, concepts and possible ways of development regarding one goal. Other
goals for geometry instruction were probably not unknown to teachers, but the professional debate did not provide the same type of tools to talk and think about these goals. Notice again that there were no specific handbooks or teaching literature regarding geometry instruction at the secondary level until the late 1950’s. Nor did the formal curriculum provide extensive descriptions of goals, content or teaching methods.

However, we can also observe how the professional debate by no means was a complete reflection of the situation of the working teachers. As I have discussed in the previous section, an important aspect of the teachers’ practice was not covered by the professional debate. Also at this point, we can talk about an amplifying filter effect.

An international comparison of the goals of geometry instruction at the lower secondary level reveals that there are similarities as well as differences between Sweden and other western countries. For example, both in England and the United States, the argument about geometry being an excellent subject for training of reasoning was pronounced in teacher journals and course plans.18 However, in both England and the U.S. other types of arguments regarding the purpose of geometry instruction were promulgated in teacher journals and other professional literature about mathematics instruction during the period 1905-1962. Hence, the professional debates in these countries were more diversified in that respect.

Regarding the arguments about teaching methods and textbook design, similar concepts and arguments appeared in Sweden as in other western countries. For example, both in England and Germany, spatial intuition, symmetry and an experimental approach were considered important among those who wanted to develop teaching methods and textbooks at the secondary level, see (Yamamoto, 2006). Thus, the attempts to develop geometry instruction in Sweden were in tune with other countries, at least on a rhetorical level. To what extent the Swedish attempts were more or less radical than in other western countries is not clear and would require a more detailed investigation.

In the Swedish professional debate about geometry instruction no explicit references were made to journals, reports or conferences organized by ICMI, an organization whose goal was to support development of mathematics instruction and cooperation between countries. Moreover, geometry instruction at the secondary level had been one of the most treated topics within ICMI since its foundation in 1908, see (Furinghetti, 2003). Of course, the Swedish debaters may have been influenced by ideas discussed within ICMI. One interesting finding is that the Swedish debaters did not refer to some sort of ICMI document to support their viewpoints. This might reflect a lack of interest in ICMI’s work and it might be related to the Swedish debaters’ background and their capital. None of the leading persons in Swedish school geometry was successful as a mathematician,

18 See (González & Herbst, 2006) for a description of the arguments about geometry instruction in the USA. See (Howson, 1982) and (Price, 2003) for a description of the arguments about geometry instruction in England.
and they all explicitly emphasized a difference between scientific mathematics and school mathematics. Moreover, in the debate, their textbooks were criticized for being unscientific. In contrast, the leading persons in ICMI were or had been successful mathematicians, see (Furinghetti, 2003). My conjecture is that the leading persons in Swedish school geometry were reluctant to associate themselves with ICMI due to ICMI's close ties to scientific mathematics; in the Swedish context that did not support their causes or strengthen their position.

However, the fact that ICMI was not mentioned in the Swedish professional debate on geometry instruction might be related to the fact that no explicit references were made to any kind of reports or policy documents. On the other hand, to my knowledge there were no extensive reports or policy documents about geometry instruction at the secondary level in Sweden or in other countries, apart from those produced within ICMI.

References
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