History of mathematics education in training teachers of mathematics: considering its potentialities

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Abstract

The purpose of this paper is to analyze the potentialities of history of mathematics education in training programmes of math teachers. A departing postulate, that teachers will tend to develop current pedagogical practice of a better quality by maintaining a historical relationship with past educational practices produced by their professional ancestors, will be put forward. Consequently, this study will answer the following questions: what history of mathematics education could be included in training of math teachers? To answer this question, the theoretical and methodological background of Cultural History will be used. Through history of mathematics education, teachers may become aware of processes and curricular models for the teaching of mathematics that historically had been circulating at a planetary scale acquiring sense at specific time and places.

Initial considerations

In his most recent studies, one of the greatest contemporary historians, Roger Chartier, analyzes the transformations that historiography has undergone since the 1990s. It is worth mentioning that the author discusses how the writing of history has changed since these recent times. His studies conclude that conceptions that treat history as a copy of the past are already outdated. On the basis of these considerations, Chartier argues that the task of the historian is now defined as follows: regardless of the area studied, it refers to the construction of representations of the past. After all, history has ceased being an attempt at reproducing the past and has become a representation of it. In this light, a first question within the specificity of this communication is valid: What type of representation of his profession’s past do the mathematics teachers have?

The analysis of several already conducted studies reveals a range that presents different forms of relationship of the mathematics teachers with his professional past. Fictional, memorial and also historical relationships can synthesize these forms.

The objective of this study is to analyze the potentials history of mathematics education in training teachers of mathematics. We will attempt to defend the idea that if the mathematics teachers maintain a historical relationship with professional practices carried out in the past he will tend to develop a better quality profession in his everyday activities today.

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1 Here it is worth citing texts like Chartier (2006), Chartier (2007), Chartier (2008).

About the transformations in writing history

In a recently published work in the Spanish language, French historian Roger Chartier calls attention to the change in course given by historical production in recent years. This change has to do with the celebration of what has come to be called ‘global history’.

In 2000, one of the main themes at the 19th International Congress on Historical Sciences held in Oslo was ‘global history’. This proposal was based on the following rejections: rejection of the nation-state framework that retrospectively delimits a social and cultural entity already present even before its political advent; rejection of the traditional cutouts of historical monographs that explore specificities of a province, region or city; and finally, rejection of the micro-historical focus, suspected of having not cared for the distant (Chartier, 2007, p. 74; our translation).

After ascertaining the trend of changing the contemporary way to write history, intending for it to become global, Chartier asks: ‘how do we construct history thought of on a global scale?’ (ibid., p. 75). Of the alternatives determined by this historian, the one that seems most consistent is above all to think of global history as the history of contacts, meetings, acculturations and miscegenation.

Comparative historical studies constitute specificity; it seems, of the more general discussion on resuming the theme of historical research of a global nature. But, it is necessary to recognize an older relationship for these studies. They date back to 1928, when Marc Bloch underscored the importance of this type of study (Haupt, 1995, p. 196). From his seminal text for comparative historians, Bloch defined what he understood by comparison: ‘For human sciences, to practice the comparative method is to research in order to explain the similarities and contrasts analogous series taken from different social circles have’ (Bloch, 1995 [1930], p. 89).

It is necessary to recall – which motivated one of the above mentioned rejections dealt with by Chartier – that historical production is traditionally produced on a national basis. Comparative historical studies raise the issue of transfer between countries, between cultures, enabling certain problems to be understood beyond what regional determinants allow. According to Clarice Nunes, historical comparison can suggest a new understanding of the territory, recognizing it not as a national territory, but rather a spatial discontinuity, which led Pierre Furter to ask whether regional differences (and therefore, intra-national) would not be as significant as any comparison between nations. He says the territory cannot a priori be treated as a homogeneous unit since it is a perceived, modeled and lived space. For this reason, a same school organization, a same curriculum, the same pedagogical and didactical options can be interpreted differently and have different impacts in a space in which it will appear as discontinuous, heterogeneous and structured on different elements. (Nunes, 2001, p. 63; our translation).
The interest in comparative history thus connects to the production of knowledge not conditioned on a conception of space taken as national territory. This is in agreement with the elements that justify the resumption of compared education studies: the reorganization of global space. The challenge of thinking about research that works without local and regional restraints. Thus, taking into account the idea of discontinuity, problems present within a transnational scope become part of historical understanding. This may be the greatest meaning given today to historical-comparative research.

Evoking philosopher Paul Ricoeur, historian Roger Chartier recalls that a return to writing a global history must be thought of as variations in scale in history: ‘At each scale, we see things not seen at another scale and each vision has its reasons’ (Ricoeur *apud* Chartier, 2007, p. 76).

Further ahead, the same historian underscores that ‘What matters is the selection of a framework for study capable of making connected histories visible, which are related to populations, cultures, economies and powers’ (Chartier, 2007, p. 78).

So, where to begin in historical research that aims at broadening one’s vision and does not become conditioned to a given country, regionally? Research related to mathematics education seems to be a privileged theme for answering this question. After all, mathematics constitutes school knowledge present in the curricula of every nation, even if it presents itself in a very different way in distinct places and even inside a country, depending on the region and on its educational politics. But, the rubric ‘mathematics’ is part of the school curricula worldwide.

**History of Mathematics and History of Mathematics Education**

The origins of the discussions on mathematics education get lost in time. However, it is possible to recall the creation of the International Commission on Mathematics Education (CIEM/IMUK), in 1908, as an important milestone: a highlight. That is when a certain distancing between mathematics and mathematics education begins to occur.

If the age of research in mathematics education is at least centenary, everything indicates the area of studies we can call the ‘history of mathematics education’ is very recent. In the editorial in its first number, the *International Journal for the History of Mathematics Education* underscores it is ‘a field which hitherto was marginally represented by the existing journals’. And that the rousing success of the Topic Study Group 29, *The History of Learning and Teaching Mathematics*, at the 10th International Congress on Mathematics Education in Copenhagen in 2004, which demonstrated the need for a permanent and stable international forum for such research. (Schubring, 2006, p. 1).

Seeking to build its own space in research, the studies on the history of mathematics education have been trying to show they cannot be reduced to the history of mathematics. In other words, the history of mathematics and the history of mathematics education do not overlap.
Obviously, due to its recent emergence, the history of mathematics education encounters resistance in establishing itself in the scientific community. This can be seen by those who until now have judged the past of elementary mathematics education as an integral part of the History of mathematics. Regardless, the affirmation of this area of research in mathematics education is represented by the process of convincing ‘that it is in conditions to dominate a series of issues that until then had been under the jurisdiction of another community’ (Gavroglu, 2007, p.187). We could even broaden these observations, adding that besides dominating a set of already existing questions, which were allocated in other scientific community, in the ambit of mathematics education there are questions about the past that had not yet been formulated. Among them, there is one that seems to be fundamental: what is the utility of the history of mathematics education?

**About the usefulness of the history of mathematics education**

It is possible to think that the first way to take into account the dialogue between different research areas is the one of responding to questions such as: What is the use of such knowledge? This is, as a matter of fact, the initial question a layman always asks a certain scientific production. Thus, before any information, coming from any fieldwork that is strange to us, it is soon brought to mind the questioning about the usefulness of certain studies. This kind of questioning can be seen not only as a way of validating the efforts in someone else’s research, only from a utilitarian and immediate conception of science. A positive dimension of this questioning must be in the need that we have to know to what extent certain specific knowledge can contribute to the transformation of problematic situations in the current life. And, maybe, a more advantageous dialogue may be established between laymen and the scientific production, as well as among researchers from different areas, most of all to enable the description of the importance of a certain investigation field in the solution of present problems. Therefore, this is a matter of, academically, asking about the relevance of such a theme or subject that has been studied. The difficulties of the so-called base research – or pure research – to participate in this dialogue are always too many. It is not rare to see that the research goes into history to look for elements for the assurance of its importance. The case of mathematics is emblematical. There have been examples mentioned by historians about this knowledge that runs through, practically, its entire trajectory. Thus, one could ask oneself, for instance: what was the purpose of the works of Apollonius about the conics? According to Browder and Mac Lane, in 1604, that is, 1800 years later, the mathematician and German physicist Johannes Kepler read the works of Apollonius and Islamic writings over the same subject, and he studied their applications in the optics domain (parabolic mirrors). In 1609 he made the brilliant statement (but impossible without the aid of the old theory) that the orbits of the planets should be described as ellipses and not as circles and epicycles. The main foundations for Newton’s gravitation theory where then launched. (Browder & Mac Lane, 1980).
How about matrices? What is their use? Once more quoting Browder and Mac Lane, we could say that possible to say that sixty years have gone by since the development of the theory of matrices, as a part of pure mathematics (1860), until its application in Physics. At first, Arthur Cayley used matrices to describe linear geometric transformations, rotations, translations and similarities. In 1925, Heisenberg used matrices as a mathematics tool («matricial mechanics») to describe atomic systems in quantum mechanics. (Browder & Mac Lane, 1980).

These and several examples taken from the history of mathematics by the mathematicians Felix Browder and Saunders Mac Lane are found in the text ‘The relevance of mathematics’. The title of the text is revealing: it will be necessary to see what Mathematics is for to be able to talk about its importance…And this use was sought in historical examples, in the relative distances that made works produced in a ‘pure’ sphere until they were used as elements for a better understanding of reality.

Well, having analyzed these examples, in the mathematical ambit, the subject of the present text can proceed. In the terms previously discussed, the task of writing the history of mathematics education is one of knowing, historically, how representations about the mathematics teaching and learning processes were built and how these representations started to have a meaning in the pedagogical practices by teachers in their most diverse contexts and time.

In the specific character of this kind of research, how to maintain the dialogue with the urgent matters of mathematics education? Otherwise said: once the teaching and learning of mathematics have been considered since long ago as something somehow problematic, what do the historical studies of mathematics education have to say? In other words, yet: What is the utility of the history of mathematics education? What is its importance? In an interview published by the magazine Sciences Humaines - SH, in the issue number 18, from September/October in 1997, one last question formulated to Roger Chartier is reproduced below, together with the answer given. The question refers to the utility of the history for present times.

SH: As a conclusion, do you think we can, from historical studies, understand the present?

CHARTIER: I think this is a rhetorical contour by historians, to justify their position, to say that the past can enlighten the present. This idea seems unfounded to me, because I think history is founded on the discontinuity. The events do not repeat themselves and there is no means of turning back time and there is no model given by history. In ancient times, the historical examples used to serve as a guide to the present. This is not the case nowadays. But, on the other hand, a work on the Templars, the Carolingian empire or on the XVII century… have a connection with the present, not by a possible homology of the situations, but because this work can lead the reader to make use of critical instruments that can be useful for the study of their own society (Chartier, 1997, p. 29; our translation).

What is the utility, to mathematics teachers, of the results of scientific production over the history of mathematics education? It is commonplace, when it
comes to answer to that question, the use of the same rhetoric built by historians, mentioned by Chartier. It can be found in many studies about the history of mathematics and/or mathematics education. Therefore, the mathematics teachers, knowing the history of mathematics education, would better understand the current stage of their occupation. Or, in a simpler way: the history of mathematics education is important to understand the problems of the present… Such statements sound somehow like a mermaid chant, in an area where the need to solve immediate problems tends to absorb proposals of extremely pragmatic character. The projects that point at the direct improvement of the teaching and learning of school mathematics, without mediations, are normally prioritized. To them, the relevant credit is granted without discussions. Thus, the rhetoric justification that the history of mathematics education would help to understand current problems… But, as Chartier points out, this is not how it works. There is not a direct transmission, linear, from the past to the present. Then, what is the purpose of the history of mathematics to the mathematics teachers?

The task of the historian of mathematics education refers to that one of construction of crossings of ingenuous, mythical, romantic relations, full of memories about the practices of the mathematics teaching performed in the past. The utility of their production – whose result is history of mathematics education – is the one which considers that, mathematics teachers who have an anti-historical relation with his professional ancestors may, with the appropriation of that history, have a less fanciful and more scientific relationship with the past. This fact tends to alter their day-to-day practices, which are then performed in a more consistent way.

Such considerations, about the relevance of the history of mathematics education to the teachers of this subject, evidently, constitute a bet on the constant change, whose core is anchored in the principle that more knowledge means better teaching practices: the alteration of the relation that the mathematics teachers have with the professional past of his or hers occupation leads, then, to a change in the quality of their practices in the present reality.

By getting to know the history of mathematics education, the mathematics teachers take benefit from what is the core of the historian’s work, which according to Chartier, inscribes in the prophecies about the future, strange as this may sound:

To better situate the greatness and miseries of the transformations of the present, it may be useful to appeal to the only competence that the historian can brag about. They have always been regrettable prophets, but, sometimes, by remembering that the present is full of sedimentary or tangled pasts, they could contribute with a more lucid diagnosis of the novelties that seduced or drove away their contemporaries (Chartier, 2008, p. 15; our translation).

Perhaps, yet, a reflection can be brought here elaborated in another work, which was about the utility of the studies on history of mathematics education. The reflection led to the writings of Bloch. He, longer before Chartier, discussed the utility of history.
Patiently and didactically, the historian Marc Bloch, in his classic work ‘Apologie pour l’histoire ou Métier d’historien’, starts the book aiming at explaining to a little boy the use of history. From this initial point, he goes on discussing the role of history with the reader. For what matters in this text, the occupation of the historian connects directly to the need for the world comprehension and, in this case, his or her task is the one of production of knowledge, through specific work, which characterizes their condition of historians. What is the meaning of considering the work of the historian – the result of what he or she produces – as knowledge? An epistemologe starting point is made necessary: all human practices represent an association – maybe it would be better to call it dialectics – between innovation and inheritance. Hence, for example, the pedagogical practices of mathematics teachers always contain a dimension from the past and another one aimed at launching into the future, towards unpublished actions. What leads to conclude that without historical knowledge of mathematics education, the possibility of a better understanding of the practices performed by mathematics teachers in their day-to-day work is lost (Valente, 2008).

For the representations built by mathematicians and teaching experts in different historical times about the mathematics that should be taught at schools, emerge the appropriations, made by teachers, of those representations. These representations – will be – responsible to guide practices that will bring meaning to didactical-pedagogical actions of teachers in the classroom. The knowledge of these representations about the past of mathematics education must enable the performance of better quality practices of teaching and learning in current times.

What history of mathematics education for teacher education?

The evocation at the outset of this communication regarding the transformations the writing of history has been undergoing represents an important theoretical and methodological framework for this study. It deals with thinking of the historical research of mathematics education within the contemporary scope of historians’ work. And this work goes back to what has been summarized by the effort to produce a ‘global history’. It would not be fitting to think of a national history of mathematics education, considering the changes and transformations in this field as having been strictly local, national. It will be necessary to think of a ‘global history’ – terms that represent an articulation between the broadest aspect of a phenomenon and its local determinants – and which has been called a ‘glocal history’, alluding to the inseparable connection of the ‘global’ with the ‘local’.

The difficulties contemporary historiography has been encountering to produce a glocal history, in its search for convenient themes for this production, find ripe ground when we deal with mathematics education. After all, as we stressed above, mathematics is a universal subject in curricula.

What glocal history of mathematics education could be present in the education of mathematics teachers? The answer involves multiple aspects contemplated by
the historical study of mathematics education. They may not need to go back too far in time. The framework to create the CIEM in 1908 can represent an interesting starting point. Another reference to be worked on with professors goes back to the period known as the Modern Math Movement in the mid-20th Century.

How do we include history of mathematics education guided by these reference points in the education of mathematics teachers? The answer to this question, in theoretical-methodological terms, can be found by historians in their work in constructing a *glocal* history. It is a matter of studying with the teachers

[...] the processes through which shared references, imposed models, texts and goods that circulate on a global scale are appropriated, to give meaning to a specific time and place (Chartier, 2007, p. 81; our translation).

Exchanges, circulation of curriculum models, local applications of international discussions. This is the dynamic that explains the transformation of mathematics education for some time now. The two historical moments that serve as emblematical examples of this dynamic are those at the beginning and middle of the 20th Century.

Having mathematics teachers realize how the mathematics curriculum that intended to teach Differential and Integral Calculus starting in 1908 through Felix Klein's interventions was formulated and given meaning seems to constitute an important point in their education. However, analyzing with the teachers how this international initiative took place in each country, and how it was given a specific meaning, favors the criticism of Platonism of the pure idea, the pure curricula, which must be equal everywhere, which seems to have been the intention of mathematicians and mathematics professors at the head of the movement at that time.

This same formative quality occurs in the analysis of the Modern Math Movement: have future mathematics teachers understand the modernistic proposal of synthesis in algebraic structures in the attempt to teach linear algebra and study what happened in their countries with the application of international documents can mean considerable advances in the formation of mathematics professors.

Such perspective is very different from that one of thinking mathematics education as unique, dealing with a school knowledge that should be uniform, international school mathematics. This, by the way, was the perspective announced by Jean Piaget in the emblematical work *L'Enseignement des mathématiques* published in 1955. In it, together with other authors, directed at mathematics education, Piaget positioned himself about mathematics teaching like this:

The problem with mathematics teaching is set today in terms that trespass frontiers. The differences due to different cultures are less important than the similarities resulting from the structure of science and the mathematics thinking (Piaget et. al. 1955, p. 8 - our translation).

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2 Schubring (2005) stresses the importance of the study of teacher's biographies as a means of understanding their own professional formation as well as the history of mathematics teaching itself.
Final Considerations

For these final considerations, there remains a discussion more focused on why to include the history of mathematics education in the education of mathematics teachers. Which takes us back to the title of this communication: what potential exists for this knowledge in teachers’ education?

More than anything else, this question points to the meaning of history as knowledge. To what point do the mathematics teachers need the history of mathematics education to better exercise his professional activity?

In reality, every field of knowledge is always interested in its history. That is why we have the history of philosophy, history of medicine, history of law, history of mathematics, etc. In this sense, recognition of the importance of the history of mathematics in the education of mathematics teachers means attesting to how significant it will be for the teachers to know how mathematics knowledge was and is being produced.

However, the formative dimension of the history of mathematics education seems to be different from the history of mathematics. It points to the professional development of the teacher, to his need to understand what his profession inherits from past times and that are present in his everyday pedagogical practice. Furthermore, the teachers of mathematics in the 21st Century are not an heir of mathematicians, but rather of teachers of mathematics from the 20th Century. Of course, this is controversial: the fact that the professional origins of this area of knowledge are not delimited by the mathematician/teachers of mathematics division. However, as pointed out by Michel de Foucault, in his criticism of the idea of origin, it is not a determinant in the unfolding of history. Thus, in professional terms, the elementary and high school teachers seem to owe much more to those teachers from schools and high schools constituted in the 19th Century and consolidated the following century, than the mathematicians, even if they worked in higher education.

There are questions regarding the representation of the past of the mathematics teachers’ profession. These are consolidated and certainly constitute a restraint to the good performance of the teachers’ professional activities. These questions go back to the lack of knowledge concerning the history of mathematics education. What can we say, for example, to mathematics teachers when they make extremely pessimistic statements about current teaching, such as? ‘school was better before’, ‘the teaching level has fallen and we no longer teach mathematics like before’, among several other representations expressed by mathematics teachers nowadays?

It will be up to the history of mathematics education to answer these. It will have the task of criticizing these representations of the past, which are fictional and memorial in reality, and deconstruct them on behalf of new representations founded on the review of documents and sources from professional practices carried out in the past. These actions certainly do not constitute activities the history of mathematics places as a priority in its research.
References