Empirical test of the predictive power of the capital asset pricing model on the European stock market
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B.Sc. of Business Administration

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Declaration of Research Work Integrity

This work has not previously been accepted in substance for any degree and is not being concurrently submitted in candidature of any degree. This thesis is the result of our own investigations, except where otherwise stated. Other sources are acknowledged by giving explicit references. A bibliography is appended.

By signing the present document, we confirm and agree that we have read RU’s ethics code of conduct and fully understand the consequences of violating these rules in regards of our thesis.

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Abstract

The purpose of this research is to examine the predictive power of the capital asset pricing model in the European stock market. The research follows the method used by Fama & MacBeth (1973) for the empirical analysis which tests stock returns on the S&P Euro index for the time-period 1998-2015. Betas are estimated for individual stocks and portfolios are then formed based on the ranked betas. From there, portfolio betas are estimated and regressed against actual portfolio returns to see if there exists a positive linear relationship between beta and average return. The results obtained from this research suggest that the predictive power of the capital asset pricing model is quite poor since the model failed to give significant positive results neither for the overall period nor any of the sub periods examined. Therefore, we cannot recommend the CAPM as a method for predicting stock returns.
This research is a thesis for a B.Sc. degree in Business Administration, with a financial emphasis, at Reykjavík University. The thesis accounts for 12 ECTS and was conducted during the period of December 2016 until May 2017. The authors of this thesis are Alexander Jónsson and Einar Sindri Ásgeirsson. In this thesis, the predictive power of the Capital Asset Pricing model is explored in the European market. The authors would like to thank the instructor of the thesis, Dr. Stefan Wendt, for all the help and good advice he provided.

Reykjavík, May 2017

_______________________________  _______________________________
Alexander Jónsson                Einar Sindri Ásgeirsson
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Introduction

Background
The rate of return from holding financial instruments is to some extent predictable over time, however the source of this predictability is subject to controversy as to whether the predictability is attributed to market inefficiencies or the result of changes in the required rates of return. Research suggests that the models with the strongest predictive power are the ones that focus on risk and required return (Ferson and Harvey, 1991).

The Capital Asset Pricing model or CAPM for short, is a widely used model in the financial sector to calculate the required rate of return of financial instruments. Additional uses include determining the price of stocks, estimate excess risk and occasionally predicting future returns in an efficient capital market (Lintner, 1965; Jensen, Black & Scholes, 1972; Fama & MacBeth, 1973).

The CAPM was first introduced in the mid-1960s, by William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) building on Markowitz (1952) modern portfolio theory. One of the attractions of the CAPM is that it offers robust and intuitively pleasing estimations about risk measurement and the connection between expected return and risk (Fama & French, 2004). However, empirical evidence against the CAPM has been presented which likely has a lot to do with the models generally simplifying assumptions and the difficulties in implementing the model. Nevertheless, the CAPM is commonly used in practice still today by many investors and CFO’s. For example, in a study by Grahm and Harvey (2001) 392 CFOs were surveyed to determine how firms calculate the cost of equity capital. They concluded that the CAPM is by far the most approved method. Accordingly, 73.5% of respondents reported either always using the CAPM or more frequently apt to using the CAPM for estimating the cost of capital, capital budgeting and capital structure.

When it comes to asset pricing, determining the cost of equity provides an important basis for estimating the required rate of return on investments. Kolouchová and Novák (2010) set out to investigate the popularity of various methods when it comes to estimating the cost of equity in practice. They found that valuation expert’s most popular model for determining the cost of equity was, in fact, the CAPM.

In his paper regarding capital market efficiency, Fama (1991) writes about the practical usefulness of the CAPM. He states that even though there exist empirical
evidence against the model, market professionals and academics still think about risk in terms of market beta and that practitioners retain the market line of the Sharpe-Lintner model as a representation of the trade-off between expected return for risk available from passive portfolios (Fama, 1991).

When making investment decisions, investors need to have as accurate information as possible. Most of the research relating to the CAPM focuses on the U.S. market so it is interesting to investigate the effectiveness of the model in other markets, for example the European market. Investing is always risky so the ability to assess this risk and evaluate different investment opportunities is extremely valuable. It is therefore advantageous for investors to know the strengths and weaknesses of the CAPM along with any useful applications. This information would be useful and interesting for all investors interested in investments in the Euro area along with the management of relevant companies.

Purpose

The purpose of our research is to investigate how accurate and reliable the model really is. The focus point will be to investigate how well the CAPM works when applied to stocks on the European market. This can be done by analysing historical data from companies in an index that is representative for the market. For the analysis of this project, we will focus on the companies in the S&P Euro Index. That choice is based on the availability and accuracy of data from that index. Historical price data from the companies in the index is gathered and analysed during an 18-year period, 1998-2015.

The CAPM has been around for a long time, but how does it fare in current times and on less researched markets such as the European market? In our analysis, we will test the predictive power of the CAPM using the method applied by Fama & MacBeth (1973) on the European market and determine whether the model is a viable option for predicting returns.

To test the validity of the CAPM we will seek to explain the following assumptions of the CAPM:

1. The relationship between the expected return on a security and its risk in any efficient portfolio is linear.
2. In a market of risk-averse investors, higher risk should be associated with higher expected return.
3. By adding the assumption that there is unrestricted riskless borrowing and lending at a known rate, \( R_f \), we have the market setting of the Sharpe (1964) and Lintner (1965) CAPM. Thus, it implies that the risk-free rate should equal the intercept.

The structure and division of the study is as follows: First we review what has already been written about the CAPM where we split the discussion into two sections. In the first section, we will take a general look and provide a basic understanding of the model touching on the models logic and assumptions. The second section will include discussing the existing research regarding the empirical tests and methodology applied and providing a discussion of the results and problems faced when testing the model. After reviewing the literature, we will discuss the development of our hypothesis. From there, we will introduce the data obtained to perform our tests and its limitations. Next, we will cover in detail, the methodology used in this research before moving on to presenting and discussing our results. Lastly, we will sum up the results in the conclusion chapter.

**Literature Review**

**The Capital Asset Pricing Model**

The Capital asset pricing model seeks to explain the relationship between expected returns and systematic risk in an efficient capital market. The importance of explaining this relationship serves two important roles. On one hand, the model gives investors an idea of the return they may expect on a given investment. It is useful for determining whether the expected return predicted by the investor is greater or less than the expected return given its systematic risk. On the other hand, the model helps investors to make an educated guess about the expected return on financial assets that have not been traded in the open market before, like IPO of a stock (Hirschey & Nofsinger, 2008; Bodie, Kane, & Marcus, 2011).

The foundation on which the CAPM is built was laid down in 1952 by Harry Markowitz’s modern portfolio theory. The CAPM was developed 12 years later in articles by William Sharpe (1964), John Lintner (1965), and Jan Mossin (1966). In his article, Harry Markowitz set up a formal model of portfolio selection giving a tangible form to diversification principles. Markowitz modern portfolio theory about mean-
variance portfolios shows how risk-averse investors can construct portfolios to optimize their expected return based on a given level of risk. Markowitz model sets out to identify the efficient frontier of risky assets. The main notion behind the efficient set of is that we are only looking at the portfolio that maximizes our expected return. Thus, the efficient frontier is constructed by portfolios that minimizes the variance for any target expected return. The methodology of Markowitz’s model allows us to determine a set of efficient portfolios, those that offer the best risk-return trade-offs. As can be seen on Figure 1 below, an investor would only want to hold a risky portfolio of assets that lies on the efficient part of the portfolio frontier, which starts at the global minimum variance portfolio (Bodie et al., 2011; Elton, Gruber, Brown, & Goetzmann, 2009; Markowitz, 1952).

![Figure 1. The minimum-variance frontier of risky assets.](Source: ReResolve asset management, n.d.)

Sharpe (1964) and Lintner (1965) added two key assumptions to Markowitz model. First is that for a given asset value over a specified period, investors agree on joint distribution of assets return. Secondly, unlimited lending and borrowing at a known risk-free rate for all investors. They proclaimed that there was a single portfolio of risky assets that is preferred to all other portfolios. Sharpe formed the following model commonly referred to as the Capital Allocation Line, CAL:

$$E(r_p) = r_f + (E(r_i) - r_f) \frac{\sigma_p}{\sigma_i}$$
Where \( r_p \) is the return of the portfolio, \( r_i \) is the return of stock \( i \), \( r_f \) is the risk-free rate, \( \sigma_i \) is the variance of stock \( i \) and \( \sigma_p \) is the variance of the portfolio.

With the slope expressing the reward-to-volatility ratio:

\[
\text{Sharpe ratio: } (E(r_i) - \frac{r_f}{\sigma_i})
\]

This tells us how much in terms of return the investor needs to be compensated for assuming an additional unit of risk. For every risky portfolio on the efficient frontier we draw the corresponding CAL. The slope of the CAL is maximized at the point of tangency between CAL and efficient frontier. The new efficient frontier is a straight line starting at the risk-free rate and passes through point A in Figure 2, which is referred to as the optimal risky portfolio. This result is very powerful and implies that all investors, regardless of risk aversion, will invest in the same portfolio of risky assets. This result is also referred to as the separation property as it tells us that the portfolio choice problem can be separated into two independent tasks:

1. Finding the optimal risky portfolio
2. Allocating funds between the optimal portfolio risky portfolio and the risk-free asset.

![Figure 2. The Capital allocation line, the optimal risky portfolio](Source: Munasca, 2015)
Now, four decades later the CAPM has become the core of modern financial economics and is still widely used in investments applications, such as asset pricing, risk evaluation and performance assessment of managed portfolios. The attraction of the CAPM is that it offers powerful and intuitively pleasing predictions about risk measurement and the relation between expected return and risk (Fama & French, 2004).

The CAPM is based on several simplifying assumptions:

1. There are many investors, each with an endowment (wealth) that is small compared to the total endowment of all investors. Investors are price-takers, in that they act as though security prices are unaffected by their own trades.
2. All investors plan for one identical holding period.
3. Investments are limited to a universe of publicly traded financial assets, such as stocks and bonds, and to risk-free borrowing or lending arrangements.
4. Investors pay no taxes on returns and no transaction costs on trades in securities.
5. All investors are rational mean-variance optimizers, meaning that they all use the Markowitz portfolio selection model.
6. All investors analyse securities in the same way and share the same economic view of the world. The result is identical estimates of the probability distribution of future cash flows from investing in the available securities; that is, for any set of security prices, they all derive the same input list to feed into the Markowitz model. This assumption is often called homogeneous expectations. (Bodie et al., 2011, page 309)

Many of these assumptions are clearly unrealistic, however, we can look at how the equilibrium will prevail in this hypothetical world of securities and investors when:

1. All investors choose to hold a portfolio of risky assets (will refer to as stocks) in proportion that duplicate representation of the assets in the market portfolio (M), which includes all traded assets. The proportion of each stock in the market portfolio equals the market value of the stock divided by the total market value of all stocks.
2. Not only will the market portfolio be on the efficient frontier, but it also will be the tangency portfolio to the optimal capital allocation line (CAL) derived by each investor. Thus, the capital market line (CML), the line from the risk-free rate through the market portfolio, M, is also the best attainable capital allocation line. All investors hold M as their optimal risky portfolio, differing only in the amount invested in it versus in the risk-free asset.
3. The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the representative investor.
4. The risk premium on individual assets will be proportional to the risk premium on the market portfolio, $M$, and the beta coefficient of the security relative to the market portfolio. Beta measures the extent to which returns on the stock and the market move together.

(Bodie et al., 2011, page 309-310)

Now we have the following standard CAPM equation that can be used to calculate expected return:

$$E(r_i) = R_f + \beta_i (E(R_m) - R_f)$$

Where $E(r_i)$ is the expected return on a security $i$, $R_f$ is the risk-free rate, $\beta_i$ is the beta of the asset $i$, and $E(R_m)$ is the return on the market.

The linear relationship between expected return and risk, measured by beta can be seen graphically as the Security market line (SML) in Figure 3, illustrates how risky securities are priced according to the CAPM. Securities that are priced correctly according to the model should plot on the SML. Thus, investors are being fully compensated for risk. However, securities that plot above the SML are under-priced and stocks below the SML are over-priced. Because the beta of the market is 1, the slope of the SML is the risk premium of the market portfolio (Bodie et al., 2011; Elton et al., 2009)

![Figure 3. The Security market line.](Source: Subach, 2010)
The risk premium on a stock or portfolio varies directly with the level of systematic risk, beta. The security market line (SML) shows you the risk or expected return trade-off with CAPM. Sharpe (1964) found that total risk of securities could be broken down into two risk types: systematic risk or market risk and unsystematic risk. The difference between the two is that, unsystematic risk is limited to each individual asset and can be diversified away unlike systematic risk that cannot be eliminated through diversification. Unlike the Capital Market Line (CML), which is drawn using standard deviation as a measure of risk, hence meant to reflect total risk, the SML is drawn using the contribution of the asset to the portfolio variance, systematic risk, which is measured by the assets beta (Hirschey & Nofsinger, 2008; Lofthouse, 2001). Meir Statman (1987) shows that 10-15 securities will diversify most of the unsystematic risk away. This effect is illustrated in Figure 4.

![Figure 4. Unsystematic risk and diversification](Source: Ordur textile and finance, n.d.).

**Testing the CAPM**

One of the main assumptions predicted by the CAPM, as stated above, is that the market portfolio is a mean-variance efficient portfolio. Consider that the CAPM treats all traded risky assets. To test the efficiency of the CAPM market portfolio, we would need to construct a value-weighted portfolio of a huge size and test its efficiency. So far, that task has not been feasible. An even more difficult problem in testing the validity of the model is that the CAPM is stated in terms of expected returns, that is, the model implies a linear relationship between expected returns and systematic risk. To be able to test
the theory we must make a shift from expected returns to actual returns, as all we can observe is actual returns. To address the problems listed above we can employ the market model, originally proposed by Markowitz (1959) and extended by Sharpe (1963) and Fama (1968). Now we can represent security returns in excess form as:

\[ R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it} \]

Where \( R_{it} \) is the excess return of the asset, \( \alpha_i \) is the intercept, \( \beta_i \) is the beta of the asset, \( R_{m} \) is the return on the market and \( \epsilon_{it} \) is the error term.

Let us now see how this framework for statistically decomposing actual stock returns meshes with the CAPM. The non-systematic risk is independent of the systematic risk, that is, \( \text{Cov}(R_m, R_i) = 0 \). Therefore, the covariance of the excess rate of return on a security with that of the market index is:

\[ \text{Cov}(R_m, R_i) = \beta_i \sigma_m^2 \]

Where \( \beta_i \) is the beta of the asset, \( R_i \) is the return on the asset, \( R_m \) is the return on the market and \( \sigma_m^2 \) is the variance of the market.

And because of that, the sensitivity coefficient, \( \beta_i \) in equation (1), which is the slope of the regression line representing the index model equals:

\[ \beta_i = \frac{\text{Cov}(R_m, R_i)}{\text{Var}(R_m)} \]

Where \( \beta_i \) is the beta of the asset, \( R_i \) is the return on the asset and \( R_m \) is the return on the market.

The index model beta coefficient turns out to be the same beta as that of the CAPM expected return-beta relationship, except we can replace the theoretical market portfolio of the CAPM with the well-specified and observable market index (Bodie, Kane & Marcus, 2011).

Black (1972) developed an extended version of the CAPM often referred to as the Zero-beta CAPM, where he relaxes the assumption of unlimited borrowing and lending at a known risk-free rate as he feels that assumption is unrealistic. He introduces another assumption that allows unrestricted short selling. If there is no riskless asset exists investors can use a portfolio of risky assets that is uncorrelated with the market portfolio instead as the riskless asset. This portfolio is called the Zero-beta portfolio.

The equation for Black’s model is much like the Sharpe-Lintner one. The only thing that changes is that the risk-free rate, \( r_f \), is replaced with the expected return on the assets uncorrelated with the market, \( E(R_{zm}) \), that is:

\[ E(r_i) = E(R_{zm}) + \beta_i (E(R_m) - E(R_{zm})) \]
Where $E(r_i)$ is the expected return on a security, $E(R_{zm})$ is the rate of return on the zero-beta portfolio, $\beta_i$ is the beta of the asset and $R_m$ is the return on the market.

**Empirical tests**

Early on, majority of the empirical tests included the use of a time series first pass regression. This estimated the betas and the use of a cross-sectional second pass regression to test the hypotheses we derived from the CAPM model (Elton et al., 2009)

**Lintner (1965) and Douglas (1967)**

John Lintner (1965) conducted the first empirical test from the CAPM. Soon after, Douglas (1967) duplicated the study in which resulted in a similar outcome to Linter’s original test. Lintner used the two-pass procedure on individual stocks. He used US data in which covered a 10-year period, 1954-1963 with one measurement per year. He looked at returns of 301 companies listed on the New York Stock Exchange and used the predecessor of the S&P 500 index was used as a proxy for the market portfolio. To estimate beta for each security Lintner used the following regression, where he regressed each security’s annual return against the return on the market index.

$$R_{it} = \alpha_i + \beta_i * R_{mt} + \epsilon_{it}$$

(i)

Where $\beta_i$ is the beta of the asset, $R_{it}$ is the return on the asset at time $t$, $R_{mt}$ is the return on the market at time $t$ and $\epsilon_i$ is the error term and $\alpha_i$ is the intercept.

After estimating the beta values the second pass cross sectional regression was conducted.

$$R_i = y_0 + y_1 \beta_i + y_2 \sigma^2(\epsilon_i) + \epsilon_i$$

(ii)

Where $y_1$ is the slope of the beta of the asset, $R_i$ is the return on the asset $i$, $y_2$ is the slope of the residual variance and $\epsilon_i$ is the error term and $y_0$ is the intercept.

If the CAPM were to hold, the coefficients should be: $y_0 = 0$, $y_1 = R_m - R_f$ and $y_2 = 0$. However, the results were not promising for the CAPM, the coefficient for the intercept, $y_0$, was much larger any acceptable risk-free rate, the coefficient for the residual risk, $y_2$, was significantly different from zero and slope of the SML, $y_1$ was significantly different from the market risk premium.
Miller & Scholes (1972)

Miller and Scholes (1972) conducted a study on the CAPM following the two-pass procedure (Lintner, 1965; Douglas, 1967) and obtained very similar results. That being, the slope was too flat and notably different from the market risk premium. The intercept and the residual variance were significantly different from zero. However, in order to examine the statistical problems from the two-pass procedure, Miller and Scholes ran a simulation test by generating random numbers. According to the CAPM model they set up numbers that accurately met the expected return beta relationship. That is, the simulated rates of return were set up to completely agree with the CAPM. However, the results were almost identical to the ones generated using real data, thus the CAPM was rejected using the two-pass technique. Evidently, this proves that there are some statistical problems with the two-pass methodology. They demonstrated that even if the CAPM is rejected using that method, the model could still be valid. They go on to argue that the methodology used in testing the model and further resulting in rejecting the model is a consequence due to an error in measuring beta for the second-pass cross sectional regression. Because of statistical problems, it is evident why Miller and Scholes concluded that the two-pass procedure is problematic. The regression equation (i) estimates the coefficients simultaneously and these estimates are dependent on each other. In estimating the intercept of a single variable regression depends on the estimate of the slope coefficient. In conclusion, if the beta estimate is biased, so will the estimate of the intercept.

They show that we must deal with the emerging statistical problems by considering betas from the first-pass regression which are estimated with a substantial sampling error. Thus, the estimated betas are not a good input for the second-pass regression and will provide misleading results, ultimately not testing the validity of the model. By using said beta, estimated with a measurement error, in the second-pass regression will lead to a downward biased coefficient for the slope, $y_1$, and an upward biased coefficient for the intercept, $y_0$. In the empirical testing of the model covered above we can see that is exactly the case, that is, the intercept being higher than predicted by the CAPM resulting and the slope being much flatter than predicted by the model. (Bodi, Kane & Marcus, 2011). Beta estimate variability is also caused by the fact that other important but unmeasured sources of common stock volatility are at work. Model specification bias distorts beta estimates because the market model fails
to include other important systematic influences on stock market volatility (Hirschey & Nofsinger, 2008).

Black, Jensen & Scholes (1972)

Even though the early empirical tests listed above reject the CAPM, it does not necessarily mean the model is defective. It could simply be due to a statistical error from measuring beta. In order to avoid the measurement error that effectively led to biased estimates of the SML, Black, Jensen and Scholes (1972) provided additional insight into the nature and structure of security returns. Rather than testing individual securities, they proposed a method to improve the precision of estimated betas by grouping securities in portfolios. In their article, they showed that in previous tests of the model the structure of the process in which appears to be generating the data causes the cross-sectional tests of significance to be misleading and therefore not providing direct tests of the validity of the CAPM. Due to aggregation, estimates of portfolio betas will be less affected by measurement error than beta estimates of individual securities, that is, it will reduce the statistical errors that may appear when estimating beta coefficient. They examined all securities on the NYSE from 1926-1965 by using an equally weighted portfolio of all securities from the NYSE as a proxy for the market index. The outline of their study began by estimating beta for each security by regressing monthly returns against the market index for the first 60 months of the time period using equation (i). Then 10 portfolios were formed by ranking securities in order of their calculated betas. Portfolio 1 was comprised of securities with the highest betas, portfolio 2 comprised of securities with the next highest betas and so on. By doing so, they established that the portfolios had a large spread in their betas. However, by forming a portfolio of securities based on their estimated beta would still lead to unbiased estimates of the portfolio beta since betas used to select portfolios would still be subject to measurement error. To overcome this problem, they used betas of individual securities estimated in a previous period to form portfolios for the following year. By doing that, they could eliminate much of the sampling variability in estimated betas for individual securities. The next step is to calculate each portfolio’s monthly returns for the following year and then the steps were repeated once a year for the entire sample period. Then, they calculated the mean monthly returns and estimated beta coefficients for each of the 10 portfolios. Lastly, they regressed mean portfolio returns against the portfolio betas using equation (ii) for the entire sample period as well as
various sub-periods. The evidence from their empirical analysis led them to reject the
classical form of the CAPM as the term for the intercept was greater than the risk-
free rate. The results, however, seemed to be consistent with the zero-beta CAPM as
the term for the slope of the SML was significant and positively linear as predicted by
the model. In general, the results for the whole testing period and sub-periods were
similar.

**Sharpe and Cooper (1972)**

Sharpe and Cooper (1972) conducted a simple test on the CAPM in the form of
simulated portfolio strategies. From 1931-1967, the stocks on the New York stock
exchange were studied and tested with the initial goal to determine if the securities with
higher betas would carry a higher return. The constructed portfolios with different betas
by, first, regressing individual asset returns against market returns based on the previous
five years using equation (i). Once a year they ranked all shares by their beta and
divided them into 10 categories. They formed an equally weighted portfolio for each
category, the highest beta stocks in one portfolio, the next highest in another portfolio
and so on. The investment strategy followed was to hold securities in one category only,
over the entire period. The results of this research showed that shares with higher betas
generate higher returns. They found that more 95% of the variation in expected return
can be explained by differences in beta hence, it showed a linear relationship between
realized average returns and their betas. However, the intercept or the risk-free rate was
5.54% which is considerably higher than the 2% it was during this period and thus not
consistent with the Sharpe-Lintner version of the model. However, these results lend
support to the zero beta CAPM.

**Fama & MacBeth (1973)**

Fama and MacBeth (1973) follow a similar methodology as conducted by Black et al.,
(1972) but added another explanatory variable, square of the beta coefficient, to the
equation to test that no nonlinearities exist in the risk-return relationship. The
importance of the linear condition had been largely overlooked in the early empirical
tests of the model. Another distinction from the BJS study is that Fama and MacBeth
use betas in one period to predict returns in a later period whereas in the BJS method,
betas and average returns were computed in the same period. The following stochastic
model for returns was used to test the validity of the CAPM:

\[ R_{pt} = y_{0t} + y_{1t}B_p + y_{2t}B_i^2 + y_{3t}\sigma^2 + \eta_{pt} \]
The subscript p refers to constructed portfolios but not individual stocks. Using equation (iii) Fama and MacBeth tested a series of hypotheses regarding the CAPM. They tested whether the expected value of the risk premium, \( y_{1t} \), which is the slope of the SML is positive, that is, \( E(y_{1t}) = E(R_{mt} - r_f) > 0 \). To test for linearity the variable for beta squared is included in the equation, thus, the hypothesis tested for that condition is \( E(y_{2t}) = 0 \). To test for risk that is unrelated to beta the hypothesis is tested, \( E(y_{3t}) = 0 \). They also test whether the intercept equals the risk-free rate, that is, \( E(y_{0t}) = r_f \). The disturbance term, \( \eta_{it} \) is assumed to have a mean of zero and to be independent of all other variables in the equation. Like Jensen, Black & Scholes (1972), Fama and MacBeth find that the intercept is greater than the risk-free rate, subsequently rejecting the Sharpe-Lintner CAPM. However, the results of their study supported the important testable implications of the zero-beta CAPM. The coefficient, \( y_1 \), was significantly different from zero, thus, the slope of the SML was positive further indicating a positive risk-return trade off. The coefficients, \( y_2 \) and \( y_3 \) were not significantly different from zero, meaning there is no evidence of nonlinearity or residual variance affecting returns. Finally, the observed fair game properties of the coefficients and residuals of the risk-return regressions are consistent with an efficient capital market where price fully reflects all available information.

**Modigliani (1972)**

Empirical tests and research is mostly focused on stocks listed in the United States. In essence, the literature is not as comprehensive when it comes to the European market. Reasoning for this could be the availability of data and a greater efficiency in the American stock market. However, in 1972, Franco Modigliani ran the first tests of the CAPM in the European stock market. Prior to his study, the CAPM had never been on other capital markets apart from the US market. The tests were conducted on eight major European markets where he replicated earlier tests made on the U.S. market (Black, Jensen & Scholes (1972), Friend & Blume (1970), and Jacob (1971)). Modigliani (1972) stated that European markets were generally believed to be less efficient than the U.S. market. If true, it would imply that the pricing of risk for European securities might be less rational than for American securities. The data used for the empirical test were daily price and dividend data of 234 common stocks from eight major European countries from March 1966 through March 1971. The data was...
corrected for all capital adjustments. Since the data came from eight different countries, the stocks were analysed separately so the regression results were presented individually for each country. Results from the U.S. market over a similar time period were used as a comparison. The outcome of the study provided some support to the hypothesis by showing that systematic risk is an important factor for the pricing of European securities. Furthermore, a positive relationship between realised return and risk was shown for seven out of the eight markets tested. Additionally, no evidence of lesser rationality or efficiency of the European stock markets was found. Still, the test period was short, spanning only five years and the sample limited (Modigliani, 1972).

**Roll’s critique (1977)**

Known today as Roll’s critique, Richard Roll (1977), criticized previous research on the Capital asset pricing model. He suggested the methodology was not accurate because tests were performed only using a proxy of the market portfolio but not the real market portfolio. He pointed out that the only testable hypothesis associated with the CAPM would be that the market portfolio is mean-variance efficient and all other implications of the model such as linear relationship between return and beta, follow from the markets portfolio’s efficiency and therefore are not independently testable. Roll claimed that the real market portfolio includes all assets which also includes human capital and real estate. He further suggests that using only a proxy of the market portfolio would not be enough to prove or disprove whether the CAPM works. He went more in depth to say that it was impossible to define the real market portfolio and therefore stated that the CAPM could never be tested in earnest.


Roll and Ross (1995) and Kandel and Stambaugh (1995) expanded Roll’s critique. They argued that the rejection of the risk-return relationship indicated by the CAPM could be the result of proxies in the market portfolio are mean-variance inefficient rather than refuting the relationship between average returns and betas of the model. They reveal that virtually any proxy for the market portfolio that is not considerable inefficient and should produce a positive linear risk-return relationship in a large sample could fail to produce a significant relationship. Kandel and Stambaugh (1995) explain how many do not view the implications of the CAPM separate since either implies the other.

The CAPM implications are embedded in two predictions:
(1) the market portfolio is efficient, and
(2) the security market line (the expected return-beta relationship) accurately
describes the risk-return trade-off, that is, alpha values are zero.

In their article, they go on to demonstrate how one implication could hold nearly perfect
while the other fails dramatically. They tested Black’s zero beta CAPM using the two-
pass method with a proxy for the market portfolio as the efficient portfolio is only
theoretical. After running the first-pass time series regression (i) they ran the following
second pass generalized least squared regression. By doing that they accounted for
correlation across residual.

\[ r_i - r_z = y_0 + y_1 \times (\text{Estimated } \beta_i) \]

Where \( r_i \) is return on stock \( i \), \( r_z \) is, \( y_0 \) is the intercept, \( y_1 \) is the beta and \( \beta_i \) is the estimated
beta.

Their conclusion where that the coefficient of the intercept and coefficient of the slope
will be biased by a term proportional to the relative efficiency of the proxy chosen to
reflect the true market portfolio. If the market index used in the regression is fully
efficient, the test will be well specified. But the second-pass regression will provide a
poor test of the CAPM if the proxy for the market portfolio is not efficient. Thus, we
still cannot test the model in a meaningful way without a reasonably efficient market
proxy.

David W. Mullins (1982)

David W. Mullins (1982) made a comprehensive assessment of the validity of the
CAPM. His findings were that the model was imperfect, but went on to explain how
the model was an important tool for investors when it comes to determining required
return of assets. Furthermore, he suggested that even if the assumptions of the model
were theoretical and unrealistic, it is important to simplify the reality in order to
construct comprehensive models to determine asset pricing.

Clare, Smith, & Thomas, (1997)

Clare, Smith, & Thomas, (1997) tested both the conditional and unconditional versions
of the CAPM on the UK stock market. Their paper provides an important link between
formal asset pricing and the evidence for the predictability of excess returns. They use
both market value ranked and dividend yield ranked portfolios to find a suitable spread
between risk and return. The results show a significant and powerful role for beta in
explaining expected returns.
Pettengill, Sundaram, & Mathur (1995)

Pettengill, Sundaram, & Mathur (1995) find a consistent and highly significant relationship between beta and portfolio returns. They make a key distinction between their test and previous empirical tests in that they explicitly recognize that the positive relationship between returns and beta as predicted by the Sharpe-Lintner-Black model is based on expected rather than realized returns. They look at the impact caused by using realized market returns as a proxy for the expected market returns. In periods where excess market returns are negative, an inverse relationship between beta and portfolio returns should exist. When they adjust for the expectations concerning negative market excess returns, they find a consistent and highly significant relationship between beta and portfolio returns for the total sample period and across subperiods. They also find a positive risk-return trade-off.

Fama & French (1992)

Fama & French (1992) wanted to examine possible factors on stocks returns besides the beta, such as book to market equity and size. The data they used came from all nonfinancial firms on the NYSE, AMEX and NASDAQ and annual industrial files of income-statement and balance-sheet data from 1962-1989. Since they had values of book to market equity, leverage and other accounting variables, they could estimate a portfolio beta and assigned that beta to each stock in the portfolio. This allowed them to use individual stocks in the Fama-MacBeth regressions. During the pre-1969 period, they acknowledged a positive relationship between beta and average returns (Black, Jensen, & Scholes, 1972; Fama & MacBeth, 1973). However, from the 1969-1990 period, their results suggest that the relationship between average return and beta disappears. In addition, they find that the beta-return relationship is weak during the 50-year period from 1941-1990. Conclusively, their tests did not support the prediction from the basic Sharpe-Lintner-Black model. Fama & French (1992) suggest that the poor results for beta could be because of other explanatory variables that are correlated with the true betas which overshadows the relationship of average returns and the estimated betas. Still, it does not explain why the beta seems to have little explanatory power during the respective time periods when used on its own. The results of Fama & French (1992) continue to determine that book to market equity, and earnings-price ratio are poor substitutes for beta. However, their main results conclude that size and
book to market equity, which are easily measured variables, seem to describe the cross-section of average returns. They summarise their results as follows:

1. When we allow for variation in beta that is unrelated to size, there is not reliable relation between beta and average return.
2. The opposite roles of market leverage and book leverage in average returns are captured well by book-to-market equity.
3. The relation between E/P and average return seems to be absorbed by the combination of size and book-to-market equity.

(Fama & French, 1992, page 445)

**Fama and French (2004)**

Further research on the CAPM includes yet another paper by Fama and French (2004) where they go over the recent and relevant research on the CAPM. They state that the CAPM presented by Sharpe (1964) and Lintner (1965) has never been an empirical success whereas the version by Black (1972) has had some positive findings. However, when the research began to incorporate variables like size, several price ratios, and momentum that also contribute to the explanation of average returns that the beta provides, the results of the model faced problems that according to Fama & French (2004) are enough to invalidate most applications of the CAPM. The Three-factor-model presented by Fama & French (1992) adds book to market equity and size to the formula, which with the beta constitute the three factors. With that model up to 90% of the diversified portfolio returns. These results are further supported by Fama & French (1993) and (1996). For the scope and purpose of our thesis we will focus on the earlier approach used by Fama & MacBeth (1973) which does not incorporate factors other than beta.

**Hypothesis development**

**Testable implications**

To test conditions of C1-C2 we must identify some efficient portfolio, Rm. Since it is unobservable we must use a proxy for the efficient portfolio. A proxy is, in general, considered good when its movements correspond relatively well to movements of the theoretically correct variable (Studenmund, 2011). If we assume that the capital market is perfect, short selling of all assets is unlimited and expectations are homogeneous, meaning that all investors derive the same and correct assessment of the distribution of
the future value of any asset or portfolio from information available without cost. Then Sharpe (1972) has shown that in a market equilibrium, the market portfolio, defined by,

\[ X_{im} = \frac{\text{Total market value of all units of asset } i}{\text{Total market value of all assets}} \]

is always efficient. Since it contains all assets in positive amounts, the market portfolio is a convenient reference point for testing the expected return-risk conditions C1-C2 of the model (Fama & MacBeth, 1973).

The CAPM model is presented in terms of expected returns but its implications must be tested with data on period-to-period security and portfolio returns. Fama and MacBeth (1973) suggest a model of period-to-period returns that allow the use of observed average returns to test the expected return conditions of C1-C2, but at the same time in as a general way as possible. For this purpose, a similar stochastic model they used in their research is used for this study:

\[ R_{it} = y_{0t} + y_{1t}B_{1} + y_{2t}B_{1}^2 + e_{i} \]  

(1)

Where \( y_{0t} \) is the intercept, \( y_{1t} \) is the slope and \( y_{2t} \) is the beta squared.

Fama and MacBeth (1973) outline the following testable implications of the CAPM:

**Condition 1**

The relationship between the expected return on a security and its risk in any efficient portfolio is linear \( \text{E}(y_{2t}) = 0 \).

**Condition 2**

In a market of risk-averse investors, higher risk should be associated with higher expected return: that is, \( \text{E}(R_{m})-\text{E}(r_{f}) > 0 \).

If we add the assumption that there is unrestricted riskless borrowing and lending at a known rate, \( R_{ft} \), then we have the market setting of the Sharpe (1964) and Lintner (1965) CAPM\(^3\) which implies that \( \text{E}(y_{0t}) = R_{ft} \)

---

\(^1\) Condition 1 will be referred to as C1 here after  
\(^2\) Condition 2 will be referred to as C2 here after  
\(^3\) The Sharp-Lintner hypothesis will be referred to as SL
The Hypotheses

For the sake of the scope of this thesis the focus will be on conditions 1 and 2. The following hypotheses are tested:

\[ C1 - H_0: E(y_{2t}) = 0 \]  \hspace{1cm} \text{Linearity}

\[ C2 - H_0: E(y_{1t}) = E(R_m) - E(r_f) > 0 \]  \hspace{1cm} \text{Positive expected risk-return trade off}

\[ SL - H_0: E(y_{0t}) = 0 \]  \hspace{1cm} \text{Sharpe – Lintner CAPM}

If the CAPM holds, the Sharpe – Lintner condition should not be rejected. The coefficient \( E(y0) \) should equal zero because the SML starts at the y-axis at the average risk free return. Condition C2 should be rejected. The slope of the SML, \( E(y_1) \) should indicate a positive price of risk. Condition C1 should not be rejected. The coefficient \( E(y_2) \) should equal zero because the effects of nonlinearities should equal zero. If the coefficient for beta squared, \( E(y_2) \) is positive that would indicate a nonlinear relationship between risk and return. High-beta securities would provide expected returns more than proportional to risk.

Data

The data for this study are monthly percentage returns for all stocks in the S&P EURO since the period January 1998 through December 2015. The data comes from a Thomson Reuters Eikon terminal DataStream. The index S&P EURO is selected for this research and is a sub-index of the S&P 350 which includes all Eurozone domiciled stocks from the parent index. The index contains constituents from developed markets within the Euro Zone only. The index is designed to be reflective of the Eurozone market. The stocks are float-adjusted market cap weighted. The dataset used for the research is the monthly historical price of the index for the last 20 years along with historical price data for each company in the index for the last 20 years. From this data, the calculations for the empirical tests are made. As we are following the methodology of Fama and MacBeth (1973) we will be using the total return data as they did opposed to raw price data that was also available. The total return index assumes dividends are reinvested in the index after the close on the ex-date.
Methodology

In this section the methodology of the empirical analysis is explained step by step with all formulas used for calculations are presented. For the empirical test the methodology approach used by Fama and MacBeth (1973) is followed.

The first step in the calculations is to calculate the actual monthly returns for each stock over the entire time horizon of available data from the historical price data in the dataset.

Then sub periods are formed, the sub periods overlap where the first sub period spans the first 9 years of the entire period (1998-2006) and then rolls forward three years so that the next sub period spans another 9 years and starts three years later than the first period (2001-2009) for a total of four sub periods. Each sub period is further split into three parts; Portfolio formation period where betas for each stock are calculated from the stock return data and portfolios formed, portfolio beta estimation period where a beta is estimated for each portfolio, and a testing period, where the test values for conditions C1 and C2 are gathered.

When the portfolios have been formed, the portfolio return for each month is calculated as the average return from the individual monthly returns of the stocks in each portfolio both for the portfolio beta estimation period and the testing period. From there the fixed beta of each portfolio from the portfolio beta estimation period is calculated. Finally, we regress monthly portfolio returns from the testing period against the fixed portfolio betas which generates 36 values for the intercept and slope for each sub period. From there the mean values for the intercept and slope are tested via t-test to see whether the slope and intercept values are significantly different from zero. In addition, we run a regression of average returns from each portfolio against their fixed betas along with the betas squared to test for linearity. The steps are repeated for each sub period. Each step is explained in further detail below.

Actual Returns

From the historic price data gathered into the dataset, actual returns are calculated for each stock and the index itself by subtracting the price at month 1 from the price at month 2 and dividing that outcome by the price at day one. By subtracting one from that outcome a percentage return for the first month has been found. The formula is then repeated for each month of the price data to get the monthly returns. In general terms:
\[ R_{it} = \frac{P_{it} - P_{it-1}}{P_{it-1}} \]  

(2)

Where \( R_{it} \) is the return of stock \( i \) at time \( t \), \( P_{it} \) is the price of stock \( i \) at time \( t \) and \( P_{it-1} \) is the price of stock \( i \) at time \( t-1 \).

The S&P Euro Index returns are calculated in the same way:

\[ R_{mkt} = \frac{I_{t} - I_{t-1}}{I_{t-1}} \]  

(3)

Where \( R_{mkt} \) is the return of the index at time \( t \), \( I_{t} \) is the price of the index at time \( t \) and \( I_{t-1} \) is the price of the index at time \( t-1 \).

**Period analysed**

To be able to give a good estimation we use data spanning the longest possible period. Preferably a time period spanning a few decades would be used however given the availability of data an 18-year period used. For the test itself, the data is analysed in four sub periods, each consisting of three stages where each stage (period) lasts three years. The first stage is the portfolio formation period where a beta for each stock in the dataset is calculated based on three year monthly returns. The stocks are then grouped into portfolios by the rank of their beta. The next three years are used for estimation of the beta of each portfolio by regressing portfolio returns against the S&P Euro Index and the final three years are used to regress the portfolio returns against the portfolio betas.

We choose the starting period, 1998, because the Euro came around that time.

Table 1. Portfolio formation, Beta estimation and testing period overview.

<table>
<thead>
<tr>
<th>Periods</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</table>

**Estimation periods**

The choice of the 3-year portfolio formation periods and 3 year periods for estimating the independent variable \( \beta_i \) in the risk-return regressions reflects a desire to balance the statistical power obtained with a large sample from a stationary process against potential problems caused by any nonconstancy of the \( \beta_i \) (Fama & MacBeth, 1973).
Within the 18-year period analysed an estimation period of three years is used to evaluate the betas for each company like how Fama and MacBeth (1973) conducted their tests where 4 and 5 years were used for the estimation, however they used data from the period 1926 until 1968 or 42 years which is a much longer period than the 18 years in this study. Therefore, a three-year period was selected which is still a multi-year period and fits better to the scope of this study. Furthermore, by using a multi-year estimation period, random fluctuations from each year are diversified making the estimate more reliable.

**Stock beta estimation**

Like has been mentioned before, testing the model presents an unavoidable “error in variables” problem, The CAPM equation is in terms of true values of the relative risk measure $B_i$, but in empirical tests we must estimate $B_i$. In this study, we estimate the betas for each stock using the following equation:

$$\beta_i = \frac{Cov(R_m, R_i)}{Var(R_m)}$$

(4)

Where $R_m$ is the return on the proxy for the market portfolio, the S&P Euro Index and $R_i$ is the return of the individual stock $i$. Blume (1970) explains that if the errors in the estimated $B_i$ are substantially less than perfectly positively correlated, the estimated Betas of portfolios can be much more precise estimates of true Betas than the estimated Betas for individual securities. To reduce the loss of information in testing the risk-return relationship caused by using portfolios rather than individual securities, a wide range of values of estimated portfolio betas is obtained by forming portfolios based on ranked values of estimated betas for individual securities. However, forming portfolios using ranked values of betas causes clustering of positive and negative sampling errors within portfolios. That could lead to large portfolio betas being overstated of the true beta and low portfolio betas being underestimated. This can be minimised to a great extent by forming portfolios using ranked betas from one period and then using a subsequent period to estimate portfolio betas. With fresh data, within a portfolio errors in the individual security beta are to a large extent random across securities, so that in a portfolio beta the effects of this sampling error is minimized. (Fama & MacBeth, 1973; Black, et.al, 1973; Blume & Friend, 1970). From the individual stock betas, the portfolios are formed based on their rank.
To illustrate the method, we now go through the steps taken for the first sub period (which lasts from 1998-2006) by calculating the beta for each stock in the dataset using monthly returns from the first portfolio formation period 1998-2000 using formula (3). Then the betas are ranked from highest to lower and six portfolios constructed, where portfolio 1 contains the stocks with the 20 highest betas and portfolio 6 contains the 22 stocks with the lowest betas. Portfolios 2,3,4 and 5 all contain 20 stocks with appropriate betas. We choose the number of securities in each portfolio based on Statman’s (1987) research where he stated that 15-20 stocks would diversify away most of the firm-specific part of returns.

**Portfolio return estimation**

From there the monthly portfolio returns are calculated for both the portfolio beta estimation period 2001-2003 and the test period itself 2004-2006, in total six years. This is done by taking the average of the individual monthly returns of the stocks in each portfolio using the formula:

\[
R_{t, \bar{x}} = \frac{1}{n} \times \sum_{i=1}^{n} x_i
\]

(5)

Where \( n \) is the number of stocks being averaged, \( x_i \) is the individual stock in the list of stocks in the portfolio and \( R_{t, \bar{x}} \) is the monthly return of the portfolio at time \( t \).

**Portfolio beta estimation**

From there the beta of each portfolio is calculated in the portfolio beta estimation period 2001-2003 by regressing portfolio monthly returns against the market index, using the following equation:

\[
R_{pit} = \alpha_p + \beta_{pi} \times R_m + e_{pi}
\]

(6)

Where \( R_{pit} \) is the return of portfolio \( i \) at time \( t \), \( \alpha_p \) is the intercept, \( \beta_{pi} \) is the beta of portfolio \( i \), \( R_m \) is the return of the market index and \( e_{pi} \) is the standard error.

**Portfolio beta estimation – Time varying**

In order to estimate time varying betas of each portfolio we conduct a similar method as used by Fama and MacBeth (1973). For the portfolio estimation period 2001-2003 we use equation (7) by regressing portfolio monthly returns against the market index to get each portfolios beta for the following month. After that we roll the period forward
month by month the next three years (2004-2006). Thus, by using a three-year window and rolling the window forward month by month allows us to estimate each portfolio’s monthly beta.

**Estimating the SML**

When the portfolio betas have been estimated, the portfolio betas and the portfolio returns are used to estimate the intercept and slope of the security market line. We estimate the ex-post security market line for each month of the testing period of 2004-2006 by regressing portfolio returns against the portfolios betas using the following equation:

\[ R_{it} = y_{0t} + y_{1t}B_1 + \varepsilon_i \]

(7)

Where \( y_{0t} \) is the intercept, \( y_{1t} \) is the slope and \( \varepsilon_i \) the standard error.

Using this equation allows us to find the intercept, \( y_0 \) and slope of the security market line, \( y_1 \) for each month of the testing period 2004-2006.

In the same time period, we also test for nonlinearity using the following equation:

\[ R_{it} = y_{0t} + y_{1t}B_1 + y_{2t}B_1^2 + \epsilon_i \]

(8)

Where \( y_{0t} \) is the intercept, \( y_{1t} \) is the slope, \( y_{2t} \) is the beta squared and \( \epsilon_i \) the standard error.

This process is then repeated for the three other sub periods, a total of four times during the 18-year time period leaving us with a total of 144 estimations of \( y_0, y_1 \) and \( y_2 \).

**T-test on coefficients**

Finally, for each equation stated above we compute the mean value for each of the coefficients using the equation:

\[ \bar{X} = \frac{1}{n} \times \sum_{i=1}^{n} x_i \]

(9)

Where \( \bar{X} \) is the mean of the coefficients, \( n \) is the number of coefficients being averaged, \( x_i \) is the individual coefficient \( i \).

The standard deviation for each coefficient is estimated by applying the following equation:
\[
\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}
\]

(10)

Where \( N \) is the number of coefficients, \( x_i \) is the coefficient \( i \), and \( \mu \) is the mean of the coefficients.

Then we run the following t-test to test whether the means are significantly different from zero at a 95% significance level.

We test for the null hypotheses

- \( \text{C1 – H}_0: E(y_{2t}) = 0 \) \hspace{2cm} \text{Linearity}
- \( \text{C3 – H}_0: E(y_{1t}) = E(R_m) - E(r_f) > 0 \) \hspace{2cm} \text{Positive expected return-risk trade off}
- \( \text{SL – H}_0: E(y_{0t}) = 0 \) \hspace{2cm} \text{Sharpe – Lintner CAPM}

Against the alternative

- \( \text{C1 – H}_1: E(y_{2t}) \neq 0 \) \hspace{2cm} \text{Non-Linearity}
- \( \text{C3 – H}_1: E(y_{1t}) \neq 0 \) \hspace{2cm} \text{Positive expected return-risk trade off}
- \( \text{SL – H}_1: E(y_{0t}) \neq 0 \) \hspace{2cm} \text{Sharpe – Lintner CAPM}

The decision rule

Critical values are found using statistical tables.

The rejection rule for regression coefficients are the P-values.

The t statistic is calculated with the formula:

\[
t = \frac{\bar{X} - 0}{\sigma \sqrt{n}}
\]

(11)

Where \( \bar{X} \) is the mean of the sample, \( \sigma \) is the standard deviation of the sample and \( n \) is the number of values in the sample. Formally the \( \bar{X} - 0 \) is included since the test is

\[4\text{ Reject } H_0 \text{ if: } t_b > t_{n-1, \alpha/2} < -t_{n-1, \alpha/2}\]
supposed to determine whether the sample mean is different from zero at the given significance level.

If the CAPM holds true, the intercept, $E(y_{0t})$ should equal zero and the slope of the security market, $E(y_{1t})$ should equal the average risk premium of the market portfolio.

**Results**

In this section, the results of the research are presented and used as a basis for comparison against the hypothesis of the study. First the results from all four testing period are combined into an overall testing period and from there results from each testing period are presented individually.

**Overall testing period**

For the overall testing period results from each of the four sub periods is combined to create a comprehensive measure of the analysis.

![Figure 5. Average monthly return versus beta for equally weighted portfolios, all four testing periods combined.](image)

As can be seen in Figure 1, the regression line is slightly downward sloping indicating a negative relationship between portfolio beta and average portfolio return suggesting that returns are lower with higher betas. This is counterintuitive and goes against the theory of the CAPM where higher betas imply higher returns.

Table 2. Summary statistics from the t-test for the overall period.
We start by checking if our hypothesis of the SL CAPM holds, that is that the intercept equals the risk-free rate. We conduct a t-test with 144 observations giving us the results shown in Table 2 above, a t-statistic of 1.867, which is lower than the critical value of 1.96. Given this value we fail to reject the null hypothesis, as the intercept is not significantly different from zero and conclude that the intercept equals the risk-free rate supporting the SL hypothesis. However, the t statistic is close to the critical value and would exceed the critical value at $\alpha = 10\%$ which is 1.645. In that case the SL hypothesis would be rejected and concluded that the intercept does not equal the risk-free rate.

According to the CAPM, higher betas should be associated with a higher level of return (Fama & MacBeth, 1973). To test whether there is a positive return risk relation as described in C2 we test the hypothesis, $E(y_{1t}) = E(R_{mt})-E(r_f) > 0$. We are testing whether the slope of the security market line, $E(y_{1t})$, which should equal to the average risk premium of the market portfolio, $E(R_{mt})-E(r_f)$ is significantly different from zero. We conduct a t-test with 144 observations and get the results shown in Table 2, a t statistic of -0.044 which does not exceed the critical value of -1.96 so we fail to reject the null hypothesis since the data suggests that the slope is not significantly different from zero. That allows us to reject the notion of the CAPM that higher risk, as noted by beta gives higher returns.

In the early empirical literature of the model, the importance of the linearity condition C1 was largely overlooked so Fama and MacBeth (1973) emphasized on testing for that. Assuming the market portfolio is efficient, if $E(y_{2t})$ in our regression equation is positive, the prices of high beta securities are on average too low, their expected returns are too high relative to those of low beta securities, while the reverse holds if $E(y_{2t})$ is negative. To test for linearity, we find squared beta for each portfolio and finally we run the cross-sectional regression using formula (8).

If the process of price formation in the capital market reflects the attempts of investors to hold efficient portfolios, then the linear relationship of the CAPM between expected return and risk must hold (Fama & MacBeth, 1973). We test for the null hypothesis that $E(y_{2t})$ equals zero.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $y_0$</td>
<td>0.009</td>
<td>0.055</td>
<td>144</td>
<td>1.867*</td>
</tr>
<tr>
<td>Slope $y_1$</td>
<td>0.000</td>
<td>0.065</td>
<td>144</td>
<td>-0.044</td>
</tr>
</tbody>
</table>
Table 3. Regression output for the four testing periods combined.

<table>
<thead>
<tr>
<th>Regression Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
</tr>
<tr>
<td>R Square</td>
</tr>
<tr>
<td>Adjusted R Square</td>
</tr>
<tr>
<td>Standard Error</td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>Coefficients</strong></td>
</tr>
<tr>
<td>Intercept</td>
</tr>
<tr>
<td>Beta</td>
</tr>
<tr>
<td>Beta^2</td>
</tr>
</tbody>
</table>

Given the result of the t-statistic for the coefficient of the squared beta $2,083$ and a P-value of 0.4959 we can reject the null hypothesis as $E(y^2)$ is significantly different from zero and conclude that there are nonlinearities in the security market line. However, this P-value is so close to the alpha level of 0.05 that the null hypothesis would not be rejected at $\alpha = 10\%$. Since the null hypothesis can be rejected, the result of this regression over the whole period is not in favour of the CAPM. We can also see that by adding squared beta as an explanatory variable the coefficients for the intercept, $E(y_0)$ becomes significantly different from zero at a 95% significance level. Therefore, giving us stronger support to reject the notion of the SL CAPM that the intercept equals the risk-free rate. By adding squared beta into the equation, the slope of the SML, $E(y_1)$ becomes significantly different from zero as opposed to the result in table 2. However, due to the t statistic for the coefficient, $E(y_1)$ being negative, $-2,132$ the notion of the CAPM that there is a positive expected risk-return trade-off is rejected. So here all our hypotheses are rejected concluding that the intercept does not equal the risk-free rate, the slope of the SML is significantly different from zero and there are nonlinearities in the SML.

To examine the effects of time the sub periods are each evaluated separately in addition to the whole period.

**Testing period 1**

Testing period 1 spans the last three years of sub period 1, the years 2004-2006.
As can be seen in Figure 2 the regression line is downward sloping indicating a negative relationship between portfolio beta and average portfolio return suggesting that returns are lower with higher betas. That is counterintuitive and goes against the theory of the CAPM where higher betas imply higher returns.

Table 4. Summary statistics from the t-tests for testing period 1.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $y_0$</td>
<td>0.024</td>
<td>0.038</td>
<td>36</td>
<td>3.865***</td>
</tr>
<tr>
<td>Slope $y_1$</td>
<td>-0.007</td>
<td>0.035</td>
<td>36</td>
<td>-1.236</td>
</tr>
</tbody>
</table>

To determine whether the SL hypothesis holds true we conduct a t-test using the values in Table 4. The model predicts that the intercept is equal to the risk-free rate. If the CAPM holds the $E(y_{0t})$ or the intercept should equal $R_{ft}$ or the risk-free rate, $E(y_{0t}) = R_{ft}$. Testing with 36 observations the results are shown in Table 4.

The intercept is significantly different from zero, thus the null hypothesis is rejected at a 10%, 5% and 1% alpha levels and it is concluded that the intercept does not equal the risk-free rate. This result allows us to reject the SL CAPM as the assumption of their model is that the intercept is the risk-free rate.

To test whether there is a positive return risk relation we test the hypothesis, $E(y_{1t}) = E(R_{mt}) - E(r_t) > 0$. Testing with 36 observations the results in Table 4 show us that the coefficient for the slope of the SML, $E(y_1)$ is not significantly different from
zero. Therefore, we fail to reject the null hypothesis and conclude that there is not a positive price of risk in the capital markets as indicated by the CAPM and thus the result for period 1 is not supportive of the CAPM model. By looking at figure 2, we can see the risk-return relationship graphically where the slope is downward sloping indicating risk return trade-off is negative which is in contrary with the CAPM prediction. For example, the slope of the SML has a mean coefficient, $E(y_1) = -0.007$. In other words, for this period the incremental return per unit of beta was -0.7% per month, so we can see that higher risk, as noted by beta gives lower returns.

To test for the linearity condition of C1 we run the following regression between average portfolio returns, calculated portfolios betas and the portfolio betas squared by using formula (8).

Table 5. Regression output for testing period 1.

<table>
<thead>
<tr>
<th></th>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.010</td>
<td>0.016</td>
<td>0.599</td>
<td>0.592</td>
</tr>
<tr>
<td>Beta</td>
<td>0.023</td>
<td>0.035</td>
<td>0.672</td>
<td>0.550</td>
</tr>
<tr>
<td>Beta^2</td>
<td>-0.015</td>
<td>0.017</td>
<td>-0.885</td>
<td>0.441</td>
</tr>
</tbody>
</table>

The results of the regression shown in table 5 give us a P-value of 0.44, we fail to reject the null hypothesis as the coefficient of the squared beta, $E(y_{2t})$ is not significantly different form zero and conclude that there is a linear relationship between expected return and beta. We can see from the results that the intercept, $E(y_{0t})$ is not significantly different from zero as it was in table 4. Therefore, adding the variable of linearity, $E(y_{2t})$ makes the intercept insignificant. The slope of the SML is still not significantly different from zero. The results support the SL hypothesis that the intercept equals the risk-free rate but rejects the notion that there is a positive risk return trade off. We can see that $R^2 = 0.716$ which means 71.6% of the variability in returns is explained by our predictors. The explanatory power of the equation is therefore very good for this period as well.
Testing period 2

Testing period 2 spans the last three years of sub period 2, the years 2007-2009.

![Figure 7. Average monthly return versus beta for equally weighted portfolios formed on 2001-2003.](image)

As can be seen in Figure 3 the regression line is downward sloping indicating a negative relationship between portfolio beta and average portfolio return suggesting that returns are lower with higher betas. Which goes against the theory of the CAPM where higher betas imply higher returns.

Table 6. Summary statistics from the t-tests for testing period 2.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept  $y_0$</td>
<td>0.006</td>
<td>0.073</td>
<td>36</td>
<td>0.453</td>
</tr>
<tr>
<td>Slope  $y_1$</td>
<td>-0.006</td>
<td>0.091</td>
<td>36</td>
<td>-0.376</td>
</tr>
</tbody>
</table>

Following the same procedure as conducted above we test the whether the intercept equals the risk-free rate as predicted by the SL version of the CAPM. Our t-test with 36 observations gives us the results shown in Table 6. Given the results we fail to reject the null hypothesis as the intercept is not significantly different from zero and the conclusion is that the intercept equals the risk-free rate. Given this result we accept the SL hypothesis, which however, contradicts the result for sub-period 1.

To test whether the slope of security market line, $E(y_1)$ shows a positive risk return relationship we conduct a t-test with 36 observations and get the results shown
in Table 6 where the slope has a t statistic of -0.376 which does not exceed the critical value of -2.021 which suggests that the slope of the SML is not significantly different from zero so, we fail to reject the null hypothesis and conclude that the slope of the security market line does not show a positive risk-return relationship. This result is in line with the result from sub-period 1 where the results were not supportive for the CAPM.

Testing for linearity in sub-period 2 we apply the same cross sectional regression as used for testing period 1 between average portfolio returns, calculated portfolio betas and portfolio squared betas, formula (8).

Table 7. Regression output for testing period 2.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.004</td>
<td>0.035</td>
<td>-0.120</td>
</tr>
<tr>
<td>Beta</td>
<td>0.013</td>
<td>0.068</td>
<td>0.195</td>
</tr>
<tr>
<td>Beta^2</td>
<td>-0.009</td>
<td>0.032</td>
<td>-0.280</td>
</tr>
</tbody>
</table>

For the coefficient of beta squared we get the t statistic -0.280 and a P-value of 0.79. We therefore, fail to reject the null hypothesis as the coefficient, E(y^2t) is not significantly different from zero. From that we can conclude that there is a linear relationship between expected returns and their betas, supporting the CAPM. We can also see that the SL hypothesis still holds and the slope of the SML is still not significantly different from zero when adding beta squared as an explanatory variable. Explanatory power of the equation not very good but not all bad, R^2 = 0.238, so 23.8% of returns explained by the model.

Testing period 3
Testing period 3 spans the last three years of sub period 3, the years 2010-2012.
As can be seen in Figure 4 the regression line is upward sloping indicating a positive relationship between portfolio beta and average portfolio return suggesting that returns are higher with higher betas. That supports the theory of the CAPM where higher betas imply higher returns.

Table 8. Summary statistics from the t-tests for testing period 3.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $y_0$</td>
<td>-0.001</td>
<td>0.059</td>
<td>36</td>
<td>-0.051</td>
</tr>
<tr>
<td>Slope $y_1$</td>
<td>0.005</td>
<td>0.078</td>
<td>36</td>
<td>0.394</td>
</tr>
</tbody>
</table>

We test the SL hypothesis that the intercept equals the risk-free rate. The coefficient for the intercept has a t statistic of -0.051 which does not exceed the critical value of -2.021. We fail to reject the null hypothesis and conclude that the intercept equals the risk-free rate. The results are supportive of the SL version of the CAPM, similar to the result in sub-period 2.

The results of the test of the slope of the SML are shown in Table 8. The slope has a t-statistic of 0.394 which does not exceed the critical value of 2.021. We therefore, fail to reject the null hypothesis and conclude that there is not a positive risk-return trade off.

Testing for linearity we run the cross-sectional regression with formula (8).
Testing the null hypothesis that the squared beta coefficient, E(y^2_t), equals zero we get the following results shown in Table 9.

Table 9. Regression output for testing period 3.

<table>
<thead>
<tr>
<th>Regression Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
<td>0.291</td>
</tr>
<tr>
<td>R Square</td>
<td>0.085</td>
</tr>
<tr>
<td>Adjusted R Square</td>
<td>-0.526</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.003</td>
</tr>
<tr>
<td>Observations</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.007</td>
<td>0.109</td>
<td>-0.063</td>
</tr>
<tr>
<td>Beta</td>
<td>0.017</td>
<td>0.204</td>
<td>0.084</td>
</tr>
<tr>
<td>Beta^2</td>
<td>-0.006</td>
<td>0.094</td>
<td>-0.058</td>
</tr>
</tbody>
</table>

From the results, we can see that E(y^2_t) is not significantly different from zero with a t statistic of -0.058 and a P-value of 0.957. We therefore reject the null hypothesis in support of the CAPM, that is that there is a linear relationship between expected returns and betas. The SL hypothesis still holds and slope of the SML does not show a positive risk-return trade-off. However, the regression gives us, R^2 = 0.085 meaning that only 8.5% of variability in returns is explained by the model so the explanatory power is poor.

**Testing period 4**

Average monthly return versus beta for equally weighted portfolios formed on 2007-2009.
As can be seen in Figure 5 the regression line is upward sloping indicating a positive relationship between portfolio beta and average portfolio return suggesting that returns are higher with higher betas. That supports the theory of the CAPM where higher betas imply higher returns.

Table 10. Summary statistics from the t-tests for testing period 4.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $y_0$</td>
<td>0.005</td>
<td>0.044</td>
<td>36</td>
<td>0.714</td>
</tr>
<tr>
<td>Slope $y_1$</td>
<td>0.007</td>
<td>0.039</td>
<td>36</td>
<td>1.075</td>
</tr>
</tbody>
</table>

Testing for the SL hypothesis we get results shown in table 10. The coefficient for the intercept has a t-statistic of 0.714 which does not exceed the critical value of 2.21. Given the result we fail to reject the null hypothesis and conclude that the intercept equals the risk-free rate, supporting the SL version of the CAPM.

Testing for the prediction that the slope of the SML shows a positive risk-return trade-off gives the results shown in Table 10. The coefficient for the slope has a t-statistic of 1.075 which does not exceed the critical value of 2.021. Once again we fail to reject the null hypothesis and conclude that there is not a positive price for risk in capital markets.

To test for linearity, we run the cross-sectional regression with formula (8) again and get the results shown in Table 11.
Table 11. Regression output for testing period 4.

<table>
<thead>
<tr>
<th>Regression Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
</tr>
<tr>
<td>R Square</td>
</tr>
<tr>
<td>Adjusted R Square</td>
</tr>
<tr>
<td>Standard Error</td>
</tr>
<tr>
<td>Observations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.023</td>
<td>0.005</td>
<td>4.850</td>
</tr>
<tr>
<td>Beta</td>
<td>-0.030</td>
<td>0.009</td>
<td>-3.236</td>
</tr>
<tr>
<td>Beta^2</td>
<td>0.017</td>
<td>0.004</td>
<td>4.033</td>
</tr>
</tbody>
</table>

The result of the regression give us the t-statistic for the coefficient of the squared beta of 4,033 and a P-value of 0.027 which is lower that the alpha level. Therefore, we reject the null hypothesis and conclude that $E(y_{2t})$ is significantly different from zero. That rejects the CAPM and states that there is not a linear relationship between expected returns and their betas. This result contradicts the results from the other three sub-periods which all failed to reject this hypothesis. We can also see here that the SL hypothesis is rejected as $E(y_{0t})$ is significantly different from zero. Furthermore, we conclude that the slope of the SML, $E(y_{1t})$ is significantly different from zero. This shows that $E(y_{0t})$ and $E(y_{1t})$ are affected by adding the squared beta, $E(y_{2t})$ to the model. Explanatory power of the equation is very high as $R^2 = 0.918$.

Time varying betas

While using fixed betas our results are mostly inconclusive so perhaps some problems might be occurring since we are using fixed betas, for example for sub-period 1 the portfolios are formed from betas estimated during the dotcom bubble, the portfolio betas are calculated during the market crash 2001-2003 and then those betas are regressed against portfolio returns when the market is bouncing back from the crash. This could contribute to some odd numbers which might affect our research. The method of using fixed betas has been criticized for not being accurate enough and not considering changes within the periods. Jagannathan & Wang, (1996) suggest using time varying betas to adjust for that error.

We compare the fixed beta results with results when using time varying betas. In Table 12 the results are shown for the same tests as are reported above only now the betas are time varying. The outcome shows that there was not significant difference...
compared to the fixed betas since all the values had the same level of significance as the calculations with fixed betas i.e. the slope of the beta coefficient was never significant and the intercept was only significant for the overall period and sub-period 1. Therefore, adjusting the betas to vary over time did not change the results.

Table 12. Summary of statistics with time varying betas.

<table>
<thead>
<tr>
<th>Overall period with time varying betas</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $y_0$</td>
<td>0.011</td>
<td>0.071</td>
<td>144</td>
<td>1.922*</td>
</tr>
<tr>
<td>Slope $y_1$</td>
<td>-0.003</td>
<td>0.084</td>
<td>144</td>
<td>-0.361</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sub period 1</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $y_0$</td>
<td>0.024</td>
<td>0.042</td>
<td>36</td>
<td>3.428***</td>
</tr>
<tr>
<td>Slope $y_1$</td>
<td>-0.007</td>
<td>0.042</td>
<td>36</td>
<td>-0.982</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sub period 2</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $y_0$</td>
<td>0.014</td>
<td>0.114</td>
<td>36</td>
<td>0.719</td>
</tr>
<tr>
<td>Slope $y_1$</td>
<td>-0.013</td>
<td>0.141</td>
<td>36</td>
<td>-0.553</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sub period 3</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $y_0$</td>
<td>0.001</td>
<td>0.053</td>
<td>36</td>
<td>0.075</td>
</tr>
<tr>
<td>Slope $y_1$</td>
<td>0.005</td>
<td>0.071</td>
<td>36</td>
<td>0.382</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sub period 4</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Observations (n)</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept $y_0$</td>
<td>0.007</td>
<td>0.049</td>
<td>36</td>
<td>0.815</td>
</tr>
<tr>
<td>Slope $y_1$</td>
<td>0.005</td>
<td>0.044</td>
<td>36</td>
<td>0.727</td>
</tr>
</tbody>
</table>

If we examine the results from table 12 in more depth. Starting with the SL hypothesis that $\text{E}(y_0) = R_f$, we can see that for the overall period the intercept is significantly different from zero at $\alpha = 10\%$, thus there is evidence allowing us to reject the null hypothesis and concluding that the intercept does not equal the risk-free rate. However, at $\alpha = 5\%$ and $\alpha = 1\%$ the results are insignificant and supportive of the SL notion of the CAPM. These results are the same as we got using fixed betas. In sub-period 1 the mean coefficient of the intercept is the same whether using time varying or fixed betas.
and significantly different from zero, rejecting the SL condition. For sub-periods 2, 3 and 4 the intercept is not significantly different from zero supporting SL condition that the intercept equals the risk-free rate. The results of the SL condition are the same using time varying betas as when using fixed betas.

Now let’s look at the results in relation with condition C2. The results for the overall period and all the sub-periods are in line with our results using fixed betas. The slope of the SML, $E(y_1)$ is not significantly different from zero in all periods, thus rejecting condition C2 that there is a positive risk-return trade-off.

Summing up these results show us that using time varying betas does not improve statistical performance of the CAPM in our research.

**Conclusions**

In this paper, empirical tests were conducted to determine the predictive power of the CAPM in the European stock market. This was done by analysing stock returns from the S&P Euro index. Summarised, our results of the CAPM are mostly negative regarding the predictive power of the model. We find no evidence that support the CAPM assertion that there is a positive trade-off between expected returns and risk as noted in condition C3. For the overall-period as for all sub-periods, the slope of the SML, $E(y_1)$ is not significantly different from zero. That suggests that there is no positive relationship between average return and beta. This result is of similar nature to the earliest empirical tests conducted by Lintner (1965b), Douglas (1967), and Miller & Scholes (1972) where they found the slope of the SML to be much flatter than indicated by the CAPM. However, Miller & Scholes (1972) pointed out some problems with the testing procedure conducted in those tests due to a measurement error in betas estimated from the first pass regression. In our test, we have accounted for these problems by forming portfolios ranked by beta estimates like Fama & MacBeth (1973), Black et al., (1972), Sharpe & Cooper (1972) and Modigliani (1972). However, those studies supported the assumption that higher beta rewards higher returns as implied by the CAPM. In more recent tests performed by Pettengill et al., (1995) and Clare et al., (1997) the results suggested a positive risk-return trade-off.

The results of this risk-return trade-off hypothesis seems to do worse when beta squared is added as an explanatory variable into the equation. Especially for the overall period and sub-period 4 where the slope is significantly different from zero, but
negative, which suggests that stocks with higher betas generate lower returns. That is counter intuitive since no risk averse investor would be willing to incur greater risk in exchange for lower returns.

When testing condition C1 we get mixed results. In the overall and sub-period 4, we reject the hypothesis that there is a linear relationship between average returns and betas, since the results suggest that there are nonlinearities present in these periods. However, in the other sub-periods the hypothesis is not rejected which supports the CAPM’s assumption that effects of nonlinearities are zero. Thus, the results do not support the positive conclusion of this hypothesis by Fama and MacBeth (1973). Therefore, we cannot assert that investors should assume that the relationship between expected returns and beta is linear, as implied by the CAPM. Pettengill et al., (1995) find results consistent with Fama and MacBeth in contrast with our results. They adjust for expectations concerning negative market excess returns and find a highly significant relationship between beta and returns.

Our results testing the SL notion of the CAPM, that is, investors can borrow and lend at a risk-free rate, vary. For the overall period the t-statistic for the intercept is significant at a 1% alpha level. For subperiod 1 the intercept is significantly different from zero and thus we reject the SL hypothesis. However, for the remaining 3 subperiods, the results are insignificant and thus supporting the SL hypothesis of the CAPM.

It is important to note here that even that the SL hypothesis is accepted we are only talking about their assumption of unlimited lending and borrowing, that is that the risk-free rate equals the intercept, the SL CAPM model would still be rejected as the slope is not significantly different from zero.

The SL hypothesis of the CAPM has not had favourable results in earlier literature as most seem to reject their version of the model (Friend & Blume, 1970; Fama & MacBeth, 1973; Black et al., 1972; Lintner, 1965; Douglas, 1967; Miller & Scholes, 1972; Sharpe & Cooper, 1972). Our results are thus not quite in line with the early tests as we cannot uniformly support the negative conclusions of the SL hypothesis. In subperiods 1 and 4 the SL hypothesis seems to be affected by adding beta squared as an explanatory variable to the model. The coefficient for the intercept becomes insignificant in period 1 and becomes significant in period 4.

The purpose of this research was to determine how effective the CAPM is in explaining the relationship between expected returns and systematic risk.
beta. Overall the validity of using the CAPM to predict returns on the S&P Euro does not withstand our empirical analysis. The slope is much flatter than predicted by the model and sometimes negative. The positive risk-return relationship is a core prediction of the CAPM and that alone is enough evidence to reject both the Sharpe-Lintner version of the CAPM and Black’s zero-beta CAPM. As we explained in the introduction, the CAPM is widely used by investors, market professionals and academics for various financial applications (Graham & Harvey, 2001; Fama, 1991; Kolouchová & Novák, 2010; Fama & French, 2004). However, our results cannot support the use of the model in determining expected returns. Our test and other empirical tests get a slope much flatter than predicted by the CAPM, so in using the model to estimate the cost of equity will deliver too high estimate of cost of equity for high beta stocks and too low estimate of cost of equity for low beta stocks (Friend & Blume, 1970)

**Limitations**

One problem in testing the validity of the CAPM is that market indexes are only imperfect proxies for the overall market. Richard Roll (1977) criticized previous empirical analysis of the model where he asserts that to test for the risk-return relationship, or in fact any other assertion of the model, one must first test for the efficiency of the market portfolio. Roll & Ross (1995) and Kandel & Stambaugh (1995) explain in their literature how mean-variance inefficient proxies of the market portfolio could lead to rejection of the CAPM even if the model perfectly explains the risk-return trade-off. Considering the results from our analysis it would be interesting to test for market efficiency and see if our results are the result of an inefficient market rather than the model being invalid. However, according to Roll (1977) the proxy for the market portfolio must include all capital market assets and no such market index exists making it impossible to test the validity of the model.

One of the assumptions of the CAPM that has been questioned is that it implies that investors behave rationally. Rational investors are generally thought to be risk averse, can fully exploit all available information, and do not suffer from psychological biases. When many investors suffer from similar biases at the same points in time, market prices can behave in ways that are not captured by traditional asset pricing model (Hirschey & Nofsinger, 2008). So CAPM might work in theory but not in real market settings if the assumptions do not hold.
However, the CAPM is only a theoretical model based on many unrealistic assumptions so maybe it was not meant to accurately explain a real world setting but could be practical in use, like being taught in finance courses. The model seeks to explain the risk-return relationship given that these assumptions hold. In the words of Fama & French (2004), “… despite its seductive simplicity, the CAPM’s empirical problems probably invalidate its use in applications” (p. 44).
References


