An Integer Programming Formulation for the
Music School Timetabling Problem

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by

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Abstract

Before every semester, music schools in Iceland face the challenge of creating timetables for students and teachers registered at the school. Many music schools create their timetables manually, which is very time consuming. In this thesis a model will be presented that is intended to solve the music school timetabling problem, assigning students and teachers to courses and classrooms.

This project is done in cooperation with the music school Tónmenntaskóli Reykjavíkur which is located in Reykjavík. The assignment process at Tónmenntaskóli Reykjavíkur is currently done manually. Before the assignment process starts the school gathers data and information from every teacher and student regarding their availability and wishes for the semester. Tónmenntaskóli Reykjavíkur has agreed to share this information and provide the required data needed to create the model.

The main goal of the project is to create an optimization model which automates the assignment process, ensuring that students and teachers are assigned to right courses considering their preferences and trying to grant all wishes for the semester. The automation should minimize the scheduling time and reduce possible errors that may occur when done manually.

The results show that an integer programming model with a two phase model approach can be used to solve the music school timetabling problem, creating a feasible schedule for a music school were students and teachers are assigned to right courses.
Heiltölubestunarlíkan fyrir stundaskráagerð tónlistarskóla

Þórhildur Gunnarsdóttir

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Útdráttur

Fyrir hvert skólaár standa starfsmenn tónlistarskóla á Íslandi frammi fyrir krefjandi verkefni, að búa til stundatöflur fyrir nemendur sína og kennara, raða kennurum og nemendum niður á námskeið og skólastofur. Flestir tónlistarskólar í dag útbúa stundatöflurnar sínar handvirk sem er krefjandi og tímafrækt. Í þessari ritgerð verður sett fram stærðfræðilíkan sem gæti hjálpað starfsmönnum tónlistarskóla við gerð þessara stundataflna.

Stundaskráargerð Tónmenntaskóla Reykjavíkur, tónlistarskóli staðsettur í Reykjavík, er gerð handvirk og hefur skólinn samþykkt ad vinna med höfundí þessarar ritgerðar. Skólinn safnar upplýsingum um alla nemendur og kennara í upphafi skólaárs varðandi óskir og þarfr fyrir komandi skólaár. Skólinn er viljugur til ad deila öllum þeim upplýsingum sem þarf til hónnunar og prófunar á módelinu.

Aðal markmið verkefnisins er ad setja fram bestunarlíkan sem býr til stundatöflur sjálfvirk, minnka tímann sem það tekur að raða kennurnum og nemendum á námskeið. Líkanið á að lágmarka villur og tryggja ad nemendum og kennura í upphafi atum á rétt námskeið og um leið uppfylla óskir þeirra og þarfir. Með sjálfvirku ferli mun úthlutnartími styttast og líkur á villum minnka

Niðurstöður sýna að heiltölubestunarlíkan þar sem tveggja fasa aðferð er notuð leysir stundatöfluvandamál tónlistarskóla, að úthluta nemendum og kennurnum á námskeið og skila löglegri stundatöflu.
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Master of Science
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Contents

Acknowledgements xi
Contents xiii
List of Figures xv

1 Introduction 1

2 Background 3
   2.1 The music school timetabling problem 3
   2.2 The structure of Tónmenntaskóli Reykjavíkur 3
      2.2.1 The current assignment process 4

3 Literature Review 7
   3.1 Integer programming 7
   3.2 Generalized assignment problem 7
   3.3 Timetabling 8
      3.3.1 The class-teacher problem 8
      3.3.2 Course scheduling 8
      3.3.3 University timetabling problem 8
      3.3.4 High school timetabling problem 9
      3.3.5 The music school timetabling problem 9
      3.3.6 Techniques used for timetabling problems 9

4 Model 11
   4.1 General features of the model 11
   4.2 Variables 12
      4.2.1 Group A variables 13
      4.2.2 Group C variables 13
      4.2.3 Group B variables 14
   4.3 Constraints 15
      4.3.1 Availability Constraints 15
      4.3.2 Singularity Constraints 16
      4.3.3 Linking Constraints 16
      4.3.4 Regulatory Constraints 18
      4.3.5 Time Constraints 19
   4.4 Objective function 19
   4.5 The model 20
   4.6 Phase I: Group course scheduling 21
      4.6.1 Variables 22
4.6.2 Constraints ................................................................. 23
  4.6.2.1 Availability Constraints ......................................... 23
  4.6.2.2 Singularity Constraints .......................................... 23
  4.6.2.3 Linking Constraints .............................................. 23
  4.6.2.4 Regulatory Constraints ......................................... 24
4.6.3 Objective function .................................................... 25
4.6.4 The model ............................................................... 26
4.7 Phase II: Private course scheduling .................................... 27
  4.7.1 Variables .............................................................. 27
  4.7.2 Constraints ............................................................ 28
    4.7.2.1 Availability Constraints ..................................... 28
    4.7.2.2 Linking Constraints ........................................... 28
    4.7.2.3 Regularity Constraints ....................................... 29
  4.7.3 Objective function .................................................. 30
  4.7.4 The model ............................................................. 30
5 Data ............................................................................... 31
  5.1 Data Collection .......................................................... 31
  5.2 Data Processing .......................................................... 31
  5.3 Case Study - Tónmenntaskóli Reykjavíkur ............................ 32
6 Results ........................................................................... 33
  6.1 Phase I model results ..................................................... 33
  6.2 Phase II model results ................................................... 34
7 Conclusions ..................................................................... 37
Bibliography ....................................................................... 39
List of Figures

4.1 Groups A, B and C containing all variables created for the model. 13
5.1 The data process. 31
6.1 The resulting timetable for all group courses. 33
6.2 The resulting timetable for student number 2. 34
6.3 The resulting timetable for teacher number 11. 35
6.4 The resulting timetable for student number 3. 35
6.5 The resulting timetable for teacher number 6. 35
Chapter 1

Introduction

Music schools are educational institutions focusing on music where they specialize in the study, training and research of music. In most elementary schools, music education is mandatory whereas within higher educational institutions such as music colleges and universities registering is optional.

In Iceland there are about 83 music schools [1]. That music schools specialize in the study, training and research of music, offering a variety of musical instruments to study on. The main focus of these music schools is music where students attend private courses, group courses, study music theory and often play in bands. Attending an music school in Iceland is not compulsory but everyone has the right to register. However, there are limited positions available. The minimum age required is preferably when starting elementary school and it is very common amongst children to begin studying music at that time.

Before every semester, music schools face the challenge of creating timetables for their students. Assigning students to courses they are registered in with the right teacher. In the beginning of each semester the schools gather information from their teachers and students regarding their availability and other preferences that need to be considered when creating the timetables. Other things taken into consideration regarding the scheduling is that there is limited classroom availability and some courses might need specific classrooms.

Music schools in Iceland are very different in size and therefore it varies between schools how the assignment process is done, the larger the school the more complex the process is. However, in most cases the scheduling is done manually. Doing the scheduling manually is a complicated process, it is time consuming and increases the likelihood of errors.

In this thesis we look at creating timetables for a music school in collaboration with Tónmenntaskóli Reykjavíkur, a music school located in Reykjavík. Today Tónmenntaskóli Reyjavíkur assigns students and teachers to classes manually.

The main goal of this project is to create a model which returns a feasible timetable for a music school. The overall aim of the project is to automate the process of creating timetables, minimizing the scheduling time and reducing the possibility of errors that can occur when done manually.

The paper is organised in the following way. In Chapter 2, the music school timetabling problem will be defined along with the structure and current assignment process at Tónmenntaskóli Reykjavíkur. Chapter 3 is a literature review. In Chapter 4, the model will be presented. In Chapter 5, the data used for the model will be described, the data collection and data processing along with a case study. Chapter 6 contains the results while Chapter 7 contains conclusions and final words.
Chapter 2

Background

In this chapter the music school timetabling problem will be defined and described in detail along with the current assignment process at Tónmenntaskólí Reykjavíkur.

2.1 The music school timetabling problem

The music school timetabling problem is the problem of assigning students and teachers to right courses. The courses are assigned to a specific day at a certain time and to a suitable classroom that fulfils all the courses requirements, being appropriate for the number of students and having the required facilities available. The details of the timetabling problem for a music school varies from one school to another based on the characteristics of the school, how the courses are taught, structured and how the schools arrange and utilise their resources.

Courses offered by music schools are private courses and group courses. Private courses are when a single student is with a teacher studying on his musical instrument. Group courses are usually the courses where students study music theory, play in bands or participate in any kind of group activities. Students are usually registered in both, private and group courses and teachers can be private and/or group course teachers.

The music schools resources are the teachers and classrooms. The allocation of these resources depends on their availability. All teachers have various preferences regarding their availability and the availability of classrooms is defined by the music school itself, depending on what courses can be taught in each classroom based on the size of the classroom and facilities available.

2.2 The structure of Tónmenntaskólí Reykjavíkur

To gain a better understanding on the structure of the music school Tónmenntaskólí Reykjavíkur, Rúnar Óskarsson the headmaster of the school was interviewed [2].

Tónmenntaskólí Reykjavíkur offers a variety of different courses. The courses offered by the music school are private and group courses. The private courses are courses where a single student is assigned to a teacher at a certain time of day studying a specific musical instrument. The private courses are: piano, violin, flute, guitar, cello, clarinet, saxophone and bassoon. Group courses are when a group of students are registered in the same course with one teacher. The type of group courses are as follows: music theory, string orchestra, playing in a big band, flute preschool and violin preschool. Collision is not allowed, meaning
each student or student group can only be assigned to one course with one teacher at specific
day and time in a certain classroom.

The basic structure of the school is that every student assigned to a private course attends
two courses a week with one teacher, where each course is scheduled for 30 minutes. All
students that attend private courses must study music theory and are assigned to groups of six
to eight people according to their age. Therefore, in each music theory course, all students
are at the same age and the number of courses is dependent on the number of students within
the same age group. The music theory courses are scheduled once a week for an hour at a
time.

Those students that are beginning their music career at a young age are first assigned to
flute preschool before they can be registered to the basic private courses. The flute preschool
is a group course of six to eight people and is scheduled twice a week for an hour at a time.
If a student’s intention is to study on the violin there is a similar preschool for the beginners
where during the first year a student is assigned to one violin preschool group course, sched-
uled for an hour at a time and one violin private course for 30 minutes each week. Playing
in a big band or the string orchestra is optional and those courses are scheduled once and
twice a week respectively.

The music school has 16 available classrooms in use with 15 teachers currently teaching.
There are only two classrooms used for group courses while the rest of them is divided
between various private courses.

There are many difficulties in controlling the students and teachers schedules because
of so many different students belonging to different courses with various availability of
classrooms making it a complex process creating the timetables.

2.2.1 The current assignment process

Currently, the assignment process at the music school is done manually. The school gathers
all information about their students through a registration process online which each student
is required to fill out, along with submitting their extra curriculum and leisure timetables.
When all students have registered, the information in the computer system regarding all
students is transferred manually on to sheets of paper, a printed out application form spe-
cially made by the school. These application forms are printed in a variety of colours. The
 technique used to arrange students to group courses is by using the coloured paper where
all students at the same age have the same colored application form. Students within the
same colored application form are then divided into group courses based on their availabil-
ity, wishes, school and any extra curriculums. The colour of the application form is used to
facilitate the categorization of students into groups depending on their age and what group
course or music theory, the student is registered in.

Students are assigned to private courses with teachers depending on what instrument
they play on or if there are special requirements from students about which teacher they
want to be assigned to. Each private teacher teaches a specific instrument and the students
are assigned to teachers according to that. Usually a student is assigned to a teacher at the
beginning of their studies and then continues with that same teacher throughout their school
attendance.

With the number of students and information on what courses they are registered to the
school gets an overview of how many courses will be taught the following semester and the
students are divided into group courses according to that. Normally a student attends two
private courses per week and one group course, studying music theory.
2.2. THE STRUCTURE OF TÓN-Menntaskóli Reykjavíkur

All information regarding the teaching staff and students is gathered at the beginning of the semester, the teachers preferences regarding what days of the week they wish to teach and all information about the students availability and wishes.
Chapter 3

Literature Review

This chapter introduces the integer programming problem, the assignment problem and its generalized version. An introduction will be given on problems included in the field of timetabling, the class-teacher problem, course scheduling, high school timetabling, the university timetabling problem and their connection to the music school timetabling problem. The final section then gives an overview of the techniques used to solve the timetabling problems.

3.1 Integer programming

An integer programming problem is a mathematical optimization or a feasibility program, maximizing or minimizing a function subjected to linear equality and inequality constraints where at least one of the variables is restricted to integer values. Integer programming is a powerful framework with great flexibility for expressing discrete optimization problems, using binary variables, choosing between two alternatives [3], [4].

The integer programming problem can be expressed as:

\[
\begin{align*}
\text{min} & \quad c'x \\
\text{s.t.} & \quad Ax \geq b \\
& \quad x \in \{1, 0\}
\end{align*}
\]

where \( c \) and \( b \) are vectors and \( A \) is a matrix [3], [4].

Bradley et al. [5] introduced the importance of integer programming models and how they play an important role in supporting managerial decisions in three areas, capital budgeting, warehouse location and scheduling.

The assignment problem [6] is a special type of linear programming problem on how to assign assignments to assignees to perform tasks. The problem does it in such a way that time or cost is minimized or the profit or sale is maximized.

3.2 Generalized assignment problem

The Generalized Assignment Problem (GAP) is a well-known optimization problem. The objective of GAP is to find the optimal assignment of \( n \) agents to perform \( m \) tasks with a fixed capacity availability [7].
The mathematical formulation of the GAP is:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

(3.4)

s.t. $$\sum_{j=1}^{n} r_{ij} x_{ij} \leq b_i$$ \quad \forall \ i = 1, ..., m$$

(3.5)

$$\sum_{i=1}^{m} x_{ij} = 1$$ \quad \forall \ j = 1, ..., n$$

(3.6)

$$x_{ij} \in \{1, 0\}$$ \quad \forall \ i, j$$

(3.7)

where \(c_{ij}\) is the cost of assigning agent \(i\) to task \(j\), \(b_i\) is the capacity of agent \(i\) and \(r_{ij}\) is the weight of agent \(i\) if assigned to task \(j\). The objective function is to minimize the total assignment cost of tasks to agents [7].

The assignment-type problem and its generalized version are powerful modeling tools. It has been shown [8] how such models are appropriate for formulating timetabling and scheduling problems where time periods are determined for activities according to specific constraints. Ferland [8] has shown how timetabling and scheduling applications are formulated as such problems. He shows how the GAP can be used to establish the schedule of lectures according to student registrations and lecturer and classroom availability. Other problems such as traineeship scheduling, preventive maintenance scheduling, sports league scheduling and nurse scheduling can also be formulated as GAP [8].

### 3.3 Timetabling

There are many problems that are included in the field of timetabling and over the last few decades many contributions to the timetabling problem have appeared. In [9] an introduction is given on some basic timetabling problems, the class-teacher problem and the course scheduling problem.

#### 3.3.1 The class-teacher problem

The class-teacher problem [9] consists of a set of students who follow the same program. The problem is to assign each course, consisting a group of students, to a time period with a teacher making sure that each teacher only gets assigned to one course at a time.

#### 3.3.2 Course scheduling

Course scheduling [9] is a problem that high schools and universities face. The school offers a variety of courses and the problem is to assign each course to some time period ensuring that no student is required to take more than one course at a time. This problem is similar to the examination scheduling problem.

#### 3.3.3 University timetabling problem

The university timetabling problem is the problem and process of assigning courses to a time period, assigning them to a specific day at a certain time and finding a classroom that meets
3.3. TIMETABLING

all the courses requirements and is appropriate for the number of students registered in each course.

In [10] an algorithm for computing a timetable for a university department is formulated where the problem is solved as an assignment problem and a linear mathematical model finds an optimal solution. In [11] an integer programming formulation is presented for the university timetabling problem, where a timetable for an Engineering Department was used as a case study and solved successfully.

3.3.4 High school timetabling problem

The high school timetabling problem is the problem of creating a weekly schedule for all courses of a high school, finding a suitable classroom and assigning courses to time slots in such a way that no teacher assigned to a course is involved in more than one course at a time. High school timetables are different from the university timetables in a way that in high schools, students are most often occupied and supervised every hour of the school day. In the high school timetabling problem a group of students, in the same class are timetabled together.

In [12] a model is presented for the high school timetabling problem creating a timetable for a real world application, an Italian high school. The problem was solved successfully and different approaches of solving the problem were compared, a genetic algorithm-based approach with various versions of simulated annealing and tabu search.

3.3.5 The music school timetabling problem

The music school problem faces the same challenges as the class-teacher problem and the course scheduling problem, assigning each course to a time period with a teacher, making sure that each teacher is only assigned to one course at a time and that no students are assigned to more than one course at a time.

The music school timetabling problem is also similar to the high school and the university timetabling problems where the music schools assign courses to appropriate time periods and classrooms. The music school problem is in a way a combination of the problems described above, the class-teacher problem the high school timetabling problem and the university timetabling problem.

3.3.6 Techniques used for timetabling problems

Many techniques have been used to solve the timetabling problem, techniques such as the graph colouring technique [13], integer and linear programming, [10], [14], the genetic algorithm approach [12], [15]–[17] and the tabu search technique [12], [18], [19].

The graph colouring technique is a well known technique, simple but limited. It is where nodes of a graph are coloured in a way that no two adjacent nodes share the same colour and the number of colours used is a minimum. In [13] the graph colouring technique is used to solve the problem studied by Cole [20] of preparing examination timetables, arranging a timetable for 34 examination papers.

Akkoyunlu [10] presents a linear algorithm for the university timetabling problem, where a mathematical model for scheduling courses is described within a single department of a university. The model was solved with all the basic requirements but it was assumed that there was not limit on the total number of available classrooms, which was not considered a problem since in America, the classrooms are usually allocated from a campus-wide pool.
In [16] a multiobjective genetic algorithm was proposed for the class-teacher timetabling problem, minimizing the violations of soft constraints and respecting the class-teacher aspect. Results were obtained with the algorithm from a real life case study, a university establishment in Portugal.

Tabu search techniques [19] is a metaheuristic method used for mathematical optimization, getting a global optimum to a combinatorial optimization problem. In [19] a new model developed with tabu search to solve the scheduling problem where the number of courses and length is not known in advance. Data from a educational institution in Yugoslavia was used to test the efficiency of the model. A new approach was developed finding a feasible course schedule. The new model is able to solve the class-teacher timetabling problem without knowing the length of the courses. In most approaches in the literature, for course scheduling problems, it is assumed that the length of the courses are fixed.
Chapter 4

Model

In this chapter a music school timetabling problem is modeled as an optimization problem using binary variables.

4.1 General features of the model

We will use an Integer Programming to formulate the model that will construct a timetable for a music school. The equations for the model may vary between schools reflected by the special requirements placed by each school. However, the basic structure remains the same.

There are six sets that define the basic structural elements for the model approach. These sets are:

- **S**: The set of students attending the school. An example of a student could be a piano student taking music theory and private courses, a student attending flute preschool group courses or a student playing in a band. An individual student is denoted as $s \in S$.

- **T**: The teachers that are teaching the courses in the timetable. For each course, either private or group courses, there needs to be at least one assigned teacher. The teachers are denoted with the letter $t \in T$.

- **C**: The courses that are scheduled for the students. The courses are denoted with the letter $c \in C$.

- **P**: The places, are the classrooms that are available for the scheduled courses. The classrooms are denoted with the letter $p \in P$.

- **D**: The days of the week, on what day a course may be scheduled. The days are denoted with the letter $d \in D$ and the set of all possible days is usually the five working days.

- **H**: The hour, at what time of day a course may be scheduled. In our case a time period is an half hour period from 14:00 until 19:00. The hours or time slots are denoted with the letter $h \in H$.

Other sets:

- **O**: The course type, every course is classified into course types depending on the nature of the course. An example of one course type would be private piano courses. The course types are denoted with $O_c$ and each course $c \in C$ has a course type $x_c \in O_c$. 
CHAPTER 4. MODEL

FT: Fixed teachers, is a set of students and teachers, indicating assignments of students to teachers. If \((s, t) \in FT\) then student \(s\) will be assigned to teacher \(t\).

TC: Teacher course, is a set of teachers and courses, indicating what courses a teacher is able to teach. If \((t, c) \in TC\) then teacher \(t\) is able to teach course \(c\).

CC: Course classroom, is a set of course types and places, indicating what course can take place in each classroom. If \((x, p) \in CC\) then course of type \(x\) can be assigned to place \(p\).

SA: Student availability, is a set of students, days and hours. If \((s, d, h) \in SA\) then student \(s\) is available on day \(d\) at hour \(h\).

TA: Teachers availability, is a set of teachers and days. If \((t, d) \in TA\) then teacher \(t\) is available on day \(d\).

To simplify the modelling, subsets of the original sets were created.

Tp: The private teachers that are teaching the private courses in the timetable. The private teachers are denoted with the letter \(Tp \subseteq T\).

 Cf: Some courses are scheduled before, having a fixed day and time. If \(c \in Cf\) then \(c\) will be assigned to day \(d_c\) at time \(h_c\). The courses that have a fixed time slot are denoted with the letter \(Cf \subseteq C\).

Parameters:

\(l_{\text{max}}\): For a course \(c \in C\), there is a limit on the number of students that can be assigned, denoted by \(l_{\text{max}}^c\).

\(f\): Student registration, the parameter \(f_{s,x}\) denotes how often student \(s \in S\) should be registered to a course of course type \(x \in O\). For example, all piano courses are of the course type piano. All music theory courses are categorized into different course types depending on the age of the students. If there are three music theory courses created for students at the age of 10 and two courses for the students at the age of 11, the first three courses would for example be of the course type music theory 10 while the second two courses of the course type music theory 11.

A piano student at the age of 11 would then be registered twice in the course type piano and once in the course type music theory 11, based on the structure of Tónmenntaskóli Reykjavíkur.

\(g\): Number of private teachers, the parameter \(g_s\) denotes how many private teachers a student \(s \in S\) is assigned to. Usually, a student is only assigned to one private teacher but there can be exceptions were students play on two musical instruments and are assigned to two private teachers.

4.2 Variables

In this section all the decision variables for the model will be formulated and a brief description of each variable will be given. Eleven variables were created for this model and partitioned into three groups, A, B and C. Group A consists of a single variable which is
very detailed, containing all possible solutions to the model. Group C variables are directly connected to the input data for the model. Group B variables are linking variables and created to simplify the linking between variables in Group C to the main variable in Group A. The three groups A, B and C which contain the eleven variables created for the model can be seen in Figure 4.1.

Figure 4.1: Groups A, B and C containing all variables created for the model.

4.2.1 Group A variables

The only variable in group A answers the question which students are assigned to which courses, which teacher is assigned to each course, when the course takes place and where. This main variable \(a\), takes the value of 1, when student \(s \in S\) is in course \(c \in C\), taught by teacher \(t \in T\), scheduled for hour \(h \in H\) on day \(d \in D\) in place \(p \in P\), 0 otherwise.

This relationship is described in Equation 4.1. The variable is very detailed and contains all feasible and infeasible solutions, making it difficult to work with directly. Because of its complexity, ten other variables were created. All variables are binary variables and the ten variables created are a subset of the main variable.

\[
a_{s,c,t,d,h,p} = \begin{cases} 
1 & \text{if student } s \in S \text{ is in course } c \in C \text{ with teacher } t \in T \\
& \text{on day } d \in D \text{ at hour } h \in H \text{ in place } p \in P \\
0 & \text{otherwise} 
\end{cases} 
\tag{4.1}
\]

4.2.2 Group C variables

There are seven variables in group C, all a subset of the variable in group A. These variables are able to be connected directly to the input data from the school. The variables in Group C contain information about the schools students, teachers and courses.

Every course has to have an assigned teacher. What courses the teachers are assigned to is determined by the variable \(x\) described in Equation 4.2.
\[ x_{t,c} = \begin{cases} 1 & \text{if teacher } t \in T \text{ teaches course } c \in C \\ 0 & \text{otherwise} \end{cases} \quad (4.2) \]

All teachers are registered with students and the variable described in Equation 4.3 determines which students the teachers teach.

\[ y_{t,s} = \begin{cases} 1 & \text{if teacher } t \in T \text{ teaches student } s \in S \\ 0 & \text{otherwise} \end{cases} \quad (4.3) \]

All teachers have preferences on what days they wish to teach. The variable \( z \) determines at what days a teacher is scheduled, described in Equation 4.4.

\[ z_{t,d} = \begin{cases} 1 & \text{if teacher } t \in T \text{ is scheduled on day } d \in D \\ 0 & \text{otherwise} \end{cases} \quad (4.4) \]

The variable \( w \) described in Equation 4.5 determines which courses the students are registered in.

\[ w_{s,c} = \begin{cases} 1 & \text{if student } s \in S \text{ is in course } c \in C \\ 0 & \text{otherwise} \end{cases} \quad (4.5) \]

All students are available at different days and hours based on their extra curriculum and leisure timetables. The variable \( i \) determines which days and the student is scheduled, described in Equation 4.6.

\[ i_{s,d,h} = \begin{cases} 1 & \text{if student } s \in S \text{ is scheduled on day } d \in D \text{ at hour } h \in H \\ 0 & \text{otherwise} \end{cases} \quad (4.6) \]

The variable described in Equation 4.7 determines which time slot a course is assigned to.

\[ j_{c,d,h} = \begin{cases} 1 & \text{if course } c \in C \text{ is on day } d \in D \text{ at hour } h \in H \\ 0 & \text{otherwise} \end{cases} \quad (4.7) \]

Every course is assigned to a classroom based on the classroom characteristics and availability. The variable \( k \) determines which classroom a course is assigned to, described in Equation 4.8.

\[ k_{c,p} = \begin{cases} 1 & \text{if course } c \in C \text{ is in place } p \in P \\ 0 & \text{otherwise} \end{cases} \quad (4.8) \]

### 4.2.3 Group B variables

There are three variables in group B which were formulated to connect the variables in group C to the variable in group A and to complete the model. All three variables in group B are a subset of the main variable in group A and a superset of the variables in group C. They were created to link the variables constructed by the input data to the main variable, to make it more easy to determine the values of the main variable and to simplify the formulation.
4.3. CONSTRAINTS

The variables in group B contain information about when a course can be taught based on the available classrooms at a specific time of day, at what mutual days the teacher and student are available and then exposing what courses both the teachers and students can attend.

A specific course cannot be taught many times a day in many classrooms. The variable \( l \) determines when a course is taught based on the available classrooms at a specific time of day, described in Equation 4.9

\[
I_{c,d,h,p} = \begin{cases} 
1 & \text{if course } c \in C \text{ is on day } d \in D \text{ at hour } h \in H \text{ in place } p \in P \\
0 & \text{otherwise}
\end{cases} \tag{4.9}
\]

The variable described in Equation 4.10 determines on which day a teacher teaches a student. The variable \( m \) prevents that the same teacher teaches the same student twice the same day.

\[
m_{t,s,d} = \begin{cases} 
1 & \text{if teacher } t \in T \text{ teaches student } s \in S \text{ on day } d \in D, \\
0 & \text{otherwise}
\end{cases} \tag{4.10}
\]

All student that are assigned to courses must have teachers. The variable \( n \) determines if a student is with a teacher in a course, described in Equation 4.11.

\[
n_{s,c,t} = \begin{cases} 
1 & \text{if student } s \in S \text{ is in course } c \in C \text{ with teacher } t \in T \\
0 & \text{otherwise}
\end{cases} \tag{4.11}
\]

4.3 Constraints

There are many rules and aspects that need to be fulfilled at the music school, therefore the model provides a large number of constraints. The constraints can be organized into five groups, availability, singularity, linking, regularity and time constraints.

4.3.1 Availability Constraints

Availability constraints determine the scheduling of students, teachers and classrooms based on their availability.

Students registered at the music school are scheduled according to their availability based on their extra curriculum and leisure timetable.

\[
i_{s,d,h} = 0 \quad \forall (s,d,h) \notin SA \tag{4.12}
\]

A teacher is scheduled for teaching at some day at the music school determined by the teachers available days.

\[
z_{t,d} = 0 \quad \forall (t,d) \notin TA \tag{4.13}
\]

All course types are scheduled at a place based on the availability of classrooms for each course.

\[
k_{x,c} = 0 \quad \forall (x,c) \notin CC \tag{4.14}
\]
4.3.2 Singularity Constraints

The singularity constraints make it certain that there will be no conflicts within the timetable, making sure that each teacher can only be assigned at most one course, with one student or group of students in one classroom at a specific time.

There shall be one classroom assigned to every course.

\[
\sum_{d \in D, h \in H, p \in P} l_{c,d,h,p} = 1 \quad \forall c \in C
\] (4.15)

For every course there must be one and only one teacher assigned.

\[
\sum_{t \in T} x_{t,c} = 1 \quad \forall c \in C
\] (4.16)

Every course shall be assigned to exactly one time period.

\[
\sum_{d \in D, h \in H} j_{c,d,h} = 1 \quad \forall c \in C
\] (4.17)

4.3.3 Linking Constraints

The linking constraints are constraints that were created to link together two variables where one of the variables is a subset of the other.

The variable \( a \), Equation 4.1, specifies whether a student \( s \) is taking course \( c \) with teacher \( t \) at day \( d \) and hour \( h \) in place \( p \). The variable \( a \) is connected to five variables, \( w, i, n, m \) and \( l \), defined in Equations 4.5, 4.6, 4.11, 4.10 and 4.9 respectively.

A student cannot be registered to a course unless there is a teacher assigned to the course at a certain day and hour in a specific place. A student cannot be in a course with a teacher at a certain day and hour in a specific place if there is no student assigned to that course.

\[
w_{s,c} \leq \sum_{t \in T, d \in D, h \in H, p \in P} a_{s,c,t,d,h,p} \quad \forall s \in S, c \in C
\] (4.18)

\[
a_{s,c,t,d,h,p} \leq w_{s,c} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P
\] (4.19)

A student cannot be registered at the music school at his available days and hours unless there is a teacher assigned to the course at that days and hour in a specific place. A student cannot be in a course with a teacher at a certain day and hour in a specific place if there is no student assigned to that course.

\[
i_{s,d,h} \leq \sum_{t \in T, c \in C, p \in P} a_{s,c,t,d,h,p} \quad \forall s \in S, d \in D, h \in H
\] (4.20)

\[
a_{s,c,t,d,h,p} \leq i_{s,d,h} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P
\] (4.21)

A student cannot be registered for a course with a teacher with no assigned place at certain day and hour. A place at a certain day and hour cannot be assigned for a course with no students and teachers.

\[
n_{s,c,t} \leq \sum_{d \in D, h \in H, p \in P} a_{s,c,t,d,h,p} \quad \forall s \in S, c \in C, t \in T
\] (4.22)
### 4.3. CONSTRAINTS

\[ a_{s,c,t,d,h,p} \leq n_{s,c,t} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P \]  
(4.23)

A teacher cannot teach a student on a certain day unless they are registered in a course at a specific place and hour. A course cannot be assigned to a place at a day and hour unless there is a teacher and student with mutual days scheduled.

\[ m_{t,s,d} \leq \sum_{c \in C, h \in H, p \in P} a_{s,c,t,d,h,p} \quad \forall t \in T, s \in S, d \in D \]  
(4.24)

\[ a_{s,c,t,d,h,p} \leq m_{t,s,d} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P \]  
(4.25)

A course cannot be assigned on a certain day and hour at a specific place if there are no students and teachers. Students and teachers cannot be registered in courses that have no place at a certain day and hour.

\[ l_{c,d,h,p} \leq \sum_{t \in T, s \in S} a_{s,c,t,d,h,p} \quad \forall c \in C, d \in D, h \in H, p \in P \]  
(4.26)

\[ a_{s,c,t,d,h,p} \leq l_{c,d,h,p} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P \]  
(4.27)

The variable \( n \), Equation 4.11, specifies whether a student \( s \) is taking course \( c \) with teacher \( t \) and is connected to three variables, \( x \), \( w \) and \( y \), defined in Equations 4.2, 4.5 and 4.3 respectively.

A teacher cannot be assigned to course with no students and students cannot be registered for a course unless there is a teacher assigned to that course.

\[ x_{t,c} \leq \sum_{s \in S} n_{s,c,t} \quad \forall t \in T, c \in C \]  
(4.28)

\[ n_{s,c,t} \leq x_{t,c} \quad \forall s \in S, c \in C, t \in T \]  
(4.29)

A student cannot be assigned to a course with no teachers and teachers cannot be registered for courses that have no students.

\[ w_{s,c} \leq \sum_{t \in T} n_{s,c,t} \quad \forall s \in S, c \in C \]  
(4.30)

\[ n_{s,c,t} \leq w_{s,c} \quad \forall s \in S, c \in C, t \in T \]  
(4.31)

A teacher cannot teach a student if they are not assigned to a course and there cannot be a course if there are no students and teachers.

\[ y_{t,s} \leq \sum_{d \in D} n_{s,c,t} \quad \forall t \in T, s \in S \]  
(4.32)

\[ n_{s,c,t} \leq y_{t,s} \quad \forall s \in S, c \in C, t \in T \]  
(4.33)

The variable \( l \), Equation 4.9 specifies whether a course \( c \) is on day \( d \) at hour \( h \) in place \( p \) and is connected to one variable \( j \), Equation 4.7.
CHAPTER 4. MODEL

A course cannot be at a certain day and hour if there is no registered place and a place can not be registered if there is no course assigned at a certain day and hour.

\[ j_{c,d,h} \leq \sum_{p \in P} l_{c,d,h,p} \quad \forall c \in C, d \in D, h \in H \]  
\[ (4.34) \]

\[ l_{c,d,h,p} \leq j_{c,d,h} \quad \forall c \in C, d \in D, h \in H, p \in P \]  
\[ (4.35) \]

The variable \( m \), Equation 4.10 specifies if a teacher \( t \) is with a student \( s \) on day \( d \) and is connected to one variable \( y \), from Equation 4.3.

A teacher cannot be assigned to a student if there are no mutual days available and a day cannot be decided unless a student is assigned to some teacher.

\[ y_{t,s} \leq \sum_{d \in D} m_{t,s,d} \quad \forall t \in T, s \in S \]  
\[ (4.36) \]

\[ m_{t,s,d} \leq y_{t,s} \quad \forall t \in T, s \in S, d \in D \]  
\[ (4.37) \]

### 4.3.4 Regulatory Constraints

The regulatory constraints make sure that all rules defined by the music school are fulfilled.

There is a limit on the number of students for each course which is predetermined by the music school.

\[ \sum_{s \in S} w_{s,c} \leq l_{c}^{\text{max}} \quad \forall c \in C \]  
\[ (4.38) \]

There are some courses that have a predefined time period, assigning that particular course to a certain day and hour.

\[ j_{c,d,h} = 1 \quad \forall c \in C^f \]  
\[ (4.39) \]

Student registration keeps track on how often a student is registered to a course of a certain course type. Each student must be assigned to the number of course types that they are registered in.

\[ \sum_{c \in C : x_c = x} w_{s,c} = f_{s,x} \quad \forall s \in S, x \in O^t \]  
\[ (4.40) \]

Each student must be assigned to the exact number of courses they are registered in.

\[ \sum_{c \in C} w_{s,c} = \sum_{x \in O^t} f_{s,x} \quad \forall s \in S \]  
\[ (4.41) \]

Each student needs to be assigned to the number of private teacher they are registered with. Most of the time this number is one but occasionally students study on more than one instrument and are assigned to two or more private teachers.

\[ \sum_{T \in T_p} y_{T_r,s} = g_s \quad \forall s \in S \]  
\[ (4.42) \]
4.4 OBJECTIVE FUNCTION

If a student is registered with a private teacher or teachers the student must be assigned with that particular private teacher.

\[ y_{t,s} = 1 \quad \forall (t, s) \in FT \]  

(4.43)

If a teacher is teaching at the music school the teacher needs to be assigned to a course, based on which courses the teacher is able to teach.

\[ x_{t,c} = 0 \quad \forall (t, c) \notin TC \]  

(4.44)

4.3.5 Time Constraints

The time constraints are constraints connecting the teacher and student availability.

A teacher cannot teach a student at some day unless the student is scheduled at a day and hour. A student cannot be taught by a teacher at some day and hour unless the teacher is scheduled at some day.

\[ m_{t,s,d} \leq \sum_{h \in H} i_{s,d,h} \quad \forall t \in T, s \in S, d \in D \]  

(4.45)

\[ i_{s,d,h} \leq \sum_{t \in T} m_{t,s,d} \quad \forall s \in S, d \in D, h \in H \]  

(4.46)

A course cannot be assigned to a certain day and hour if there is no student scheduled on that day and hour. A student cannot be in a course at a certain day and hour unless there is a course scheduled on that day and hour.

\[ j_{c,d,h} \leq \sum_{s \in S} i_{s,d,h} \quad \forall c \in C, d \in D, h \in H \]  

(4.47)

\[ i_{s,d,h} \leq j_{c,d,h} \quad \forall s \in S, d \in D, h \in D \]  

(4.48)

A student cannot take a course unless the student is scheduled at some time period and if a student is assigned to a course the student needs to be scheduled at some time period.

\[ w_{s,c} \leq \sum_{d \in D, h \in H} i_{s,d,h} \quad \forall s \in S, c \in C \]  

(4.49)

\[ i_{s,d,h} \leq \sum_{c \in C} w_{s,c} \quad \forall s \in S, d \in D, h \in H \]  

(4.50)

4.4 Objective function

The objective function of the model is to find a feasible solution, satisfying all the problem’s constraints. If a feasible solution is not possible, penalties could be added to the soft constraints, changing the objective function to minimizing the weighted sum of the penalties. Most of the constraints for this model are hard constraints that are required to be satisfied. Penalties could be added to constraints regarding the availability of teachers and students, preferably the teachers. This could not be done without the knowledge of the teachers and students.
4.5 The model

When everything is compiled, we get the following model:

\[
\begin{align*}
\text{min} & \quad 0 \\
\text{s.t.} & \quad i_{s,d,h} = 0 \quad \forall (s, d, h) \notin SA \quad (4.51) \\
& \quad z_{t,d} = 0 \quad \forall (t, d) \notin TA \quad (4.52) \\
& \quad k_{c,p} = 0 \quad \forall (x_c, p) \notin CC \quad (4.53) \\
& \quad \sum_{d \in D, h \in H, p \in P} l_{c,d,h,p} = 1 \quad \forall c \in C \quad (4.54) \\
& \quad \sum_{t \in T} x_{t,c} = 1 \quad \forall c \in C \quad (4.55) \\
& \quad \sum_{d \in D, h \in H} j_{c,d,h} = 1 \quad \forall c \in C \quad (4.56) \\
& \quad w_{s,c} \leq \sum_{t \in T, d \in D, h \in H, p \in P} a_{s,c,t,d,h,p} \quad \forall s \in S, c \in C \quad (4.57) \\
& \quad a_{s,c,t,d,h,p} \leq w_{s,c} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P \quad (4.58) \\
& \quad i_{s,d,h} \leq \sum_{t \in T, c \in C, p \in P} a_{s,c,t,d,h,p} \quad \forall s \in S, d \in D, h \in H \quad (4.59) \\
& \quad a_{s,c,t,d,h,p} \leq i_{s,d,h} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P \quad (4.60) \\
& \quad n_{s,c,t} \leq \sum_{d \in D, h \in H, p \in P} a_{s,c,t,d,h,p} \quad \forall s \in S, c \in C, t \in T \quad (4.61) \\
& \quad a_{s,c,t,d,h,p} \leq n_{s,c,t} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P \quad (4.62) \\
& \quad m_{t,s,d} \leq \sum_{c \in C, h \in H, p \in P} a_{s,c,t,d,h,p} \quad \forall t \in T, s \in S, d \in D \quad (4.63) \\
& \quad a_{s,c,t,d,h,p} \leq m_{t,s,d} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P \quad (4.64) \\
& \quad l_{c,d,h,p} \leq \sum_{t \in T, s \in S} a_{s,c,t,d,h,p} \quad \forall c \in C, d \in D, h \in H, p \in P \quad (4.65) \\
& \quad a_{s,c,t,d,h,p} \leq l_{c,d,h,p} \quad \forall s \in S, c \in C, t \in T, d \in D, h \in H, p \in P \quad (4.66) \\
& \quad x_{t,c} \leq \sum_{s \in S} n_{s,c,t} \quad \forall t \in T, c \in C \quad (4.67) \\
& \quad n_{s,c,t} \leq x_{t,c} \quad \forall s \in S, c \in C, t \in T \quad (4.68) \\
& \quad w_{s,c} \leq \sum_{t \in T} n_{s,c,t} \quad \forall s \in S, c \in C \quad (4.69) \\
& \quad n_{s,c,t} \leq w_{s,c} \quad \forall s \in S, c \in C, t \in T \quad (4.70) \\
& \quad y_{t,s} \leq \sum_{d \in D} n_{s,c,t} \quad \forall t \in T, s \in S \quad (4.71) \\
& \quad n_{s,c,t} \leq y_{t,s} \quad \forall s \in S, c \in C, t \in T \quad (4.72) \\
& \quad j_{c,d,h} \leq \sum_{p \in P} l_{c,d,h,p} \quad \forall c \in C, d \in D, h \in H \quad (4.73) \\
& \quad l_{c,d,h,p} \leq j_{c,d,h} \quad \forall c \in C, d \in D, h \in H, p \in P \quad (4.74) \\
& \quad y_{t,s} \leq \sum_{d \in D} m_{t,s,d} \quad \forall t \in T, s \in S \quad (4.75)
\end{align*}
\]
4.6 Phase I: Group course scheduling

Let \( m_{t,s,d} \leq y_{t,s} \quad \forall t \in T, s \in S, d \in D \) (4.76)

\[
\sum_{s \in S} w_{s,c} \leq m_{\text{max}} \quad \forall c \in C
\] (4.77)

\[
\sum_{c \in C : x_c = x} w_{s,c} = f_{s,x} \quad \forall s \in S, x \in O^t
\] (4.79)

\[
\sum_{T \in T} y_{T \in T, s} = g_s \quad \forall s \in S
\] (4.81)

\[
y_{t,s} = 1 \quad \forall (t, s) \in FT
\] (4.82)

\[
x_{t,c} = 0 \quad \forall (t, c) / \in TC
\] (4.83)

\[
m_{t,s,d} \leq \sum_{h \in H} i_{s,d,h} \quad \forall t \in T, s \in S, d \in D
\] (4.84)

\[
i_{s,d,h} \leq \sum_{t \in T} m_{t,s,d} \quad \forall s \in S, d \in D, h \in H
\] (4.85)

\[
j_{c,d,h} \leq \sum_{s \in S} i_{s,d,h} \quad \forall c \in C, d \in D, h \in H
\] (4.86)

\[
i_{s,d,h} \leq j_{c,d,h} \quad \forall s \in S, d \in D, h \in D
\] (4.87)

\[
w_{s,c} \leq \sum_{d \in D, h \in H} i_{s,d,h} \quad \forall s \in S, c \in C
\] (4.88)

\[
i_{s,d,h} \leq \sum_{c \in C} w_{s,c} \quad \forall s \in S, d \in D, h \in H
\] (4.89)

a, x, y, z, w, i, j, k, l, m, n \in \{1, 0\}

This model implementation turned out to be intractable and could not be solved, therefore the problem will be split into two phases, Phase I and Phase II for simplification.

In Phase I a model will be constructed to solve the scheduling of all group courses, assigning a student to a course at a day and hour while in Phase II a model will be created to solve the scheduling of all private courses for the music school timetabling problem, assigning a student to a private teacher at a specific day and hour.

4.6 Phase I: Group course scheduling

There are four sets that define the basic structural elements for the Phase I model approach. These four sets are partly the same sets as created for the first model approach, \( S \): The set of students, \( C \): The set of courses, \( D \): The days of the week and \( H \): The hour, at what time of day.

To simplify the group course scheduling the teachers and classrooms are ignored in the Phase I model formulation. It is assumed that each group course is assigned to a teacher beforehand. Therefore, each course has a predefined teacher and can only be taught at a specific day and hour based on the teachers availability. Knowing the number of classrooms that are suitable for the group courses, a constraint is used to determine the upper limit on how many courses can be taught at the same day and hour.
CHAPTER 4. MODEL

Other sets:

\textbf{SA: Student availability}, is a set of students, days and hours. If \((s, d, h) \in SA\) then student \(s\) is available at day \(d\) and hour \(h\).

\textbf{CA: Course availability}, is a set of courses and days. If \((c, d, h) \in CA\) then course \(c\) is available at day \(d\) and hour \(h\).

\textbf{SCP: Student course possibilities}, is a set of students and courses, listing what possible courses a student can be assigned to. If \((s, c) \in SCP\) student \(s\) can be assigned to course \(c\).

\textbf{PS: Preschool courses}, is a set of courses. If \((c_1, c_2) \in PS\) then course \(c_1\) and \(c_2\) are preschool courses. The preschool courses are scheduled twice a week so if a student \(s\) is assigned to course \(c_1\) then the student should also be assigned to \(c_2\).

\textbf{CC: Class conflict}, is a set of courses that may not be scheduled at the same time because they are taught by the same teacher. If \((c_1, c_2) \in CC\) then \(c_1\) and \(c_2\) courses may not be scheduled at the same time.

Parameters:

\textbf{\(l_{\text{max}}\):} For a course \(c \in C\), there is a limit on the number of students that can be assigned, denoted by \(l_{\text{max}}^c\).

\textbf{A:} Most group courses take two time slots because they are scheduled for an hour at a time.

To be able to schedule a course for two time slots a copy was taken of all courses. If a course \(c\) is scheduled at hour \(h\) the course \(c + A\) needs to be scheduled at \(h + 1\).

\textbf{SCC: Student course count}, each student \(s \in S\) can be assigned to a couple of courses. Most group courses are scheduled once a week, then for a student \(s\), \(SCC = 1\). If a student is assigned to a flute preschool course, \(SCC = 2\).

4.6.1 Variables

Four variables where created described in Equations 4.90, 4.91, 4.92 and 4.93. The variable \(q\), Equation 4.90, is the main variable and returns the solution to the Phase I model, answering the question which students are assigned to which group courses and at what day and time it is registered. The variable \(q\) takes the value of 1 when student \(s \in S\) is in course \(c \in C\) scheduled for hour \(h \in H\) on day \(d \in D\), 0 otherwise.

\[
q_{s,c,d,h} = \begin{cases} 
1 & \text{if student } s \in S \text{ is in course } c \in C \text{ on day } d \in D \text{ at hour } h \in H \\
0 & \text{otherwise} 
\end{cases} \quad (4.90)
\]

\[
i_{s,d,h} = \begin{cases} 
1 & \text{if student } s \in S \text{ is scheduled on day } d \in D \text{ at hour } h \in H \\
0 & \text{otherwise} 
\end{cases} \quad (4.91)
\]

\[
j_{c,d,h} = \begin{cases} 
1 & \text{if course } c \in C \text{ is on day } d \in D \text{ at hour } h \in H \\
0 & \text{otherwise} 
\end{cases} \quad (4.92)
\]

\[
w_{s,c} = \begin{cases} 
1 & \text{if student } s \in S \text{ is in course } c \in C \\
0 & \text{otherwise} 
\end{cases} \quad (4.93)
\]
The variable \( i \) determines on which days and time the student is scheduled. The variable \( j \) determines which time slot a course is assigned to and the variable \( w \) which courses the students are registered in, described in Equations 4.91, 4.92 and 4.93 respectively. The three variables are all a subset of the main variable \( q \) and return the value 1 if true and 0 otherwise.

### 4.6.2 Constraints

The constraints for the Phase I model can be organized into four groups, availability, singularity, linking and regularity constraints.

#### 4.6.2.1 Availability Constraints

Availability constraints determine the scheduling of students and courses.

Students registered at the music school are scheduled according to their availability based on their extra curriculum and leisure timetable.

\[
i_{s,d,h} = 0 \quad \forall (s, d, h) \notin SA \quad (4.94)
\]

A course is scheduled at some day at the music school, determined by the availability of the teacher teaching the course.

\[
j_{c,d,h} = 0 \quad \forall (c, d, h) \notin CA \quad (4.95)
\]

#### 4.6.2.2 Singularity Constraints

Every course shall be assigned to one time period.

\[
\sum_{d \in D, h \in H} j_{c,d,h} = 1 \quad \forall c \in C \quad (4.96)
\]

#### 4.6.2.3 Linking Constraints

The variable \( q \), Equation 4.90, specifies whether a student \( s \) is taking course \( c \) at day \( d \) and hour \( h \). The variable \( q \) is connected to three variables \( w, i \) and \( j \), Equations 4.93, 4.91 and 4.92

A student cannot be registered to a course unless the course is assigned to a certain day and hour. A student cannot be assigned to a course with no scheduled days and hours.

\[
w_{s,c} \leq \sum_{d \in D, h \in H} q_{s,c,d,h} \quad \forall s \in S, c \in C \quad (4.97)
\]

\[
q_{s,c,d,h} \leq w_{s,c} \quad \forall s \in S, c \in C, d \in D, h \in H \quad (4.98)
\]

A student cannot be registered at the music school at his scheduled days and hours unless there is a course at that certain day and hour. A course cannot be assigned to a certain day and hour if there are no students.

\[
i_{s,d,h} \leq \sum_{c \in C} q_{s,c,d,h} \quad \forall s \in S, d \in D, h \in H \quad (4.99)
\]
CHAPTER 4. MODEL

\( q_{s,c,d,h} \leq i_{s,d,h} \quad \forall s \in S, c \in C, d \in D, h \in H \) \hspace{1cm} (4.100)

A course cannot be assigned on a certain day and hour if there are no students and students cannot be assigned to a course that have no registered days and hours.

\[ j_{c,d,h} \leq \sum_{s \in S} q_{s,c,d,h} \quad \forall c \in C, d \in D, h \in H \] \hspace{1cm} (4.101)

\[ q_{s,c,d,h} \leq j_{c,d,h} \quad \forall s \in S, c \in C, d \in D, h \in H \] \hspace{1cm} (4.102)

The variable \( j \), Equation 4.92, specifies if a course is scheduled on day \( d \) at hour \( h \). The variables \( j \) is connected to the variable \( i \), Equation 4.91. A course cannot be assigned to a day and hour if there is no student scheduled at that day and hour. A student cannot be scheduled at a certain day and hour unless there is a course on that specific day and hour.

\[ j_{c,d,h} \leq \sum_{s \in S} i_{s,d,h} \quad \forall c \in C, d \in D, h \in H \] \hspace{1cm} (4.103)

\[ i_{s,d,h} \leq j_{c,d,h} \quad \forall s \in S, c \in C, d \in D, h \in H \] \hspace{1cm} (4.104)

4.6.2.4 Regulatory Constraints

The regulatory constraints make sure that all rules defined by the music school are fulfilled.

There is an upper limit on the number of students for each group course, predetermined by the music school.

\[ \sum_{s \in S} w_{s,c} \leq p^{\text{max}} \quad \forall c \in C \] \hspace{1cm} (4.105)

A student can only be assigned to a certain number of courses. If a student is assigned to a music theory course once a week, \( SCC = 1 \)

\[ \sum_{(s,c) \in SCP} w_{s,c} = SCC \quad \forall (s, c) \in SCP \] \hspace{1cm} (4.106)

There are two classrooms at the school scheduled for group courses. Therefore, only two courses can be scheduled at any time.

\[ \sum_{c \in C} j_{c,d,h} \leq 2 \quad \forall d \in D, h \in H \] \hspace{1cm} (4.107)

The preschool courses are scheduled twice a week. Students registered in preschool courses must take both courses.

\[ w_{s,c_1} = w_{s,c_2} \quad \forall s \in S, (c_1, c_2) \in PS \] \hspace{1cm} (4.108)

The flute preschool courses can not be scheduled on the same day.

\[ \sum_{h \in H} j_{c_1,d,h} + \sum_{h \in H} j_{c_2,d,h} \leq 1 \quad \forall (c_1, c_2) \in PS, d \in D, h \in H \] \hspace{1cm} (4.109)
4.6. PHASE I: GROUP COURSE SCHEDULING

Most group courses are scheduled for an hour at a time, taking two time slots. To schedule a course for two time slots a copy was taken of all courses, creating the parameter \( A \). If a group course is scheduled for two time slots then \( c \) is scheduled at \( h \) and \( c + A \) must be scheduled at \( h + 1 \).

\[
\forall c \in C, d \in D, h \in H \quad j_{c+A,d,h+1} = j_{c,d,h}
\]  

If a student is assigned to a course with two time slots it needs to be assigned to both time slots.

\[
\forall s \in S, c \in C \quad w_{s,c+A} = w_{s,c}
\]

Courses taught by the same teacher may no be scheduled on the same day and hour.

\[
\forall (c_1, c_2) \in CC, d \in D, h \in H \quad j_{c_1,d,h} + j_{c_2,d,h} \leq 1
\]

4.6.3 Objective function

The objective function of the model is to find a feasible solution, satisfying all the problem’s constraints. If a feasible solution is not possible, penalties could be added to the soft constraints, changing the objective function to minimizing the weighted sum of the penalties. Most of the constraints for this model are hard constraints that are required to be satisfied. Penalties could be added to constraints regarding the availability of teachers and students, preferably the teachers. This could not be done without the knowledge of the teachers and students.
4.6.4 The model

When everything is compiled, we get the following model:

\[
\begin{align*}
\text{min} & \quad 0 \\
\text{s.t.} & \quad i_{s,d,h} = 0 \quad \forall (s,d,h) \notin SA \\
& \quad j_{c,d,h} = 0 \quad \forall (c,d,h) \notin CA \\
& \quad \sum_{d \in D, h \in H} j_{c,d,h} = 1 \quad \forall c \in C \\
& \quad w_{s,c} \leq \sum_{d \in D, h \in H} q_{s,c,d,h} \quad \forall s \in S, c \in C \\
& \quad q_{s,c,d,h} \leq w_{s,c} \quad \forall s \in S, c \in C, d \in D, h \in H \\
& \quad i_{s,d,h} \leq \sum_{c \in C} q_{s,c,d,h} \quad \forall s \in S, d \in D, h \in H \\
& \quad q_{s,c,d,h} \leq i_{s,d,h} \quad \forall s \in S, c \in C, d \in D, h \in H \\
& \quad j_{c,d,h} \leq \sum_{s \in S} q_{s,c,d,h} \quad \forall c \in C, d \in D, h \in H \\
& \quad j_{c,d,h} \leq \sum_{s \in S} i_{s,d,h} \quad \forall c \in C, d \in D, h \in H \\
& \quad i_{s,d,h} \leq j_{c,d,h} \quad \forall s \in S, d \in D, h \in H \\
& \quad \sum_{s \in S} w_{s,c} \leq l_{max} \quad \forall c \in C \\
& \quad \sum_{(s,c) \in SCP} w_{s,c} = SCC \quad \forall (s,c) \in SCP \\
& \quad \sum_{c \in C} j_{c,d,h} \leq 2 \quad \forall d \in D, h \in H \\
& \quad w_{s,c_1} = w_{s,c_2} \quad \forall s \in S, (c_1, c_2) \in PS \\
& \quad \sum_{h \in H} j_{c_1,d,h} + \sum_{h \in H} j_{c_2,d,h} \leq 1 \quad \forall (c_1, c_2) \in PS, d \in D, h \in H \\
& \quad j_{c+d,d,h+1} = j_{c,d,h} \quad \forall c \in C, d \in D, h \in H \\
& \quad w_{s,c_A} = w_{s,c} \quad \forall s \in S, c \in C \\
& \quad j_{c_1,d,h} + j_{c_2,d,h} \leq 1 \quad \forall (c_1, c_2) \in CC, d \in D, h \in H \\
& \quad q, w, i, j \in \{1, 0\}
\end{align*}
\]
4.7 Phase II: Private course scheduling

In Phase II a model will be created to solve the scheduling of all private courses in accordance with Phase I, where the results from Phase I will be used as input data along with other required data.

There are four sets that define the basic structural elements for the Phase II model approach. These four sets are partly the same sets as created for the first model approach, \( S \): The set of students, \( T \): The set of teachers, \( D \): The days of the week and \( H \): The hour, at what time of day.

To simplify the scheduling of all private courses the courses and classrooms are ignored in the Phase II formulation. For the private course scheduling there is no need to assign students to courses, they are only assigned to teachers which indicate the musical instrument. There are fourteen classrooms available at the school for private courses and each classroom is scheduled for a specific course. In this model formulation it is assumed that all private courses can take place in whatever classroom, despite the course type. A constraint is used to determine the upper limit on how many teachers can be scheduled for each time slot.

Other sets:

\( ST \): Student teacher, is a set of students and teachers, indicating which teacher a student is assigned to. If \((t, s) \in ST\) then the student \(s\) is assigned with teacher \(t\).

\( SA \): Student availability, is a set of students, days and hours. If \((s, d, h) \in SA\) then student \(s\) is available at day \(d\) and hour \(h\).

\( TA \): Teachers availability, is a set of teachers, days and hours. If \((t, d, h) \in TA\) then teacher \(t\) is available at day \(d\) and hour \(h\).

Parameters:

\( STC \): Student teacher count, is a number of how often a student \(s \in S\) should be assigned to a teacher \(t\). Private courses are most often taught twice a week, assigning the student twice with the same teacher.

4.7.1 Variables

Four variables where created \( u \), \( v \), \( y \) and \( i \) described in Equations 4.132, 4.133, 4.135 and 4.134. The variable \( u \), Equation 4.132, is the main variable and returns the solution to the Phase II model, answering the question which students are assigned to which teachers and at what day and time. The variable \( u \) takes the value of 1 when student \(s \in S\) is in course \(c \in C\) scheduled for hour \(h \in H\) on day \(d \in D\), 0 otherwise.

\[
u_{s, t, d, h} = \begin{cases} 
1 & \text{if student } s \in S \text{ is with teacher } t \in T \text{ on day } d \in D \text{ at hour } h \in H \\
0 & \text{otherwise} 
\end{cases} \tag{4.132}
\]

The variable \( v \) described in Equation 4.133 determines on which days and time a teacher is scheduled.

\[
v_{t, d, h} = \begin{cases} 
1 & \text{if teacher } t \in T \text{ is available on day } d \in D \text{ at hour } h \in H \\
0 & \text{otherwise} 
\end{cases} \tag{4.133}
\]
\[ i_{s,d,h} = \begin{cases} 1 & \text{if student } s \in S \text{ is scheduled on day } d \in D \text{ at hour } h \in H \\ 0 & \text{otherwise} \end{cases} \quad (4.134) \]

\[ y_{t,s} = \begin{cases} 1 & \text{if teacher } t \in T \text{ teaches student } s \in S \\ 0 & \text{otherwise} \end{cases} \quad (4.135) \]

The variable \( i \) determines on which days and time the student is scheduled while the variable \( y \) defines if a student is scheduled with a teacher, described in Equations 4.134 and 4.135. The variables \( v, y, \) and \( i \) are all a subset of the main variable \( u \) and return the value 1 if true and 0 otherwise.

### 4.7.2 Constraints

The constraints for the Phase II model can be organized into three groups, availability, linking and regularity constraints.

#### 4.7.2.1 Availability Constraints

Availability constraints determine the scheduling of students and teachers.

Students registered at the music school are scheduled according to their availability based on their extra curriculum and leisure timetable.

\[ i_{s,d,h} = 0 \quad \forall (s, d, h) \notin SA \quad (4.136) \]

A teacher is scheduled at some day at the music school, determined by the availability of the teacher.

\[ v_{t,d,h} = 0 \quad \forall (t, d, h) \notin TA \quad (4.137) \]

#### 4.7.2.2 Linking Constraints

The variables \( u \), Equation 4.132, specifies if a student \( s \) is scheduled with a teacher \( t \) on day \( d \) at hour \( h \). The variables \( u \) is connected to the variables \( v, y, \) and \( i \), Equation 4.133, 4.135 and 4.134

A teacher cannot teach a student if they are not scheduled at a day and hour and there cannot be days and hours scheduled if there are no students and teachers.

\[ y_{t,s} \leq \sum_{d \in D, h \in H} u_{s,t,d,h} \quad \forall t \in T, s \in S \quad (4.138) \]

\[ u_{s,t,d,h} \leq y_{t,s} \quad \forall s \in S, t \in T, d \in D, h \in H \quad (4.139) \]

A teacher cannot be scheduled at a day and hour if there are no students and a student cannot be scheduled with a teacher unless the teacher is scheduled at that day and hour.

\[ v_{t,d,h} \leq \sum_{s \in S} u_{s,t,d,h} \quad \forall t \in T, d \in D, h \in H \quad (4.140) \]
4.7. PHASE II: PRIVATE COURSE SCHEDULING

\[ u_{s,t,d,h} \leq v_{t,d,h} \quad \forall s \in S, t \in T, d \in D, h \in H \] (4.141)

A student cannot be scheduled at a day and hour unless there is a teacher and a teacher cannot be scheduled if there is no student scheduled at that day and hour.

\[ i_{s,d,h} \leq \sum_{t \in T} u_{s,t,d,h} \quad \forall s \in S, d \in D, h \in H \] (4.142)

\[ u_{s,t,d,h} \leq i_{s,d,h} \quad \forall s \in S, t \in T, d \in D, h \in H \] (4.143)

The variable \( v \), Equation 4.133, specifies if a teacher \( t \) is scheduled on day \( d \) and hour \( h \). The variable \( v \) is connected to the variable \( i \), Equation 4.134.

A teacher cannot be assigned to a certain day and hour if there is no student scheduled on that day and hour. A student cannot be assigned to a day and hour unless there is a teacher scheduled on that day and hour.

\[ v_{t,d,h} \leq \sum_{s \in S} i_{s,d,h} \quad \forall t \in T, d \in D, h \in H \] (4.144)

\[ i_{s,d,h} \leq v_{t,d,h} \quad \forall s \in S, d \in D, h \in H \] (4.145)

4.7.2.3 Regularity Constraints

The regulatory constraints make sure that all rules defined by the music school are fulfilled.

If a student is assigned to a teacher that teacher must teach that specific student.

\[ y_{t,s} = 1 \quad \forall (t, s) \in ST \] (4.146)

Each student registered in private courses cannot be assigned to more than one course each day.

\[ \sum_{h \in H} i_{s,d,h} \leq 1 \quad \forall s \in S, d \in D \] (4.147)

Each student should be with a teacher for a certain number of times each week. Normally a student is scheduled twice a week with a teacher in a private course.

\[ \sum_{t \in T, d \in D, h \in H} u_{s,t,d,h} = STC \quad \forall s \in S \] (4.148)

Each teacher can only be registered with one student at most, for each time slot.

\[ \sum_{s \in S} u_{s,t,d,h} \leq 1 \quad \forall t \in T, d \in D, h \in H \] (4.149)

There can only be fourteen teachers assigned to each time slot because of the number of classrooms suitable for private courses.

\[ \sum_{t \in T} v_{t,d,h} \leq 14 \quad \forall d \in D, h \in H \] (4.150)
4.7.3 Objective function

The objective function of the model is to find a feasible solution, satisfying all the problem’s constraints. If a feasible solution is not possible, penalties could be added to the soft constraints, changing the objective function to minimizing the weighted sum of the penalties. Most of the constraints for this model are hard constraints that are required to be satisfied. Penalties could be added to constraints regarding the availability of teachers and students, preferably the teachers. This could not be done without the knowledge of the teachers and students.

4.7.4 The model

When everything is compiled, we get the following model:

\[
\begin{align*}
\text{min} & \quad 0 \\
\text{s.t.} & \quad i_{s,d,h} = 0 \quad \forall (s, d, h) \notin SA \quad (4.151) \\
& \quad v_{t,d,h} = 0 \quad \forall (t, d, h) \notin TA \quad (4.152) \\
& \quad y_{t,s} \leq \sum_{d \in D, h \in H} u_{s,t,d,h} \quad \forall t \in T, s \in S \quad (4.153) \\
& \quad u_{s,t,d,h} \leq y_{t,s} \quad \forall s \in S, t \in T, d \in D, h \in H \quad (4.154) \\
& \quad v_{t,d,h} \leq \sum_{s \in S} u_{s,t,d,h} \quad \forall t \in T, d \in D, h \in H \quad (4.155) \\
& \quad u_{s,t,d,h} \leq v_{t,d,h} \quad \forall s \in S, t \in T, d \in D, h \in H \quad (4.156) \\
& \quad i_{s,d,h} \leq \sum_{t \in T} u_{s,t,d,h} \quad \forall s \in S, d \in D, h \in H \quad (4.157) \\
& \quad u_{s,t,d,h} \leq i_{s,d,h} \quad \forall s \in S, t \in T, d \in D, h \in H \quad (4.158) \\
& \quad v_{t,d,h} \leq \sum_{s \in S} i_{s,d,h} \quad \forall t \in T, d \in D, h \in H \quad (4.159) \\
& \quad i_{s,d,h} \leq v_{t,d,h} \quad \forall s \in S, d \in D, h \in H \quad (4.160) \\
& \quad y_{t,s} = 1 \quad \forall (t, s) \in ST \quad (4.161) \\
& \quad \sum_{h \in H} i_{s,d,h} \leq 1 \quad \forall s \in S, d \in D \quad (4.162) \\
& \quad \sum_{t \in T, d \in D, h \in H} u_{s,t,d,h} = STC \quad \forall s \in S \quad (4.163) \\
& \quad \sum_{s \in S} u_{s,t,d,h} \leq 1 \quad \forall t \in T, d \in D, h \in H \quad (4.164) \\
& \quad \sum_{t \in T} v_{t,d,h} \leq 14 \quad \forall d \in D, h \in H \quad (4.165) \\
& \quad u, v, y, i, \in \{1, 0\}
\end{align*}
\]
Chapter 5

Data

In this chapter a brief description will be given on how and what data was collected for the model along with describing the data processing.

In order to check the capabilities of the model presented above data from the music school, Tónmenntaskóli Reykjavíkur was used. In the final section of this chapter the case study will be described.

5.1 Data Collection

The data required for the model consists of information about students, teachers, courses and classrooms. Regarding the students, the required information is the number of students, what instrument they play on and what teacher they are assigned to, what group courses they are registered in, along with their time availability to see when the student is available to attend courses.

Data regarding the teachers consists of what courses or musical instrument they are able to teach along with their preferences concerning when they are available.

Data required regarding the courses is what courses are available for the semester, their capacity and if they have a fixed time slot or not. Not all courses can be taught in every classroom, it needs to be clear how many classrooms are available at the school and what courses can be taught in each one, depending on the size and equipment available.

All the data was gathered manually from the head of the music school who printed and scanned sheets of paper containing all the information needed for the model as stated above.

5.2 Data Processing

![Diagram](image.png)

Figure 5.1: The data process.

The data was collected, structured and written manually into five Excel files. The Excel files were created to organize all information required for the model and to convert the data from being on sheets of paper to a computerized form.
The first Excel file contains information about all students, the second file contains information about the teachers availability while the third one is a list of all teachers and the courses they are able to teach. The fourth file is a list of all the group courses that have a fixed time slot while the fifth file is a summary file created to manage and have an overview of all other files.

All five Excel files were converted to a comma-separated values (CSV) file. The CSV file is readable and easy to use where there is one line for each record and a comma used to separate each field. The CSV files were created to transfer the database between applications.

The programming language Python was used to read all the data from the CSV files, collecting all the information together in one place. A Python code was created returning a .dat file for the Phase I model, storing and structuring all the specific information needed for the model. A .mod file was created based on the Phase I model formulation. A .dat file is a data file, storing information and a .mod file is a file that can store models along with other projects. The .dat file and the .mod file were used as input files into The GNU Linear Programming kit (GLPK), a package which is intended to solve large scale linear programming and mixed integer programming problems [21]. GLPK was used to transfer the data to a specific format readable for Gurobi 7.5.1, a mathematical programming solver, that was used to solve the Phase I model. A Python code was created to read the results from Gurobi, returning the solution from Phase I to the timetable.

The same process was used to solve the Phase II model except the solution from Phase I was used as additional data along with the data from the five Excel files. The data process can be seen in Figure 5.1

5.3 Case Study - Tónmenntaskóli Reykjavíkur

Tónmenntaskóli Reykjavíkur has 124 students registered in group and private courses, offering 22 group courses and 206 private courses. These 228 courses are assigned to 15 teachers, 11 private teachers, four group course teachers and one teacher teaching both group and private courses for the fall of 2017.

There are 16 available classrooms, available at any day and time of the week unless they are registered by the school itself. All group courses are taught in two of the schools classrooms. There are five classrooms that are preferably used for piano lessons, one for violin lessons while the rest of the classrooms are used for all other private lessons.
Chapter 6

Results

In this chapter the model results from Phase I and Phase II will be presented. Both models were solved with Gurobi 7.5.1.

For the Phase I model, the number of constraints were 1,279,678 and the number of variables were 427,012. After the presolve, the number of constraints were reduced to 75,472 and the number of variables were 26,781. The total time it took to solve the model was 525.84 seconds.

The Phase II model is more simple and easier to solve. The number of constraints were 335,180 and the number of variables were 112,057. After the presolve, the number of constraints were only 95 and the number of variables were 47. It only took 2.35 seconds to solve the model.

6.1 Phase I model results

The Phase I model returns a feasible solution, where all of the 124 students registered in group courses were assigned to 22 group courses scheduled for the five working days, satisfying all the problem constraints. The resulting timetable can be seen in Figure 6.1 where each group course is scheduled for two time slots and only two courses can be scheduled for each time slot. All courses are music theory courses except for course 21 which is a violin preschool course. Courses 17 and 18 are flute preschool courses that cannot be scheduled at the same day. Students registered in either one need to be registered in both courses, the same applies for courses 19 and 20. No courses are scheduled on Fridays and only two courses are scheduled on Wednesdays.

<table>
<thead>
<tr>
<th>Time</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>14:00</td>
<td>Course 1</td>
<td>Course 17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14:30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15:00</td>
<td>Course 17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15:30</td>
<td>Course 2</td>
<td>Course 20</td>
<td>Course 12</td>
<td>Course 10</td>
<td>Course 15</td>
</tr>
<tr>
<td>16:00</td>
<td>Course 20</td>
<td>Course 13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16:30</td>
<td>Course 21</td>
<td>Course 13</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17:00</td>
<td>Course 9</td>
<td>Course 21</td>
<td>Course 12</td>
<td>Course 10</td>
<td>Course 15</td>
</tr>
<tr>
<td>17:30</td>
<td>Course 9</td>
<td>Course 21</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18:00</td>
<td>Course 5</td>
<td>Course 11</td>
<td>Course 14</td>
<td>Course 7</td>
<td>Course 6</td>
</tr>
<tr>
<td>18:30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19:00</td>
<td>Course 11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.1: The resulting timetable for all group courses.
6.2 Phase II model results

The Phase II model returned a feasible solution after fixing some of the data to satisfy all the problems constraints. The data used for the case study was not sufficient for the model to return a feasible solution. The problem was regarding the availability of teachers and students. In some cases a teacher was only registered available for teaching at one day of the week, violating the constraint that each student registered with a teacher in a private course is most often scheduled twice a week, not on the same day. In other cases the teachers and students availability did not match, where a teacher was for example available on Mondays and Tuesdays while the student was only available on Mondays and Wednesdays, not being able to schedule the student twice with the same teacher. This is obviously something the music school needs to consider, having the data correct.

The solution from Phase II was found based on the results from Phase I. If a student was scheduled in a group course that student could not be scheduled in a private course at that same time. To obtain a feasible solution the data regarding the teachers availability was modified, adding extra available days to some of the teachers, to satisfy all the problems constraints.

Timetables were created for each student registered in private courses, where 124 students were assigned to 206 courses. In Figure 6.2 an example of a timetable can be seen for student number 2, assigned to Teacher 11 on Mondays and Fridays and scheduled in a group course, Course 10, on Thursdays. The red cells are time slots that student number 2 cannot be assigned to. In Figure 6.3 an example of a timetable for a teacher number 11 can be seen which teaches student number 2. The red cells are time slots that teacher 11 cannot be assigned to. In Figure 6.4 and in Figure 6.5 the resulting timetables for student number 3 and teacher number 6 can be seen.

<table>
<thead>
<tr>
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Figure 6.2: The resulting timetable for student number 2.


### 6.2. PHASE II MODEL RESULTS

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Figure 6.3: The resulting timetable for teacher number 11.

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Figure 6.4: The resulting timetable for student number 3.

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</table>

Figure 6.5: The resulting timetable for teacher number 6.
Chapter 7

Conclusions

In this thesis a mathematical model was presented to solve the music school timetabling problem, assigning teachers and students to courses and classrooms. An integer programming model was proposed where the model implementation turned out to be intractable. The problem was split into two phases and two integer programming models were proposed.

In Phase I a model will be constructed to solve the scheduling of all group courses, assigning a student to a course at a day and hour while in Phase II a model will be created to solve the scheduling of all private courses for the music school timetabling problem, assigning a student to a private teacher at a specific day and hour.

The timetable for the music school Tónmenntaskóli Reykjavíkur was used as a case study and solved successfully. The results show that the model could be a good tool to help with the scheduling where the problem has only been solved manually at Tónmenntaskóli Reykjavíkur, requiring a lot of hand- and paperwork.

To improve the solution of the model the objective function could be modified to scheduling all courses as early as possible. Minimizing the total hours for each day, the time from where the first course is scheduled until the end of the last course scheduled.

The running time for the model is short, making it suitable for additions. The music school can use the solution provided by the models, modify and make a personal touch on the solutions based on their knowledge and other information.

The goal of the project is achieved, automation is now possible. An optimization model can be created to solve the music school timetabling problem, assigning students and teachers to the right courses.

The process of solving the model is not user friendly, future work could be integrating the whole process and making it easy to use for music schools.
Bibliography


