Forecast Modelling for the Icelandic Automotive Market

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Thesis of 30 ECTS credits submitted to the School of Science and Engineering at Reykjavík University in partial fulfillment of the requirements for the degree of Master of Science (M.Sc.) in Engineering Management

June 2018

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Abstract
In all business where substantial investments are made in inventory, it is important to be able to predict future demand. Bad inventory management can cause inventory costs to surge or potential sales opportunities to be lost. To prevent such a situation, it is often possible to develop a model that utilizes historical data and quantitative methods to predict future demand. A market where sizable amounts are tied up in inventory is, for example, the automotive market. In this thesis, six different quantitative methods were used to predict automotive sales, excluding car rental sales. The methods are simple moving average, simple exponential smoothing, Holt-Winters exponential smoothing, multiple linear regression, support vector regression and random forest. The first three methods only use historical registration numbers from ICETRA to predict the size of the market while the latter three methods use selected exogenous variables as well. Two questions are presented in the paper: Do machine learning algorithms return smaller forecasting errors for the Icelandic automotive market compared to traditional time series analysis methods? Which collection of data points, monthly or quarterly, is more suitable for Icelandic dealerships and returns lower forecasting errors? These questions were answered by creating a model for each method where actual data was used for validation. The Icelandic automotive market is relatively volatile, making it difficult at times to predict the size of the market correctly. With respect to that, the results of the forecasting models are considered reasonable. The most accurate models implemented utilized regression analysis and quarterly data. The method that yielded the lowest forecasting error for a two-year forecasting period was support vector regression.

Keywords: Icelandic automotive market, forecast modelling, support vector regression, multiple linear regression, Holt-Winters exponential smoothing, random forest
Spágerð fyrir íslenska bílamarkaðinn

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Júni 2018

Útdráttur

Lykilorð: Íslenski bílamarkaðurinn, spálfíkön, stoðvigna aðhvarfsgreining, margþætt línuleg aðhvarfsgreining, Holt-Winters veldisjöfnun, ræktun slembiskóga
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Master of Science
Acknowledgements

I would like to express my gratitude to my supervisors, Dr. Jón Guðnason and Dr. Hlynur Stefánsson for their guidance, support, and valid inputs during the writing of this thesis. Weekly meetings with Dr. Jón Guðnason were of great help through the whole semester in addition to his comments on my final draft. Dr. Hlynur Stefánsson became a co-supervisor at a later stage in the semester and delivered his contribution through instructive feedback on the final draft in addition to good pointers on the thesis defense presentation.
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<th>Description</th>
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<tbody>
<tr>
<td>ANFIS</td>
<td>Adaptive Network-Based Fuzzy Inference System</td>
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<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
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<td>ARIMA</td>
<td>Autoregressive Integrated Moving Average</td>
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<td>CCI</td>
<td>Consumer Confidence Index</td>
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<td>CPI</td>
<td>Consumer Price Index</td>
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<tr>
<td>CSV</td>
<td>Comma Separated Values</td>
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<td>ES</td>
<td>Exponential Smoothing</td>
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<tr>
<td>FX</td>
<td>Foreign exchange</td>
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<td>ICA</td>
<td>Independent Component Analysis</td>
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<td>ISK</td>
<td>Icelandic Krona</td>
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<tr>
<td>LCV</td>
<td>Light Commercial Vehicle</td>
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<tr>
<td>MAE</td>
<td>Mean Absolute Error</td>
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<td>MAPE</td>
<td>Mean Absolute Percentage Error</td>
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<td>MLR</td>
<td>Multiple Linear Regression</td>
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<td>MSE</td>
<td>Mean Squared Error</td>
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<td>PC</td>
<td>Passenger Car</td>
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<td>RBF</td>
<td>Radial basis function</td>
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<tr>
<td>RMSE</td>
<td>Root Mean Squared Error</td>
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<tr>
<td>SES</td>
<td>Simple Exponential Smoothing</td>
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<tr>
<td>SMA</td>
<td>Simple Moving Average</td>
</tr>
<tr>
<td>SSE</td>
<td>Sum of Squared Error</td>
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<tr>
<td>SVM</td>
<td>Support Vector Machine</td>
</tr>
<tr>
<td>SVR</td>
<td>Support Vector Regression</td>
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<tr>
<td>VIF</td>
<td>Variance Inflation Factor</td>
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Chapter 1

Introduction

On June 20th, 1904, the first automobile arrived at Iceland’s shore. The car was of the make Cudell and imported by the Icelandic merchant Ditlev Thomsen. Roughly ten years later, the first mass-produced automobile exited Henry Ford’s factory, and Iceland’s car park began to grow. By the middle of the 20th century, the number was closing in on 10,000 cars and with new regulations on free import in 1960, the volume quadrupled in only ten years (Fornbílaklúbbur Íslands, 2018). It was announced in an annual publication of the Icelandic Federation for Motor Trades and Repairs, that by the end of 2016 the number of registered vehicles in Iceland had exceeded the country’s population for the first time with 344,664 vehicles (Karlsson & Þorsteinsson, 2017). In international comparison, Iceland's car ownership per capita has become one of the highest in the world.

Due to the size of the market, the automotive industry has begun turning over remarkable amounts. In 2016, turnover of automotive trades including car sales, repairs, spare parts sales, and accessories totaled 160 billion ISK which was 6.6% of Iceland’s gross domestic product. The largest segment, sales of new cars, totaled 121 billion ISK, increasing by almost a quarter from the year before (Karlsson & Þorsteinsson, 2017). When this thesis was written, no results were in for the 2017 turnover. However, with a record-breaking year in registrations, it is somewhat likely that the total turnover increased again between years. According to ICETRA, 23,269 new cars were registered in 2017. Hereafter, all sales figures mentioned are obtained from ICETRA.

The Icelandic economy has been in an expansion for some years now, but in only 2013 the total number of new registrations did not reach 8,000 cars. The market has nevertheless seen it worse. Following the collapse of the Icelandic banks in October 2008, merely 2,515 cars were registered in 2009. To emphasise the impact of the financial crisis, you would have to rewind to the 1960s to find a number of similar magnitude. Understandably, these low selling years can have significant impacts on businesses in the industry, especially for dealerships with deficient inventory management.
Two types of inventory management strategies are used among Icelandic dealerships, make to order and make to stock. Make to order revolves around customizing the product to the customer's needs, giving him the freedom of choosing between various colors, interior designs, and accessories. As a result, the relevant car does not go into production until the customer has fully made up his mind. The production process itself varies between manufacturers, but in general, it can take between 3-6 months to get the car manufactured and shipped to Iceland. The latter strategy, make to stock, revolves around matching production and inventory with consumer demand forecasts. This strategy is primarily used in many kinds of retail where future predictions are often based on past data. The most significant advantage of making the product to stock is the ability to minimize the customers waiting time, while the biggest disadvantage is how reliant it is on the company’s predictive abilities. With no proper forecasting method in place, the risk increases of either underestimating or overestimating the future demand, leading to possible losses in sale or high inventory costs.

Both strategies have their advantages and disadvantages. Companies implementing make to order are willing to sacrifice additional waiting time for the customer to minimize inventory costs. Meanwhile, companies applying make to stock are willing to increase their inventory cost to minimize the customers waiting time. For Icelandic car dealerships, it usually depends on two factors which strategy is chosen. The country’s economic health and the product itself. While higher end, low selling models are typically made to order, more traditional ones are made to stock due to their high turnover ratio. However, during an economic depression, circumstances can arise where dealerships primarily go by the make to order strategy to limit cost and uncertainty in their operations. During the financial crisis in 2008-09, no cars were, for example, made to stock since most of the dealerships were struggling with selling their inventory at the time. In the end, some dealerships even exported their cars back to Europe while others sold them with unusually high discounts (Thorlacious, 2010).

As has been implied already, the Icelandic automotive market can be quite volatile. For that reason, top management at Icelandic dealerships must monitor the development of the market closely, so their inventory size always reflects the demand of the market. By accurately predicting the future demand of the market, dealerships would be able to adjust their orders accordingly and maintain an optimal inventory size.

Predictive methods can be divided into two different categories, qualitative methods, and quantitative methods. Qualitative methods are used in an environment of high uncertainty with limited data available while quantitative methods are used in a more stable
environment where historical data is available (Martinovic & Damnjanovic, 2006). As far as the author knows, qualitative methods have primarily been used by dealerships to predict future demand where top management of the company is gathered together to discuss the potential heading of the market. In this thesis, however, numerous quantitative methods will be examined to conclude which one of them might be considered useful for predicting the Icelandic automotive market. These methods are first mentioned in chapter 1.2 and later explained in more detail in chapter 3.

1.1 The Icelandic Automotive Market

When working with data about the Icelandic automotive market, registrations are divided into different vehicle and usage segments. These segments are categorized by ICETRA (Samgöngustofa). The vehicle segments investigated in this paper are passenger cars (PC) and light commercial vehicles (LCV). Generally, registrations are classified into two different usage segments, public use, and car rental. Public use covers all types of registrations except for car rental registrations (Samgöngustofa, 2018). In this thesis, all car rental registrations will be excluded from prediction modelling for two main reasons. First, other factors drive the car rental market than the general market. Second, car rental vehicles are usually not as well equipped as cars intended for the public, meaning that dealerships do not order these vehicles without having a written agreement beforehand. Therefore, these cars do not usually cause inventory problems.

By reviewing total registrations of new automobiles back to 1990, the volatility of the market is apparent. This can be seen in Figure 1. With a closer look, one might detect that every time the market dwindles, the country is experiencing a recession of some sort. The most prominent low selling periods take place in 1993-95, 2001-02 and 2008-11. During these periods, Iceland was going through a recession which meant that fewer individuals were capable of renewing their cars. In fact, Einarsson et al. managed to identify over twenty instances of financial crises of different types in Iceland spanning the years 1875-2013 (Einarsson, Gunnlaugsson, Ólafsson, & Pétursson, 2015).
Although 2017 was a record-breaking year in total registrations, dealerships have often sold more cars to the public, as shown in Figure 2. Almost 40% of all registrations were devoted to car rentals in 2017. The landscape for Icelandic dealerships has therefore changed a lot in recent years with increased tourism. Back in 2010, Iceland started to get more and more attention in media all over the world following the eruption of Eyjafjallajökull, which led to a considerable increase in tourism a year later. The growth in the number of tourists was measured to be 15.7% between years in Iceland which was significantly higher than the average growth worldwide. This eruption established a growth period in tourism which has no parallel in Iceland’s history and made it the important industry that it is today (Bender, et al., 2016). This high tourism growth opened doors for a lot of businesses, especially in the car rental industry. Between 2003 and 2014, the number of issued licenses for car rental operations tripled from 51 to 151 licenses (Bender, et al., 2016). As shown in Figure 3, the ratio of car rental registrations used to fluctuate between 4-13% prior to 2008, while the ratio almost doubles in 2008 when public registrations plunge down to similar numbers to the 2001-02 crisis. The first few years following the eruption of Eyjafjallajökull, car rental registrations increased steadily as the car rental market got more competitive. Meanwhile, the economy was still in recovering mode, and public registrations only increased slightly year by year. As a result, the landscape of the automotive market has changed dramatically, and for the past eight years, car rental registrations have been around 40% of the total market. The ratio has however never been as high as in 2010 when 56% of all new registrations were to car rentals.
Figure 2: Total registrations of passenger cars and light commercial vehicles, excluding car rental vehicles.

Figure 3: Breakdown of new public (blue) and car rental (grey) registrations.
1.2 Background

The ideology of this thesis is to build six different models which can be used to forecast the Icelandic automotive market. The setup will start out with a simple benchmarking model and slowly increase the complexity of subsequent models. Two different fields of data processing are examined, traditional time series analysis and machine learning. The first four models belong to traditional time series analysis, while the last two belong to machine learning.

The first model will be based on the explicit method of simple moving average (SMA). Model two and three will be variations of exponential smoothing (ES); simple exponential smoothing (SES) and Holt-Winters exponential smoothing. In model four, exogenous variables are first introduced, and multiple linear regression (MLR) is implemented. For the last two models, machine learning algorithms called support vector regression (SVR) and random forest are applied using same exogenous variables as MLR. The first two methods, SMA and SES, will mostly be used as simple benchmarks in this thesis and are therefore not discussed in much detail in Results and Discussion.

Finally, seven different exogenous variables were gathered as predictors, all believed to possess some predictive ability of the automotive market. The variables chosen were the consumer price index (CPI), unemployment rate, purchasing power and consumer confidence index (CCI). There were three different foreign exchanges, ISK to US dollar, Euro and the Japanese Yen. These variables are analysed and explained in more detail in chapter 3.2.

1.3 Research Aim and Objectives

This thesis aims to find an effective way to predict the Icelandic automotive market with the help of historical data and quantitative methods. In Iceland, there are nine different car dealerships which all need to send regular sales forecasts to their manufacturers, whether they are on a monthly or a yearly basis. To estimate future sales of their brands, they first need to lay stress on the current situation of the market and its possible heading over the planning horizon. By only estimating sales of their brand/s, the message to the manufacturer/s may be unclear whereas market share can, for example, fall even though volume increases. More importantly, with a reliable estimation of future demand, Icelandic dealerships would be able to optimize their inventory level, avoiding problems such as under/overstocking. The methods generally used by dealerships to predict the Icelandic
market are qualitative methods, where predictions are mostly based on employees’ intuition and experience in the profession. In this paper, however, six different quantitative methods will be considered to predict the Icelandic automotive market; simple moving average, exponential smoothing, Holt-Winters exponential smoothing, multiple linear regression, support vector regression and random forest.

The following research questions were developed:

1. Do machine learning algorithms return smaller forecasting errors for the Icelandic automotive market compared to traditional time series analysis methods?
2. Which collection of data points, monthly or quarterly, is more suitable for Icelandic dealerships and returns lower forecasting errors?

Which included composing the following objectives:

- Study existing literature and state of the art forecasting methods
- Apply appropriate statistical methods to analyse the data
- Develop a model for each method that is considered
- Apply the models to real data and compare results
- Draw a conclusion about which method is considered most suitable for predicting the Icelandic automotive market

1.4 Limitations and Assumptions

All data processing is limited to the amount of data that could be obtained. Monthly registration numbers obtained from ICETRA did not go further back in time than 1990. For SMA, SES and Holt-Winters, the entire dataset can be used for modelling since only sales figures are needed as input. For MLR, SVR and random forest, however, exogenous variables are used as predictors, meaning that input data is limited by the variable that has the shortest available history. Three exogenous variables limit the dataset, they are; CCI, unemployment rate and foreign exchange of Euro to ISK. A complete list of variables and their time intervals are visible in Table 1.

The reliability and accuracy of the models implemented are largely dependent on the reliability of the dataset itself. With dirty data, i.e., inconsistency in the data obtained from the data sources, the performance of the models will be unreliable (Al-Mudimigh & Ullah,
The size of the dataset can also limit the reliability of the results. However, it differentiates between methods whether more data can improve model accuracy or not. For simple methods such as SMA, SES, and Holt-Winters, more data does not necessarily improve the model’s predictive power. While methods based on pattern recognition such as MLR, SVR, and random forest, usually improve their accuracy with more data since the relationship between the dependent and independent variable is studied in more detail and with more model parameters. However, when methods based on pattern recognition are implemented, one must be careful that the ratio between exogenous variables chosen and observed values in the dataset is not too high (Raudys & Jain, 1990). Random forest is an exception to that statement since one of its strengths is to be able to handle datasets which include more exogenous variables than number of observed values (Kouwayé, 2016).

Finally, in this thesis, a two-year forecast of the market is generated by using six different methods. Three of those methods, MLR, SVR and random forest use exogenous variables as predictors. In reality, these exogenous variables would have to be estimated, adding an underlying error to the model. For hypothetical purposes, the assumption is made that all exogenous variables are estimated correctly for the forecasting period.

1.5 Thesis Outline

This thesis starts with a short introduction of the Icelandic automotive market and its history, following the problem at hand. The six forecasting methods examined and applied in this thesis are introduced, along with the research objectives, limitations, and assumptions. Chapter 2 includes a literature review of numerous state of the art methods used for sales forecasting in recent years. In chapter 3, the dataset is introduced and analysed, in addition to the methods used for forecasting. Chapter 4 reveals results and comparison of the selected models. Chapter 5, contains discussions and speculations of the results, in addition to future work and the conclusion of the thesis.
Chapter 2

State of the Art

As far as the author knows, it is uncommon for employees in the Icelandic automotive industry to use quantitative methods to forecast the future market demand. That, however, has not been the case elsewhere. Automobile sales forecasting has received notable attention in the last three decades, and numerous different methods have been implemented. Recent articles about the subject mostly focus on various econometrics models and machine learning methods.

In 2009, Hülsman et al. built a time series model consisting of trend, seasonal, calendar and error additive components. The three latter components were estimated univariately while the trend component was estimated multivariately with multiple linear regression (MLR) and support vector regression (SVR). Macroeconomic and market-specific influences were chosen as predictors by a feature selection and a prediction made for the German automotive market. The data consisted of yearly, quarterly and monthly registrations of the years 1992 to 2007, in addition to ten influencing factors. Their results showed that SVR returned more accurate forecasts compared to MLR. Additionally, quarterly models were considered superior to monthly and yearly models even though yearly models returned slightly lower error estimates. They reasoned that the yearly dataset was too small, with limited information content. (Brühl, Hülsmann, Borscheid, Friedrich, & Reith, 2009).

Two years later, Hülsman et al. wrote another article presenting more enhanced models which increased both the accuracy as well as the explicability to some extent. In the latter instance, both the German and the American automotive market were considered. The methodology used in the second article mainly consisted of time series analysis and classical data mining algorithms which were used on macroeconomic and market-specific exogenous variables on a yearly, quarterly and monthly base. Six different methods were implemented; ordinary least squares, quantile regression, SVR, decision trees, k-nearest neighbour, and random forest. Results showed that the most suitable method considered was decision trees,
being both accurate as well as explicable (Hülsmann, Borscheid, Friedrich, & Reith, 2011).

In 2011, Yuan and Lee developed an SVR model to predict future sales of the Taiwan automotive market. Their data consisted of monthly car registrations from 2005 until 2009, in addition to numerous economic factors such as CPI, CCI, foreign exchanges and gasoline prices which were used as predictors. In their article, they put a lot of emphasis on the importance of optimizing the free parameters of SVR with genetic algorithms to improve model performance. To validate their model, they compared their method with three other methods; backpropagation neural network, least-mean square algorithm and SVR without genetic algorithms. Their results showed that SVR with genetic algorithms achieved the best forecasting accuracy of them all (Lee & Yuan, 2012). Despite those findings, mean absolute percentage errors (MAPE) of all models were somewhat high. This is most likely a consequent of limited data availability. According to Liu et al., one of SVR’s strengths is the ability to work with smaller training samples though the statement is not quantified (Liu, Yin, Gao, & Tan, 2008). Perhaps five years of monthly data is not enough. Another decision making, that caught the author’s attention, was the choice of not adding a seasonal factor to the model. Considering the distribution of sales in Taiwan, a seasonal factor might increase the accuracy of the model.

Wang, Chang, and Tzeng also developed a model to predict future sales of the Taiwan automotive market except with an adaptive network-based fuzzy inference system (ANFIS). Their model used several coincident indicators, leading indicators, wholesale price indices, independent indices and exchange rates as input variables where stepwise regression was applied to select the most influential ones. Their dataset consisted of monthly sales over a period of 78 months, where 60 months were used for training and 18 for testing. Results of ANFIS modelling was compared with two other models based on autoregressive integrated moving average (ARIMA) and artificial neural network (ANN). Three contributions were established in their study. First, that economic variables were considered good predictors of Taiwan’s automotive market. Second, that ANFIS modelling combined with stepwise regression outperformed many traditional methods, including ARIMA and ANN modelling. Third, that forecasting models can provide valuable insights of the market to manufacturers (Wang, Chang, & Tzeng, 2011).

In 2015, Yang and Li proposed a model based on Seasonal Index and radial basis function (RBF) neural network to forecast auto sales in China. By building a hybrid model, they managed to increase the accuracy of the forecasts compared to using single models. Their data consisted of monthly auto sales in China from January 2010 to April 2014 where the last 4 months were used to test the model. For the seasonal index method, the time series
was decomposed into long term trend, seasonal component, cyclical component and an irregular component to predict future sales. An RBF network consists of three layers; input layer, hidden layer, and an output layer. In their case, the input layer consisted of 12 neurons, representing auto sales of 12 months prior to the predicted value, in addition to three neurons in the hidden layer and one in the output layer. The activation function used for the hidden layer was a normalized radial basis function while an identity function was used for the output layer. Their results showed that the hybrid model outperformed both single models in accuracy, leading to the conclusion that hybrid models are well applicable for forecasting the automotive market (Yang & Li, The Combination Forecasting Model of Auto Sales Based on Seasonal Index and RBF Neural Network, 2016). A mentionable downside to their model is basing all predictions on historical values only. That way, the model would most likely overpredict future sales with the economy heading into a recession.

Based on recent literature acknowledged, most proposed automobile sales forecasting models use economic indicators in addition to historical sales to predict the future market demand. The reason for that ideology is not necessarily that those indicators have the highest correlation with the market but more because of their persistence. Factors such as number of adverts in play or active sales promotions going each time can impact the market quite heavily, but in the long-run, the impact of those factors usually fade out while economic indicators are considered better long-term predictors (Dekimpea, Hanssensb, & Silva-Rissocd, 1998).
Chapter 3

Methods

This chapter is divided into four parts. The first part describes the data in general as well as the data sources used. The second part addresses all data processing done before modelling for a better understanding of the data and its behavior. Finally, the last two parts describe the six methods chosen for modelling which are split up into traditional time series analysis and machine learning.

3.1 Data

The dataset used in this thesis includes historical registrations of new automobiles, as well as numerous economic indicators. The type of registrations under examination is limited to passenger and light commercial vehicles while excluding all car rental registrations as mentioned in 1.1. Since numerous different methods are implemented in this thesis, the ideology is to move slowly from simple traditional time series analysis methods over to more complex and state-of-the-art solutions. The first three methods only use historical sales as input, while the latter three use economic indicators in addition to historical sales to hopefully improve the accuracy of the models. Historical values of seven different indicators were gathered, all believed to possess some predictive ability of the automotive market. The indicators chosen were the consumer price index (CPI), unemployment rate, purchasing power and consumer confidence index (CCI). In addition to three different foreign exchanges, Icelandic Krona to US dollar, Euro and the Japenese Yen. Other indicators were considered as well, especially the gross domestic product (GDP). However, because GDP is only available on a yearly basis, it was not included.

The data obtained for this thesis was acquired from multiple sources. As mentioned in chapter 1, all registration data was acquired from ICETRA (Samgöngustofa). Historical values of CPI and purchasing power were obtained from Statistics Iceland (Hagstofa
The unemployment rate was gathered from the Directorate of labour (Vinnumálastofnun), CCI from GI rannsóknir ehf., and finally, all foreign exchanges from The Central Bank of Iceland (Seðlabanki Íslands). Definitions of exogenous variables are visible in Table 1, in addition to the available time interval.

<table>
<thead>
<tr>
<th>EXOGENOUS VARIABLE</th>
<th>TIME INTERVAL</th>
<th>VARIABLE DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSUMER PRICE INDEX</td>
<td>Jan. 1990 – Dec. 2017 (monthly values)</td>
<td>“The Consumer Price Index (CPI) is a measure that examines the weighted average of prices of a basket of consumer goods and services, such as transportation, food and medical care. It is calculated by taking price changes for each item in the predetermined basket of goods and averaging them” (Investopedia, 2018). CPI is measured monthly by Statistics Iceland as a 12-month percentage change.</td>
</tr>
<tr>
<td>UNEMPLOYMENT RATE</td>
<td>Feb. 2000 – Dec. 2017 (monthly values)</td>
<td>“Unemployment rate is the share of the labor force that is jobless, expressed as a percentage. When the economy is in poor shape and jobs are scarce, the unemployment rate can be expected to rise. When the economy is growing at a healthy rate and jobs are relatively plentiful, it can be expected to fall” (Investopedia, 2018). The unemployment rate is measured monthly by the Directorate of labour.</td>
</tr>
<tr>
<td>PURCHASING POWER</td>
<td>Jan. 1990 – Dec. 2017 (monthly values)</td>
<td>“Purchasing power is the value of a currency expressed in terms of the amount of goods or services that one unit of money can buy. Purchasing power is important because, all else being equal, inflation decreases the amount of goods or services you would be able to purchase” (Investopedia, 2018). Purchasing power is measured monthly by Statistics Iceland as a 12-month percentage change.</td>
</tr>
<tr>
<td>CONSUMER CONFIDENCE INDEX</td>
<td>March 2001 – Dec. 2017 (monthly values)</td>
<td>“The Consumer Confidence Index (CCI) survey is an index that measures how optimistic or pessimistic consumers are with respect to the economy in the near future. The Consumer Confidence Index (CCI) is based on the concept that if consumers are optimistic, they tend to purchase more goods and services. This increase in spending inevitably stimulates the whole economy” (Investopedia, 2018). CCI is measured monthly by GI rannsóknir ehf., on the scale 0-200, 100 being a neutral opinion.</td>
</tr>
<tr>
<td>ISK/EUR FOREIGN EXCHANGE</td>
<td>Jan. 1999 – Dec. 2017 (monthly values)</td>
<td>“Foreign exchange is the exchange of one currency for another or the conversion of one currency into another currency” (Investopedia, 2018). In this case, conversion of ISK to Euro, using the average monthly selling rate from The Central Bank of Iceland.</td>
</tr>
</tbody>
</table>
As can be seen in Table 1, some data discrepancy is present in the time intervals of the exogenous variables. Therefore, the data will be split into two different time intervals, 1990-2017 and 2001-2017. The second time interval is confined by three predictors, with historical records going back to January 1999, February 2000 and March 2001. Those predictors are the foreign exchange of ISK to Euro, unemployment rate, and the consumer confidence index. In the following chapter, the predictive power of those variables will reveal why it is worth sacrificing 11 years of historical registrations.

After having obtained the data from the various sources, the dataset was split into two different comma separated values (CSV) Excel files, each consisting of separate time intervals (monthly, quarterly). All data processing was carried out in the open source software R.

### 3.2 Data Analysis

More attention is given to the time interval 2001-2017 in this thesis because of the focus of comparing machine learning to traditional time series analysis. Because SMA, SES and Holt-Winters only need historical sales as input, those methods will be implemented over both time intervals while MLR, SVR and random forest are only tested on the latter time interval.

To validate the methods used in this thesis, the two time intervals mentioned above are split into training and test periods. Generally, the optimal ratio between the training and test period is close to 2/3 towards training and 1/3 towards testing. That ratio differs however between datasets since it is dependent on various factors such as the total number of observations and the complexity of the data. Smaller datasets for example, usually need a larger proportion for training to reach optimal results (Dobbin & Simon, 2011). Due to

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISK/USD FOREIGN EXCHANGE</td>
<td>Jan. 1990 – Dec. 2017 (monthly values)</td>
<td>“Foreign exchange is the exchange of one currency for another or the conversion of one currency into another currency” (Investopedia, 2018). In this case, conversion of ISK to US dollar, using the average monthly selling rate from The Central Bank of Iceland.</td>
</tr>
<tr>
<td>ISK/JPY FOREIGN EXCHANGE</td>
<td>Jan. 1990 – Dec. 2017 (monthly values)</td>
<td>“Foreign exchange is the exchange of one currency for another or the conversion of one currency into another currency” (Investopedia, 2018). In this case, conversion of ISK to Japanese Yen, using the average monthly selling rate from The Central Bank of Iceland.</td>
</tr>
</tbody>
</table>
restrictions of the three predictors previously mentioned and the size of the dataset for the second time interval, the ratio will be 87.5% towards training and 12.5% towards testing.

3.2.1 Graphical Analysis

Graphical analysis should always be one of the first steps to go to when analysing data. By looking at the data visually, outliers become more easily detected as well as trends and seasonalities, if present. Therefore, graphical analysis can quickly reveal valuable insights of the dataset. The very first analysis done in R was to perform a decomposition where the data series was split up into three categories; random, seasonal and trend. The trend component is determined first by subtracting a 12-month moving average from the time series. Next, the seasonal component is computed by dividing the time series with the 12-month moving average, which is then averaged over all identical seasonal periods. Finally, the noise in the data is determined by solving for Random in the decomposition formula (Equation 1). This was done by taking advantage of the package “stats” in R (R Core Team, 2017). Results of a classical seasonal decomposition by moving averages can be seen in Figure 4. This is the only pre-modelling analysis done on the whole dataset since most data processing was performed for the latter time interval, 2001-2017. Figure 4 reveals that seasonality is undoubtedly present in the data, sales being higher over the summer months.

\[ \text{Time series} = \text{Trend} \times \text{Seasonal} \times \text{Random} \]  

(1)

Figure 4: Decomposition of multiplicative time series where monthly sales are split into three factors; random (ratio), seasonal (ratio) and trend (no. of registrations)
### 3.2.2 Correlation, Time Lag, and Logarithmic Transformation

Correlation is a measurement that indicates how strongly a pair of variables interrelate. It is a dimensionless quantity that can be useful when comparing the linear relationship between pairs of variables in different units (Montgomery & Runger, 2002). The correlation was used to compare the linear relationship between the automotive market and the chosen exogenous variables, in addition to the relationship between the variables themselves. The Pearson correlation coefficient is defined as:

\[
p = \frac{\text{cov}(X,Y)}{\sqrt{\text{var}(X)\text{var}(Y)}} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}
\]  
(2)

where X and Y are random variables, \(\sigma_{XY}\) the covariance of the random variables, \(\sigma_X\) the standard deviation of X and \(\sigma_Y\) the standard deviation of Y.

Because exogenous variables used for modelling are not believed to have an immediate impact on the automotive market, a time-lagged grid search was performed on the training set. Every exogenous variable was therefore lagged up to the maximum of 10 months to find the highest correlation with the market. The results of the grid search can be seen in Table 2.

<table>
<thead>
<tr>
<th>EXOGENOUS VARIABLE</th>
<th>MONTHLY TIME LAG</th>
<th>CORRELATION</th>
<th>QUARTERLY TIME LAG</th>
<th>CORRELATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>10</td>
<td>-0.45</td>
<td>4</td>
<td>-0.48</td>
</tr>
<tr>
<td>UNEMPLOYMENT RATE</td>
<td>0</td>
<td>-0.72</td>
<td>0</td>
<td>-0.74</td>
</tr>
<tr>
<td>PURCHASING POWER</td>
<td>9</td>
<td>0.49</td>
<td>3</td>
<td>0.53</td>
</tr>
<tr>
<td>CCI</td>
<td>4</td>
<td>0.82</td>
<td>1</td>
<td>0.85</td>
</tr>
<tr>
<td>FX ISK-EUR</td>
<td>0</td>
<td>-0.77</td>
<td>0</td>
<td>-0.79</td>
</tr>
<tr>
<td>FX ISK-USD</td>
<td>0</td>
<td>-0.86</td>
<td>0</td>
<td>-0.89</td>
</tr>
<tr>
<td>FX ISK-JPY</td>
<td>0</td>
<td>-0.79</td>
<td>0</td>
<td>-0.82</td>
</tr>
</tbody>
</table>

Table 2: Grid search results where highest correlation is detected between dependent and independent variables

Similar results were obtained after running a grid search on both monthly and quarterly data, that is, a 10-month lag of CPI correlates to a 4-quarter delay of CPI et cetera. These variables are likely to add value to the model since a relatively high correlation is achieved for at least five variables (unemployment rate, CCI, FX ISK-EUR, FX ISK-USD, FX ISK-JPY). It differentiates between factors whether the correlation is positive or negative. Two of them have a positive correlation (purchasing power, CCI), while the other five have
a negative one. A negative correlation with the automotive market suggests that when a factor such as unemployment rate goes up, the size of the market decreases. A positive correlation has the contrary effect, so when CCI goes up, meaning the public is optimistic for the coming months, it is likely that car dealerships will benefit. Finally, this relatively high correlation for all foreign exchanges can be explained with vehicle price changes. When the Icelandic Krona either weakens or strengthens, the purchasing price in ISK changes accordingly since all cars are imported.

Having implemented a time-lagged grid search on the exogenous variables, the correlation between the chosen variables and the market are fairly high for most variables. When a linear relationship between variables is not present, there are methods available to increase linearity, such as performing a logarithmic transformation on the data. Such transformations are often employed to stabilize the variance of a series while decreasing the likelihood of heteroscedasticity (Lütkepohl & Xu, 2012). After examining every dependent and independent variable with and without a logarithmic transformation. It was decided to perform a logarithmic transformation on the dependent variable, as well as on all independent variables except purchasing power since that indicator is not strictly positive. In Figure 5, a correlation matrix after a logarithmic transformation can be seen, including correlation coefficients and distributions of the variables. The figure was generated by using the R package “psych” (Revelle, 2017). An identical figure for quarterly data can be found in the Appendix.

Figure 5: Correlation matrix of monthly independent variables (2002-2015) including their distributions
The most notable result in Figure 5, is that correlation between the market and four independent variables (CPI, unemployment, purchasing power, CCI) increase with the logarithmic transformation. Meanwhile, it becomes clear that it does not add value to use all three foreign exchanges since strong dependency exists between those variables.

### 3.2.3 Multicollinearity

When building a model with multiple exogenous variables, one needs to be careful that multicollinearity is not present in the model. Multicollinearity exists in a model where a strong dependency is present among the regressor variables. Dependencies among the regressor variables can have severe effects on the regression coefficients, as well as the relevancy of the model (Montgomery & Runger, 2002).

To determine whether multicollinearity is present, it is possible to use the variance inflation factor (VIF), a measure that can detect multicollinearity. If the VIF coefficient exceeds 1, some level of multicollinearity is detected in the model though it is not considered to be a problem until it exceeds the value of 10 (Montgomery & Runger, 2002). VIF is defined as:

\[
VIF(\beta_j) = \frac{1}{(1-R_j^2)} \quad j = 1,2,...,k
\]  

where \( \beta_j \) represents the expected change in response Y per unit change in \( x_j \), when all remaining regressors \( x_i(i \neq j) \) are held constant. \( R_j^2 \) is the coefficient of multiple determination from regressing \( x_j \) on the other \( k-1 \) independent variables (Montgomery & Runger, 2002).

The independent variables chosen for modelling were all tested for multicollinearity using the package “faraway” in R (Faraway, 2016), which calculates the VIF coefficient. Multicollinearity was detected for all foreign exchanges with VIF being close to 10 or above. Not surprising results, since all of them are dependent on the Icelandic Krona. To eliminate this multicollinearity in the model, only the foreign exchange of Euro to ISK was used for modelling since the majority of Icelandic dealerships pay their manufacturer in Euros. By eliminating those two variables from the model, every independent variable except for the CCI had a VIF of 4 or lower, VIF of CCI being just over 5.
3.2.4 Variable Selection

An important part of modelling lies in selecting the right regressor variables to use in the model. The objective of variable selection is to find a combination of regressors that will increase the accuracy of a model while minimizing the number of regressor variables needed. One of the most widely used techniques of variable selection is called stepwise regression. A method where a sequence of regression models is performed with independent variables either being added or removed after each regression to obtain the best possible combination of regressors (Montgomery & Runger, 2002). This method was implemented by using the “MASS” package in R (Venables & Ripley, 2002). Results of the stepwise regression were that the CPI, unemployment rate, purchasing power, CCI and the foreign exchange of Euro to ISK, should all be included in the model.

3.2.5 Validation Methods

According to Chai and Draxler, a combination of metrics is often required to assess all aspects of model performance (Chai & Draxler, 2014). Therefore, the following methods are used to validate the results; root-mean-squared error (RMSE), mean absolute error (MAE), maximum error and mean absolute percentage error (MAPE). RMSE is considered a useful method when comparing different methods on the same dataset since it is on the same scale as the data. The method’s weakness, however, is being more sensitive to outliers compared to other validation methods, such as MAE. The maximum prediction error is monitored to easily spot and compare possible weaknesses in the model. Finally, MAPE is calculated because of its advantage of being scale-independent. Because of that feature, it has been a frequent method to compare forecast performances across different datasets. The four measures are defined as:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}}
\]

(4)

\[
MAE = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{n}
\]

(5)

\[
Max\ Error = \max(|y_i - \hat{y}_i|)
\]

(6)

\[
MAPE = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{\sum_{i=1}^{n} |y_i|} * 100
\]

(7)

where \(y\) is the observed value, \(\hat{y}\) the predicted value and \(n\) the number of observations.
3.3 Traditional Time Series Analysis

In traditional time series analysis, interpretable models are built by making assumptions about the model and data in question. By analysing historical data, certain trends, seasonalties or other factors, can be found that describe the behavior of the data. Those findings can then be used to build a model used for information or prediction purposes. Three traditional methods are implemented in this thesis; simple moving average, exponential smoothing and multiple linear regression. In the following subchapters, the theory behind each method is briefly explained.

3.3.1 Simple Moving Average

In time series analysis, the simple moving average method is considered one of the most straightforward methods available to use for forecasting. A simple moving average is an equally weighted average of the previous n data points in a time series (Hansun, 2013). SMA is defined as:

\[ SMA = \frac{P_M + P_{M-1} + \cdots + P_{M-(n-1)}}{n} \]  

(8)

where \( P_M \) stands for the data point at time M and n stands for the number of data points used in the calculation.

3.3.2 Exponential Smoothing

In 1956, simple exponential smoothing was first suggested in literature by Robert Goodell Brown (Brown, 1956). In 1959, Brown’s theory was expanded by Charles C. Holt (Holt, 2004) and finally a year later, Robert Winters, a student of Holt’s, enhanced his method for capturing seasonality which is called the Holt-Winters method (Winters, 1960). Simple exponential smoothing and Holt-Winters were implemented on both time series intervals and are addressed in the next two subchapters.

3.3.2.1 Simple Exponential Smoothing

Simple exponential smoothing is a method that uses historical sales to project future demand. Projecting future demand by using historical sales does obviously not always apply, such as when a new model arrives, or a radical change is made to a previous one (Brown, 1956). When projecting demand for a relatively stable market, however, the results are usually good. The formula for basic exponential smoothing is defined as:
\[ \tilde{S}_t = \alpha S_t + (1 - \alpha) \tilde{S}_{t-1} \quad 0 \leq \alpha \leq 1 \]  \hspace{1cm} (9)

where \( S_t \) stands for actual sales during time period \( t \), \( \tilde{S}_t \) for forecast of expected sales in time period \( t \) and \( \alpha \) for the smoothing factor. A small value of \( \alpha \) means that older values are weighted more heavily, while values near 1 give the latest values more weight. For implementation, the package “stats” in R was used (R Core Team, 2017).

### 3.3.2.2 Holt-Winters

For products with stable sales and small seasonal fluctuations, the simple exponential smoothing method proves quite satisfactory. Many products, however, experience trends in sales when they are first introduced or when competitors upgrade their product range, making trend a valuable factor for a model. Another valuable factor for some products is seasonality, whereby some products only sell well during a particular time of year. For this reason, Charles Holt and Robert Winters extended Brown’s theory of simple exponential smoothing by considering long-run trends and seasonal effects. It is possible to develop a Holt-Winters forecasting model with either a multiplicative or an additive seasonal effect (Holt, 2004) (Winters, 1960). In this thesis, the multiplicative model is chosen since the time series of automobile registrations possesses a seasonal pattern which is proportional to the level of sales instead of being a definite number. The model for multiplicative Holt-Winters is defined as:

The forecast of sales for \( T \) periods in the future would be obtained by:

\[ S_{t,T} = (\tilde{S}_t + TR_t) F_{t-L+T} \]  \hspace{1cm} (10)

Where \( \tilde{S}_t, F_t \) and \( R_t \) are given by:

\[ \tilde{S}_t = \alpha \frac{S_t}{F_{t-L}} + (1 - \alpha)(\tilde{S}_{t-1} + R_{t-1}) \]  \hspace{1cm} (11)

\[ F_t = \gamma \frac{S_t}{\tilde{S}_t} + (1 - \gamma) F_{t-L} \]  \hspace{1cm} (12)

\[ R_t = \beta (\tilde{S}_t - \tilde{S}_{t-1}) + (1 - \beta) R_{t-1} \]  \hspace{1cm} (13)

where actual sales in period \( t \) is given by \( S_t \). The estimate of the smoothed and seasonally adjusted sales in period \( t \) is given by \( \tilde{S}_t \), using \( L \) periods of seasonal effect (12 months or 4 quarters in this thesis). \( F_t \) is the estimate of the seasonal factor for period \( t \) and \( R_t \) the estimate of the trend. Finally, the mean squared error (MSE) is minimized by optimizing the weights \( \alpha, \beta, \gamma \), which function in the same way as \( \alpha \) does in SMA, except \( \beta \) denotes to trend and \( \gamma \) to seasonality. This method was implemented in R by using the package “stats” where starting
values for $\hat{S}_t, F_t$ and $R_t$ are inferred by performing a decomposition in trend and seasonal component for the first two years using moving averages (R Core Team, 2017).

### 3.3.3 Multiple Linear Regression

Multiple regression analysis is one of the most widely used statistical procedures for both scholarly and business research today because of its applicability to varied problems and ease of interpretation. Regression analysis is used primarily for two purposes, either building forecasting models or drawing conclusions about individual predictor variables and their relationship with the dependent variable (Mason & Perreault, 1991). Regression analysis generates an equation that describes the relationship between independent variables and the dependent variable by minimizing the sum of squared error (SSE). This generated equation includes the intercept of the regression line added to the multiplication of the regression coefficients with the independent variables. Therefore, the regression coefficients can be interpreted as the change in the mean of the dependent variable for a unit change in the independent variable (Montgomery & Runger, 2002).

As concluded in chapter 3.2, the five exogenous variables selected for modelling are CPI, unemployment rate, purchasing power, CCI and the foreign exchange of Euro to ISK. Also noted in 3.2 and shown in Figure 4, is that seasonality is undoubtedly present in the sales of new automobiles in Iceland. Therefore, seasonal dummy variables are added to the model to improve prediction accuracy. At last, because both monthly and quarterly forecasts are implemented, two multiple regression models were formulated in the following manner:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \gamma_1 m_1 + \gamma_2 m_2 + \cdots + \gamma_{11} m_{11} + \varepsilon \quad (14)$$

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \gamma_1 q_1 + \gamma_2 q_2 + \gamma_3 q_3 + \varepsilon \quad (15)$$

where $\hat{y}$ is the estimated value of $y$ (the observed value), $\beta_0$ the estimated intercept of the regression line, $\beta_1 - \beta_5$ the estimated regressor coefficients, $\gamma_1 - \gamma_{11}$ the estimated seasonal regressor coefficients, $m_1 - m_{11}$ the monthly seasonal factors, $q_1 - q_3$ the quarterly seasonal factors and $\varepsilon$ the error term. The dummy variables $m$ and $q$ are binary variables with the value of 1 if present and 0 otherwise. The reason for only using dummy variables for 11 months and 3 quarters in the formulas is due to the dummy variable trap (Grotenhuis & Thijs, 2015). This method was implemented in R by using the package “stats” (R Core Team, 2017).
3.4 Machine Learning

While traditional time series analysis methods rely on rule-based programming, machine learning is based on algorithms that can learn from data. As processing power of computers increased in the late 1990s, data scientists stopped building finished models and began training computers to develop their own models based on techniques formulated in the 1930s and 1940s such as neural networks. The difference between these machine learning algorithms and statistical methods is that machine learning is unconstrained by the preset assumptions of statistics. As a result, machine learning algorithms can yield insights into data that human analysts are not able to detect and make predictions with a higher degree of accuracy. What these algorithms do extraordinarily well, is creating rules by repeatedly going through any amount of data while trying out every combination of variables to fit a model or a behavior (Pyle & José, 2015).

Two machine learning algorithms, support vector regression and random forest, are implemented in this thesis. The main reason for choosing SVR is due to the algorithm’s good generalization abilities (Xu & Suzuki, 2011) (Juang & Hsieh, 2009) (Bi, Tsimhoni, & Liu, 2011). Applications of this algorithm are utilized in various industries and can be applied for both classification and regression. It has for example been used to forecast CPI in China (Wang, Wang, & Zhang, 2012), the stock prices of several global companies (Trafalis & Ince, 2000) and travel time in Taiwan (Wu, Ho, & Lee, 2004). The reason for choosing random forest is mainly because of its robust ability to deal with outliers and noisy data (Guo, Ma, Cukic, & Singh, 2004). Next two subchapters go briefly through the theory of SVR and random forest.

3.4.1 Support Vector Regression

Support vector machine (SVM) is an algorithm developed by Vladimir Vapnik and his colleagues at the AT&T Bell laboratories in 1995. Originally, SVM was intended for classification problems, such as face and text identification, but soon they found ways to apply the method to regression and time series prediction problems as well (Adhikari & Agrawal, 2013). The method of support vector regression was first published in 1996 as a new regression technique based on Vapnik’s concept of support vectors (Drucker, Burges, Kaufman, Smola, & Vapnik, 1996). The objective of SVR was well explained by Smola and Schölkopf back in 2003: “our goal is to find a function f(x) that has at most ε deviation from the actually obtained targets y_i for all the training data, and at the same time is as flat as possible. In other words, we do not care about errors as long as they are less than ε, but will
CHAPTER 3: METHODS

Applications of support vector regression have been popular for regression problems for the reasons of needing smaller training samples, having good generalization abilities and being powerful when working with non-linear relationships (Liu, Yin, Gao, & Tan, 2008). This ability to model complex non-linear relationships is thanks to the assistance of kernel functions. Kernel functions can transform an input space with non-linear relationships, over to a higher dimensional space where the former non-linear relationship becomes linear (Üstün, Melssen, & Buydens, 2006). Four kernel functions were examined for modelling: linear, polynomial, radial basis and sigmoid. The kernel function returning the best results and chosen for modelling was the radial basis function. The principle of SVR is described below:

Input variables \( x \) are mapped into a higher dimensional feature space with the non-linear mapping \( \phi \). This mapping translates a low dimensional non-linear problem into a linear high dimensional problem with the help of a kernel function. With this mapping, the regression function can be defined as:

\[
    f(x) = w \phi(x) + b
\]

where \( w \) is a weight vector and \( b \) is a bias term. Instead of minimizing the sum of squared error as in traditional regression, SVR minimizes structural risk which helps the algorithm to generalize better. Structural risk minimization involves minimizing empirical risk as well as the VC (Vapnik-Chervonenkis) dimension, a measure of the model’s complexity (Basak, Pal, & Patranabis, 2007). Structural risk minimization can be defined as:

\[
    \min \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{N} (\xi_i + \xi_i^*)
\]

s. t. \((w \phi(x_i) + b) - y_i \leq \varepsilon + \xi_i \; \forall i = 1, 2, \ldots, N\)

\((y_i - (w \phi(x_i) + b) \leq \varepsilon + \xi_i^* \; \forall i = 1, 2, \ldots, N\)

\(\xi_i \geq 0; \forall i = 1, 2, \ldots, N\)

\(\xi_i^* \geq 0; \forall i = 1, 2, \ldots, N\)

where \( C \) is a cost function measuring empirical risk, \( \xi_i \) and \( \xi_i^* \) are slack variables and \( \varepsilon \) is the tolerance error. Because of the mapping \( \phi \) into a higher dimensional feature space, it is simpler to solve the optimization problem above in its Lagrange dual formulation (Maldonado & Weber, 2010), which can be written as:

\[
    \min \frac{1}{2} \sum_{i,j=1}^{N} (\alpha_i^* - \alpha_i)(\alpha_j^* - \alpha_j)K(x_i, x_j) + \varepsilon \sum_{i=1}^{N} (\alpha_i^* - \alpha_i) - \sum_{i=1}^{N} y_i(\alpha_i^* - \alpha_i)
\]
\[ s.t. \sum_{i=1}^{N} (\alpha_i^* - \alpha_i) = 0 \]
\[ 0 \leq \alpha_i \leq C; \forall i = 1, 2, ..., N \]
\[ 0 \leq \alpha_i^* \leq C; \forall i = 1, 2, ..., N \]

where \( K(x_i, x_j) \) is the kernel function and \( \alpha_i^*, \alpha_i \) are Lagrange multipliers. Only nonzero coefficients of \( \alpha_i^*, \alpha_i \) contribute to the regression model where the corresponding input vectors \( x_i \) are called support vectors. These support vectors along with the corresponding Lagrange multipliers give the values of the weight vector (Bi, Tsimhoni, & Liu, 2011):

\[ w = \sum_{i=1}^{N} (\alpha_i^* - \alpha_i) x_i \]  

(19)

Finally, the following function can be used to predict new values depending only on the support vectors:

\[ f(x) = \sum_{i=1}^{N} (\alpha_i^* - \alpha_i) \varphi(x_i) \varphi(x) + b \]  

(20)

This method was implemented in R by using the package “e1071” which applies the dual representation of the model (Meyer, et al., 2017). Radial basis function was chosen as the kernel where \( \gamma \) defines how far the influence of a single training example reaches. Radial basis function is defined as:

\[ K(x_i, x_j) = \exp(-\gamma \|x_i - x_j\|^2); \gamma > 0 \]  

(21)

An important factor in applying SVR is to choose appropriate tuning parameters of \( \varepsilon, C, \) and \( \gamma \) for each dataset. SVR is very sensitive to the choice of parameters, and small deviations in parameter selection can cause significant differences in performance. A common way of resolving this issue is to run a grid search for various combinations of the parameters on the training dataset with the goal of maximizing cross-validation accuracy (Hsu, Chang, & Lin, 2016). This approach was implemented by using the package “e1071” with 10-fold cross-validation. Brief definitions of the parameters and their possible impact on the model can be seen below:

- **Parameter of loss function (\( \varepsilon \))**: The approximation accuracy placed on the training data points. As \( \varepsilon \) gets bigger, the precision on the training data points decreases.
- **Regularization parameter (\( C \))**: Determines the trade-off between the flatness of the function \( f \) and the amount up to which deviations larger than \( \varepsilon \) are tolerated. When \( C \) is too high or low, the generalization ability of the model becomes weaker.
- **Gamma (\( \gamma \))**: Defines how far the influence of a single training example reaches. With
a high value of $\gamma$, the points closest to the decision boundary can carry a lot of weight on the boundary. Meanwhile, with a low value of $\gamma$, the decision boundary is influenced by more points, leading to a more linear boundary.

### 3.4.2 Random Forest

The originator of the random forest algorithm is Tim Kam Ho. He proposed a method to build multiple decision trees using the random subspace method and called it Decision forests (Ho, 1998). Contrary to the methodology of traditional decision trees where the best split is chosen at each node among all features. The random subspace method randomly selects only a subset of components from the feature vector and chooses a best split among the selected features. By randomly selecting only a number of features and combining results from multiple trees, Ho was able to increase the generalization accuracy of the decision trees algorithm while preserving the accuracy on the training data (Ho, 1998). Later, Leo Breiman extended Ho’s algorithm by adding his idea of bagging to the method and named it Random forests (Breiman, Random Forests, 2001). In his paper, he describes his method of building a forest of uncorrelated trees to the maximum size using CART methodology (Breiman, Friedman, Olshen, & Stone, 1984) by implementing bagging with replacement and Ho’s random subspace method on the dataset. Since Breiman published his paper about Random forest, it has been widely used across numerous industries, for both classification and regression purposes. One of the strengths of random forest is its power to handle large datasets with high dimensionalities. The algorithm can handle thousands of input variables and detect which of the variables are considered better predictors than others. Another strength of random forest is its ability to generalize. Leo Breiman states in his paper that because of the law of large numbers, the algorithm should not overfit data (Breiman, Random Forests, 2001). The downside of random forest, however, is its interpretability. Despite the mechanism of the procedure appearing simple, its mathematical properties remain largely unknown and for that reason, it is often referred to as a black box method (Palczewska, Palczewski, Robinson, & Neagu, 2013). The procedure behind the algorithm is as follows:

1. Assuming a training set of size N with M features. A bootstrap sample is randomly taken from N with replacement to grow n number of trees.
2. At each node, m number of features are selected at random from M, with $m < M$. The best split on these m is used to split the node, m being a constant while growing the forest.
3. Each tree is grown to the largest extent possible without pruning, with a minimum size of terminal nodes k.

4. A new value is predicted by averaging all predictions over the n trees.

In this thesis, the random forest algorithm was implemented by using the package “randomForest” in R (Liaw & Wiener, 2002). All relevant parameters can be tuned in the package, such as the number of trees grown (n), number of variables randomly sampled at each split (m) and the minimum size of terminal nodes (k). The parameters were adjusted by trial and error.
Chapter 4

Results

This chapter covers the results of all implemented models. The chapter is split into four subchapters. First, a comparison between SMA, SES, and Holt-Winters on the whole dataset is examined, for monthly and quarterly models. Next, a comparison is made between all models for the latter time interval where exogenous variables are used for three of the methods. Third, models returning the best results are explored in more detail to evaluate the goodness of fit. Same exogenous variables and time lags are used when comparing MLR and SVR models in 4.3.2 - 4.3.3 and 4.3.5 - 4.3.6. Meanwhile, a grid search procedure is performed on time lags of the variables in 4.3.7 to minimize the training error of the models. Finally, the best method is chosen to predict yearly values in chapter 4.4.

4.1 Modelling without Exogenous Variables

Three methods were implemented on the whole dataset of 1990-2017, SMA, SES and Holt-Winters. Monthly test results can be seen in Table 3 for the three methods noted. The year 1990 was excluded from error estimation for all models since it differentiates between methods how many observations are needed for evaluation purposes before generating the first prediction. Results of SMA and SES are not surprising since the forecast levels out rather quickly and predicts the same value for most of the 24 months forecasted. Holt-Winters however, returns relatively good results considering the simplicity of the model and the volatility of the market. Optimal parameters selected for $\alpha$, $\beta$ and $\gamma$ indicate that a bit more emphasis is imposed on the latest values for level and seasonality while small emphasis is imposed on the latest trend factor.
Quarterly test results can be seen in Table 4. Again, the Holt-Winters model returns the best results among the three methods. Because RMSE, MAE and maximum error are validation methods that are scale-dependent, it is not valid to compare those measures between the monthly and quarterly models. As mentioned in 3.2.5 however, MAPE is a scale-independent measure that can be used to compare performances of models using different datasets. By comparing MAPE results between Table 3 and Table 4, it is interesting to see that forecasting error goes down for all methods by changing the time interval from monthly to quarterly. At the same time, it can be argued whether the decrease in accuracy is significant enough to sacrifice the detail of the monthly models. Another noticeable change between the monthly and quarterly models is the selections of smoothing variables which minimize the MSE in Holt-Winters. The trend of the series is still mostly based on older values, but the majority of level and seasonality is now based on the latest value. In fact, with γ of 1, no weight at all is put on older values when evaluating the seasonality factor.
4.2 Modelling with Exogenous Variables

For the time interval of 2001-2017, all methods were implemented. Monthly test results can be seen in Table 5. Again, the first year was excluded from error estimation for all models due to same reasons as noted in 4.1, in addition to the time-lag procedure mentioned in 3.2.2. The model returning the best monthly results for the second time interval is MLR, though Holt-Winters follows close behind with almost identical error measures of RMSE and MAE. It is intriguing that the forecasting error between the two models does not deviate more since Holt-Winters only bases its forecast on historical sales while MLR uses exogenous variables as well. This difference is examined more closely in chapter 4.3.2, where a simple example verifies the importance of adding exogenous variables to the model for certain cases. The test results of the SVR model are disappointing but understandable at the same time since the accuracy of machine learning algorithms usually improves with more data. In 4.3.1, the SVR model is examined in more detail, including parameter selection and possible overfitting. Random forest returns very similar results to SVR and would most likely be improved as well with more data.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAX ERROR</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMA</td>
<td>Window size = 6</td>
<td>436</td>
<td>366</td>
<td>880</td>
<td>30,4%</td>
</tr>
<tr>
<td>SES</td>
<td>$\alpha = 0,844$</td>
<td>569</td>
<td>517</td>
<td>1032</td>
<td>44,7%</td>
</tr>
<tr>
<td>HOLT-WINTERS</td>
<td>$\alpha = 0,708$</td>
<td>198</td>
<td>163</td>
<td>421</td>
<td>15,1%</td>
</tr>
<tr>
<td></td>
<td>$\beta = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma = 0,594$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MLR</td>
<td></td>
<td>195</td>
<td>161</td>
<td>398</td>
<td>14,7%</td>
</tr>
<tr>
<td>SVR</td>
<td>Kernel = RBF $\epsilon = 0,1$</td>
<td>335</td>
<td>260</td>
<td>791</td>
<td>22,0%</td>
</tr>
<tr>
<td></td>
<td>$\gamma = 0,04$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C = 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RANDOM FOREST</td>
<td>$n = 120$</td>
<td>314</td>
<td>259</td>
<td>634</td>
<td>21,9%</td>
</tr>
<tr>
<td></td>
<td>$m = 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$k = 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Results of monthly models using historical sales and exogenous variables as input

Quarterly test results for time interval two can be seen in Table 6. This time, the forecasting error for the MLR model is considerably lower than for any other model. The second-best results are returned by SVR while random forest only outperforms the two benchmarking methods. By comparing MAPE between monthly and quarterly models, the MAPE of quarterly models are again markedly lower for all models except for Holt-Winters.
which increases its forecasting error slightly. It is safe to say that quarterly models return more accurate results over monthly models since 8 out of 9 models have a lower MAPE for quarterly data.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAX ERROR</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMA</td>
<td>Window size = 2</td>
<td>1182</td>
<td>1014</td>
<td>2056</td>
<td>28,5%</td>
</tr>
<tr>
<td>SES</td>
<td>$\alpha = 0.943$</td>
<td>1297</td>
<td>1152</td>
<td>2190</td>
<td>32.9%</td>
</tr>
<tr>
<td>HOLT-WINTERS</td>
<td>$\alpha = 0.852$</td>
<td>659</td>
<td>558</td>
<td>1041</td>
<td>16.3%</td>
</tr>
<tr>
<td></td>
<td>$\beta = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma = 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MLR</td>
<td></td>
<td>411</td>
<td>332</td>
<td>708</td>
<td>10.5%</td>
</tr>
<tr>
<td>SVR</td>
<td>Kernel = RBF</td>
<td>681</td>
<td>559</td>
<td>1282</td>
<td>15.7%</td>
</tr>
<tr>
<td></td>
<td>$\epsilon = 0.1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma = 0.04$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C = 4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RANDOM FOREST</td>
<td>$n = 40$</td>
<td>770</td>
<td>648</td>
<td>1411</td>
<td>18.5%</td>
</tr>
<tr>
<td></td>
<td>$m = 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$k = 5$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Results of quarterly models using historical sales and exogenous variables as input

4.3 Model Comparison

In the following subchapters, monthly and quarterly models are analysed in more detail for the best overall methods. Training period error estimates will be addressed for each model, and the residuals examined to validate the goodness of fit.

4.3.1 Monthly Holt-Winters

Holt-Winters is one of three methods which were implemented on monthly data for both time intervals. Because the model for time interval one returned better results, that model is examined better. In Table 7, results of RMSE and MAPE can be seen for both the training and test period.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOLT-WINTERS</td>
<td>$\alpha = 0.665$</td>
<td>206</td>
<td>178</td>
</tr>
<tr>
<td></td>
<td>$\beta = 0.009$</td>
<td>18.3%</td>
<td>13.1%</td>
</tr>
<tr>
<td></td>
<td>$\gamma = 0.526$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Results of a monthly Holt-Winters model for time interval 1991-2017
CHAPTER 4: RESULTS

The error for the test period is considerably lower than for the training period which is mostly believed to be caused by the evaluation of the seasonal factor. If the number of cars sold in a single month is unusually high one year, that same month will likely be poorly fit the following year since the model will generate a high seasonal factor. This can be observed by analysing Figure 6.

The model does rather well on the test dataset (excluding the summer of 2016) since the factors of level, trend and seasonality are quite stable in the years before and throughout the forecasting period. The weakness of forecasting with Holt-Winters however, is that it does not do well when circumstances change. This is due to the setup of the forecasting function, i.e., using fixed values for level, trend, and seasonality throughout the forecasting period. In the case of predicting sales for the years of 2016-17, this drawback does not affect the forecast, but an example will be shown in 4.3.2 where it does.

Finally, the validity of the model was analysed graphically with residual plots (Figure 7), and no unwanted trends were spotted.

![Holt-Winters](image)

**Figure 6:** Results of a monthly Holt-Winters model including a two-year forecast
4.3.2 Monthly MLR

Training and test error estimates for the monthly MLR model can be seen in Table 8. The training error is somewhat higher than the test error, but by analysing Figure 8, the model does better following the financial crisis in 2008. Two eye-catching peaks in sales in 2004 and 2006 skew the training error of the model significantly. No underlying reasons were found explaining those peaks but dealership campaigns and high discounts are plausible.

<table>
<thead>
<tr>
<th>TRAING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>METHOD</td>
<td>RMSE</td>
</tr>
<tr>
<td>MLR</td>
<td>214</td>
</tr>
</tbody>
</table>

Table 8: Results of a monthly MLR model for time interval 2002-2017

All estimated coefficients of the regression model were examined, and a detailed summary of the regression results are accessible in the Appendix. Every coefficient in the model was considered statistically significant, with a significance level of 5% or lower. The values of the t-statistic implied that the most valuable coefficients in the model were for the foreign exchange, unemployment rate and the dummy variables June, July, and May. By analysing the residual plots in Figure 9, no unwanted trends were spotted though the variance was not as stable as for the Holt-Winters model.
In Figure 10, the potency of adding exogenous variables to the model is demonstrated by comparing the monthly Holt-Winters model to the MLR model. For this case the forecasting period was expanded to 108 months, making the year of 2009 the first year of forecasting. Because the Holt-Winters model experiences an extreme down-trend in 2008, it estimates that this trend will continue for all subsequent forecasting periods, dragging the number of sales down to zero by the year of 2011. Meanwhile, the MLR model picks up that the financial crisis eventually passes by and sales start increasing again.
Figure 10: Comparison between a monthly Holt-Winters and MLR model when forecasting period is increased to nine years

### 4.3.3 Monthly SVR

Training and test error estimates for the monthly SVR model can be seen in Table 9. For this model, the training error is considerably lower than the test error, indicating possible overfitting.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVR</td>
<td>Kernel = RBF, $\varepsilon = 0,1$, $\gamma = 0,04$, $C = 4$</td>
<td>156</td>
<td>14,3%</td>
</tr>
</tbody>
</table>

Table 9: Results of a monthly SVR model for time interval 2002-2017

The fit of the SVR model is highly dependent on parameter selection. By tuning the parameters in a way that minimizes the training error, the model will be able to fit the training dataset exceptionally well. However, a near perfect fit on the training data will almost definitely lead to overfitting. That is, the model will begin picking up random fluctuations in the data to model them in some way. When this happens, it usually leads to the model not being able to generalize well on new unseen data. For illustration purposes, the parameters $\varepsilon$ and $C$ were tuned to the extreme to get the best possible fit on the training dataset. Those results can be seen in Table 10. By tuning the parameters this way, the training error collapses almost to zero, while the model does substantially worse at generalizing on new unseen data.
### Table 10: Results of a monthly SVR model for time interval 2002-2017 when parameters are tuned to minimize training error

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVR</td>
<td>Kernel = RBF, $\varepsilon = 0$, $\gamma = 0.04$, $C = 1000$</td>
<td>0.18, 0.01%</td>
<td>567, 43.1%</td>
</tr>
</tbody>
</table>

In Figure 11, a visual representation of the training and test error from Table 9 can be seen for the monthly SVR model. Finally, residual analysis was performed which concluded with no unwanted trends and relatively constant variance of the errors (Figure 12).
4.3.4 Quarterly Holt-Winters

Training and test error estimates for the quarterly Holt-Winters model and time interval one can be seen in Table 11. The quarterly model returns the second best MAPE test results of all models while the training error is relatively high.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOLT-WINTERS</td>
<td>α = 0.869</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>β = 0.041</td>
<td>RMSE 671</td>
<td>MAPE 20.3%</td>
</tr>
<tr>
<td></td>
<td>γ = 1</td>
<td>RMSE 467</td>
<td>MAPE 11.7%</td>
</tr>
</tbody>
</table>

Table 11: Results of a quarterly Holt-Winters model for time interval 1991-2017

By analysing Figure 13, the reason for this high training error is evident. Sales in quarter two are overestimated for the majority of the years prior to the financial crisis in 2008. Other quarters are fitted relatively well. Residual plots in Figure 14 indicate that the model tends to overpredict the biggest sales months.

Figure 13: Results of a quarterly Holt-Winters model including a two-year forecast
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4.3.5 Quarterly MLR

Training and test error estimates for the quarterly MLR model can be seen in Table 12. The lowest test MAPE is recorded for all models while the training error is almost doubled the test error.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>MAPE</td>
</tr>
<tr>
<td>MLR</td>
<td>551</td>
<td>19,6%</td>
</tr>
</tbody>
</table>

Table 12: Results of a quarterly MLR model for time interval 2002-2017

All estimated coefficients of the regression model were examined, and a detailed summary of the regression results are accessible in the Appendix. There were two coefficients which were not considered statistically significant, CCI and the dummy variable representing the first quarter of the year. Neither one of them were excluded from the model since both training and test error rose by excluding CCI, and the dummy variable could not have been left out. The values of the t-statistic implied that the most valuable coefficients in the model were for the foreign exchange, unemployment rate, CPI and the dummy variable representing quarter two. By analysing the residuals plots in Figure 15, no decisive trend was spotted, i.e., more data points would at least be needed to verify it.
Quarterly SVR

Training and test error estimates for the quarterly SVR model can be seen in Table 13. The lowest training MAPE is recorded for all models while the test error is slightly higher than for Holt-Winters and MLR.
### Chapter 4: Results

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVR</td>
<td>Kernel = RBF, $\varepsilon = 0.1$, $\gamma = 0.04$, $C = 4$</td>
<td>RMSE 466, MAPE 13.9%</td>
<td>RMSE 681, MAPE 15.7%</td>
</tr>
</tbody>
</table>

Table 13: Results of a quarterly SVR model for time interval 2002-2017

In Figure 17, a visual representation of the training and test error can be seen for the quarterly SVR model. Residual analysis was concluded with no unwanted trends and relatively constant variance of the errors (Figure 18).

![Support Vector Regression](image1.png)

**Figure 17:** Results of a quarterly SVR model including a two-year forecast

![Training Fit and Test Fit](image2.png)

**Figure 18:** Residual analysis of a quarterly SVR model (logarithmic scale)
4.3.7 Variable Grid Search

For comparison reasons, same exogenous variables and time lags have been used for MLR and SVR so far, based on the analysis addressed in chapter 3.2. The ideology of running a grid search to find the highest possible correlation between the dependent and independent variable was primarily chosen to simplify the comparison between models. Another implementation would be to run multiple regressions with the goal of minimizing training error while trying out every possible combination of time lags from 0 to 10 months. A lot of combinations would presumably overfit the training data with this ideology, while some useful and rational combinations could be determined at the same time. In Table 14 and Table 15, top results of the grid search can be seen for monthly and quarterly models.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>TIME LAG</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td></td>
<td>CPI = 7 months</td>
<td>RMSE</td>
<td>MAPE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Unemployment = 0 months</td>
<td>208</td>
<td>21,2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Purchasing Power = 0 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CCI = 4 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Foreign Exchange = 0 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SVR</td>
<td>Kernel = RBF</td>
<td>CPI = 10 months</td>
<td>143</td>
<td>11,3%</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon = 0.1$</td>
<td>Unemployment = 0 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma = 0.04$</td>
<td>Purchasing Power = 0 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C = 4$</td>
<td>CCI = 7 months</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Foreign Exchange = 0 months</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 14: Results of monthly MLR and SVR models following a grid search of time-lagged exogenous variables (two-year forecast)

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>TIME LAG</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td></td>
<td>CPI = 4 quarters</td>
<td>RMSE</td>
<td>MAPE</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Unemployment = 0 quarters</td>
<td>544</td>
<td>18,3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Purchasing Power = 3 quarters</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CCI = 0 quarters</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Foreign Exchange = 0 quarters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SVR</td>
<td>Kernel = RBF</td>
<td>CPI = 4 quarters</td>
<td>338</td>
<td>11,2%</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon = 0.1$</td>
<td>Unemployment = 0 quarters</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\gamma = 0.04$</td>
<td>Purchasing Power = 0 quarters</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C = 4$</td>
<td>CCI = 3 quarters</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Foreign Exchange = 0 quarters</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 15: Results of quarterly MLR and SVR models following a grid search of time-lagged exogenous variables (two-year forecast)
A noticeable decrease is detected for all error estimates in Table 14 and Table 15, even though no substantial time lag changes were made from the values chosen in 3.2.2. With these small changes, SVR manages to generate a function that does markedly better over both the training and test period. Additionally, the difference between error estimates of the training and test period decreases, suggesting that the model seems to be generalizing better. To validate that severe overfitting was not taking place, the forecasting period was increased to five years or roughly 30% of the dataset. In Table 16 and Table 17, monthly and quarterly results for both methods are visible for a test period of five years. Finally, in Figure 19 and Figure 20, monthly and quarterly results of the SVR method are plotted using forecasting periods of two years and five years. By analysing the two figures, the models seem to be generalizing adequately well for the longer forecasting period as well, implying that the algorithm does not seem to be overfitting the training data severely. Nevertheless, preventing the algorithm from modelling all noise in the data is difficult since events such as dealership campaigns and discounts are not part of the model. One clear case of overfitting was at least detected by the author where the algorithm fits the data suspiciously well considering the circumstances at the time. The time in question was the summer of 2009 when dealerships offered customers outstanding deals solely to cut down their stock during the crisis.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>TIME LAG</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLR</td>
<td></td>
<td>CPI = 7 months&lt;br&gt;Unemployment = 0 months&lt;br&gt;Purchasing Power = 0 months&lt;br&gt;CCI = 4 months&lt;br&gt;Foreign Exchange = 0 months</td>
<td>233</td>
<td>25,0%</td>
</tr>
<tr>
<td>SVR</td>
<td>Kernel = RBF&lt;br&gt;ε = 0,1&lt;br&gt;γ = 0,04&lt;br&gt;C = 4</td>
<td>CPI = 10 months&lt;br&gt;Unemployment = 0 months&lt;br&gt;Purchasing Power = 0 months&lt;br&gt;CCI = 7 months&lt;br&gt;Foreign Exchange = 0 months</td>
<td>158</td>
<td>12,8%</td>
</tr>
</tbody>
</table>

Table 16: Results of monthly MLR and SVR models following a grid search of time-lagged exogenous variables (five-year forecast)
Table 17: Results of quarterly MLR and SVR models following a grid search of time-lagged exogenous variables (five-year forecast)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>CPI = 4 quarters</th>
<th>Unemployment = 0 quarters</th>
<th>Purchasing Power = 0 quarters</th>
<th>373</th>
<th>12,9%</th>
<th>298</th>
<th>9,9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVR</td>
<td>Kernel = RBF</td>
<td>ε = 0.1</td>
<td>γ = 0.04</td>
<td>C = 4</td>
<td>CCI = 3 quarters</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Unemployment = 0 quarters</td>
<td>Purchasing Power = 0 quarters</td>
<td>373</td>
<td>12,9%</td>
<td>298</td>
<td>9,9%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 19: Comparison between monthly SVR models in tables 14 and 16

Figure 20: Comparison between quarterly SVR models in tables 15 and 17
4.4 Results Summary

Six different methods were implemented in this thesis and one method clearly separated itself from others in terms of forecasting accuracy. The machine learning algorithm support vector regression returned the lowest forecasting errors for both monthly and quarterly data, in addition to the lowest training errors. Because quarterly data returned lower MAPE values for all SVR models, it would be recommended to use the quarterly SVR model for long term predictions. In Table 18, the quarterly predictions from the SVR model implemented in Table 15 have been summed up to yearly values, leading to a two-year forecasting error of 4.2%. Finally, a visual representation of the results in Table 18 can be seen in Figure 21.

<table>
<thead>
<tr>
<th>METHOD</th>
<th>PARAMETERS</th>
<th>TIME LAG</th>
<th>TRAINING ERROR</th>
<th>TEST ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVR</td>
<td>Kernel = RBF, $\epsilon = 0.1$, $\gamma = 0.04$, $C = 4$</td>
<td>CPI = 4 quarters, Unemployment = 0 quarters, Purchasing Power = 0 quarters, CCI = 3 quarters, Foreign Exchange = 0 quarters</td>
<td>RMSE = 790, MAPE = 6.7%</td>
<td>RMSE = 538, MAPE = 4.2%</td>
</tr>
</tbody>
</table>

Table 18: Yearly results after summing up quarterly predictions from the SVR model in table 15

![Figure 21: A visual representation of the training and test error in table 18](image)
Chapter 5

Discussion

The objective of this study was to formulate a model which could help Icelandic car dealerships to predict future demand for new automobiles in Iceland. A reliable forecasting model could be of great value to dealerships, helping them make difficult decisions about when it is safe to invest heavily in inventory. Two different approaches were used for modelling with the goal of determining which one was more suitable. That is, comparing traditional time series analysis methods to machine learning. Methods of simple moving average and exponential smoothing were used as benchmarks, while Holt-Winters exponential smoothing, multiple linear regression, support vector regression and random forest were compared in more detail. Holt-Winters exponential smoothing returned surprisingly good results considering its simplicity. However, as illustrated in Figure 10, the method falls short when dealing with future shifts in trend. Meanwhile, MLR, SVR and random forest use the predictive power of their exogenous variables to forecast, meaning that shifts in trend are possible over the forecasting period. Five exogenous variables were considered valuable for forecasting after an analysis performed in 3.2.4 (CPI, unemployment rate, purchasing power, CCI, foreign exchange). Those variables were lagged appropriately in 3.2.2 to maximize their correlation with the market. Finally, MLR, SVR and random forest models were built and compared using same variables and time lags. This approach led to MLR models being ranked higher than SVR and random forest models regarding accuracy. However, a second approach was implemented as well, where a grid search procedure was performed on the time-lagged variables. This led to better results for the SVR models, in addition to outperforming the MLR models. Ultimately, the SVR machine learning algorithm returned more accurate forecasts for the Icelandic automotive market while interpretability of the MLR models were greater. This brings up the Occam dilemma: “Accuracy generally requires more complex prediction methods. Simple and interpretable functions do not make the most accurate predictors” (Breiman, Statistical Modelling: The
Two Cultures, 2001). Leo Breiman stated that though complex predictors might be unpleasant, the soundest path was to go for predictive accuracy first, before trying to understand why.

For every method implemented, forecasts were generated using monthly and quarterly data. Majority of the results established in this thesis indicate that quarterly data returns more accurate forecasts. Nevertheless, one needs to decide whether the improvement in accuracy of the quarterly models is enough to sacrifice the detail of the monthly models. Through the perspective of Icelandic dealerships, some might only want to use forecasting models as a tool for evaluating the size of next year’s market, while others might like to get an estimation of the coming months.

Overall, the results of the MLR and SVR forecasting models are regarded reasonably accurate, especially considering the volatility of the Icelandic automotive market and the economy as a whole. There are examples of others getting better results using similar methods (Brühl, Hülsmann, Borscheid, Friedrich, & Reith, 2009) (Hülsmann, Borscheid, Friedrich, & Reith, 2011). However, that is understandable since their datasets consisted of markets of multiple magnitude compared to Iceland, and in more stable economies.

In this thesis, the assumption was made that economic indicators would be correctly estimated and therefore actual values used over the forecasting period. In reality, these estimates contain some margin of error themselves which would skew the results of the models. Finally, the models using exogenous variables as predictors can only detect changes which can be identified through the selected variables. Hence, important external factors such as various legislative changes made by the government would not be recognized by the models.

5.1 Future Work

Some further work is still needed so Icelandic car dealerships can rely on the outcome of the SVR or MLR model. A vital factor in improving the accuracy and reliability of the models is to attain more data. With more data, the algorithms should be able to model the relationship between the dependent and independent variables more accurately, leading to lower forecasting errors. This applies especially to SVR and random forest, being machine learning algorithms that learn from data instead of relying on rule-based programming like MLR. Another valuable feature of having larger datasets is the ability to increase the size of the test period. With larger test periods, it becomes easier to validate the reliability of models
since it would most likely have to deal with more variations of test data than before. The risk behind small datasets such as the one used in this thesis is the fact that the test data might simply be unusually easy to predict, resulting in misleading findings.

If there is no way of collecting more data about the Icelandic market, an equally interesting idea would be to apply the same methodology used in this thesis to a bigger foreign automotive market such as Germany. With that transition, the accuracy of the models would be expected to increase, especially with Germany having one of the strongest economies in the world.

Final remarks on possible future work would be to examine new predictor variables which might add to the predictive power of the models. As noted in chapter 4, none of the models, for example, include any event variables such as dealership campaigns which might be a valuable addition if formulated correctly. Last of all, it would be worth considering hybrid models since others have managed to improve their results by combining predictions from several models (Yang & Li, The Combination Forecasting Model of Auto Sales Based on Seasonal Index and RBF Neural Network, 2016).

5.2 Conclusion

This thesis proposed that quantitative methods could be used to improve inventory management of Icelandic car dealerships by building forecasting models that predict the future demand of the market. While qualitative methods can often return good estimates of the future, they are highly dependent on good intuition of the team members. Therefore, an accurate forecasting model can be used to either support or second guess the initial intuition of the team. Because Iceland’s economy is rather volatile, the automotive market can fluctuate tremendously between years. For that reason, Icelandic dealerships need to monitor the market closely to prevent circumstances of having large and expensive inventories heading into a recession. On the contrary, dealerships would like to enlarge their inventories just before an expansion for the competitive advantage over their competitors. This is where a good forecasting model can help.

Six different forecasting methods were implemented in the thesis; simple moving average (SMA), simple exponential smoothing (SES), Holt-Winters exponential smoothing, multiple linear regression (MLR), support vector regression (SVR) and random forest. The first two methods were used as benchmarks, while the latter four were examined in more detail. Holt-Winters models returned surprisingly good results considering their simplicity,
while more complex models with exogenous input variables were acknowledged as being more suitable for the Icelandic market. Random forest models returned relatively high error estimates for both monthly and quarterly data and were therefore not examined in much detail in chapter 4. More data would most likely be needed to improve the accuracy of the random forest algorithm. The final results of the MLR and SVR models were considered reasonable given the volatility of the market. Results showed that MLR models tend to be more consistent to SVR models when applied to limited data while being more interpretable as well. On the contrary, SVR can formulate more complex linear relationships through its Kernel function, leading to lower error estimates. The risk of applying SVR on such small datasets, however, is the possibility of overfitting the data. To avoid overfitting, the tuning parameters of $\varepsilon$, $\gamma$ and $C$ need to be cautiously selected. Overall, lower forecasting errors for the Icelandic automotive market can be achieved with machine learning algorithms while traditional time series analysis methods are still valid contenders, especially because of their interpretability.

A comparison was made between using monthly and quarterly data for the models. In majority of the cases, the quarterly models returned lower error estimates for both training and test data. Therefore, it would be suggested to use quarterly models for long-term forecasting while monthly models could be exploited for a rough estimation of the coming months. It is important to recognise that results of all models using exogenous variables in this thesis are based on the premise that the future economic indicators are estimated correctly. As a result, those models are highly dependent on the accuracy of economic predictions. To conclude, it is worth pointing out that forecasts should always be interpreted with great care, especially forecasts for highly volatile markets such as the Icelandic automotive market.
Bibliography


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https://www.mbl.is/greinasafn/grein/1317171/


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Yang, L., & Li, B. (2016). The Combination Forecasting Model of Auto Sales Based on Seasonal Index and RBF Neural Network. *International Journal of Database Theory and Application, Vol. 9, No. 1*, pp. 67-76.
Appendix

This appendix includes a correlation matrix of independent variables for quarterly data after a logarithmic transformation, in addition to summary results from R of several MLR models.

![Correlation Matrix of Independent Variables](image)

Figure 21: Correlation matrix of quarterly independent variables (2002-2015) including their distributions

Summary from R of the MLR monthly model results using time-lags from 3.2.2:

Residuals:
Min 1Q Median 3Q Max
-1.18881 -0.12908 0.03222 0.15825 1.25892

Coefficients:

|                     | Estimate | Std. Error | t value | Pr(>|t|)  |
|---------------------|----------|------------|---------|----------|
| (Intercept)         | 7.26423  | 1.08434    | 6.699   | 3.89e-10 *** |
| Inflation           | -0.22815 | 0.05827    | -3.915  | 0.000136 *** |
| Unemployment        | -0.33308 | 0.06578    | -5.064  | 1.18e-06 *** |
| PP                  | 4.08144  | 1.40220    | 2.911   | 0.004152 **  |
| CCI                 | 0.36210  | 0.12911    | 2.780   | 0.005701 **  |
| ISK.EUR             | -1.02625 | 0.13227    | -7.759  | 1.19e-12 *** |
| jan                 | 0.41088  | 0.11766    | 3.492   | 0.000629 *** |
| feb                 | 0.34529  | 0.11798    | 2.927   | 0.003956 **  |
Summary from R of the MLR quarterly model results using time-lags from 3.2.2:

Residuals:
```
Min  1Q Median  3Q Max
-0.84372 -0.13545  0.04476  0.15194  0.57040
```

Coefficients:
```
Estimate Std. Error t value  Pr(>|t|)
(Intercept)  8.96133    1.89640   4.725 2.11e-05 ***
Inflation  -0.24128    0.09191  -2.625  0.01165 *
Unemploy    4.56970    2.26841   2.014  0.04970 *
CCI         0.26796    0.23162   1.157  0.25315
ISK.EUR    -1.02879    0.21749  -4.730 2.08e-05 ***
QTR1        0.17316    0.10521   1.646  0.10645
QTR2        0.30125    0.10201   2.953  0.00490 **
QTR3        0.28864    0.10202   2.829  0.00684 **
```

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.269 on 47 degrees of freedom
Multiple R-squared: 0.9119, Adjusted R-squared: 0.8969
F-statistic: 60.8 on 8 and 47 DF, p-value: < 2.2e-16

Summary from R of the MLR monthly model results using time-lagged values from Table 14:

Residuals:
```
Min  1Q Median  3Q Max
-0.84389 -0.17186  0.01383  0.15488  1.49438
```

Coefficients:
```
Estimate Std. Error t value  Pr(>|t|)
(Intercept)  6.04305    0.93359   6.473 1.27e-09 ***
Inflation  -0.24634    0.05281  -4.664 6.77e-06 ***
Unemploy    2.72986    0.98923   2.760 0.006504 **
CCI         0.40490    0.12394   3.267 0.001346 **
ISK.EUR    -0.89597    0.12570  -7.128 3.92e-11 ***
jan         0.42381    0.11744   3.609 0.000418 ***
feb         0.35708    0.11735   3.043 0.002765 **
mar         0.47378    0.11755   4.030 8.80e-05 ***
apr         0.38298    0.11754   3.258 0.001385 **
```

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.269 on 47 degrees of freedom
Multiple R-squared: 0.9119, Adjusted R-squared: 0.8969
F-statistic: 60.8 on 8 and 47 DF, p-value: < 2.2e-16
Summary from R of the MLR quarterly model results using time-lagged values from Table 15:

Residuals:
    Min     1Q    Median     3Q     Max
-0.61960 -0.11606  0.00594  0.13876  0.67473

Coefficients:
  Estimate Std. Error t value Pr(>|t|)
(Intercept)  7.77040    1.71369   4.534 3.98e-05 ***
Inflation  -0.22709    0.08867  -2.561 0.013700 *
Unemploy    0.35102    0.09053   3.877 0.000327 ***
PP           4.04309    2.05236   1.970 0.054750 .
CCI         0.40670    0.18061   2.252 0.029052 *
ISK.EUR    -0.90132    0.20797  -4.334 7.66e-05 ***
QTR1        0.13311    0.09968   1.335 0.188195
QTR2        0.26935    0.09916   2.716 0.009212 **
QTR3        0.27353    0.09849   2.777 0.007849 **

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2592 on 47 degrees of freedom
Multiple R-squared:  0.9182,  Adjusted R-squared:  0.9043
F-statistic: 65.94 on 8 and 47 DF,  p-value: < 2.2e-16