Bondi-Hoyle-Lyttleton Accretion of Cold and Self-Interacting Dark Matter

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2019
BONDI-HOYLE-LYTTLETON ACCRETION OF COLD AND SELF-INTERACTING DARK MATTER

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16 ECTS thesis submitted in partial fulfillment of a B.Sc. degree in Physics

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Reykjavík, May 2019
Abstract

The accretion of a compact object is explored in a Bondi-Hoyle-Lyttleton (BHL) accretion geometry where the surrounding media consists of dark matter (DM). Theoretical expectations for both cases where the medium is collisionless and collisional are examined to obtain a sense of the properties of the BHL wake that forms in each case, including the size of the Hoyle-Lyttleton radius and magnitude of the accretion rate. N-body simulations of the BHL accretion geometry are performed for the case of cold dark matter (CDM), which is collisionless, and self-interacting dark matter (SIDM), which for large cross sections should approach the collisional case. The results of the simulations are analyzed and show significant differences between the two DM models. The results paint a picture of a denser accretion column (wake) close to the compact object in the case of SIDM and a higher fraction of bounded particles which results in a larger accretion rate than in the case of CDM. The CDM wake is in contrast more disperse whereas for SIDM the velocity dispersion is lower and the velocity dispersion profile flatter. These differences can be explained by collisions of the SIDM particles on their trajectory to form the wake, which results in a net loss of kinetic energy of the wake particles. The larger the cross section, the stronger these differences are.
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1 Introduction

1.1 Dark Matter Evidence

Dark matter plays a central role in our understanding of modern cosmology. In this section, some key pieces of evidence that support the existence of dark matter are discussed.

1.1.1 Galaxy Clusters

One of the earliest evidence for the existence of dark matter came from studying the masses and velocity dispersions of galaxy clusters. In 1933, the famous astronomer Fritz Zwicky studied the dynamics of multiple galaxy clusters and discovered a large variance in the apparent velocities of eight galaxies located within the Coma Galaxy Cluster [1]. Zwicky applied the virial theorem to estimate the mass of the Coma cluster based on the kinematics of the galaxies and compared it with the estimate based on the observed mass. Zwicky’s calculations showed that 800 galaxies of $10^9$ solar masses in a sphere with a radius of $10^6$ light years should exhibit a velocity dispersion of approx. 80 km/s. However, observations showed that the line-of-sight average velocity dispersion amounted to approx. 1000 km/s. From this huge discrepancy in the velocity dispersion, Zwicky concluded that the amount of dark matter prevalent in the cluster should greatly exceed that of luminous matter [1].

In 1937, Zwicky refined his analysis further by assuming that the Coma cluster contained 1000 galaxies in a sphere with a radius of $2 \times 10^6$ light years. He obtained a lower limit on the mass of the cluster which he found to be $4.5 \times 10^{13} M_\odot$ which corresponds to an average mass per galaxy of $4.5 \times 10^{10} M_\odot$. Assuming an average absolute luminosity of $8.5 \times 10^7 L_\odot$ for cluster galaxies, resulted in a mass-to-light ratio of about 500. It is now known that Zwicky overestimated the mass-to-light ratio by a factor of 8.3 because he used an incorrect value for the Hubble constant. Regardless of this correction, the velocity dispersion of the Coma cluster results in a very large mass-to-light ratio which points to the existence of an unknown type of matter that produces negligible emission of electromagnetic radiation, which Zwicky referred to as "dark matter". Figure 1.1 shows a summary of the dark matter problem in the 1950s.
1 Introduction

Figure 1.1: Evidence for dark matter in the 1950s. Distance, mass, luminosity, and mass-to-light ratio of several galaxies and clusters of galaxies. From Schwarzschild, 1954 [2].

1.1.2 Rotation Curves

The rotation curves of galaxies are an important piece of evidence for the existence of dark matter. The galaxy rotation curves are circular velocity profiles of the visible gas and stars in a galaxy as a function of radial distance from the galaxy’s center and from them one can infer the distribution of mass within the galaxy. Figure 1.2 shows rotation curves for the galaxies M31, M101 and M81.

The theoretically predicted behaviour of galaxy rotation curves based on photometric observations was not in agreement with the measured rotation curves of galaxies from observations of the 21-cm hydrogen line. Observations showed many galaxies having almost flat rotation curves at large radial distances from the center of the galaxies while the expectation based on the visible matter were sharply declining rotation curves. In 1970, based on these comparisons of predicted and measured rotation curves, the idea of Zwicky re-emerged, that there had to be some additional, undetected matter in the outer parts of some galaxies. The unobserved matter’s mass had to be at least comparable to the mass of the observed galaxy and its distribution had to be different from the optical part of the galaxy.

The evidence for the fact that the gravitational mass present in galaxies has to increase beyond their optical region started to accumulate as more and more galaxies exhibited large mass-to-light ratios and their rotation curves were observed to be flat out to their largest observed radius. The idea that dark matter was present in large amounts in the outer regions of galaxies started to gain traction within the scientific community.
1.1.3 Cosmic Microwave Background

As evidence for the existence of dark matter in galaxy clusters and galaxies gradually piled up, astronomers began to wonder what was the composition of this mysterious material. The simplest explanation was that the matter was made up of so called “MACHOs” (massive astrophysical compact halo objects) which are compact objects that are too faint to be detected such as black holes and white dwarfs (see below).

Further evidence for dark matter comes from measurements on cosmological scales of the angular power spectrum of the temperature anisotropy of the cosmic microwave background (CMB). The CMB is the remnant electromagnetic radiation from the hot early days of the universe dating to the epoch of recombination, at a redshift of $z = 1100$ or roughly 378,000 years after the Big Bang. The CMB power spectrum of temperature fluctuations is a measure of the amplitude of fluctuations shown at different angular scales on the sky. Measurements of the angular scale and height of the peaks of the oscillatory pattern measured in the power spectrum give information on cosmological parameters, including the cosmological baryon density parameter and the ratio of the dark matter to baryonic matter.

When measurements from the Wilkinson Microwave Anisotropy Probe (WMAP) experiment and more recently, the Planck Collaboration, are compared to the total matter density one finds that less than 20% of the total matter in the Universe is baryonic which leaves little room for baryonic MACHOs [1]. This is strong indirect evidence that dark matter consists of nonbaryonic material.

Additionally, there are robust theoretical and observational arguments that imply that the large scale structure of the Universe could not have formed by the
present time without dark matter. Before recombination the universe was ionized and baryons were tied to photons. Since dark matter does not couple to baryons and photons strongly, it can clump together early on to provide the gravitational potential needed for ordinary matter to cluster at the time of recombination to form massive astrophysical structures [4]. Models of the formation and evolution of structures without dark matter failed to create potential wells that are deep enough at early enough times to give rise to the broad range of dense structures that are observed today, from the large scale filamentary structure, to galaxy clusters and galaxies.

1.2 Dark Matter Candidates

1.2.1 Classical Classification of Dark Matter Candidates

Dark matter candidate particles are classified into three families by their typical velocities in the early Universe: hot, warm and cold dark matter. These dark matter families have been modeled extensively and influence cosmological structure formation each in a different way. Hot dark matter (HDM) models, with large primordial thermal velocities and a dark matter particle mass of around $30 \text{ eV}$, form superclusters at first which were originally thought to fragment into smaller galaxies in a “top-down” approach. In cold and warm dark matter (CDM and WDM) models, small structures form at first which grow by merging and through accretion to create bigger systems in a “bottom-up” approach. Dark matter structures significantly smaller than galaxies can form in CDM models, but not in WDM models. CDM models with a particle mass of around $100 \text{ GeV}$ can form the smallest dark matter halo with a mass roughly corresponding to that of the Earth. The smallest structure WDM can form, with a particle mass of around $2 \text{ keV}$, corresponds to the size of a dark halo of a dwarf galaxy. Hence, the smallest structure which can form directly is radically different in each family.

Neutrinos were the prototype candidates for HDM particles and are the only DM candidates that have been shown to exist. Early on, the neutrino seemed to be a promising DM candidate. However, numerical simulations have shown that the “top-down” hierarchical structure scenario that is exhibited by HDM is not compatible with galaxy clustering constraints [5]. This discrepancy resulted in a decrease of interest in HDM models and the neutrino as a DM candidate. In contrast, the “bottom-up” structure formation of CDM and WDM is largely in agreement with current observations.
1.2 Dark Matter Candidates

1.2.2 WIMPs and Axions

With seemingly no viable DM candidates within the Standard Model (SM) of elementary particles, new particle physics theories have been proposed that predict novel particles which are considered to be DM candidates. Examples of such candidates are weakly interacting massive particles (WIMPs) which interact via gravity and the weak nuclear force. Another candidate is the extremely light axion associated with the solution to the so-called strong CP problem in quantum chromodynamics.

Interactions between the SM and WIMPs or axions might potentially be large enough to be detected in the laboratory. However, on larger scales the particles behave as CDM and the interactions are negligible. Neither axions nor WIMPs have been detected so far. For a review on current dark matter searches in particle accelerators such as the LHC see [6].

1.2.3 Problems With the CDM Model

The structure of the Universe is largely consistent with cold and collisionless DM particles that purely interact with each other and other SM particles through gravity. However, some inconsistencies between observations and CDM predictions still remain.

High-resolution dark matter simulations show that the density profiles of CDM halos are “cuspy”, i.e., the density increases steeply toward the center at lower radii. Observationally, however, a large number of galaxies exhibit internal kinematics that are consistent with having a constant “cored” density profile. This is known as the core–cusp problem.

Another discrepancy between the CDM predictions (without baryonic physics) is known as the missing satellites problem. Simulations show that CDM halos grow via hierarchical mergers of smaller halos and are therefore rich with substructure with multiple smaller halos orbiting larger ones. In contradiction, the number of small galaxies observed in the Milky Way are an order of magnitude fewer than the number of predicted subhalos.

Other issues include the Too-big-to-fail (TBTF) problem: The most luminous satellites in the Milky Way (MW) are expected to inhabit the most massive DM subhalos. However, predictions from CDM simulations show that these subhalos have central densities that are too large to host the observed satellites, given their observed internal kinematics. The implication would then be that in CDM, these massive subhalos should not host any galaxy, but these subhalos are simply too big to fail to form stars, which is a conundrum. For a review on the CDM challenges and their possible solutions see [7].
1.2.4 Self-Interacting Dark Matter (SIDM).

In order to resolve the core-cusp and missing satellites problems, Spergel and Steinhardt proposed an alternative to collisionless CDM [8]. This alternative is known as self-interacting dark matter (SIDM). This new promising form of DM particles involve self-interactions between the particles via elastic scattering.

SIDM and CDM have some noticeably different predictions for the inner structure of dark matter halos. In the standard analogy, the classical hydrostatic equilibrium of a self-gravitating sphere of gas where the pressure and temperature of the gas counteracts gravity, is replaced by a self-gravitating collection of collisionless particles kept in equilibrium due to their velocity dispersion. The velocity dispersion of CDM halos decreases towards the center of the halo. This is necessary to keep the halo in equilibrium given the cuspy CDM density profile. The inclusion of self-interactions introduces a heat conduction mechanism, which results in the transfer of heat from the hotter outer region to the inner cooler region of the halo. As a result, SIDM halos have a more uniform velocity dispersion as a function of radius than their CDM counterparts, i.e., they develop an isothermal core. CDM halos have a universal density profile as a consequence of their hierarchical structure formation. If collisions are added to the simulations, the cusps in the density profiles are converted into cores when particles are heated deep within the inner halo. SIDM halos also take a more spherical shape as collisions isotropize the particle velocities. If self-interactions are frequent enough, they can also affect the abundance of substructure, reducing subhalo masses (due to collisions between particles in the subhalo and the more energetic particles in the halo) in a way that could address the missing satellites problem [9].

The scattering rate of SIDM is proportional to the DM density, thus at large radii where the collision rate is negligible the differences between the structure of CDM and SIDM halos are minuscule. In order for the collisions to have a noticeable effect on the local dark matter distribution of a given structure, each particle has to scatter at least once over the lifetime of that particular structure, e.g. a halo. For dark matter halos today, this corresponds to a cross section per unit mass of at least $\sigma/m \sim 1 \, \text{cm}^2/\text{g}$ so that the size of the core generated by self-interactions is large enough relative to the galaxy within. Relevant galactic cores are of order of 1 kpc. Therefore, the SIDM model has different predictions on small scales in dense inner regions of halos but remains consistent with the large-scale structure formation predicted by CDM models.
1.2.5 SIDM Constraints

After Spergel and Steinhardt’s proposal of SIDM, a plethora of N-body simulations were performed to probe the effects of self-interactions on DM halos. From these simulations emerged various constraints on the self-scattering cross section. These are summarized in fig. 1.3. The positive observations specify challenges to the CDM predictions that can be resolved by adding self-interactions as well as the cross section required to resolve each problem.

For instance, dwarf and low surface brightness (LSB) galaxies require cross sections of at least 1 cm²/g in order to explain the large galaxy cores that are observed. Fine-tuning the cross section is not necessarily required, a cross section as large as 50 cm²/g results in density profiles that are consistent with observation on dwarf galaxy scales. To resolve the core–cusp problem and TBTF issues on small scales while retaining consistency with larger scale constraints the required cross section is around \( \sigma/m \sim 0.5 - 1 \) cm²/g [9].

Another issue is that SIDM models that assume a constant cross section seem to violate several astrophysical constraints such as the observed ellipticity of the mass distribution of galaxy clusters. In order to avoid such constraints, a promising solution was explored by adding a velocity-dependence to the cross section. Generally, most plausible SIDM models favor a cross section that is mildly velocity-dependent on dwarf galaxy scales and decreases strongly towards larger velocities and scales. This behaviour is expected in many particle models with SIDM candidates. More massive halos will typically have DM particles with larger velocities for scattering, due to the deeper gravitational potential. Hence, the cross section constraints should be viewed as a function of halo mass (meaning the virial mass).

<table>
<thead>
<tr>
<th>Positive observations</th>
<th>( \sigma/m )</th>
<th>( v_{\text{rel}} )</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cores in spiral galaxies</td>
<td>( \geq 1 ) cm²/g</td>
<td>30-200 km/s</td>
<td>Rotation curves</td>
</tr>
<tr>
<td>Milky Way</td>
<td>( \geq 0.6 ) cm²/g</td>
<td>50 km/s</td>
<td>Stellar dispersion</td>
</tr>
<tr>
<td>Local Group</td>
<td>( \geq 0.5 ) cm²/g</td>
<td>50 km/s</td>
<td>Stellar dispersion</td>
</tr>
<tr>
<td>Cores in clusters</td>
<td>( \sim 0.1 ) cm²/g</td>
<td>1500 km/s</td>
<td>Stellar dispersion, lensing</td>
</tr>
<tr>
<td>Abell 3827 subhalo merger</td>
<td>( \sim 1.5 ) cm²/g</td>
<td>1500 km/s</td>
<td>DM-galaxy offset</td>
</tr>
<tr>
<td>Abell 520 cluster merger</td>
<td>( \sim 1 ) cm²/g</td>
<td>2000-3000 km/s</td>
<td>DM-galaxy offset</td>
</tr>
<tr>
<td>Constraints</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Halo shapes/ellipticity</td>
<td>( \leq 1 ) cm²/g</td>
<td>1300 km/s</td>
<td>Cluster lensing surveys</td>
</tr>
<tr>
<td>Substructure mergers</td>
<td>( \leq 2 ) cm²/g</td>
<td>( \sim 500-4000 ) km/s</td>
<td>DM-galaxy offset</td>
</tr>
<tr>
<td>Merging clusters</td>
<td>( \leq ) few cm²/g</td>
<td>2000-4000 km/s</td>
<td>Post-merger halo survival (Scattering depth ( \tau &lt; 1 ))</td>
</tr>
<tr>
<td>Bullet Cluster</td>
<td>( \leq 0.7 ) cm²/g</td>
<td>4000 km/s</td>
<td>Mass-to-light ratio</td>
</tr>
</tbody>
</table>

Figure 1.3: Constraints and positive observations (i.e., observations that can be explained by SIDM) on self-interaction cross section per unit mass, \( \sigma/m \). From Tulin and Yu [9].
1 Introduction

1.3 Bondi-Hoyle-Lyttleton Accretion in a DM medium: the impact of the DM nature

Bondi accretion, named after Hermann Bondi is spherical accretion onto a compact object traveling through the interstellar medium. The rate of accretion when the compact object moves through a uniform gas cloud was studied thoroughly by Bondi, Hoyle and Lyttleton. I will refer to this particular type of accretion as “Bondi–Hoyle–Lyttleton” (BHL) accretion. The results from the combined work of Bondi, Hoyle and Lyttleton have been applied to a variety of astrophysical problems at different scales. The objective of this project is to probe and compare the difference between CDM and SIDM as the surrounding DM accretion media in BHL accretion of a compact object. This thesis will cover both analytical expectations of the accretion as well as the results of DM N-body simulations.
2 Analytical Expectations

2.1 Hoyle and Lyttleton Analysis

Bondi, Hoyle and Lyttleton investigated the rate of accretion of a point mass traversing a uniform gas cloud. Specifically, Hoyle and Lyttleton [10] considered the case of a star traversing an infinite gas cloud at a constant speed, $v_\infty$. To simplify the problem, the cloud is assumed to have a uniform density and to be free of self-gravity. To describe this situation, we use the the coordinate system where the star is stationary. The star’s gravity causes a flux of gas particles that form a wake behind the star, a part of which is then accreted by the star (geometry shown in figure 2.1).

![Figure 2.1: Sketch of the Bondi–Hoyle–Lyttleton accretion geometry. From Edgar [11].](image)

If pressure effects are ignored then the trajectory of a gas streamline with impact parameter $\zeta$ can be represented as a single particle moving in a ballistic orbit. The equations of motion for the conservation of particle and angular momentum are:

\[ \ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2} \]  
(2.1)

\[ r^2\ddot{\theta} = \zeta v_\infty \]  
(2.2)

Where $M$ is the mass of the star, $v_\infty$ is the speed at infinity and $r$, $\theta$ and $\zeta$ are defined as is shown in fig. 2.1. Analytical solutions for the flow field are the following
2 Analytical Expectations

(derived from Bisnovatyi-Kogan et al. [12]):

\[ v_r = \sqrt{v_\infty^2 + \frac{2GM}{r} - \frac{\varsigma^2 v_\infty^2}{r^2}} \]  \hspace{1cm} (2.3)

\[ v_\theta = \frac{\varsigma v_\infty}{r} \]  \hspace{1cm} (2.4)

\[ r = \frac{\varsigma^2 v_\infty^2}{GM(1 + \cos \theta) + \varsigma v_\infty^2 \sin \theta} \]  \hspace{1cm} (2.5)

\[ \rho = \frac{\rho_\infty \varsigma^2}{r \sin \theta(2\varsigma - r \sin \theta)} \]  \hspace{1cm} (2.6)

where \( \rho_\infty \) is the density at infinity. At the \( \theta = 0 \) axis the radius of the streamline is given by

\[ r(\theta = 0) = \frac{\varsigma^2 v_\infty^2}{2GM} \]  \hspace{1cm} (2.7)

where the tangential velocity (along \( \theta \)) goes to zero, while the radial velocity goes to \( v_\infty \). The density goes to infinity when the material approaches the \( \theta = 0 \) axis. Particles are assumed to be accreted through an infinitely thin column with infinite density on this axis. Particles that are gravitationally bound to the star at the \( \theta = 0 \) axis, i.e., particles that fulfill the inequality

\[ \frac{1}{2} v_\infty^2 - \frac{GM}{r} < 0 \]  \hspace{1cm} (2.8)

are assumed to get accreted eventually. This condition defines a critical impact parameter

\[ \varsigma_{HL} = \frac{2GM}{v_\infty^2}. \]  \hspace{1cm} (2.9)

Particles with a smaller impact parameter will be accreted by the star. From eqs. 2.7 and 2.9 we define the Hoyle-Littleton radius (which has the same magnitude as \( \varsigma_{HL} \))

\[ r_{HL} = r(\varsigma = \varsigma_{HL}, \theta = 0) = \frac{2GM}{v_\infty^2}. \]  \hspace{1cm} (2.10)

The Hoyle-Lyttleton accretion rate is then given by

\[ \dot{M}_{HL} = \pi \varsigma_{HL}^2 v_\infty \rho_\infty = \frac{4\pi G^2 M^2 \rho_\infty}{v_\infty^3}. \]  \hspace{1cm} (2.11)
2.2 Bondi and Hoyle Analysis

The analysis was continued by Bondi and Hoyle [13] to incorporate the accretion column. The accretion column is assumed to take the form of a wake which forms as material encounters the $\theta = 0$ axis at a distance $r$ from the star along this axis (note that we only consider distances $r$ where $\theta = 0$ for the rest of this section). The wake is assumed to be thin, i.e. the cross-section radius of the accretion column, $s = s(r)$, is much smaller than $r$ ($s \ll r$). The geometry of the analysis is sketched in fig 2.2.

Figure 2.2: Sketch of the geometry for the Bondi–Hoyle analysis. Here we are only considering distances $r$ where $\theta = 0$. From Edgar [11].

Thus, the mass flux between the distances $r$ and $r+dr$ is

$$2\pi s \frac{1}{2}\frac{\rho_\infty v_\infty}{v_\infty} dr = \Lambda dr \hspace{1cm} (2.12)$$

where $\Lambda = 2\pi GM\rho_\infty/v_\infty$ is the constant mass flux (independent of $r$). Multiplying the mass flux, by the tangential velocity along the accretion axis, $v_{\theta}(\theta = 0) = (2GM/r)^{\frac{1}{2}}$ and dividing by the approximate area of the wake, $2\pi sd r$, gives us the momentum flux into the wake in the interval $r$ to $r+dr$. The momentum flux gives us an estimate of the pressure, $P$, in the wake:

$$P \approx \frac{\Lambda dr \cdot v_{\theta}(\theta = 0)}{2\pi sd r} = \frac{\Lambda}{2\pi s} \sqrt{\frac{2GM}{r}}. \hspace{1cm} (2.13)$$

Particles that encounter the wake at a distance $r$ will take a time of about $r/v_\infty$, which is of the order of $GM/v_\infty^2$, to get accreted. Thus, the mass per unit
length or the linear density of the wake can be estimated from the accretion rate to be

\[ m \approx \Lambda \frac{GM}{v_\infty^3}. \tag{2.14} \]

\section{2.3 Expectations of BHL accretion with DM as the ambient medium}

The purpose of revising the analysis of Bondi, Hoyle and Littleton is to form analytical expectations of BHL accretion onto a compact object when the surrounding media consists of CDM or SIDM. This serves the purpose of gaining a better understanding of the system that will be simulated in section 3. Even though the calculations have many simplified assumptions, they give an estimate of the amplitude of the effects and the characteristic scale of the system, which will improve the choice of the initial conditions and simulation parameters.

\subsection*{2.3.1 Density profile of the BHL wake: the case of collisionless (CDM) particles}

The expectation is that the ballistic approximation for the BHL accretion described in section 2.1 would be applicable to the case of collisionless CDM particles. Solving eq. 2.5 for \( \varsigma \) as a function of \( r \) and \( \theta \) gives us

\[ \varsigma(r, \theta) = \frac{1}{2} \left( r \sin \theta + \sqrt{r^2 \sin^2 \theta + \frac{1}{v_\infty^2} \frac{4rGM(1 + \cos \theta)}{4rGM(1 + \cos \theta)}} \right) \tag{2.15} \]

so eq. 2.6 gives us the density ratio \( \rho / \rho_\infty \) as a function of \( r \) and \( \theta \):

\[ \frac{\rho(r, \theta)}{\rho_\infty} = \frac{\varsigma(r, \theta)^2}{r \sin \theta(2\varsigma(r, \theta) - r \sin \theta)}. \tag{2.16} \]

Eq. 2.16 will be used to estimate the density of the wake for the collisionless case.
2.3.2 Density profile of the BHL wake: the case of collisional particles

BHL accretion with SIDM as its accretion medium can be expected to be approximate to the case where the ambient medium is fully collisional (fluid approximation), in the limit where the self-interaction cross section is very large. We are interested in the cross-sectional density profile of the accretion column, i.e., the wake, for the collisional case which can be expressed in terms of the linear density \( m \):

\[
\rho(r) = \frac{m}{\pi s(r)^2}
\]

where \( s(r) \) is the cross-section radius of the accretion column at a distance \( r \) as before. To simplify the problem, the fluid is assumed to be polytropic. The adiabatic exponent \( \gamma = c_P/c_V \), the ratio of specific heats at constant pressure and volume respectively, is \( \gamma = 5/3 \) for a monatomic ideal gas. The sound velocity is given by [14]

\[
c_s = \sqrt{\frac{\gamma P}{\rho}}.
\]

The pressure \( P \) of a cross-section of the accretion column at a distance \( r \) is estimated from the momentum flux (eq. 2.13). To compare with the collisionless case we need to express the pressure of the wake as a function of \( r \) and \( \theta \) for small angles. We modify eq. 2.13 by replacing \( r \) with the projection of \( r \) onto the \( \theta = 0 \) axis \( r \cos \theta \) to get

\[
P(r, \theta) \approx \frac{\Lambda}{2\pi s} \sqrt{\frac{2GM}{r \cos \theta}}
\]

where we recover eq. 2.13 when \( \theta = 0 \). From eqs. 2.17 and 2.19 we eliminate \( s \) to achieve an estimate of the cross-sectional density in terms of the pressure of the wake:

\[
\rho \approx \frac{2\pi mr \cos \theta}{\Lambda^2 GM} P^2.
\]

From eqs. 2.14, 2.18 and 2.20 and plugging in for \( \Lambda \) (given below eq. 2.12), we ultimately end up with the density ratio \( \rho/\rho_\infty \) for the collisional case as a function of \( r \) and \( \theta \) (small \( \theta \)):

\[
\frac{\rho(r, \theta)}{\rho_\infty} = \frac{GM}{r \cos \theta} \frac{v_\infty^2 \gamma^2}{c_s^4}.
\]
2.3.3 Comparison between the collisional and collisionless cases

We are curious to find out at what point the collisional and collisionless cross sectional density profiles are comparable. Figure 2.3 shows the densities as a function of $\theta$ for $v_\infty = c_s = 30$ km/s at $r = 0.01 \ r_{HL}$ and $r = 0.1 \ r_{HL}$.

![Graph showing comparison between collisional and collisionless cases](image)

**Figure 2.3:** $\frac{\rho}{\rho_\infty}$ as a function of $\theta$ for the collisionless and collisional case at $r = 0.01 \ r_{HL}$ (top panel) and $r = 0.1 \ r_{HL}$ (bottom panel); $r_{HL}$ is the Hoyle-Littleton radius defined in eq. 2.10.
2.3 Expectations of BHL accretion with DM as the ambient medium

For the collisionless case, the densities are large close to the $\theta = 0$ axis but decrease as you move away from the wake. The collisional wake densities remain constant at a fixed distance close to the $\theta = 0$ axis as a result of the estimated constant pressure of a wake cross section at that distance. Both densities increase with decreasing $r$ and become comparable in size at different angular distances from the wake depending on the value of $r$. 
3 N-body Simulations

3.1 Purely Gravitational N-body Simulations

3.1.1 System Dynamics

N-body simulations tackle the problem of calculating the dynamics of a system of N particles. The relevant dynamics when the particles interact purely via their mutual gravitational attraction is relatively simple. The equations of motion for N non-relativistic particles interacting gravitationally are given by Newton’s law of gravitation, i.e., a particle $i$ with mass $m_i$ positioned at $r_i$ is affected by the force

$$F_i = -\sum_{j \neq i} G \frac{m_i m_j (r_i - r_j)}{|r_i - r_j|^3}.$$  \hspace{1cm} (3.1)

Thus we have a problem which consists of a set of non-linear second order ordinary differential equations which relate the positions of the particles in the system with the acceleration $\frac{F_i}{m_i} = \frac{d^2 r_i}{dt^2}$ for all $i$.

3.1.2 Softening Length

Already for N=3, no general analytical closed solutions exist for arbitrary initial conditions. Therefore, numerical methods are required to integrate the equations of motion. Caution must be exercised to ensure both efficiency and accuracy. A couple of issues arise from the equation of the gravitational force. Equation 4.1 diverges when the distance between two particles approaches zero. This can lead to some numerical artifacts such as arbitrarily large relative velocities (and hence, arbitrarily small timesteps, which makes the computation unfeasible) in close encounters between particles.

One way to avoid this problem is to modify small scale gravitational interaction by introducing a softening or smoothing length, which represents a characteristic distance below which the gravitational interaction becomes constant at very small
3 N-body Simulations

distances. An example of a softened force is:

\[
F_i = -\sum_{j \neq i} G \frac{m_i m_j (r_i - r_j)}{|r_i - r_j|^2 + \epsilon^2)^{3/2}}
\]  

(3.2)

where \( \epsilon > 0 \) is the softening length.

Although the gravitational N-body problem is not complicated on a conceptual basis, it can be difficult to solve numerically. The number of operations required for the force calculation for all particles scales as \( O(N^2) \), since evaluating the force on each particle requires taking the contributions from all the other members of the system into account. This implies that simulations with large \( N \), i.e., \( N \geq 10^6 \), become very computationally demanding [15, 16].

The expensive evaluation of the force has led to the development of a wide number of numerical algorithms and techniques with the goal of obtaining a numerical method that is efficient and reliable. The tree code method [17] is an example of a fast, general algorithm when the force contributions from very distant particles do not need to be computed at very high accuracy. A hierarchical spatial tree is used to define localized groups of particles. The force on each particle is evaluated using the approximated force of each group. The direct pairwise force calculation is only employed for nearby particles. The method reduces the number of operations to \( O(N\log(N)) \) with the minor drawback of introducing some (small) errors due to the long range force approximation.

3.2 SIDM Simulations

N-body simulations have been essential tools to explore the effects of dark matter self-interactions on astrophysical structure formation and evolution. Initial SIDM simulations employed smoothed particle hydrodynamics to model DM collisions. For simplicity, most simulations assume a velocity-independent and isotropic scattering with a constant self-interacting cross section per unit mass, \( \sigma/m \). This is the scenario we will follow.

The SIDM simulations employed in this work follow a standard Monte Carlo approach to account for DM self-interactions [18]. The self-interactions are assumed to be elastic and isotropic. The scattering probability for each particle \( i \) with its neighbour \( j \), \( P_{ij} \), for \( k = 38 \pm 5 \) nearest neighbours is determined in a time step \( \Delta t_i \) as:

\[
P_{ij} = \frac{\sigma}{m} W(r_{ij}/h_i) m_i v_{ij} \Delta t_i
\]  

(3.3)

where \( \sigma/m \) is the SIDM cross section per unit mass, \( m_i \) is the simulation particle mass, \( v_{ij} \) is the relative velocity between particles \( i \) and \( j \), \( r_{ij} \) is the distance between
3.3 Initial Conditions

In order to explore the development of a BHL wake created by a compact object with the surrounding media consisting of CDM and SIDM, the following setup was

\[ \Delta t_i \] is chosen small enough to avoid multiple scatterings during a single time step. The total probability for a particle to interact with any of its \( k \) nearest neighbours is given by

\[ P_i = \frac{1}{2} \sum_j P_{ij} \] (3.5)

All scattering events are between two particles, therefore it is required that the factor \( 1/2 \) be present in eq. 3.5 to produce the correct number of events, as opposed to the number of particles that scatter.

A collision between particle \( i \) and one of its \( k \) nearest neighbours takes place when \( x \), a uniformly distributed random number drawn from the interval \((0, 1)\), is less than or equal to \( P_i \), i.e. \( x \leq P_i \). To determine the neighbour \( j \) that collides with particle \( i \), the neighbours are sorted by distance to particle \( i \) and the first neighbour \( l \) that fulfills \( x \leq \sum_j P_{ij} \) is chosen.

When an elastic collision occurs between two particles, \( i \) and \( l \), the velocities of the particles are updated to

\[ \mathbf{v}_i = \mathbf{v}_{cm} + \left( \frac{v_{il}}{2} \right) \mathbf{e}, \]
\[ \mathbf{v}_l = \mathbf{v}_{cm} - \left( \frac{v_{il}}{2} \right) \mathbf{e}, \]

where \( \mathbf{e} \) is a unit vector in a random direction and \( \mathbf{v}_{cm} \) is the center-of-mass velocity of the particle pair. In this case, angular momentum is not conserved, however, both energy and linear momentum are.

3.3 Initial Conditions

3.3.1 General Setup

In order to explore the development of a BHL wake created by a compact object with the surrounding media consisting of CDM and SIDM, the following setup was

\[^1\] The kernel function and the smoothing length describes the way in which the distribution of matter is smoothed out given the discrete representation with particles.
arranged. The initial conditions consist of a periodic simulation cube with a side length $l_{\text{box}} = 15$ kpc. The cube encloses $N^3$ high resolution DM particles within a tube of radius $R_{\text{tube}} = 1$ kpc and length $l_{\text{cube}} = 15$ kpc with a central axis, which we refer to as the x-axis. Beyond the tube there are $(N/2)^3$ low resolution particles which occupy the rest of the space within the cube. The particles are generated with velocities that follow a Maxwellian distribution with a velocity dispersion of 30 km/s. The masses of the low and high resolution particles, $m_{\text{low}}$ and $m_{\text{high}}$, are chosen so that they satisfy the following relations:

$$m_{\text{high}} = \frac{\rho(0)}{N^3} V_{\text{tube}}$$  \hfill (3.6)$$

$$m_{\text{low}} = \frac{\rho(0)}{(N/2)^3} (V_{\text{cube}} - V_{\text{tube}})$$  \hfill (3.7)$$

where $V_{\text{tube}} = \pi R_{\text{tube}}^2 l_{\text{cube}}$ and $V_{\text{cube}} = l_{\text{cube}}^3$ which ensures an initial density of $\rho(0) = \Delta c \times \rho_c$ where $\Delta c$ is the overdensity and $\rho_c$ is the critical density, $\rho_c = 3H_0^2/8\pi G$, where $H_0$ is the Hubble constant and $G$ is Newton’s gravitational constant.

The mass of the compact object, in this case a supermassive black hole, was chosen to be $M_{\text{BH}} = 5 \times 10^7 M_\odot$. The black hole is positioned at the center of the cube within the high resolution tube with an initial velocity $v_{\text{BH}}(0) = 30$ km/s in the direction of the x-axis, see fig. 3.1. The overdensity was chosen as $\Delta c = 10^4$ so the density of the surrounding DM media is $\rho(0) = 10^4 \rho_c$.

The softening lengths for the low and high resolution particles were chosen as

$$\epsilon_{\text{low}} = l_{\text{cube}} \sqrt{m_{\text{low}}/M_{\text{BH}}}$$
and
\[ \epsilon_{\text{high}} = l_{\text{cube}} \sqrt{m_{\text{high}}/M_{\text{BH}}} \]
to prevent strong discreteness effects. The choice of softening comes from setting the maximum stochastic acceleration from close encounters to be less than the minimum acceleration in the simulation [19].

### 3.4 Simulation Results

#### 3.4.1 Lower Resolution Tests

The Arepo code [20] was used to run the simulations in the University of Iceland high-performance computing cluster, Garpur. Lower resolution simulations with \(N=64\) and \(N=128\) were used to perform tests. The results that follow however, refer mostly to the higher resolution case with \(N=256\). Setups with both CDM and SIDM with a cross section of \(\sigma = 200 \text{ cm}^2/\text{g}\) were run. After a simulation time of approx. 0.4-0.8 Gyr had elapsed, a wake could clearly be seen by looking at density maps and differences in the accretion column between the CDM and SIDM simulations were apparent, see fig. 3.2.

![Figure 3.2: Accretion column behind the compact object (black hole) forming in CDM (left panel) and SIDM (right panel) for \(N=256\) after a simulation time of approx. \(t \sim 0.12 \text{ Gyr}\) has elapsed. Projected density map (integrated along the z-axis) of high resolution particles. The black hole location is beyond the range shown in the plots, it is approximately located at \(x = 2.7 \text{ kpc}\).](image)

The black hole (BH) slows down with time because of dynamic friction as more and more mass gradually concentrates in the wake. The speed of the BH approaches zero, and takes a time of approx. 0.12 Gyr to stop to a halt.
For this setup, the Hoyle-Lyttleton radius is initially \( r_{\text{HL}}^{(0)} = \frac{2GM_{\text{BH}}}{(v_{\text{BH}}^{(0)})^2} = 0.48 \text{ kpc} \) and for the SIDM case the mean free path of the SIDM particles is given by \( \lambda = \frac{1}{\rho \sigma} \). When using a cross section of \( \sigma = 200 \text{ cm}^2/\text{g} \) the mean free path is comparable to the initial Hoyle-Lyttleton radius for densities of the order of \( 18 \times \rho^{(0)} \) and larger, \( \lambda \left( \rho = 18 \times \rho^{(0)} \right) = 0.48 \text{ kpc} \).

### 3.4.2 Higher Resolution Analysis

Higher resolution simulations with \( N=256 \) were carried out to obtain more detailed results. SIDM simulations were run for the different cross sections, \( \sigma = 200, 10^3 \) and \( 10^4 \text{ cm}^2/\text{g} \). Densities and velocity dispersions were analyzed and compared at different regions of the accretion column for all simulations. The densities at a specific region of the wake are estimated by dividing the total particle mass within a cylinder of radius \( R \), which encircles the x-axis, by the volume of the cylinder. Density profiles of two regions for all high resolution simulations can be seen at figures 3.3.a and 3.3.b after a simulation time of \( t \sim 0.08 \text{ Gyr} \) has elapsed. The former region stretches from the location of the BH to 0.5 kpc to the left of it and the latter region encloses the densest part of the wake, from the location of the BH to 0.1 kpc to the left of it.

In general, the densities increase closer to the BH and near the x-axis as expected (as can be seen by comparing the amplitude of the densities between figs. 3.3.a and 3.3.b). Adding collisions seems to have the effect of drastically amplifying the wake densities close to the BH. This enhancement increases with larger cross sections. The densities of the SIDM wake overpower that of the CDM one at almost all regions of the accretion column except when near the x-axis and far away from the BH, see figure 3.3.c. In fig. 3.3.c the region begins at 0.5 kpc to the left of the BH’s position and ends at a distance of 0.1 kpc left of it, thus excluding the densest part of the wake, near the BH. In these regions, SIDM densities are lower than that of CDM. Although figures 3.3.a-c are taken after a simulation time of \( t \sim 0.08 \text{ Gyr} \) has elapsed, these plots are representative of the behaviour of the wake since the trend does not alter significantly at later times.

The velocity dispersion of a specific region within the wake is estimated similarly to the density. The velocity dispersion, \( \sigma_{\text{vel}}^2 = \frac{\sum (v - \bar{v})^2}{N_c} \), is calculated using all \( N_c \) particles within a cylinder of radius \( R \) which envelopes the x-axis. Velocity dispersion profiles of two regions can be seen at figures 3.4.a and 3.4.b after a time of \( t \sim 0.08 \text{ Gyr} \) has elapsed. The former region stretches from the BH’s position to 0.5 kpc to the left of it and the latter region encloses the densest part of the wake, from the BH’s location to 0.1 kpc to the left of it, i.e., they are the same regions as in figs. 3.3.a and 3.3.b.
3.4 Simulation Results

Figure 3.3: Density profiles of high resolution DM simulations (for CDM and different SIDM models as given in the legend) after a time $t \sim 0.08$ Gyr has elapsed, of the configuration shown in fig. 3.1. Estimated densities within a cylinder from the $x$-axis (coincident with the linear trajectory of the BH) in units of the initial ambient density, $\rho(0)$, as a function of the radius of the cylinder, $R$. The cylinder stretches from: a) the BH’s location to 0.5 kpc to the left of it, b) the BH’s location to 0.1 kpc to the left of it and c) 0.5 kpc to 0.1 kpc to the left of the BH’s location.
Figure 3.4: Velocity dispersion profiles of high resolution DM simulations (for CDM and different SIDM models as given in the legend) after a time: a) and b) $t \sim 0.08$ Gyr and c) $t \sim 0.12$ Gyr has elapsed, of the configuration shown in fig. 3.1. Estimated velocity dispersions within a cylinder from the $x$-axis (coincident with the linear trajectory of the BH) as a function of the radius of the cylinder, $R$. The cylinder stretches from: a) and c) the BH’s location to 0.5 kpc to the left of it and b) the BH’s location to 0.1 kpc to the left of it.
In general, the velocity dispersion increases near the BH and as you approach the x-axis (as can be seen by comparing figs. 3.4.a and 3.4.b). The velocity dispersions in the CDM case are larger than in the SIDM case in all regions within the accretion column. The most pronounced difference can be seen in the region close to the BH. The velocity dispersion decreases and the profile flattens out with increasing cross section. Figures 3.4.a-b are from $t \sim 0.08$ Gyr. The trend remains relatively consistent at later times but eventually the CDM velocity dispersion starts dropping off slowly, see fig. 3.4.c.

Only a part of the particles in the ambient medium are bounded to the BH and will eventually get accreted by it. Particles that satisfy the relation

$$\frac{1}{2}v^2 - \frac{GM_{BH}}{r} < 0$$

where $r$ is the distance from the particle to the BH and $v$ is the velocity of the particle in the stationary frame of the BH, are bounded while others are unbounded. Table 3.1 shows the number of bounded particles, $N_b$, and the fraction of bounded particles, $N_b/N_c$, in two regions within the wake at $t \sim 0.08$ Gyr.

<table>
<thead>
<tr>
<th>$\sigma$ [cm$^2$/g]</th>
<th>a)</th>
<th>b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N_c$</td>
<td>$N_b$</td>
</tr>
<tr>
<td>CDM</td>
<td>3421</td>
<td>3070</td>
</tr>
<tr>
<td>200</td>
<td>5144</td>
<td>4719</td>
</tr>
<tr>
<td>$10^3$</td>
<td>9384</td>
<td>8846</td>
</tr>
<tr>
<td>$10^4$</td>
<td>13374</td>
<td>12814</td>
</tr>
</tbody>
</table>

The table shows that very near the BH most of the particles are bounded, and as expected, the fraction of bounded particles decrease with distance from the BH. The number of bounded particles and the fraction of bounded particles within the wake is larger for SIDM than for CDM, especially close to the BH. This effect increases with cross section. This trend is consistent at later times.
3 N-body Simulations

On the other hand, the total mass of the bounded particles within the wake, $M_{\text{bound}}$, can be estimated as the total mass of the bounded particles within a cylinder of radius $R = 0.5 \text{ kpc}$ that stretches from the location of the BH to 0.5 kpc to the left of it. The accretion rate can then be estimated as

$$\dot{M}_{\text{accr}} = \frac{M_{\text{bound}}}{\Delta t_B}$$

where

$$\Delta t_B = \frac{r_{\text{HL}}^{(0)}}{\sqrt{(v_{\text{BH}}^{(0)})^2 + \sigma_{\text{vel}}^2}}.$$ 

The Hoyle-Littleton accretion rate for this simulation setup is

$$\dot{M}_{\text{HL}} = \frac{4\pi G^2 M_B^{(0)} \rho^{(0)}}{(v_{\text{BH}}^{(0)})^3} \sim 6.1 \times 10^4 M_\odot/ \text{Myr}.$$ 

Fig. 3.5 shows the estimated accretion rate, in units of the Hoyle-Littleton accretion, for the simulations after a time of $t \sim 0.08 \text{ Gyr}$ has elapsed as a function of cross section, (which is zero for CDM). The accretion rate increases with cross section.

![Figure 3.5: Estimated accretion rate, $\dot{M}_{\text{accr}}$ in units of $\dot{M}_{\text{HL}}$ after a time of $t \sim 0.08 \text{ Gyr}$ has elapsed as a function of cross section, $\sigma$ (CDM has cross section $\sigma = 0 \text{ cm}^2/\text{g}$).](image)
3.4 Simulation Results

Finally, figures 3.6.a and 3.6.b show the average density ratio of the CDM and SIDM wake, $\frac{\rho_{\text{SIDM}}}{\rho_{\text{CDM}}}$, close to the x-axis, after a time of $t \sim 0.08$ Gyr has elapsed, as a function of cross section. The cylinder has radius $R = 0.01$ kpc and in fig. 3.6.a it stretches from 0.5 kpc to 0.1 kpc to the left of the BH and in fig. 3.6.b it stretches from the BH to 0.5 kpc to the left of the BH. Figure 3.6.a shows that far away from the BH and close to the x-axis, it is the CDM wake that is the most dense and the density decreases with increasing cross section. However, in other regions, especially close to the BH, the opposite occurs (fig. 3.6b), with the average density in the wake increasing with the cross section. The trend for both the accretion rate and average density ratios remain consistent at later times, so these plots are representative of the general trends across time.

![Figure 3.6: Average density ratio of the CDM and SIDM wake, $\frac{\rho_{\text{SIDM}}}{\rho_{\text{CDM}}}$, close to the x-axis, after a time of $t \sim 0.08$ Gyr has elapsed as a function of cross section. The cylinder has a radius of $R = 0.01$ kpc and stretches from: a) 0.5 kpc to 0.1 kpc to the left of the BH and b) 0.1 kpc to the left of the BH to the BH’s location.](image)
3.5 Discussion

If we assume that particles in both the SIDM and CDM simulations follow a ballistic trajectory towards the wake, then collisions in SIDM simulations might alter these paths. A possible explanation for the larger fraction of bounded particles to the supermassive black hole for the SIDM cases is that collisions on their trajectory might result in a net loss of kinetic energy of the particles that end up in the wake. These particles might then impact the wake at a position that is much closer to the BH than their original trajectory would have led them otherwise; the particles would also have lower characteristic velocities due to the net loss of kinetic energy. This would explain the increased SIDM densities close to the BH, as more and more particles would impact the wake closer to the BH, see fig. 3.2. The velocity dispersion of the particles within the region of the SIDM wake would also be lower relative to CDM due to the net loss of kinetic energy. Notice that since the density profile within the cylinder increases towards the axis of the cylinder (for regions near the BH), collisions are more frequent closer to it, but since particles close to the wake’s axis are also hotter (larger velocity dispersion) due to the dynamics of the generation of the wake (clear in the CDM case), self-collisions redistribute the energy from the inside out. This net "heat transfer" from the center of the wake to the outside, results in a lower velocity dispersion and a flatter velocity dispersion profile as a function of distance from the x-axis as we can see in fig. 3.4.b.

In contrast, the CDM accretion column is more disperse, the velocity dispersion is larger and wake densities aren’t concentrated as heavily towards the region close to the BH as in the SIDM case due to a lack of collisions. Following from this interpretation, larger SIDM cross sections, which result in an increase in the number of collisions, would be expected to amplify this difference which is consistent with the results and picture painted so far. We note that far from the BH, the density within the SIDM wake is actually smaller than for the CDM case (see fig. 3.3.c). The different behaviour seems to indicate that since the gravity of the BH is weaker here, the wake is less dense and collisions are less frequent in the SIDM case. They do however and they seem to be acting as a form of pressure that "puffs up" the wake. This is in fact, the analytical expectation we had based on the results of section 2 (figure 2.3). The fact that this expectation breaks down close to the BH indicates that the gravity of it is strong enough to keep on replenishing the wake with more particles than in the CDM case due to the lowering of the kinetic energies of the incoming particles caused by collisions. A more dedicated study would be needed to establish this picture more firmly.
4 Conclusions

This work studies the differences between the impact of the presence of a compact object (supermassive black hole) in an ambient medium made of cold (CDM) or self-interacting dark matter (SIDM). To do this, we first presented simple analytical expectations of the two cases based on the classical Bondi-Hoyle-Lyttleton (BHL) accretion for collisionless particles and a collisional (polytropic) gas. We then performed numerical N-body simulations in the CDM and SIDM cases. The simulation results turned out to be somewhat in the direction of the simplified theoretical expectations with a few major discrepancies between the theoretical collisional case and the SIDM simulations. Examples of key results are a significant density enhancement of the SIDM wake close to the black hole and the fact that the accretion rate seems to increase with larger cross sections. Very large cross sections were needed for the difference between the two DM models to be noticeable. SIDM simulations were performed using a constant cross section, whereas, in general, self-interacting cross sections are dependant on the relative velocity of the interacting particles. This is something that could be studied in the future.

Other interesting ideas that could be explored in the future are dynamic friction and including tracer particles in the simulations to better understand the ballistic trajectories of the particles as they approach the black hole, specifically for the SIDM simulations, where the particle trajectories could be significantly altered due to collisions.

The applications of BHL accretion are numerous, they include: binary systems, protostellar clusters and galaxy clusters to name a few. For example, wind accretion in binary systems is a popular application of the BHL geometry [11]. The inclusion of DM accretion to these systems might prove to be interesting, particularly to constrain or find signatures of dark matter self-interactions in the denser regions within halos, where BHL accretion into massive black holes might be possible. For instance, an interesting result that is counterintuitive at first sight is that we have found that the accretion rate is actually enhanced in SIDM, although the cross sections need to be very large for the effect to be noticeable.
References


REFERENCES


