Towards the relativistic regime of the gravothermal fluid approximation in SIDM halos

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TOWARDS THE RELATIVISTIC REGIME OF THE GRAVOTHERMAL FLUID APPROXIMATION IN SIDM HALOS

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16 ECTS thesis submitted in partial fulfillment of a
Baccalaureus Scientiarum degree in Physics

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Abstract

In this work we examine the evolution of self-interacting dark matter (SIDM) halos using the gravothermal fluid approximation. The self-similar solution for the development of a halo's core presented by Balberg, Shapiro and Inagaki (2002) is analyzed along with the gravothermal collapse phase occurring subsequently in the core's evolution. Parametric models were developed in order to describe this late-stage evolution of the core, as well as computing the point in time at which the particles in the core transition into the relativistic regime of the gravothermal collapse phase and finding its relation to the SIDM cross-section.

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Acknowledgments

I would like to thank my parents, for their support and encouragement, and express my gratitude to my advisor, Jesús Zavala Franco, for all the help and support he provided me with while working on this project.

1 Introduction to dark matter

One of the greatest mysteries in modern cosmology is the nature of dark matter (DM). It is called 'dark' because it does not appear to interact with ordinary matter, it does not emit electromagnetic radiation and only its gravitational effects are seen. Its presence is implied by various astrophysical observations, most notably the discrepancies in mass estimates of galaxies/clusters made when comparing measurements of their internal dynamics (e.g. velocity dispersion of galaxies in a cluster) with their expected gravitational potential given their luminosities.

In 1933 the Swiss astronomer Fritz Zwicky published a paper on the Coma cluster of galaxies. In it he showed that the measured velocity dispersion of the cluster's members was so high that in order to keep the system stable, the average mass density in it would need to be much greater than what calculations based on visible, luminous matter indicated. He ascribed this result to the presence of undetected, dark matter. Soon after that the Swedish astronomer Erik Holmberg, who studied systems of galaxies, and the American physicist Sinclair Smith, who studied the mass of the Virgo cluster, obtained similar results. Not much attention was given to these issues until the 1970s with the pioneer work of Vera Rubin when they were looked at in the context of the discrepancy between measured and predicted rotation curves of galaxies.

Rotation curves are important tools to study the kinematics of rotationally supported galaxies, as they show the orbital velocity of the gas and stars within them as a function of distance to the galactic center. After such rotation curves had been computed for a number of galaxies an unexpected feature became increasingly evident, which was that rotation curves tend to be flat in the outskirts of spiral galaxies. This was surprising as the velocity of a rotating disk of gas and stars was predicted to decline beyond the radial distance within which most mass is contained. The flat rotation curves indicated that there was more gravity and therefore more mass than expected from estimates based on the galaxies' observed light. Many different phenomena were suggested to try to explain these issues but with further developments in cosmology the dark matter hypothesis eventually came out on top. In this hypothesis, galaxies and galaxy clusters are embedded in massive self-gravitationally bound structures, called halos, which extend well beyond the edge of the visible galaxies (de Swart, Bertone & van Dongen, 2017).

The great amount of dark matter required to match observations indicate that it is not composed of normal baryonic[1] matter. Of the total mass-energy

[1]Baryonic matter in the context of the physics of galaxies is used to refer to all ordinary matter, not just baryons, i.e., it includes leptons as well. The reason for the name is that baryons dominate by mass, and thus the gravity of the ordinary matter.

content of the Universe, around 5% is baryonic matter, 26% is dark matter and the remaining 69% are consistent with a cosmological constant Λ associated with dark energy. The current cosmological paradigm that contains dark matter and dark energy as essential components is called Lambda cold dark matter (ΛCDM). The 'cold' here means that the dark matter particles had low thermal velocities in the early Universe, as opposed to other known dark particles such as neutrinos. ΛCDM is extremely successful for explaining the large scale (distances greater than $\sim$Mpc) structure of the Universe, but there are several challenges which arise at smaller scales. Among those is the discrepancy between density profiles obtained from simulations of CDM halos, which tend to be cuspy, and observed rotation curves of a fraction of dwarf galaxies that indicate a density profile with a core. Another one is that only a couple of dozens of satellite galaxies have been observed in the Milky Way, but CDM simulations predict a number of dark matter halos which is more than 10 times higher. It is possible however that these challenges are related to the complexity of the baryonic physics that creates the luminous galaxies. It is also possible that they point to a fundamental problem with the CDM hypothesis. Since our knowledge of baryonic physics remains uncertain and incomplete, and since the nature of dark matter remains a mystery, the CDM challenges remain an open avenue of research.

A promising suggestion that might offer a solution to these problems is self-interacting dark matter (SIDM), where the dark matter particles undergo elastic scattering after collisions. This leads to drastically different results for inner halo structure compared to CDM predictions, especially as SIDM halos tend to form cored density profiles with reduced central density. They are also expected to have an isothermal velocity dispersion, as initially the velocity dispersion in the inner region of the halo decreases towards the center but self-interactions drive it to become uniform with radius (Tulin & Yu, 2018).

Another interesting aspect of SIDM halos are their hypothesized potential to form black holes via the so-called gravothermal catastrophe. After an isothermal central core has formed, the core will transfer both mass and energy to the outer regions of the halo through the flow of particles and heat. As the core evolves and shrinks in size and mass, its density and temperature will increase. The core will contract until it reaches a critical point when the velocities of the particles within it become relativistic, at which dynamical instability sets in that eventually leads the core to collapse catastrophically to form a black hole. This mechanism has been suggested as a possible origin of quasars and supermassive black holes which reside in the centers of most galaxies Shapiro, 2018.

In this work, models of SIDM halos based on the gravothermal fluid approximation will be analyzed in pursuit of obtaining parametric solutions for when this critical point is reached and how the development of the core towards it changes with the cross-section of the SIDM.

2 The self-similar solution for SIDM halos

If we treat the halo as a spherically symmetric ideal gas in hydrostatic equilibrium with conductivity κ then the following equations apply (Balberg et al., 2002):

$$\frac{\partial M}{\partial r} = 4\pi r^2 \rho, \tag{2.1}$$

$$\frac{\partial\left(\rho v^2\right)}{\partial r} = -\frac{GM\rho}{r^2}, \tag{2.2}$$

$$\frac{L}{4\pi r^2} = -\kappa \frac{\partial T}{\partial r}, \tag{2.3}$$

$$\frac{\partial L}{\partial r} = -4\pi\rho r^2 v^2 \left(\frac{\partial}{\partial t}\right)_M \ln\left(\frac{v^3}{\rho}\right), \tag{2.4}$$

where $M(r)$ is the mass within a radius r, $\rho(r)$ is the density, $v(r)$ is the one-dimensional velocity dispersion and $L(r)$ is the luminosity through a sphere with radius r. Equation (2.1) describes the integrated mass in terms of density, equation (2.2) describes hydrostatic equilibrium for an ideal gas, equation (2.3) is Fourier's law of heat conduction and equation (2.4) is analogous to the first law of thermodynamics (see Lynden-Bell and Eggleton (1980) for further details).

There are two physical length scales in the system; the mean free path for SIDM which is given by

$$\lambda = \frac{1}{\rho\sigma}, \tag{2.5}$$

and the gravitational scale height of the system, given by

$$H = \sqrt{\frac{v^2}{4\pi G\rho}}. \tag{2.6}$$

The relevant timescale for the system is the relaxation time, which for SIDM is defined as

$$t_r = \frac{1}{a\rho v\sigma} \tag{2.7}$$

where σ is the cross-section per unit mass and the constant a, for particles interacting elastically like hard spheres with a Maxwell-Boltzmann velocity distribution, is $a = 4/\sqrt{\pi} \approx 2.26$.

Equation (2.3) can also be written in a more detailed form. As the conductivity κ is not known for a gas in the SIDM regime, which is between a collisionless and fully collisional fluid, it has to be expressed in terms of an effective impact parameter, which can be found empirically to be $b \approx 25\pi/(32\sqrt{6}) \approx 1.002$. To account for both the short mean free path (SMFP) and long mean free path (LMFP) limits we use a combination of the dominant factors determining particle collisions in each of the limits, $1/\lambda$ and vt_r/H^2 respectively. Equation (2.3) can thus be written as

$$\frac{L}{4\pi r^2} = -\frac{3}{2} b\rho v \left(\frac{1}{\lambda} + \frac{vt_r}{H^2} \right)^{-1} \frac{\partial v^2}{\partial r}. \tag{2.8}$$

Substituting in equations (2.5) and (2.6) gives

$$\frac{L}{4\pi r^2} = -\frac{3}{2} abv\sigma \left(a\sigma^2 + \frac{4\pi G}{\rho v^2} \right)^{-1} \frac{\partial v^2}{\partial r}. \tag{2.9}$$

If we assume the LMFP limit, i.e. that the entire halo is dilute enough so that the relation $\lambda \gg H$ holds everywhere, we can neglect the $a\sigma^2$ term within the parentheses in equation (2.9). This simplifies the system to a purely gravitational one, where collisions are gravitational encounters (as opposed to the SMFP limit where the system is a self-gravitating sphere where collisions are frequent and conductivity is determined by the mean free path). Following the derivation of Balberg et al. (2002) further, it is possible to describe the time-dependent evolution of the system in the LMFP limit. This is the so-called self-similar solution (i.e. independent of spatial scales) developed originally by Lynden-Bell and Eggleton (1980). The equations can be separated into the time evolution of a central density core and spatial mass/energy profiles that have a shape which is independent of spatial scales. We will concentrate only on the equations that characterize the time evolution of the core which can be described by a single parameter ζ, given by

$$\frac{dE_c}{dt} = \zeta \frac{E_c}{M_c} \frac{dM_c}{dt}, \tag{2.10}$$

where E_c is the total energy of the core, which is assumed to be isothermal. The following relations for the core mass, radius, density and circular velocity dispersion can thus be derived assuming the equations for an isothermal gas sphere:

$$\frac{d\log r_c(t)}{d\log M_c(t)} = 2 - \zeta, \quad \frac{d\log v_c^2(t)}{d\log M_c(t)} = -(1-\zeta),$$
$$\frac{d\log \rho_c(t)}{d\log M_c(t)} = -(5 - 3\zeta). \tag{2.11}$$

The time dependence of these core variables are given by the self-similar solution in terms of the collapse time, which is the time from $t = 0$ until the core density goes to infinity; $t_{coll}(0) = \theta t_{r,0}/\xi_e$, where $\theta = 2(11 - 7\zeta)^{-1}$ and ξ_e is a mass evaporation parameter defined by the relation

$$\frac{dM_c(t)}{dt} = -\xi_e \frac{M_c(t)}{t_r(t)}, \tag{2.12}$$

which parameterises the mass flux between the core and the boundaries. Using the results from Balberg et al. (2002) we obtain $\zeta = 0.7655$, $\theta = 0.3545$ and $\xi_e = 1.21 \times 10^{-3}$. We also have the equation for the time dependence of the core mass, which is

$$\frac{M_c(t)}{M_c(0)} = \left(1 - \frac{t}{t_{coll}(0)}\right)^{\theta}, \tag{2.13}$$

and by using equations (2.11) and (2.13) the relations for the density, radius and circular velocity can be derived, giving

$$\frac{\rho_c(t)}{\rho_c(0)} = \left(1 - \frac{t}{t_{coll}(0)}\right)^{\theta(3\zeta-5)} \tag{2.14}$$

$$\frac{r_c(t)}{r_c(0)} = \left(1 - \frac{t}{t_{coll}(0)}\right)^{\theta(2-\zeta)} \tag{2.15}$$

$$\frac{v_c^2(t)}{v_c^2(0)} = \left(1 - \frac{t}{t_{coll}(0)}\right)^{\theta(\zeta-1)}. \tag{2.16}$$

3 A phenomenological model to account for self-interactions

As stated earlier, the basis for the derivation of the self-similar solution is the assumption that the entire halo resides in the LMFP regime and thus the $a\sigma^2$ term in equation (2.9) could be ignored. It is clear however that as the density of the core rises, $\lambda = (\rho\sigma)^{-1}$ decreases and thus the core evolves towards the SMFP regime. As time gets closer towards the collapse time $t_{coll}(0)$, the development of the core deviates from the self-similar solution and becomes dominated by the thermal evolution of a system that is fully in the fluid regime. Eventually, the assumption is that the core will become dynamically unstable and undergo relativistic collapse. (Balberg et al., 2002)

Finding a complete analytical description that shows the time-evolution of the core density as a function of the cross-section is not a trivial matter, but finding the profile it tends towards in its later stages is possible. The self similar solution has a vertical asymptote at $t = t_{coll}(0)$ but for values of σ which are greater than zero, the set of differential equations (2.1)–(2.4) needs to be solved numerically. The asymptotic solution for the density profile has a simple form, which was found to be described by the following equation:

$$\frac{\rho_c(t)}{\rho_c(0)} = e^{\beta_1 t/t_0 + \beta_2}, \tag{3.1}$$

where $\beta_1 = 0.108733\,(a\hat{\sigma}^2)^{-1.00233}$, $\beta_2 = -31.5299\,(a\hat{\sigma}^2)^{-1.00173}$, $t_0 = t_{r,0}(b\hat{\sigma})^{-1}$ and $\hat{\sigma} = \sigma/\sigma_0$ with $\sigma_0 = 4\pi r_c^2(0)/M_c(0)$. These equations for β_1 and β_2 were obtained by fitting equation (3.1) to the data from the numerical calculations performed by Balberg et al. (2002). In figure 3.1 both the curve given by the self-similar solution and lines representing the profile given by equation (3.1) for selected values of $a\hat{\sigma}^2$ are displayed.

In figure 3.2 a simplified density profile of the halo can be seen, with a constant density core and where $\rho \propto r^{-2.19}$ in the outer regions Balberg et al., 2002. The profile of the core is displayed for selected times late in the halo's evolution and its tendency towards high densities and low radii can clearly be observed.

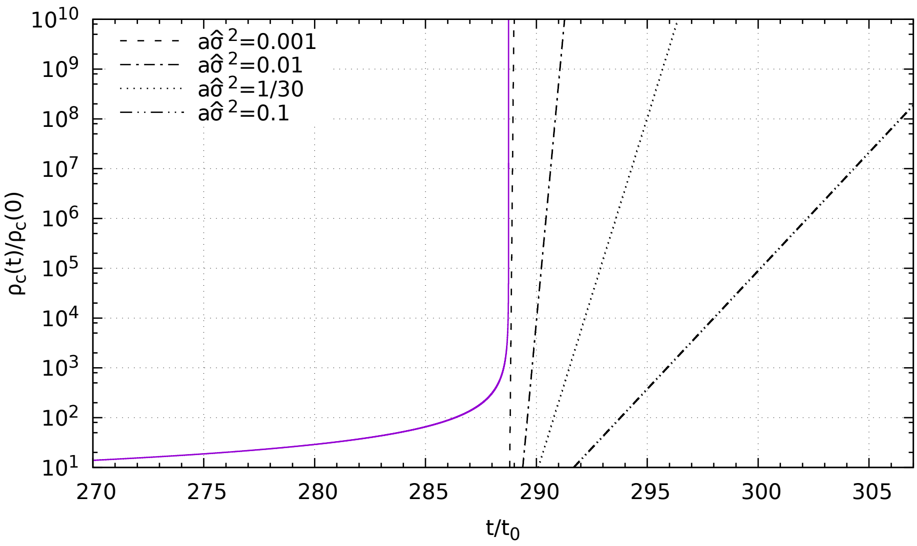

Figure 3.1: The time-evolution of the core density in a system described by the gravothermal fluid approximation. The solid curve shows the self-similar solution (see equation 2.14) and the dotted lines represent the asymptotes of the fits (see equation 3.1) for different values of $a\hat{\sigma}^2$.

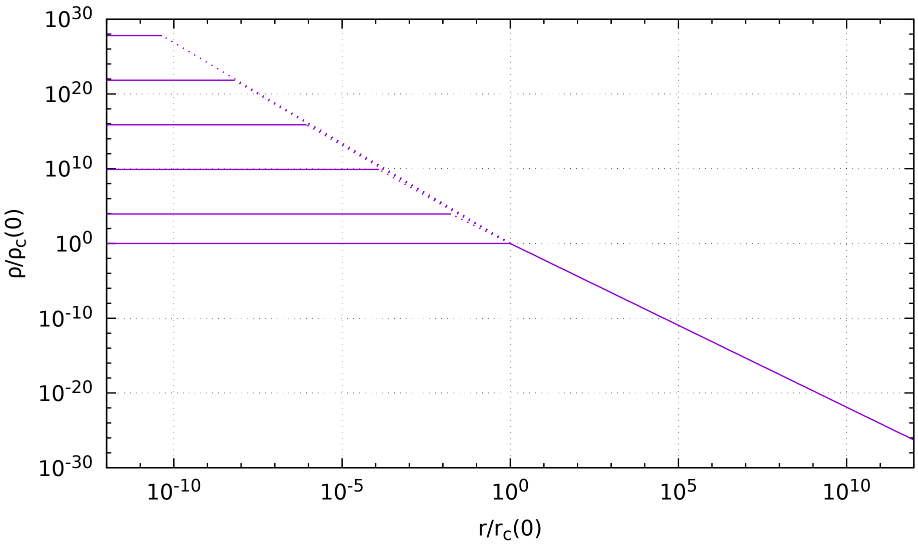

Figure 3.2: The simplified density profile of the halo at $t = 0$ along with the density profile of the core at selected times late in its evolution, when $a\hat{\sigma}^2 = 0.01$. The core profiles are drawn at $t/t_0 \in \{0.0,\ 290.0,\ 291.25,\ 292.5,\ 293.75,\ 295.0\}$ and the dotted lines only serve to bridge the gap between the profiles of the core and the outer region.

3.1 A simple model for the evolution of the core

To be able to determine when the core has transitioned into the relativistic regime a model for the typical velocities within the core needs to be developed. To get an idea for how such a model should look like, the following relations for a spherical system can be used:

$$M_c(t) = \rho_c(t)\frac{4}{3}\pi r_c^3(t), \quad v_c^2(t) = \frac{GM_c(t)}{r_c(t)}. \tag{3.2}$$

If we assume that the mass within the core is conserved,

$$\frac{M_c(t)}{M_c(0)} = \frac{\rho_c(t)\frac{4}{3}\pi r_c^3(t)}{\rho_c(0)\frac{4}{3}\pi r_c^3(0)} = \frac{\rho_c(t) r_c^3(t)}{\rho_c(0) r_c^3(0)} = 1, \text{ thus } \frac{\rho_c(t)}{\rho_c(0)} = \left(\frac{r_c(0)}{r_c(t)}\right)^3. \tag{3.3}$$

Inserting this result into the circular velocity equation (3.2) then yields

$$\frac{v_c^2(t)}{v_c^2(0)} = \frac{GM_c(t) r_c(0)}{GM_c(0) r_c(t)} = \frac{r_c(0)}{r_c(t)} = \left(\frac{\rho_c(t)}{\rho_c(0)}\right)^{1/3}. \tag{3.4}$$

Now, since the core mass is not actually conserved (the core loses mass to the outer halo as it shrinks) this simple equation is not sufficient to describe the typical velocities within the core, given approximately by the circular velocity. Again, fitting to the data from the full numerical solution given in Balberg et al. (2002) gives a more accurate model for the late-term evolution:

$$\frac{v_c^2(t)}{v_c^2(0)} = \left(\frac{1}{\varepsilon}\frac{\rho_c(t)}{\rho_c(0)}\right)^{1/3.5735} = \left(\frac{1}{\varepsilon}e^{\beta_1 t/t_0 + \beta_2}\right)^{1/3.5735} \tag{3.5}$$

where $\varepsilon = 14.1763(a\hat{\sigma}^2)^{-0.639864}$ and equation (3.1) was used to obtain the right hand side of equation (3.5). This velocity profile can be seen in figure 3.3 for selected values of $a\hat{\sigma}^2$, along with the curve given by the self-similar solution.

In figure 3.3 a horizontal line is also marked at $v_c^2(t)/v_c^2(0) = 3.7 \times 10^7$ which corresponds to $v_c(t) = 0.1c$, assuming that $r_c(0) = 1\,\text{kpc}$ and $v_c(0) = 50\,\text{km/s}$ (these are the typical size and velocity within dwarf galaxies that are inferred to have a core). Using this condition and equation (3.5) we can find the time at which the circular velocity becomes relativistic as a function of the cross-section. Denoting that time with t_{rtv} and solving for $v_c^2(t_{rtv})/v_c^2(0) = 3.7 \times 10^7$ yields

$$t_{rtv} = \frac{t_0}{\beta_1}\left(3.5735 \ln\left(3.7 \times 10^7\right) + \ln(\varepsilon) - \beta_2\right) \tag{3.6}$$

Figures 3.4 and 3.5 both show the time at which the circular velocity becomes relativistic, t_{rtv}, as a function of σ. By comparing the results for $t_{rtv}(\sigma)$ to the age of the Universe (13.8×10^9 years) we can get an estimate of what the cross-section of the SIDM would have to be in order for a halo with $r_c(0) = 1\,\text{kpc}$ and

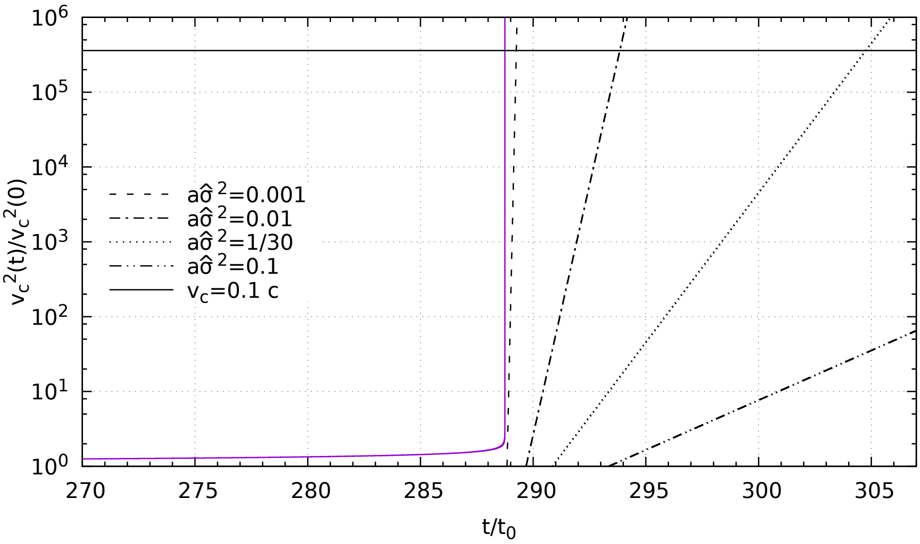

Figure 3.3: The circular velocity evolution in the core of a system described by the gravothermal fluid approximation. The solid curve shows the self-similar solution and the dotted lines represent the asymptotes of the numerical solution for different values of $a\hat{\sigma}^2$, which are well-fitted by equation (3.5). The solid horizontal line shows where $v_c(t) = 0.1c$, when $r_c(0) = 1\,\mathrm{kpc}$ and $v_c(0) = 50\,\mathrm{km/s}$.

$v_c(0) = 50\,\mathrm{km/s}$ to have been able to reach relativistic circular velocities in the core. The results, as can be seen in figure 3.5, for $t_{rtv}(\sigma) < 13.8 \times 10^9$ years is that $23.3\,\mathrm{cm}^2/\mathrm{g} < \sigma < 100.3\,\mathrm{cm}^2/\mathrm{g}$.

Now, it would be useful to generalize these results and get a better idea for what this cross-section range would be for halos with other values of $r_c(0)$ and $v_c(0)$. Using results from cosmological simulations performed by Springel et al. (2008) we obtain the following relation for Cold Dark Matter haloes[1]

$$r_{max} = 0.0344 v_{max}^{1.426}, \tag{3.7}$$

where v_{max} is the maximum circular velocity and r_{max} is the radius where this maximum is reached. If we assume that $v_c(0) \approx v_{max}$ and $r_c \approx 0.1 r_{max}$ we can compute the $t_{rtv}(\sigma)$ curve for more sets of values for $r_c(0)$ and $v_c(0)$, as is done in figure 3.6. There we can see, that for particles within the core of a SIDM halo to be able to reach relativistic velocities in a time period which is less than the age of the Universe, the initial circular velocity could not have been greater then $\sim 100\,\mathrm{km/s}$. Using this method we also appear to obtain the condition for all of the curves that for $t_{rtv}(\sigma) < 13.8 \times 10^9$ years than $\sigma \gtrsim 20\,\mathrm{cm}^2/\mathrm{g}$.

[1] Although these are not SIDM simulations, we note that the value of v_{max} and r_{max} do no change significantly between CDM and SIDM since the effect of self-interactions are only relevant in the inner regions of the halo, well within r_{max}.

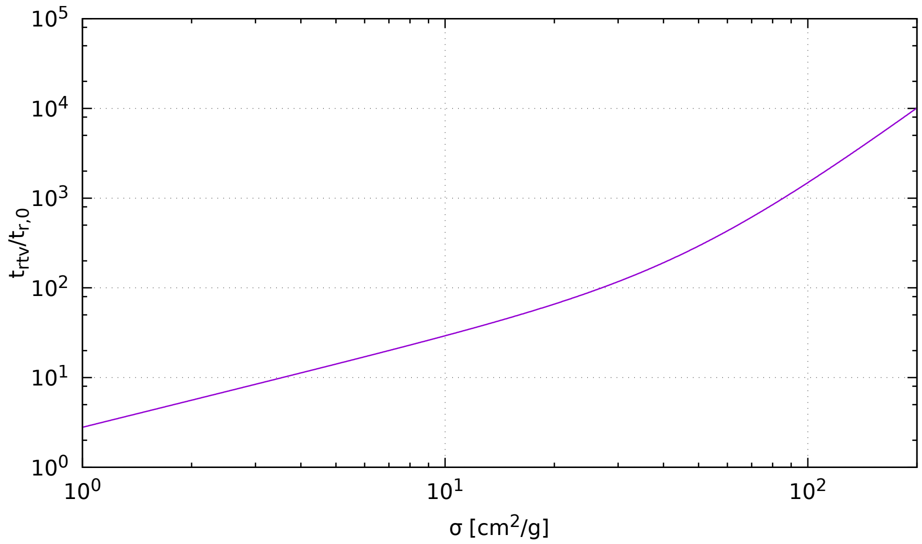

Figure 3.4: The time at which the circular velocity in the core becomes relativistic, in terms of the initial relaxation time, as a function of the cross section when $r_c(0) = 1\,\mathrm{kpc}$ *and* $v_c(0) = 50\,\mathrm{km/s}$.

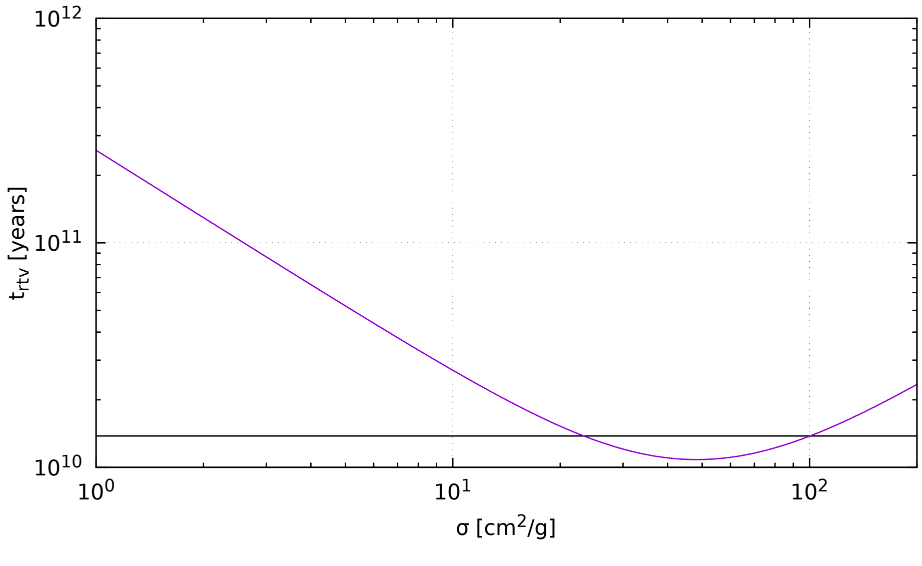

Figure 3.5: The time at which the circular velocity in the core becomes relativistic as a function of the cross section, when $r_c(0) = 1\,\mathrm{kpc}$ *and* $v_c(0) = 50\,\mathrm{km/s}$. *The solid black line represents the estimated age of the Universe,* 13.8×10^9 years.

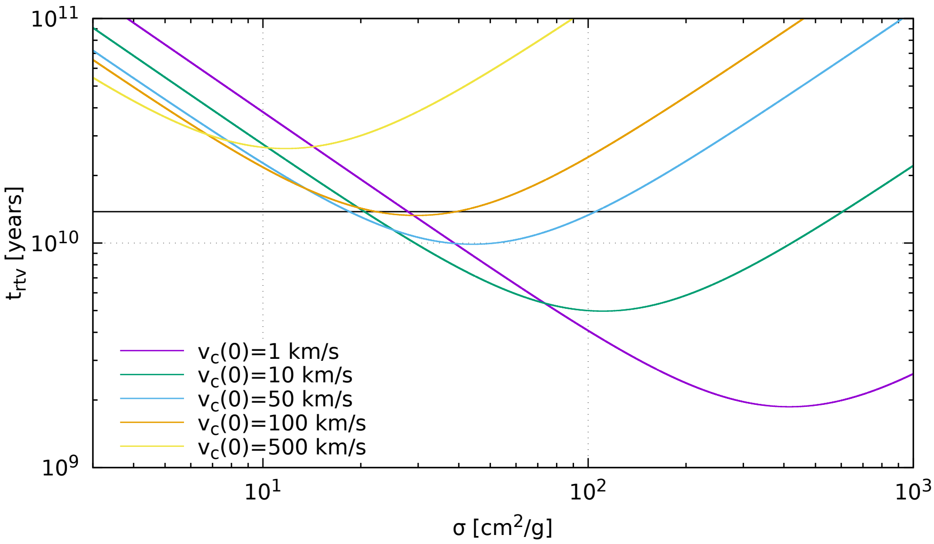

Figure 3.6: The time at which the circular velocity in the core becomes relativistic as a function of the cross section, for selected values of $v_c(0)$. *The solid black line represents the estimated age of the Universe,* 13.8×10^9 years.

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