Asset Price Bubbles

Kári Steinar Pétursson

Thesis of 30 ECTS credits
Master of Science in Financial Engineering

May 2019

Asset Price Bubbles

by

Kári Steinar Pétursson

Thesis of 30 ECTS credits submitted to the School of Science and Engineering
at Reykjavík University in partial fulfillment
of the requirements for the degree of
Master of Science (M.Sc.) in Financial Engineering

May 2019

Supervisor:

Dr. Sverrir Ólafsson, Supervisor
Professor, Reykjavík University, Iceland

Examiner:

Dr. Bjarki A. Brynjarsson, Examiner
Managing Partner, Sundaborg ehf, Reykjavík, Iceland

Asset Price Bubbles

Kári Steinar Pétursson

May 2019

Abstract

According to traditional finance theory, financial securities such as stocks have an intrinsic value defined as the discounted expected future dividends. An asset price bubble emerges when the price exceeds this intrinsic value. This thesis aims to mathematically formalize and analyse this phenomenon through several models which assume that the bubble is either rational or irrational. Rational bubble models assume that all market participants are rational and have rational expectations. Irrational bubble models allow for behavioural biases such as herd behaviour and overconfidence. A review of testing for bubbles empirically is included along with an analysis of bubble indicators such as a high P/E ratio, high CAPE ratio and low dividend yield, these ratios are applied on indexes from both the Icelandic and U.S. stock markets. A variance bound test is also applied on the S&P500. The main findings indicate that risk shifting, credit expansion and decreased interest rates help fuel bubble growth. The analysis implies that psychological factors can have a significant effect of bubble emergence and growth. The analysis also implies that bubbles can exist even though rational traders know that the asset's price is too high. This is due to the dispersion of opinion of the bursting time which leads to long lasting deviations from fundamentals.

Verðbólur

Kári Steinar Pétursson

maí 2019

Útdráttur

Hefðbundin fjármálafræði skilgreinir virði eignar t.d. hlutabréfs sem væntar, núvirtar arðgreiðslur frá eigninni. Þegar verðið fer umfram þetta hefur verðbóla myndast á eigninni. Þessi ritgerð setur myndun verðbóla fram stærðfræðilega ásamt því að greina hverjir undirliggjandi áhrifaþættir eru með hjálp líkana. Fræðilega eru verðbólum og líkönum þeirra skipt í tvo flokka, annars vegar rökréttar (e. rational) og hins vegar órökréttar (e. irrational). Fyrri flokkurinn gerir ráð fyrir að allir fjárfestar hafi rökréttar væntingar og að sálfræðilegir þættir hafi ekki áhrif á ákvarðanatöku þeirra. Seinni flokkurinn leyfir hins vegar áhrif frá sálfræðilegum þáttum eins og hjarðhegðun. Framkvæmd er greining á þáttum sem benda til þess að verðbóla sé til staðar eins og hátt V/H hlutfall (e. P/E ratio), hátt CAPE hlutfall og lágt A/V hlutfall (e. dividend yield), þessum hlutföllum ásamt variance bound test eru beitt á hlutabréfavísitölur á Íslandi og í Bandaríkjunum. Helstu niðurstöður benda til þess að 'risk shifting' eða tilhneiging stjórnenda til þess að ráðast í áhættusamari fjárfestingar með það að markmiði að hámarka virði hlutafés en auka þannig áhættu lánveitenda, 'credit expansion' eða aukið aðgengi fjárfesta að lánsfé ásamt lágu vaxtastigi hjálpi verðbólum að myndast og vaxa. Niðurstöður benda einnig til þess að sálfræðilegir þættir geti haft áhrif á verðbólur. Að lokum benda niðurstöður til þess að verðbólur geti myndast þrátt fyrir að rökréttir fjárfestar viti að verð eignar er of hátt. Þetta er mögulegt vegna ólíkra skoðana á því hvenær verðbólan springur sem leiðir til langtíma fráviks frá innra virði eignar.

Asset Price Bubbles

Kári Steinar Pétursson

Thesis of 30 ECTS credits submitted to the School of Science and Engineering
at Reykjavík University in partial fulfillment of
the requirements for the degree of
Master of Science (M.Sc.) in Financial Engineering

May 2019

Student:

....
Kári Steinar Pétursson

Supervisor:

....
Dr. Sverrir Ólafsson

Examiner:

....
Dr. Bjarki A. Brynjarsson

....
date

....
Kári Steinar Pétursson
Master of Science

"Price is what you pay; value is what you get"
Warren E. Buffett (2008)

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Background

Asset price is dependent on supply and demand and changes as market participants react to new information entering the market. This new information could for example be new income statements, legislation, competition or new reports on the state of the economy. A financial security such as a stock has some intrinsic value which according to traditional finance theory depends on the dividends the owner expects to receive from holding the stock. The expectation is a key factor which each investor has to estimate and changes as new information enters the market. When an asset's price increases above what is assumed to be it's intrinsic or fundamental value, we say that a bubble has formed on the asset's price.

For centuries asset prices have exhibited bubble behaviour, these bubbles typically have three distinct phases. The first phase is often defined by some sort of financial liberalization or a decision by the central bank to increase lending. This lending increase results in increased asset prices, such as stocks or real estate. This can continue for several years as the bubble inflates. During the second phase, the bubble bursts and asset prices collapse in a short time period. The third phase includes defaults of firms or individuals who have bought assets at inflated prices. (Allen & Gale, 2000)

When attempting to model bubbles in asset prices we make distinctions between rational and irrational bubbles. The former assumes all market participants act rationally and have rational expectations, these concepts will be expanded on later. The latter does not make this assumption and allows for human behaviour such as herd behaviour or overconfidence to affect market prices. It is however difficult to identify a bubble before it bursts as expected future dividends are difficult to estimate. Several attempts have been made to model bubbles in order to understand their nature and behaviour, we will analyse several examples under both rationality and irrationality of investors.

A bubble bursting can lead to a recession and the severity of such a recession depends on the size of the bubble when it bursts. Historical examples of this are given in chapter 2.

1.2 Rational Expectations Hypothesis

The Rational Expectations Hypothesis (REH) states that the expectation of the future realisation of an economic variable is rational if the agent uses all the available and useful information to predict it. It was first proposed by Muth (1961). According to REH the expectation at time t of the realisation of x at time $t + 1$ can be written as the mathematical expectation of x_{t+1} conditional on the information set available at time t. Under REH, economic agents form unbiased forecasts of future values of an economic variable and their forecast errors are random with mean zero. More formally, the agents subjective distribution of expectations corresponds to the objective probability distribution of the true model describing the economy. This does not argue that the agents are always correct in their expectations of future variables. It does however state that the forecast error is random with a mean of zero, which means that on average rational expectations will be correct.

1.3 Efficient Market Hypothesis

In the 1960's, Eugene Fama developed the The Efficient Market Hypothesis (EMH). It states that asset prices fully reflect all available information. Thus it should be impossible to "beat the market" consistently since market prices should only react to new information. The EMH requires agents to have rational expectations, which means that on average the population is correct even if no one person is. It is however not expected for agents to be rational, when new information is introduced, some investors may overreact and others underreact. Reactions from investors should be random and normally distributed so that the net effect on prices cannot be consistently exploited to make profits. Thus, any one person may be wrong about the market, but the market as a whole is always right.

This hypothesis has faced criticism from investors and researchers who have disputed the EMH empirically and theoretically. Some claim the imperfections of financial markets are a combination of irrational human behaviour such as overconfidence, overreaction and other human error in reasoning and information processing which lead to psychological feedback mechanisms. (Malkiel, 2003)

1.4 Utility

To explain consumer behaviour, economics relies on the fundamental that people choose those goods and services they value most highly. For economists to describe how consumers choose between different consumption possibilities, the notion of utility was developed. Utility describes the satisfaction a consumer obtains from consuming one unit of the consumption good. Another way to look at it is how consumers rank different goods and services. It is however not a psychological function or feeling that can be observed or measured but a scientific construct that economists use to understand how rational consumers make decisions. Consumer demand functions are derived from the assumption that people maximize their utility when choosing between consumption goods.

Marginal utility is the additional utility gained from consuming one additional unit of the consumption good or the first derivative of the total utility. Marginal utility declines as the consumed amount of the consumption good increases. (Samuelson & Nordhaus, 2010)

1.5 Liquidity

Market liquidity describes how big the trade-off between the time it takes to sell an asset and the price it can be sold for. If the market is liquid, the selling of an asset is fast with a small price reduction. In a relatively illiquid market, selling the asset quickly will require some price reduction. Money or cash is the most liquid asset as there is no trade-off between time and its value. Real estate however is not liquid as the time it takes to sell the asset is highly dependent on its sales price. Given an unchanged price, it would take a long time to sell the asset, however given a large price reduction it could be sold quickly. Liquidity in the stock market has a similar meaning of how easily assets (stocks) can be converted to cash. The market for a stock is said to be liquid if one can easily sell the share and it's sale has a small impact on the stock's price. Another way is to look at the bid-ask spread where liquid stocks have a small spread while illiquid typically have larger spreads.

1.6 Objective

The objective of the thesis will focus on if and how financial theory can explain the existence and behaviour of asset price bubbles, both under a rationality framework where all investors act rationally and under a non-rational framework, where investors can make irrational decisions. The main focus will be on the theoretical side. Model analysis and simulations along with empirical testing through valuation metrics such as P/E, CAPE ratio and dividend yield will be done to better understand the behaviour of asset price bubbles along with applying variance bound tests on stock indexes.

1.7 Structure of thesis

The thesis will start by introducing some important concepts such as Bubbles in general, Rational Expectations and Efficient markets. Then in chapter 2, some well known historical bubbles will be discussed such as the crash of 1929 and the US housing bubble. Chapter 3 introduces asset pricing and how rational bubbles can form. The chapter will also include models that introduce stochastic bubbles through utility maximization arguments and credit models along with looking at pricing by no-arbitrage. Chapter 4 introduces irrational investor behaviour and how bubble formations can occur under the irrationality of human behaviour. Behavioural models will also be analysed and simulated.

Chapter 2

Historic Bubbles

Throughout history, markets all over the world have experienced bubbles in asset prices, these large price increases which are often a result of pure speculation can cause a severe recession affecting millions of people. This chapter provides a short summary of a few well known and extreme incidents of such events.

2.1 Tulip mania

The Dutch tulip mania is often considered to to be the first major financial speculative bubble which took place in the 17th century. Tulips were introduced into Europe from Turkey in 1550 and the vividly coloured flowers became a popular and costly item. Prices for individual bulbs began to rise to unwarranted heights in northern Europe with everyone dealing in bulbs and speculating on its price which was believed to have no limit. This price increase craze reached its height in Holland during 1633–37 with a single bulb selling for more than ten times the annual pay of a skilled craftsman. Homes, estates, and industries were mortgaged and livestock sold so that bulbs could be bought for later resale at higher a price. The crash came in early 1637, sweeping away fortunes and leaving behind financial ruin for people throughout the Netherlands. (French, 2007)

2.2 Crash of 1929

The twenties, the decade that followed World War I was a time of great wealth and excess in the US. Post war optimism, a growing industrial sector and new technologies led to rising stock prices. This rise encouraged more people to invest, often taking highly leveraged positions. This, along with the belief that stock prices would continue to rise lead to a speculative bubble.

Through the twenties share prices rose to unprecedented heights, with the Dow Jones Industrial Average increasing from 63 in 1921 to its peak in 1929 at 381. (National Bureau of Economic Research, 2018) This boom ended in a crash in October 1929, and by mid November the Dow had lost almost half its value. This downturn continued until 1932 when the Dow closed at 41. The following years are known as the great depression, which affected most developed countries at the time.

2.3 The Market Crash of 1987

The market crash of 1987 or "Black Monday" refers to Monday, October 19, 1987 when stock markets all over the world fell significantly. The Dow Jones Industrial Average had more than tripled in the preceding 5 years. As a result, valuations were excessive with price to earnings ratios exceeding 20. Then the Dow plunged 22% in a single day. This quickly affected markets around the world with most major stock markets seeing declines exceeding 20%. The selloff halted the next day due to the Fed stepping in. Some say that a speculative bubble was in place and that the price drops were simply a return to normalcy. Speculation still remains as to the causes of this extremely rapid crash, many point to the lack of trading curbs which have been implemented into markets today. Another cause is the automated trading programs where human decision making is taken out of the equation with buy and sell orders generated automatically. After the crash, circuit breaker rules were implemented by the SEC which halt trading on a particular stock if it drops by more than a predetermined percentage within a predetermined time frame.

While the automated programs contributed to the selloff, the majority of trades at that time went through a very slow process which often included several phone calls per trade, which is very slow compared to today's trades.

2.4 Dot-com bubble

In the 1990s rapid technological advancements occurred, computer ownership rose significantly and as a result, internet usage increased. Immediately, businesses saw the internet as an opportunity for significant profits and many new companies like Yahoo, Ebay and Amazon were starting up. This lead to many online business founders, shareholders and employees becoming very wealthy almost overnight.

This, along with low interest rates lead to significant speculative investments in "Dot-Com" companies at any valuation. Investors had confidence that the companies would turn future profits and thus overlooked traditional metrics, such as the P/E ratio. This was also fed with easy access to capital, lowering of the U.S. Capital Gains Tax in 1997 and the huge amounts to be made when companies went through IPO's. From 1995 to 2000, the Nasdaq-100 Composite index rose 400%. By early 2000, the reality started to sink in and investors realised that the valuations of these companies had exceeded any possible future earnings and a speculative bubble had emerged. By the end of 2002, the index had dropped to 1,114 from 4,700 at its peak. (Yahoo, finance, 2018)

2.5 United States housing bubble

In the years leading up to the 2008 crash, banks had increased lending to people with poor credit and a high probability of defaulting on their mortgages. This is what is called a sub-prime mortgage. This caused property prices to rise significantly, peaking in 2006. These mortgages were pooled together and sold as a single tradeable security known as a collateralized debt obligation or CDO which had a much higher rating classification than the underlying because of diversification. When these loans started to default, a chain effect started in the economy which resulted in several banks and lending institutions going bankrupt. This caused the housing bubble to burst and had significant economic consequences around the world. Figure 2.1 shows the Housing Price Index in the U.S. from 1987 - 2019, a clear drop in home prices in 2008 is apparent. (Financial Crisis Inquiry Commission, 2010)

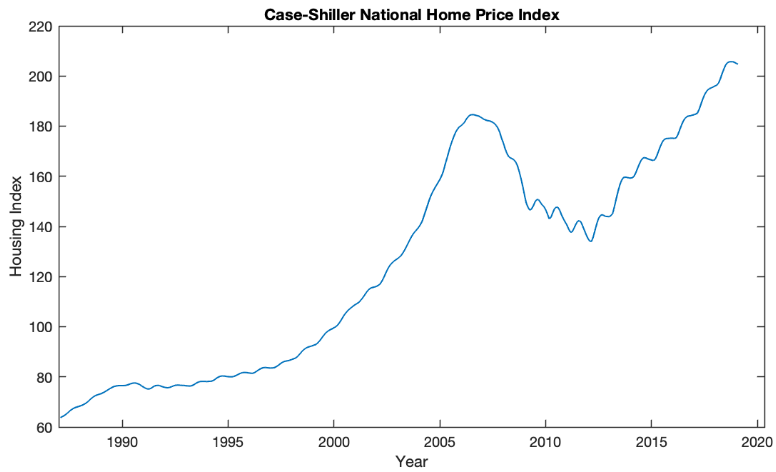


Figure 2.1: Case-Shiller U.S. National Home Price Index 1987-2019

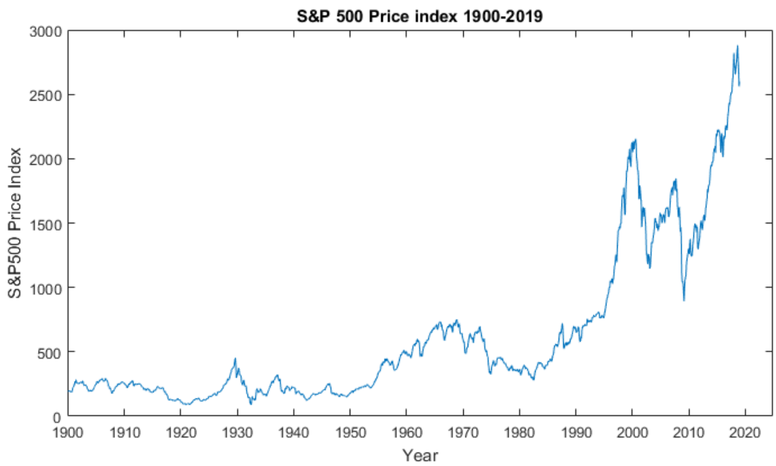


Figure 2.2: S&P500 Index 1900- 2019

Figure 2.2 shows the S&P500 index from 1900 to 2019 adjusted for inflation using the consumer price index. Here, many of the crashes described above are notable, the crash of 1929, crash of 1987, the dot-com bubble in the years leading up to 2000 and the housing bubble in 2008.

Chapter 3

Rational Bubbles

3.1 Definition

Rational bubble models assume that market participants act rationally and have rational expectations of an asset's price. In standard financial asset pricing theory, the value of an asset should equal the present value of future dividends the owner expects to receive. This is the systematic force driving the asset price. If prices follow this expectation, the rational expectations equilibrium is driven by fundamentals. However, in principle other rational expectations equilibria are possible. Prices can exceed the fundamental but still be rational deviations, this is called a rational bubble. We assume that bubbles exist in an infinite time horizon, there are no arbitrage opportunities and investors have symmetrical information, these concepts will be discussed in detail later.

Blanchard and Watson (1982) concluded that given both rationality of behaviour and of expectations, it is not implied that the price of an asset should necessarily be equal to it's fundamental value. They then proposed a model where bubbles are consistent with rationality along with testing for bubbles statistically. Diba and Grossman (1988b) concluded that the existence of a rational bubble would reflect a self confirming belief that an assets price depends on a variable (or combination of variables) which is intrinsically irrelevant, not part of the market fundamentals. For this to be a rational expectations solution, the bubble's value today must equal the discounted value of next periods expected value while being independent of the systematic force driving the asset price. Thus the asset price$^1P_t{}^0$ has two components, the fundamental$^1v_t{}^0$ and the bubble$^1B_t{}^0$ or $P_t = v_t + B_t$

An example of this is Fiat money, which is a currency not backed by gold such as the US dollar. Fiat money has value without having intrinsic value, along with having an infinite time horizon, its value comes purely from demand and supply. (Goldberg, 2005)

Tirole (1982, 1985) suggested what kind of assets could be subject to asset price bubbles. The conclusion is that the asset must be durable because the expectation of a resale value must be present to generate a resale value. Scarcity also plays a role as an asset which can be easily produced if a bubble on its price forms, drives down the price. Another conclusion is that for a bubble to form, an active market must be in place and a social mechanism for coordinating the common belief that the price will continue to grow. Tirole (1982) also shows that different information between traders and restrictions on short selling does not affect the theoretical possibility of asset price bubbles.

3.2 Rational Bubbles - Theory

To describe how rational bubbles in asset prices can occur, we start by looking at how assets are generally priced. By simply taking the definition of net return

$$r_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1 \tag{3.1}$$

where P_t and D_t represent the price and dividend payment at time t. If we rearrange, take rational expectations and assume that r is constant or $E_t[r_{t+1}] = r, \forall t$. The price at time t is,

$$P_t = \frac{1}{1+r} E_t [P_{t+1} + D_{t+1}] \tag{3.2}$$

So, the price at t is the discounted expected future price and dividend payment at time $t+1$. If the difference equation 3.2 is solved forward by replacing P_{t+1} with $E_{t+1}[P_{t+2} + D_{t+2}]/(1+r)$ and then P_{t+2}, etc. Using the law of iterated expectations, we get

$$P_t = E_t \left[\sum_{i=1}^{T-t} \frac{1}{(1+r)^i} D_{t+i} \right] + E_t \left[\frac{1}{(1+r)^{T-t}} P_T \right] \tag{3.3}$$

Thus, the equilibrium price is given by the expected discounted value of future dividends paid to the owner plus the expected discounted value at T. If we first examine assets with finite maturity. Intuitively if an asset has a finite maturity such as fixed income bonds, the price after maturity is zero $(P_T = 0)$. Which means that the asset price is unique and is simply the expected future discounted dividend stream until time T, often called the fundamental value (v_t). If the asset has infinite maturity, $T \to \infty$, the unique solution of the price only equalling the discounted dividend stream holds if the transversality condition is fulfilled, which states the following

$$\lim_{i \to \infty} E_t \left[\frac{1}{(1+r)^i} P_{t+i} \right] = 0 \tag{3.4}$$

Intuitively the transversality condition states that as the time horizon approaches infinity the expected discounted price approaches zero. If however, equation 3.4 is not fulfilled, the fundamental value solution is only one solution of many to equation 3.3. Among them, any price $P_t = v_t + B_t$, where B_t is a bubble component which follows

$$B_t = E_t \left[\frac{1}{(1+r)} B_{t+1} \right] \tag{3.5}$$

Thus, we can write the asset price as the sum of expected discounted dividends term and a B_t bubble term. (Brunnermeier, 2008)

$$P_t = E_t \left[\sum_{i=1}^{\infty} \frac{1}{(1+r)^i} D_{t+i} \right] + B_t \tag{3.6}$$

Now we examine how B_t can evolve. The simplest path B_t can follow would be that of a deterministic bubble, define $\beta = (1+r)^{-1}$ and the bubble evolves according to $B_t = B_0 \beta^{-t}$. In this case deviations grow exponentially. For this to be rational, the increase must continue forever, which makes this type of bubble implausible.

Another path B_t could follow is that in each period the bubble follows

$$B_t = (\pi)^{-1}(1+r)B_{t-1} + e = \frac{(1+r)}{\pi} B_{t-1} + e_t \qquad \text{with probability } \pi$$
$$B_t = e_t \qquad \text{with probability } 1-\pi \tag{3.7}$$
$$\text{with } E(e_t) = 0$$

Here the bubble term grows at a higher rate than r to compensate for the risk of it crashing. Increasing π decreases the rate at which B_t grows at and vice versa. The probability that a bubble such as this remains in n periods is π^n. This probability converges to zero as n grows larger. Thus rational bubbles can exist even though traders know they will burst at some point in time. The probability π may be some function of how far the price has deviated from the fundamental or how long the bubble has lasted. The bubble component may also be stochastic. An example would be when the bubble component is assumed to be deterministically related to a stochastic dividend process. (Blanchard & Watson, 1982)

The case of $B_t < 0$ cannot occur on a limited liability asset. The reason being that a growing negative bubble, conditional on the bubble surviving would mean that the asset price has to become negative at some point in the future. This is not possible due to the limited liability of shareholders. (Brunnermeier & Oehmke, 2012) What this also implies is that if a bubble bursts at any point in time, it cannot restart. This also implies that a bubble on the asset price must be present when it starts trading. These are also conclusions reached by Diba and Grossman (1988b) which will be reviewed in chapter 3.3.

3.2.1 Simple Numerical Example

Suppose we have a company which has recently begun trading on an exchange for 85 per share at time t. The last dividend paid was 1 per share. The firm is expected to grow its dividend payments at the rate shown in table 3.1 for the first 4 years. For the 5th year onwards, the growth is expected to be stable at a rate of 7%.

Table 3.1: Dividend structure years 1-5

Year	Growth Rate	Dividend	PV of Dividend
1	20%	1.20	1.12
2	18%	1.42	1.24
3	15%	1.63	1.33
4	10%	1.79	1.37
5	7%	1.92	1.37

We start by calculating the sum of PV Dividends for the first 4 years

$$P_t = \sum_{t=1}^{4} \frac{1}{(1+r)^t} D_t = 5.05$$

However, the price must also include the terminal value which is the discounted dividend stream from year 5 onwards. To estimate this, we use the Gordon growth model (equation 3.8), which is derived in Gordon and Shapiro (1956).

$$P_t = \frac{E(D_{t+1})}{r - g} \tag{3.8}$$

With g being the stable growth rate and r the appropriate required rate of return for the asset. r can be estimated using Capital Asset Pricing Model or CAPM

$$E(r) = r_f + \beta(E(r_m) - r_f) \tag{3.9}$$

CAPM describes the relationship between the expected return (r) and risk of investing in a security (β). The basis is that investors need to be compensated for the systemic or non-diversifiable risk involved with investing. This is done through the risk premium factor $(E(r_m) - r_f)$ and adjusted by the β coefficient, which measures the stock volatility relative to the market. A security with $\beta = 1$ follows the market volatility, any lower or higher values represents volatility deviations from the market. r_f is the risk free rate, often the long term government bond rate and r_m is the expected market return. Using a beta coefficient of $\beta = 1.25$, $r_f = 3\%$ and $r_m = 7\%$, we estimate a required return of

$$r = 3\% + 1.25 \cdot (7\% - 3\%) = 8\%$$

Now, we can use equation 3.8 to estimate the price per share

$$P_4 = \frac{1 \cdot (1 + 7\%)}{(8\% - 7\%)} = 107 \tag{3.10}$$

As P_4 is calculated at the end of year 4 we need to discount it to present. $107/(1 + 7\%)^4 = 76.3$. The intrinsic stock value is

$$P_I = 5.05 + 76.3 = 81.35 \tag{3.11}$$

As the stock trades at 85, this suggests that a bubble of $B_t = 3.65$ might exist. This is a very simple and limited example as many firms do not pay consistent dividends. While others pay consistently but occasionally retain dividends for a certain time period due to investments, mergers etc.

3.3 Rational Bubbles - Utility Maximization

The properties of rational bubbles using utility maximization arguments were put forward by Diba and Grossman, 1988b and Cochrane, 2000. If a stock is bought at time t, the payoff next period is the stock price plus the dividend paid, $x_{t+1} = p_{t+1} + d_{t+1}$. x_{t+1} is a random variable. Assume that and individual maximizes his expected utility over current and future values of consumption,

$$u(c_t, c_{t+1}) = u(c_t) + \beta E_t[u(c_{t+1})] \tag{3.12}$$

c_t is a stochastic process which represents consumption of a single perishable good. $u(\cdot)$ represents a utility function which is increasing and concave, utility is gained from consuming c_t, hence $u(c_t)$. β is a discount factor for future consumption and E_t represents an expectation value which is based on an information set that contains current and past values of the variables entering the model.

The investor can freely buy or sell as much of the payoff x_{t+1} at a price p_t in units s as he wishes. Further assumptions are that each share pays a dividend d_t units of the consumption good and that each period the investor receives an endowment y_t which can be interpreted as the original consumption level if the investor bought none of the asset. This leads to a maximization problem

$$\max_{\{s\}} u(c_t) + E_t\left[\beta u(c_{t+1})\right] \tag{3.13}$$

such that

$$c_t = y_t - p_t s$$
$$c_{t+1} = y_{t+1} + x_{t+1} s$$

Substituting the constraints into the objective function along with setting the derivative with respect to s equal to zero, we get the first order condition for the maximization problem.

$$p_t u'(c_t) = E_t\left[\beta u'(c_{t+1}) x_{t+1}\right] \tag{3.14}$$

The left hand side of equation 3.14 represents the loss in utility if the investor buys another unit of the asset. The right hand side of the same equation represents the increase in the discounted, expected utility gained from the extra payoff at $t+1$. Thus, the investor continues to buy or sell the asset until the marginal loss is equal to the marginal gain. Equation 3.14 can be written as

$$p_t = E_t\left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1}\right] \tag{3.15}$$

This is the central asset pricing formula. Given the payoff x_{t+1} and the investors consumption choice c_t, c_{t+1} it gives the price p_t. (Cochrane, 2000). Diba and Grossman (1988b) extend this by having a market clearing condition which states that consumption equals the received endowment plus the dividend or,

$$c_\tau = y_\tau + d_\tau \quad \forall \tau \geq t \tag{3.16}$$

Substituting 3.16 into 3.14 gives the following asset pricing equation

$$E_t q_{t+1} - \beta^{-1} q_t = -E_t\left[u'(y_{t+1} + d_{t+1}) d_{t+1}\right] \tag{3.17}$$

with

$$q_t = u'(y_t + d_t) p_t$$

Equation 3.17 is a first order expectational difference equation. The forward looking solution, denoted by F_t and often called the market fundamentals solution to 3.17 is

$$F_t = \sum_{j=1}^{\infty} \beta^j E_t\left[u'(y_{t+j} + d_{t+j}) d_{t+j}\right] \tag{3.18}$$

This equation sets the current product of the stock price and marginal utility of consumption equal to the discounted sum of future dividends and the marginal utility of consumption. The general solution to equation 3.17 is a sum of the fundamental, F_t and a rational bubble component B_t.

$$q_t = F_t + B_t \tag{3.19}$$

With B_t being the solution to

$$E_t B_{t+1} - \beta^{-1} B_t = 0 \tag{3.20}$$

A B_t value greater than zero would imply that a rational bubble exists at time t and that q_t does not only depend on the fundamental component F_t. Any solution to 3.20 would follow

$$E_t B_{t+j} = \quad {}^{j} B_t; \quad 8\ j > 0 \tag{3.21}$$

Given the existence of a non zero rational bubble component at time t, equation 3.21 implies that the the expected value at time t+j increases or decreases at a rate of $\ ^{1}$. A solution to 3.20 should also satisfy

$$B_{t+1} \quad {}^{1} B_t = z_{t+1} \tag{3.22}$$

The random variable z_{t+1} represents new information which is available at date t+1 and is generated by a stochastic process which follows

$$E_{t\ \ j}\, z_{t+1} = 0 \quad 8\ j \quad 0 \tag{3.23}$$

This information can be unrelated to F_{t+1} and thus intrinsically irrelevant. It can however have an impact on relevant variables, such as d_{t+1} through parameters which are not present in F_{t+1}. The general solution to 3.22 for t 0 is

$$B_t = \quad {}^{t} B_0 + \sum_{=1} \quad {}^{t} z \tag{3.24}$$

The first term of equation 3.24 expresses the product of $\ ^{t}$ and the rational bubble component at $t = 0$. The second term consists of a weighted sum of realizations of z_t for $= 1{:}{:}t$. The terms are weighted in such a way that the contribution of z_t to B_t increases exponentially with the difference between t and .

Diba and Grossman (1988b) also show that if a rational bubble does not exist at time t, with t 0, it cannot exist at date $t + n$ ¹$n > 0$º. This is based on the argument that negative rational bubbles cannot exist due to free disposal of shares, which means that a rational bubble component must satisfy equation 3.22 and satisfy B_{t+1} 0. Which gives the following inequality,

$$z_{t+1} \quad {}^{1} B_t \quad 8\ t \quad 0 \tag{3.25}$$

This states that the realization of z_{t+1} must be large enough to ensure that equation 3.22 gives a non-negative value for B_{t+1}. If we assume that B_t equals zero, then equation 3.25 implies that z_{t+1} 0. Equation 3.23 states that the expectation value of z_{t+1} is zero. Thus if $B_t = 0$, then $z_{t+1} = 0$ with probability 1. This result states that if a rational bubble does not exist at time t, t 0, it cannot start at time $t+n$ ¹$n > 0$º. Which means that if a rational bubble exists now, it must have started at time t = 0. Hence, the stock must have been overvalued in relation to its fundamental at every past day of trading since its inception.

Diba and Grossman (1988b) then argue that, given the impossibility of a negative rational bubbles component a new independent positive rational bubbles component cannot start after an existing positive rational bubbles component bursts.

What this analysis shows is that similar to chapter 3.2, theoretically a rational bubble can exist or an assets price can be dependent on both market fundamentals and a bubble term. This also imposes theoretical restrictions on the existence of rational bubbles while directly ruling out negative ones. These theoretical restrictions are, as mentioned above that a rational bubble can only start on the first day of the stock trading. Which implies that the stock has been overvalued relative to market fundamentals since the first day of trading. The second restriction is that a positive rational bubble cannot restart once it bursts.

3.4 Rational Bubble Model with Credit

Allen and Gale (2000) developed a model in which they show how asset prices are related to the amount of credit and how uncertainty about asset payoffs can lead to an asset price bubble. This bubble formation occurs through two main phenomena, risk shifting and credit expansion.

- Risk shifting occurs when managers make overly risky investment decisions in order to maximize profits and shareholder value at the expense of the firms debtholders. As the debtholders payoff is independent of profits, assuming the firm can make the debt payments the risk is shifted from shareholders to debtholders. The claim stockholders have on a leveraged firm can be viewed as a call option on the firms asset value. As the risk for equity holders is limited to the equity of the firm, managers have an incentive to substitute safe assets/investments with risky ones. This raises potential upside of the call option a shareholder holds. When investors borrow capital to invest in preexisting assets, risk shifting can increase the assets price above its fundamental value.

- Credit expansion occurs when the amount of capital in the economy increases. This often occurs when governments borrow to finance their activities. Credit expansion interacts with risk shifting in two ways. It can help encourage investor to fund their risky investments at the current date, thus credit expansion has an effect on asset prices. The anticipation of credit expansion in the near future can also increase current asset prices and have greater effect on the likelihood of an eventual crisis.

This model has two versions. The first version only has one source of randomness, real asset returns. The second version is extended to include the dynamic effects of credit expansion.

3.4.1 Asset Pricing with Uncertainty Generated by the Real Sector

- There are two dates, $t = 1, 2$. Each date has a single consumption good.

- An investor can choose between two assets. A safe asset X_S in variable supply and a risky asset X_R in fixed supply.

  - The safe asset pays a fixed return r. If at date 1, x units of the consumption good are invested in the safe asset the return at date 2 is rx.

  - At date 1 there is 1 unit of risky asset. If at date 1 $x \geq 0$ units of the consumption good are invested in the risky asset, Rx units are received at date 2. R is a random variable which has a continuous positive positive density $h(R)$ on $[0, R_{MAX}]$ and mean $\overline{R}$.

Return on the safe asset is determined by the marginal product of capital in the economy. For $x \geq 0$ units of the consumption good at date 1, there are $f(x)$ units at date 2. With, $f'(x) > 0$; $f''(x) < 0 \; \forall \; x$ and $f'(0) = \infty$; $f'(\infty) = 0$. A cost $c(x)$ is associated with investing in the risky asset. This cost is incurred at $t = 1$. With, $c'(x) > 0$; $c''(x) > 0 \; \forall \; x$ and $c(0) = c'(0) = 0$.

There is a number of risk neutral investors which have no wealth of their own and finance all their investments through credit from banks. There is also a number of banks, each one having $B > 0$ units of the consumption good to lend. Banks cannot distinguish between valuable and worthless assets, thus they must lend to investors. All loan contracts have the same fixed rate r and the same terms, independent of loan size and customer.

The investors portfolio must consist of units of the consumption good in both X_S and X_R, the payoff or liquidation value at $t = 2$ becomes

$$rX_S + RX_R - r(X_S + PX_R) = RX_R - rPX_R \tag{3.26}$$

Which is the return on the holdings in the safe and risky assets with the loan repayment subtracted. The amount invested in the safe asset drops out as can be seen in equation 3.26. The investors maximization problem becomes

$$\max_{X_R \geq 0} \int_{R^*}^{R_{MAX}} (RX_R - rPX_R)h(R)dR - c(X_R) \tag{3.27}$$

Here, $R^* = rP$ is the critical value of return on the risky asset, if return on the risky asset is below R^* the investor defaults as the loan rate is r. As there is only one unit of the risky asset, the market clearing condition for the risky asset X_R is

$$X_R = 1 \tag{3.28}$$

The market clearing condition for the credit market is

$$X_S + P = B \tag{3.29}$$

As the total amount borrowed is equal to the amount invested in the safe asset plus the market value of the risky asset. The market clearing condition for capital goods is

$$r = f'(X_S) \tag{3.30}$$

since the return on the safe asset is the marginal product of capital. An equilibrium for the model solves equation 3.27 while not violating the market clearing conditions. To determine the demand for credit, the first order condition for the maximization problem in equation 3.27 equates the expected net return on a unit of X_R to the marginal cost of investment, this ensures that the investors make zero profits on the last dollar borrowed and in turn determines the demand for credit. The condition that the return on the safe asset is equal to the marginal product of capital determines its equilibrium amount. By substituting $X_R = 1$ into equation 3.27, the decision problem becomes

$$\int_{R^*}^{R_{MAX}} (R - rP)h(R)dR = c'(1) \tag{3.31}$$

By substituting the budget constraint $X_S = B - PX_R$, the market clearing condition for the capital market becomes,

$$r = f'(B - P) \tag{3.32}$$

Equations 3.31 and 3.32 define the equilibrium. In this model, all investors are risk neutral. As a result, we define the fundamental value of the risky asset as the price an investor is willing to pay for one unit if there is no risk shifting. The maximization problem an investor who has wealth B faces is to choose a portfolio which consists of (X_S, X_R) and solves the following problem, this defines the fundamental value

$$\max_{(X_S, X_R) \geq 0} \int_0^{R_{MAX}} (rX_S + RX_R)h(R)dR - c(X_R) \quad \text{subject to} \quad X_S + PX_R \leq B \tag{3.33}$$

By using the first order condition for equation 3.33 we find the fundamental price $\bar{P}$. Which is the price an investor would be willing to pay for one unit of X_R if he were to use his own capital.

$$\bar{P} = \frac{1}{r}\left[R - c'(1)\right] \tag{3.34}$$

Equation 3.34 defines the fundamental value as the discounted price of net returns. Our goal is to show that the equilibrium price is higher than the fundamental or $P > \bar{P}$, this would indicate a bubble. We can rearrange equation 3.31 to

$$P = \frac{1}{r}\left[\frac{\int_{R^*}^{R_{MAX}} Rh(R)dR - c'(1)}{Pr(R \geq R^*)}\right] \tag{3.35}$$

An important component of the model is the risk shifting and the fact that the risky asset is in fixed supply. Borrowers buy the risky asset because they receive the surplus if the return is high but the bank bears the loss if the return is low. This leads to borrowers bidding up the price of the risky asset above the fundamental value.

Proposition 1 states that the equilibrium asset price P is at least as high as the fundamental price $\bar{P}$. It also states that P is strictly higher if the probability of bankruptcy is positive $(Pr(R < R^*) > 0)$.
This is proven as follows, rewrite equation 3.35

$$\begin{aligned} rP &= \frac{\int_{R^*}^{R_{MAX}} Rh(R)dR - c'(1)}{Pr(R \geq R^*)} \\ &= \frac{\int_{0}^{R_{MAX}} Rh(R)dR - c'(1) - \int_0^{R^*} Rh(R)dR}{Pr(R \geq R^*)} \\ &= \frac{r\bar{P} - \int_0^{R^*} Rh(R)dR}{Pr(R \geq R^*)} \end{aligned} \tag{3.36}$$

Now, using

$$\int_0^{R^*} Rh(R)dR \leq R^* Pr(R < R^*) \tag{3.37}$$

together with $R^* = rP$ yields

$$rP \geq \frac{r\bar{P} - rPPr(R < R^*)}{Pr(R \geq R^*)} \tag{3.38}$$

This along with $Pr(R \geq R^*) = 1 - Pr(R < R^*)$ gives

$$P \geq \bar{P}$$

Now if $Pr(R < R^*) > 0$, the inequality in equation 3.37 is strict which leads to $P > \bar{P}$. This proposition indicates that because of the risk shifting that occurs and due to the possibility of defaults, prices do increase above their fundamental values, $P > \bar{P}$. When $R < R^*$ all investors default because they are identical, this can be interpreted as a financial crisis. In more realistic models, only a portion would default. This proposition shows how shocks from the real sector can trigger a financial crisis for example through a low realisation of R after risk shifting leads to overinvestments in a risky asset.

Proposition 2 states that if we let (r, P) and (r', P') be the equilibrium interest rate and price of the risky asset before and after a mean preserving spread in the distribution of R. Then one of two possibilities occur,

a. $(r', P') = (r, P)$ and the equilibrium stays unchanged.

b. $r' > r$ and then $P' > P$, which leads to a fall in the fundamental value $\overline{P'} < \overline{P}$. This leads to an increase in the bubble size as $P' - \overline{P'} > P - \overline{P}$, along with the probability of default increasing.

3.4.2 Asset Pricing with Uncertainty Generated by the Financial Sector

From the previous section we can see that a bubble can exist in the sense that the price of the risky asset P is higher than its fundamental value $\overline{P}$, which is defined as the discounted value of net returns. In this section the time horizon of the model is extended along with showing how uncertainty about future credit expansion in the economy can increase the magnitude of the bubble.

There are now three dates $t = 0, 1, 2$, and to implement uncertainty about future credit expansion, we assume that the amount of credit available for lending by banks to investors B is partially controlled by the central bank.

- At $t = 0$, B_1 is a random variable with a positive density function $k(B)$ on $[0, B_{1,MAX}]$.
- If x is invested at time $t = 0, 1$, then the safe asset pays $r_t x$ at time $t + 1$.
- We assume that the risky asset has a certain return $\overline{R}$. If an investor holds a risky asset at time $t = 2$, it pays $\overline{R}x$ given $x \geq 0$.
- Investors can borrow at $t = 0, 1$.

The equilibrium price of the risky asset at time $t = 1$ is

$$P_1 = \frac{1}{r_1}[\overline{R} - c'(1)] \tag{3.39}$$

Let $P_1(B_1)$ represent the equilibrium value of the risky assets price at date 1, with available credit B_1. Now the equilibrium investor decision problem becomes,

$$\max_{X_{0,R} \geq 0} \int_{\underline{B}_1}^{B_{1,MAX}} [P_1(B_1)X_{0,R} - r_0 P_0 X_{0,R}] k(B_1) dB_1 - c(X_{0,R}) \tag{3.40}$$

With P_0 being the price of the risky asset at date 0 and (X_{0S}, X_{0R}) being the portfolio chosen at $t = 0$. r_0 is the borrowing rate at $t = 0$ and $\underline{B}_1$ the value of B_1 where the investor is on the verge of default at $t = 1$.

$$P_1(\underline{B}_1) = r_0 P_0 \tag{3.41}$$

Market clearing conditions are

$$X_{0R} = 1, \tag{3.42}$$

$$X_{0S} + P_0 X_{0R} = B_0, \tag{3.43}$$

$$r_0 = f'(X_{0S}) \tag{3.44}$$

Now, there exists a unique equilibrium if $E[P_1(B_1)] > c'(1)$. The equilibrium conditions are now three.

$$\int_{\underline{B}_1}^{B_{1,MAX}} [P_1(B_1) - P_1(\underline{B}_1) - c'(1)]k(B_1)dB_1 = 0 \tag{3.45}$$

$$r_0 = f'(B - P_0),$$

$$P_1(\underline{B}_1) = r_0 P_0$$

Consider an investor who holds B units of wealth and we ask what price $\overline{P}_0$ he is willing to pay for one unit of the risky asset. This is what we call the fundamental value. The investor now faces the following decision problem in which he must choose (X_{0S}, X_{0R}) to solve

$$\max_{(X_{0S},X_{0R}) \geq 0} \int_0^{B_{1MAX}} [r_0 X_{0S} + P_1(B_1)X_{0R}]k(B_1)dR - c(X_{0R}) \tag{3.46}$$
$$\text{subject to} \quad X_{0S} + PX_{0R} \leq B$$

As with the former section, we use the first order conditions to find the fundamental value of the risky asset

$$\overline{P}_0 = \frac{1}{r_0}\{E[P_1(B_1)] - c'(1)\} \tag{3.47}$$

From equation 3.45 we get the expression for the equilibrium price

$$P_0 = \frac{1}{r_0}\left[\frac{\int_{\underline{B}_1}^{\overline{B}_{1MAX}} P_1(B_1)k(B_1)dB_1 - c'(1)}{Pr(B_1 \geq \underline{B}_1)}\right] \tag{3.48}$$

This leads to **Proposition 3** which states that if we let $(r_0, P_0, \underline{B}_1, X_{0S}, X_{0R})$ be the equilibrium values in an intermediated economy along with $\overline{P}_0$ being the fundamental price for the risky asset. Then $P_0 \geq \overline{P}_0$ and the inequality is strict given that the probability of bankruptcy $Pr(B_1 < \underline{B}_1)$ is positive.

Intuitively, Propositions 1 and 3 are similar. With Proposition 3 the uncertainty about the realisation of B takes the place of the uncertainty of R in Proposition 1. There is often a great amount of uncertainty about the course of credit expansion an thus how large the bubble may become and when it will collapse.

3.4.3 Financial Fragility

The propositions above show how asset prices can go above their fundamentals in financial systems often leading to a subsequent financial crisis. However, we have not yet shown if and how credit policies can exacerbate a financial crisis. We do this by looking at the conditions that need to be satisfied at date 1 to avoid default by investors. If we examine equation 3.45

$$\int_{\underline{B_1}}^{B_{1,MAX}} [P_1(B_1) - P_1(\underline{B_1})]k(B_1)dB_1 = c'(1) \quad (3.49)$$

As $c'(1) \rightarrow 0$ the left hand side must also vanish, this can only occur if $\underline{B_1} \rightarrow B_{1MAX}$. As investment costs become less, competition for the risky asset drives its price up and increases the incentive for risk shifting. This is best seen through a numerical example.
Assume that:

B_1 is uniformly distributed on [0,2]
$B_0 = 1,\ f(X_S) = 4X_S^{0.5}$ and $\bar{R} - c'(1) = 4.$

By only examining positive prices, we can see that $P_1(B_1)$ becomes

$$P_1(B_1) = 2[(1 + B_1)^{0.5} - 1]$$

If we now substitute this into equation 3.49 and vary the marginal cost of investment, $c'(1)$, a number of interesting cases are generated.

Table 3.2: Financial Fragility Numerical Example

$c'(1)$	$\underline{B_1}$	Probability of a crisis	Equilibrium P_0	Fundamental $\overline{P_0}$	Bubble $P_0 - \overline{P_0}$
0.2	0.9	0.45	0.31	0.25	0.06
0.1	1.21	0.61	0.38	0.27	0.11
0.02	1.74	0.87	0.47	0.29	0.18

In each case in table 3.2 the expected level of credit next period is the same as the current level of credit. When $c'(1) = 0.2$, the financial system is robust because if available credit remains the same, $B_0 = B_1 = 1 > \underline{B_1} = 0.9$ a crisis will be avoided. In the second row, the financial system is more fragile as credit must be expanded to $B_1 \geq \underline{B_1} = 1.21$ if a financial crisis is to be avoided. It is not sufficient to hold credit expansion steady or increase it slightly. In the third row, the financial system is extremely fragile and a large increase in credit $B_1 > 1.74$ is sufficient to avoid defaults.

What this analysis also shows is that with increasing probabilities of a crisis, both the equilibrium and the fundamental prices increase. However, the equilibrium price increases at a faster rate than the fundamental and thus the bubble term $P_0 - \overline{P_0}$ increases.

The conclusion from this numerical example is that a high probability of a crisis can occur without high probability of credit contraction. In other words, a crisis is able to occur even when credit is expanded.

3.4.4 Discussion

The first part of this model attempts to explain the formation of asset price bubbles through risk shifting which occurs due to the inability of lenders to observe how risky the borrower's investments are. Risk shifting occurs when borrowers invest in overly risky assets due to them not bearing the loss if they receive a low return, the bank bears the loss. If however the return is high they receive the surplus and the bank only receives it's promised return. This risk shifting problem along with the risky asset being in fixed supply bids up the asset's price above it's fundamental value. The interpretations of the risky assets could be real estate, stocks or any asset where the supply reacts slowly with price changes.

The model implies that bubbles occur when there is uncertainty about asset payoffs or credit expansion in the economy. Financial liberalisation is often a large factor leading to such uncertainty. If a bubble is to be avoided, central banks and governments need to take into account possible impacts on asset prices when making policy changes and consider that not only does the amount of credit play a role, but the uncertainty of future levels of credit.

The impact of a financial crisis on the economy depends on its severity. If a crisis is moderate, the equity capital and reserves of the banking system are depleted due to defaults and various costs. For the banking system to restore these buffers to their optimal level, lending is reduced. This may have an even further negative effect on the prices of fixed supply assets such as real estate or stocks.

For a severe crisis, in which banks could go bankrupt both shareholders and bank depositors suffer losses and the stability of the banking sector is threatened. In this case lending is cut back considerably and a severe recession can follow.

The first model version describes how asset price bubbles can form in an intermediated financial system due to risk shifting. The second version describes the relationship between the amount of credit supplied by the banking system and asset prices. And finally, the model identifies that under certain market conditions the central bank must increase credit by a critical amount in order to avoid a financial crisis. Although a complete and formal model was not developed this analysis does give insight into the formation and behaviour of bubbles in asset prices.

3.5 Information Bubbles

When traders have different information, it is possible for prices to deviate from their fundamental or intrinsic value, these deviations can be rational and are called "information bubbles". The intrinsic value of an asset is conditioned on information available to all traders. If the price of an asset does not reveal all information, then prices deviate from their intrinsic value and an information bubble forms. In theory, prices reveal information because uninformed traders can learn what insiders know, if they know the relationship between information and prices.

However, having prices perfectly aggregate information is paradoxical, because if prices reveal all information then investors have no incentive to invest in information gathering. Most resolutions to this paradox suggest that prices deviate from their intrinsic value in the form of an information bubble, these bubbles are typically small in size and temporary. Hellwig (1980) suggests a solution which involves some sort of noise, for example liquidity motivated trading which creates imperfections in the relationship between information and prices.

Camerer and Weigelt (1988) describe a specific type of information bubble which they call a "mirage". It can arise when no trader has any information, but traders do not know that nobody has any information. Trades then make incorrect assumptions about information from prices. This leads to further disequilibrium trading by traders who think they have learned more information from the price, which in turn leads to even more incorrect assumptions. This leads to traders thinking they have acquired inside information, even if there is no information to acquire, hence the name "mirage" as traders think they see information which is not there.

Information bubbles are a difficult subject to test for empirically as we often do not know what information traders had at certain time points in the past. There have however been tests done on public announcement information. An example is Huberman and Schwert (1985) who tested whether bond and stock prices responded to public announcements of the Consumer Price Index. They found that prices were somewhat affected by the announcement, which means information was not completely aggregated.

3.6 Commodity Bubbles

Tangible assets such as fuels, metals and foods are called commodities. These assets are traded both in their original form and in various derivative forms on exchanges such as the Chicago Mercantile Exchange and the London Metal Exchange. Corporations use derivatives on commodities to mitigate their price fluctuation risks, such as airlines purchasing crude oil futures to hedge themselves against fuel prices rising.

Commodity prices are based on several factors, such as supply and demand trends, seasonality, stocks, trade policy and speculative trading but they also have carrying costs. Commodity markets are expected to derive a fair price for a certain commodity as there are a large number of investors, both private and institutional along with companies that actually trade or consume these tangible assets active on the market. Since the value of a commodity is meant to be consumed, its price evolution differs from traditional financial assets and tends to be a mean reverting process. Commodities are considered to have low correlation with traditional financial assets.

In the commodity market, there is no theoretical fundamental value as there is in the securities markets. This is the main reason why testing for bubbles in commodities is very difficult. Studies on commodity bubbles are scarce while bubbles in the traditional financial markets have been studied and tested for empirically for years.

However there have been some studies on whether bubbles have emerged for certain commodity prices and time periods. Areal, Balcombe, and Rapsomanikis (2014) tested for bubbles in food prices by defining the fundamental price as being the price which is purely driven by supply and demand and all deviations from that speculation. They used an augmented Dickey-Fuller test to several price indices and 28 agricultural commodity prices. They focused on the 2008, 2010 and 2011 food price surges and found that few food commodities exhibited bubble behaviour in 2008 and the surges of 2010 and 2011 being due to the forces of supply and demand.

3.7 Bubble Limitations

A possible way to rule out rational bubbles would be to assume that expectations are not rational, since the possibility of a rational bubble works under the assumption of rational expectations. If expectations are adaptive instead of rational, bubbles can also be ruled out. For example, given the information set at time t (I_t) and that $E(P_t|I_{t-1}) = P_{t-2}$. Prices will converge to the discounted dividend level, independent of the initial expectation $E(P_t|I_0)$ (Lucas, 1986).

Restrictions on the scope of a market is another reason for rational bubbles not to exist. For example if assets have a finite lifetime of T periods, rational bubbles should not occur, by backward induction. At time $T-1$ a trader with rational expectations would not pay more than the discounted terminal value at time T, put simply, a rational bubble cannot start if all traders anticipate it's ending.

Tirole (1982, 1985) shows how wealth constraints must lead to a stop in the bubble's growth at some point. Although, since agents can borrow capital these wealth restrictions may not be very important. He also concludes that bubbles cannot form if the market consists of a finite number of infinitely lived participants. They can however form if a market is composed of successive overlapping generations of participants and if the long run real interest rate is less than the population growth rate.

3.8 Derivatives and Pricing By No Arbitrage

This section considers derivative securities with an underlying risky asset. Derivatives are generally priced using no arbitrage arguments and thus a bubble term on their price should not be able to form even though their underlying assets may develop bubbles. Arbitrage is the possibility of a risk-free profit, often gained by taking advantage of a price difference between two or more markets. This chapter focuses on standard derivatives contracts such as forwards, futures and options under the no arbitrage argument.

3.8.1 Stochastic Differential Equation for Asset Prices

Asset prices can be modelled using a simple stochastic differential equation, option pricing is based on this evolution of an assets price. This equation consists of the asset price today, a fixed drift parameter μ, a volatility parameter σ and the stochastic parameter dW.

$$dS_t = \mu S_t dt + \sigma S_t dW \tag{3.50}$$

S_t is the price of the asset at time t, dS_t is the price change over each time step, dt is the timestep used. μ is the fixed drift term and σ is the volatility. dW is a wiener process or a stochastic variable which follows $N(0, dt)$ or a normal distribution with mean zero and variance dt. This variable is a martingale, defined as $\mathrm{E}(X_{n+1}|x_1, \ldots, x_n) = X_n$. From this we can derive an equation for S_T. Assume $F = F(S)$ is a continuous differentiable function of S. Then by Taylor expanding,

$$dF = \left(\frac{\partial F}{\partial t} + \mu S \frac{\partial F}{\partial S} + \frac{1}{2}\sigma^2 \frac{\partial^2 F}{\partial S^2}\right) dt + \sigma S \frac{\partial F}{\partial S} dW \tag{3.51}$$

Using Ito's Lemma and $F(t) = log(S_t)$ we get

$$\int_t^T d\log S_t = \int_t^T \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dW \tag{3.52}$$

Integrate and apply exponential on both sides gives

$$S_T = S_t \exp\left(\quad - \frac{^2}{2}\right)(T \quad t) + (W(T) \quad W(t))\right) \tag{3.53}$$

This gives the assumed evolution of an assets price. With the expectation value

$$E(S_t) = S_0 e^{\quad t} \tag{3.54}$$

and variance

$$Var(S_t) = S_0^2 e^{2 \quad t}(e^{\quad ^2 t} \quad 1) \tag{3.55}$$

We can simulate this asset price evolution using the numerical values in table 3.3. The asset price was calculated using equation 3.53 and daily returns were calculated using log returns,

$$Return_{daily} = ln\left(\frac{S_t}{S_{t \quad 1}}\right) \tag{3.56}$$

The expectation value of returns is

$$\left(\quad - \frac{^2}{2}\right) dt \tag{3.57}$$

and the variance

$$^2 dt \tag{3.58}$$

This can be seen in figure 3.1. As we are using daily returns, $d_t = 1/252$. We assume a fixed drift term $= 10\%$ and volatility of $= 15\%$. This is summarized in table 3.3.

Table 3.3: Parameter values for simulation of Stochastic Asset Price evolution

S_0	Iterations			T [Years]	d_t
100	500	0.1	0.15	5	1/252

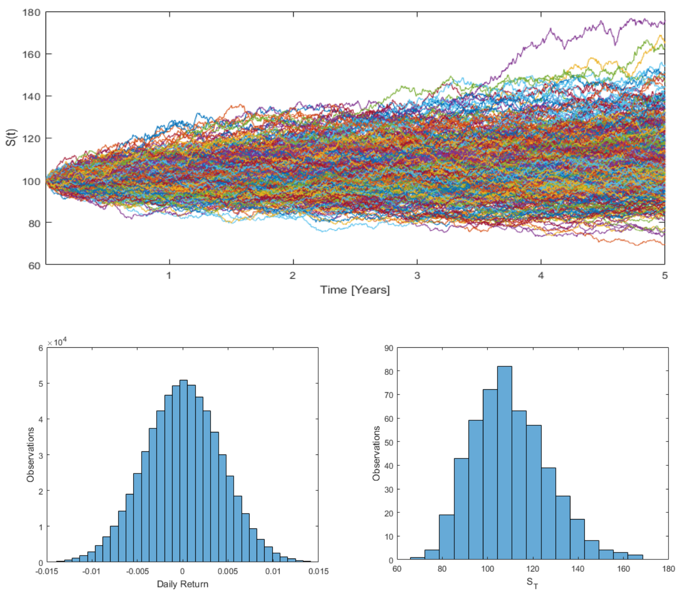


Figure 3.1: Asset price evolution, simulation with 500 iterations, 5 years.

Equation 3.50 can be modified to model exchange rate evolution. We define $S_{d;f}$ as the exchange rate or number of domestic currency units in one unit of foreign currency. For example $S_{ISK;GBP}$ = 151 ISK per GBP. We also define r_d and r_f to be the domestic and foreign interest rates respectively. Then the asset price evolution follows

$$dS_{d;f}(t) = (r_d - r_f)S_{d;f} + \sigma S_{d;f}(t)dW \tag{3.59}$$

Using similar operations as equations 3.51 to 3.52 we get

$$S_{d;f}(T) = S_{d;f}(t)\exp\left(\left(r_d - r_f - \frac{\sigma^2}{2}\right)(T - t) + \sigma(W(T) - W(t))\right) \tag{3.60}$$

3.8.2 Valuing Forwards and Futures

A forward contract is an agreement to buy or sell an asset at a predetermined future time T for a predetermined price S_T. There are two parties to a forward contract, a long position holder who agrees to buy the underlying asset at time T for a price S_T and a short position holder who agrees to sell the asset on the same date for the same price.

Just like a forward contract, a futures contract is an agreements between two parties to buy or sell an asset at a certain time in the future for a certain price. They however differ in several ways. Futures contracts are traded on an exchange and are standardized. Futures also include margin requirements, daily settlements and clearing houses to minimize potential losses if one party is unable to hold up his end of the agreement.

If we consider a forward contract on an investment asset with price S_t that provides no income, maturity at T and risk-free rate r_f. What we aim to find is the fair price F_t, which is set today and is to be paid at T while eliminating arbitrage opportunities.

We demand that a perfectly hedged position i.e. a risk free position, should return to us the risk free rate r_f. A long position in the underlying asset and a short position in the forward contract is a perfect hedge and therefore should pay the risk free rate r_f if the contract matures one period from now. This can be expressed as

$$\frac{(F_t - S_t)}{S_t} = r_f \tag{3.61}$$

Which we rewrite as

$$F_t = S_t(1 + r_f)$$

Or for T periods in exponential compounding,

$$F_t = S_t e^{r_f T} \tag{3.62}$$

Equation 3.62 states that the forward price which is set today and is to be paid at T must equal the spot price today calculated forward at the risk free rate for T periods. If $F_t > S_t e^{rT}$ an arbitrageur could buy the asset, short forward contracts on the asset and realize a profit. If $F_t < S_t e^{rT}$, the arbitrageur could short the asset, enter into a long forward contract on the asset and realize a profit. Thus, an assets forward/futures price is fixed according to equation 3.62, ensuring that the value of the contract is zero when it is entered into and that a bubble is not possible for the forward price. A bubble can however exist in the price of the underlying asset. The value or payoff from a long forward contract at time T is

$$V(t;T) = S(T) - F(t;T) = \begin{cases} > 0 \text{ if } S(T) > F(t;T) \\ < 0 \text{ if } S(T) < F(t;T) \end{cases} \tag{3.63}$$

Similarly the value of a short position is

$$V(t,T) = F(t,T) - S(T) = \begin{cases} < 0 \text{ if } S(T) > F(t,T) \\ > 0 \text{ if } S(T) < F(t,T) \end{cases} \tag{3.64}$$

The following figure plots the payoff from both long and short positions in a forward contract with the strike price F(t,T) set at 50 and the underlying S_T ranging from 0 to 100.

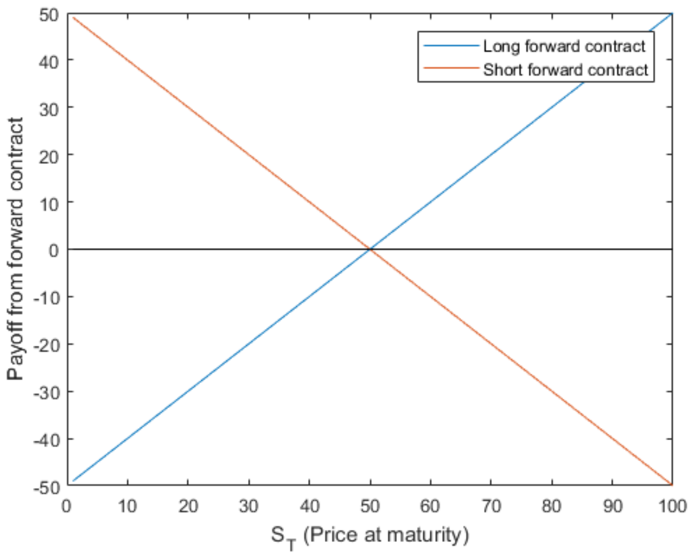


Figure 3.2: Long and short position payoff from a forward contract

Similar no-arbitrage arguments can be constructed for foreign exchange forward rates. Suppose we have at time t, Q_d units of our domestic currency, we invest those units at the domestic risk free rate r_d for T years. At time T, Q_d has become $Q_d \times e^{r_d(T-t)}$. Another option would be to invest the same amount Q_d in a foreign economy at the foreign risk free rate r_f. Suppose that $S_{f;d}$ represents the spot exchange rate or the number of foreign currency units per unit of the domestic currency. Consequently, $S_{d;f}$ represents the number of domestic currency units per unit of the foreign currency. At time t, we would purchase $S_{f;d} \times Q_d = Q_f$ units of the foreign currency, invest at r_f and enter into a forward exchange rate contract $F_{d;f}(t,T)$. For there not to be arbitrage, the following must hold

$$(Q_f e^{r_f(T-t)}) \times F_{d;f}(t,T) = Q_d \times e^{r_d(T-t)} \tag{3.65}$$

Equation 3.65 states that you should not be able to make a risk free profit by investing in a higher interest rate economy while having a forward contract to buy your domestic currency back at time T. The left hand side of equation 3.65 represents the foreign economy investment and the right hand side represents the domestic investment. Isolating $F_{d;f}(t,T)$, which is the forward exchange rate set at time t, yields,

$$F_{d;f}(t,T) = S_{d;f}(t)e^{(r_d - r_f)(T-t)} \tag{3.66}$$

Equation 3.66 gives the no arbitrage fair forward exchange rate at time T, set at time t. This exchange rate is the spot rate today calculated forward using continuous compounding by a factor which is the difference between the interest rates of the two economies.

3.8.3 Valuing Options

Options are traded on both exchanges and between two parties over the counter. Vanilla options can be one of two types. A call option gives the holder the right, but not the obligation to buy the underlying asset at a predetermined price and date. A put option gives the holder the right, but not the obligation to sell the underlying asset at a predetermined price and date. American options can be exercised at any time during the life of the option, however European options can only be exercised at maturity. There are two parties to each options contract, a long position holder and a short position holder with the former paying a premium to the latter. European options are priced using the Black-Scholes model. Black-Scholes assumes asset prices are modelled using equation 3.50, there are no dividends, risk free rate and volatility are constant. It then uses the argument that a fully hedged portfolio, with all risk eliminated, will grow at the risk free rate and Ito's Lemma to arrive at the following differential equation which describes how positions in the underlying asset can be taken in just the right way to eliminate risk. This implies that there exists only one correct price for the option.

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0 \tag{3.67}$$

A solution to this equation is the price of a European call and put option. With S_t = Price of asset today, K = strike price, r = risk free rate and T = time of expiry. The price of a call option is

$$c(t) = S(t)N(d_1) - Ke^{-r(T-t)}N(d_2) \tag{3.68}$$

And the price of a put option is

$$p(t) = Ke^{-r(T-t)}N(-d_2) - S_t N(-d_1) \tag{3.69}$$

With

$$d_1 = \frac{ln(S_t/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \tag{3.70}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} exp\left(-\frac{y^2}{2}\right)dy$$

The σ parameter is the volatility of the underlying asset, represented as standard deviation. It can be extracted from historical data of the stock price. In practice, traders usually work with implied volatility. The market quotes option prices and the implied volatility is the value of σ that, when substituted into equation 3.68 or 3.69 gives the quoted price.

In the market, prices of call and put options with the same strike price and maturity follow the so called put-call parity. Consider the following two portfolios,

- Portfolio A: one European call option plus a zero-coupon bond that provides payoff K at time T.

- Portfolio B: one European put option plus one share of the stock.

We assume that the stock pays no dividends. The call and put options have the same strike price K and same maturity T. At time T, both portfolios should have an equal value independent of the stock price. The following table summarizes the value at T.

Table 3.4: Portfolio value at T

		$S_T > K$	$S_T < K$
Portfolio A	Call option	$S_T - K$	0
	Zero-coupon bond	K	K
	Total	S_T	K
Portfolio B	Put option	0	$K - S_T$
	Stock	S_T	S_T
	Total	S_T	K

For portfolio A, the zero-coupon bond always pays K at maturity but payoffs from the call option will differ depending on the stock price at T. The call option is exercised if the stock price $S_T > K$ and the portfolio value will be $(S_T - K) + K = S_T$. If $S_T < K$ the call option will expire and the portfolio will also be worth K. For portfolio B, the value of the stock is S_T at T. If however $S_T < K$, the put option will be exercised and the portfolio will have a value of $(K - S_T) + S_T = K$. Since the portfolios have identical values at time T, they must have identical values today. If this were not the case, an arbitrage opportunity exists and one could buy the less expensive portfolio and short the more expensive one, making a risk-free profit. Portfolio A is worth c(t) and Ke^{-rT} today and portfolio B is worth p(t) and S_t today. Hence,

$$c(t) + Ke^{-rT} = p(t) + S_t \tag{3.71}$$

Thus, the relationship between prices of put and call options are based on a no arbitrage argument and if the market prices of options move away from this equilibrium the market quickly responds by taking advantage of the disequilibrium and thus correcting the prices.

The profit from European options can be expressed using the following.

A long call option

$$P_{LongCall} = \max(S_T - K, 0) - c(t) \tag{3.72}$$

A short call option

$$\begin{aligned} P_{ShortCall} &= -\max(S_T - K, 0) + c(t) \\ &= \min(K - S_T, 0) + c(t) \end{aligned} \tag{3.73}$$

A long put option

$$P_{LongPut} = \max(K - S_T, 0) - p(t) \tag{3.74}$$

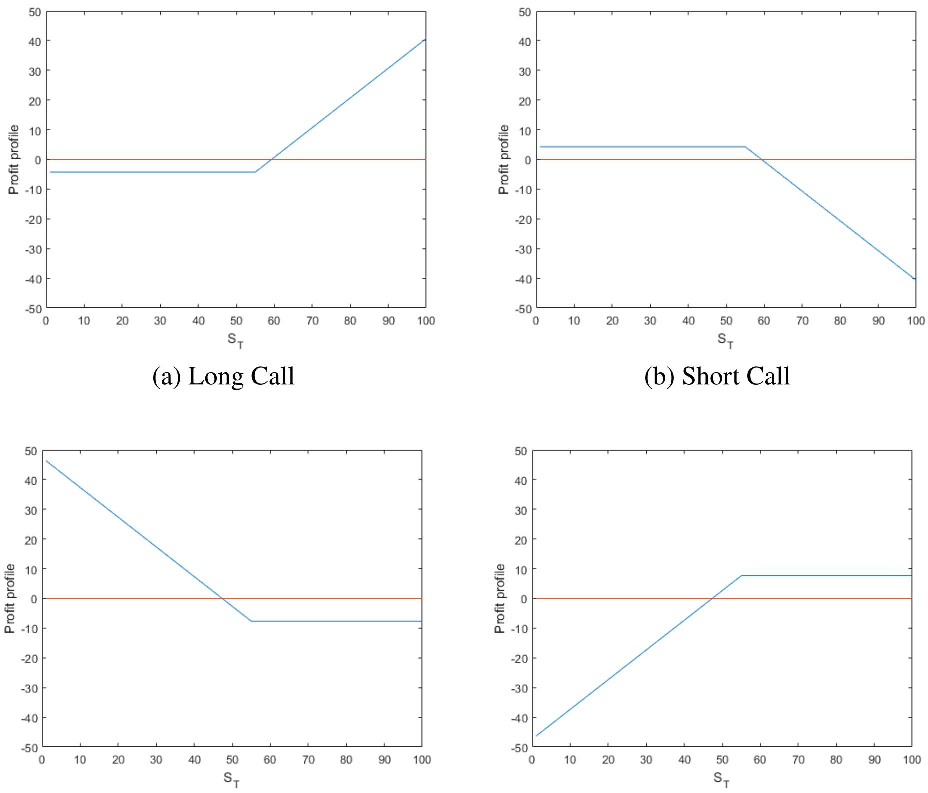
A short put option

$$\begin{aligned} P_{ShortPut} &= -\max(K - S_T, 0) + p(t) \\ &= \min(S_T - K, 0) + p(t) \end{aligned} \tag{3.75}$$

To graph the profit profiles we use a numerical example with the following parameters. Prices of call/put options are calculated using equations 3.68 and 3.69.

Table 3.5: Parameter values for European options

K		r_f	T [Years]	c(t)	p(t)
55	20%	1.5%	2	4.28	7.65



(a) Long Call (b) Short Call

(c) Long Put (d) Short Put

Figure 3.3: Profit profiles from option positions.

3.8.4 Fixing Swap Rates

Swap contracts are multi-payment contracts where one party pays a fixed price and receives a floating payment from the counterparty. This payment generally depends on the price of an asset. Let the forward prices for the asset set at t for maturity at T_i be defined as $F(t, T_i)$ and the discount factors defined as $D(t, T_i)$, with $i = 1, 2, \ldots, n$. Assume a firm needs to buy Q_i units of an asset at future times $T_1, T_2, \ldots, T_n$. They can enter a swap contract where the present value of floating payments can be written as

$$PV_{FL}(t) = \sum_{k=1}^{n} Q_k \cdot F(t, T_k) \cdot D(t, T_k) \tag{3.76}$$

and the present value of fixed payments as

$$PV_{FX}(t) = S(t, T_n) \sum_{k=1}^{n} Q_k \cdot D(t, T_k) \tag{3.77}$$

The fixed leg swap rate, $S(t, T_n)$, is generally fixed by setting the contract up in such a way that the initial net value is zero. This is done by equating the present values of the fixed and floating payments, $PV_{FL}(t) = PV_{FX}(t)$ and solving for $S(t, T_n)$. This yields,

$$S(t, T_n) = \frac{\sum_{k=1}^{n} Q_k \cdot F(t, T_k) \cdot D(t, T_k)}{\sum_{k=1}^{n} Q_k \cdot D(t, T_k)} \tag{3.78}$$

Which is the fixed payment set at t to be paid at each time period while receiving the floating payment. Since a swap contract is an exchange of future payments at a price agreed upon today, it can be valued as a portfolio of forward contracts and the same no arbitrage arguments as described in chapter 3.8.2 apply.

Interest rate swaps are contracts where one party pays a fixed interest rate, the swap rate on some nominal sum and the other party pays a floating rate, usually the LIBOR rate on the same nominal sum. The interest rate is fixed using a similar argument as before by equating the present values of the fixed and floating payments. The present value of the floating leg is

$$PV_{FL}(t) = Q \sum_{k=1}^{N} \delta_{fl}(T_{k-1}, T_k) \cdot F(t_0, T_{k-1}, T_k) \cdot D(t_0, T_k) \tag{3.79}$$

and the present value of the fixed leg is

$$PV_{FX}(t) = Q \sum_{k=1}^{N} \delta_{fx}(T_{k-1}, T_k) \cdot S(t_0, T_0, T_N) \cdot D(t_0, T_k) \tag{3.80}$$

with Q being the nominal, $\delta_{fl}(T_{k-1}, T_k)$ the time period from T_{k-1} to T_k in years, $F(t_0, T_{k-1}, T_k)$ the floating rates which are here based on the forward rates from T_{k-1} to T_k set at time t, however the actual floating payments will be based on the LIBOR rate in the future time intervals. $S(t_0, T_0, T_N)$ is the swap rate set at time t_0, we assume that $t_0 = T_0$ and $D(t_0, T_k)$ is the discount factor. If we set $PV_{FL}(t) = PV_{FX}(t)$ and solve for $S(t_0, T_0, T_N)$, we get

$$S(t_0, T_0, T_N) = \frac{\sum_{k=1}^{N} \delta_{fl}(T_{k-1}, T_k) \cdot F(t_0, T_{k-1}, T_k) \cdot D(t_0, T_k)}{\sum_{k=1}^{N} \delta_{fx}(T_{k-1}, T_k) \cdot D(t_0, T_k)} \tag{3.81}$$

Which is the swap rate or the interest rate for the fixed leg of the swap, ensuring that no arbitrage is possible when entering the swap. (S. Olafsson, 2018)

3.9 Liquidity and Asset Prices

As discussed in 1.5, liquidity describes how big the trade-off between the time it takes to sell an asset and the price it can be sold for. A liquid asset can be sold relatively fast with a small price reduction. In order to sell an illiquid asset relatively quickly, a larger price reduction is needed. Cash is an example of a very liquid asset and real estate is an illiquid asset.

Overall market liquidity refers to the amount of capital available for investment in an economy. This is governed by interest rates among other factors. When interest rates are low, capital is easily available and individuals and businesses choose risky investments such as stocks over fixed income securities because bond yields are low. This can drive up prices of risky assets given that the demand increases enough. This is often followed by increased inflation. Central banks can manage this by increasing interest rates, thus slowing down the economy and decreasing inflation. Central banks also control the amount of capital in the market by increasing or decreasing bank reserve requirements along with decreasing the capital by selling treasury securities.

Market liquidity affects asset prices and their expected returns. The analysis by Amihud and Mendelson (1986) suggests that investors require higher returns on assets with low market liquidity to compensate for the higher cost of trading these illiquid assets. This means that for an asset with a given cash flow, as it's market liquidity increases, it's price increases and it's expected return decreases. Furthermore, if the asset's market-liquidity risk is greater, risk-averse traders require higher expected return on the asset, this involves the assets sensitivity to shocks in the overall market liquidity.

This leads to a term called liquidity premium, it describes the compensation for an investor who invests in securities with low liquidity or the additional return an investor requires for holding an asset that may be difficult to convert to cash. Liquidity premium is one of the factors which explain the differences in bond yields. A firm that issues two bonds with different maturities. The shorter maturity bond would be much easier to sell and thus a more liquid investment, however the longer maturity bond would hold a higher yield to compensate the investor.

Chapter 4

Irrational Bubbles

4.1 Definition

The models introduced in chapter 3 assumed all investors acted rationally and argued that prices deviated from the fundamental while still being rational, these deviations are called rational bubbles. What these models did not incorporate were psychological factors and irrational human behaviour which can contribute to asset prices. Models which include speculation and human behaviour in asset pricing have been developed to attempt to explain these irrational prices for assets.

Irrational bubbles or fads are defined by an asset's price rising rapidly which is based on exaggerated beliefs about potential earnings created by social forces due to some underlying factor, such as new technology or organizational structure. This sharp rise is then followed by a collapse. Camerer (1989) describes three theoretical types of fads. The first one being that an assets price may fluctuate because the utility a person gets from holding the asset varies over time. Second, mass changes in beliefs about future intrinsic value. And the third one being price fluctuations because of fads in expected returns.

Irrational bubbles or deviations from the intrinsic value are mean reverting. These deviations are caused by psychological forces like fashions in asset markets, such as markets for cars, houses and technology. Irrational bubbles or fads (F_t) can be defined as a deviation between prices and the assets intrinsic value, which slowly reverts to its mean of zero.

$$P_t = \sum_{i=1}^{\infty} \frac{E(D_{t+i})}{(1+r)^i} + F_t \tag{4.1}$$

where

$$F_{t+1} = CF_t + e_t$$

Here, the C parameter measures the convergence speed of the fad decay. e_t is an independent error term with mean zero. If C is close to one, the fad can decay so slowly that investors can have trouble profiting by betting on its decay and disappearance. A C value of zero would mean the fad would disappear immediately. (Camerer, 1989)

4.2 Irrational Investor Behaviour

As opposed to traditional economic frameworks, psychological factors can and do influence investors, in practice they are subject to several cognitive biases. This section includes some well known examples from Ritter (2003), which describe why human behaviour can help fuel irrational asset price bubbles.

4.2.1 Herd behaviour

When an individual mimics the actions of a larger group, whether those actions are rational or not. Often these are decisions that an individual would not necessarily make on their own. This often happens in financial markets as investors buy or sell in the direction as the market trend. Investors buy stocks on the rise which further pushes their price even higher and sell off stocks when it starts to fall, leading to a sharp fall.

As to why human tendency is this way there are several answers. One reason being that there is social pressure afforded to conformity. Most people are social and have a natural desire to be accepted by a group rather than being branded as an outcast. Mimicking the behaviour of a group is a natural way of becoming a member of said group. Another reason could be the rationale that the more people buy into an investment decision, the probability of that decision being incorrect should go down. Even if an individual feels it is an irrational decision, the individual is more likely to be convinced of it's rationality if others have made the decision.

Chiang and Zheng (2010) analyzed herding behaviour in global markets empirically by applying daily data from 1988-2009. They found evidence of herding in advanced stock markets except the US. No evidence of herding was found in Latin American markets. They also concluded that a crisis triggers herding activity in the crisis country of origin and this spreads to neighbouring countries. During a crisis, they found supportive evidence for herding in US and Latin American markets.

4.2.2 Overconfidence

Overconfidence describes a behavioural bias which can have dangerous consequences when investing in financial markets. Overconfidence is when an investor overestimates his knowledge, abilities to control events and the precision of his forecasts while underestimating risks. Causes of this could be self-serving bias, which states that people attribute their successes on their own abilities but their losses on bad luck. Another reason could be the Illusory Superiority, which causes people to overestimate their abilities in general in relation to other peoples abilities.

Another form of overconfidence is too little diversification, because of the tendency to invest in things that we are familiar with. For example, investing in local businesses or real estate. Trading behaviour is another manifestation of overconfidence. Barber and Odean (2001) studied this and concluded that the more people traded, the worse they did.

4.2.3 Heuristics

Heuristics, which are often called rules of thumb is a strategy many people adopt. This can lead to suboptimal investment decisions. When a person is faced with N choices to invest an amount, many choose to allocate using the 1/N rule. If there are 4 stocks or other investment options, a quarter will go into each. If two of the 4 options are stock funds, half will go into equities. Benartzi and Thaler (2001) did a study on this and found that this 1/N rule has a place in reality and many people adhere to this when being presented with investment options.

4.2.4 Anchoring

Anchoring occurs when investors base their decisions on figures or statistics which are irrelevant. For example, when an investor buys stocks which have dropped significantly over a short period of time. The investor is most likely anchoring to a resent high point and believes that there could be profit from buying the stock. Although it is possible that the investor could profit from short term volatility, the stock could also be reacting to a change in fundamental value. Another example would be an investor who holds an asset that has lost value because he has anchored his fair value estimate to the original price rather than the fundamental value.

4.2.5 Framing

Framing refers to a behavioural bias which causes people to draw different conclusions based on how they are "framed" or presented, even though the payoff is the same. Or put another way, an individuals mind reacts differently to information based on the way it is presented. The perception changes as a function of some variation in framing. For example, agreeing to the proposals portrayed as risky gains, and rejecting the ones portrayed at risky losses. This leads to investment decisions being inaccurate and clouded by bias. Prospect Theory by Daniel Kahneman and Amos Tversky describe framing as the investor always looking at the decision from an angle that presents gain. Because a loss is more significant than the equivalent gain, a sure gain is favoured over a probabilistic gain and a probabilistic loss is preferred over a definite loss.

4.3 Behavioural Finance Bubble Model

J. Bradford DeLong (2009) introduced a simple behavioural model in which he attempts to model a stock market with bubbles, manias, crashes and panics. In each period a fraction of agents buy stocks and the rest buy bonds. There are no rational investors, each period investors keep the same investment strategy as they did last period. However, they do randomly encounter each other and compare returns at a rate of . The amount of investors that switch from the lower to the higher performing strategy is times the difference in rates of return. This is an attempt to model herd behaviour and thus this is an irrational model. Stocks pay a stochastic dividend d_t with probability , which is serially uncorrelated. We assume bonds pay a fixed rate r.

DeLong provides us with a difference equation for the price of stocks at time $t + 1$, p_{t+1}.

$$p_{t+1} = p_t + \;\; p_t{}^1 1 \;\;\; p_t{}^0 \;\; \frac{p_t \;\;\; p_{t \;\; 1} + d_t}{p_{t \;\; 1}} \;\;\; r \tag{4.2}$$

Where,

$$d_t = \begin{cases} \;\; ; & \text{with probability } \;\; : \\ 0; & \text{with probability } 1 \;\;\; : \end{cases}$$

So the price of stocks at time $t + 1$ is dependent on the price at time t, the comparison factor and the difference between stock and bond returns. Because there is a single unit of investors and stocks p_t can represent three things, the price of stocks, wealth invested in stocks or the number and share of agents investing in stocks. Next, we take unconditional expectations:

$$E^1 \;\; p_{t+1}{}^0 = \;\; p_t{}^1 1 \;\;\; p_t{}^0 \;\; \frac{E^1 \;\; p_t{}^0 + }{p_{t \;\; 1}} \;\;\; r \tag{4.3}$$

This gives

$$E^1 \;\; p_{t+1}{}^0 = 0 \;\; if \;\; E^1 \;\; p_t{}^0 = 0 \;\; and \;\; p_{t \;\; 1} = \frac{}{r} \tag{4.4}$$

With $p_{t \;\; 1}$ being the fundamental value of p, called p . By implementing this model in Matlab we can simulate an example with the numerical values shown in table 4.1.

4.3.1 Simulation of Behavioural Model

The following is a numerical example using deLong's behavioural model.

Table 4.1: Parameter values for deLong's model simulation

Parameter	Value
	0.5
	1.25
	0.05
T	100
r	0.05
p_0	0.3
p	0.5

Figure 4.1 shows one simulation of deLongs behavioural model using the parameters from table 4.1. The fundamental value is

$$P = \frac{\quad}{r} = \frac{0{:}5 \quad 0{:}05}{0{:}05} = 0{:}5$$

Figure 4.1: Simulations of deLong's model, xed $= 1{:}25$ parameter.

In this simulation the initial value $p_0 = 0{:}3$ is below the fundamental $p = 0{:}5$. When the price exceeds the fundamental value ($p_t > p$) a bubble exists. This simulation shows that manias do occur. As ($p_0 < p$) we see a rapid initial increase in prices until the price exceeds the fundamental by almost 0.25. This is due to to the return comparison between investors and conversion from bonds to stocks. This conversion of additional investors to stocks increases demand for stocks and thus stock prices and returns increase. This positive feedback leads to even more investors turning to stocks which drives prices even higher. Eventually the market tops out as high prices lead to a low dividend yield, then stocks cannot compete with bonds. Then the price takes a downturn as investors start to convert to bonds. Then we see the same cycle again when stock prices are below the fundamental and investors start to convert to stocks from bonds.

(a) = 1:25 (b) = 1:0

(c) = 0:75 (d) = 0:5

Figure 4.2: Simulations of deLong's model, varying parameter.

Figure 4.2 shows how the parameter controls the investor encounters and thus the price movements. A high value has volatile price movements, lower values however have less positive feedback on stock prices and thus prices are more stable.

This model has several limitations as its title suggests. It does not include panics and crashes although it does have manias. We see price values uctuate signi cantly above and below the fundamental. However downturns do not seem to be more rapid than the upswings. As this model is symmetrical it cannot model a panic or crash which is an asymmetrical movement. To implement panics and crashes several complications would need to be added. Stock holdings would need to become more sensitive as their price increases so that a small price decline could trigger a large price downturn. This along with implementations of margin calls, asymmetry, portfolio insurance, stop loss orders, capital requirements and other mechanisms. Other changes could also be made in order to make the model more realistic, such as making investors rational, changing the comparison time frame from one period to multiple (historical data) and changing the dividend payment structure from binary to non binary.

4.4 Abreu-Brunnermeier Model

Abreu and Brunnermeier (2003) proposed a model which contains both irrational or 'behavioural' agents who are subject to fads, fashions, overcon dence along with other psychological biases and rational arbitrageurs who understand that the market will eventually collapse but would like to pro t from the bubble as long as it grows. The analysis aims to challenge the E cient Market Hypothesis as it states that "If there are many sophisticated traders in the market, they may cause these bubbles to burst before they really get under way." The ideal time for the rational agents to exit would be just before the bubble collapses, that is however a di cult task. What allows the bubble to grow is the fact that rational arbitrageurs will come up with di erent solutions to the optimal timing problem and thus exit timings will vary and a consequent lack of synchronization appears.

We assume that the price begins to exceed the fundamental at some random point in time t_0, arbitrageurs become sequentially aware of this and do not realize whether they learnt this early or late relative to others. The bubble only bursts if at least a fraction sell out, however arbitrageurs are competitive as at most a fraction 1 can sell prior to the crash.

Prior to $t = 0$ the stock price p_t equals it's fundamental value and grows at the risk-free rate r, $p_t = e^{rt}$. After $t = 0$, p_t grows at $g > r$ or $p_t = e^{gt}$. This higher growth can be viewed as a series of unusual positive shocks which make investors more optimistic about potential future prospects. t_0, or the point in time where the price begins to deviate from the fundamental is exponentially distributed on $0, 1$ with a cumulative distribution function $t_0 = 1 \quad e^{t_0}$. From t_0 onwards, only a fraction $1 \quad 1$ of the price is justi ed by fundamentals and a fraction represents the bubble component. The price $p_t = e^{gt}$ is kept above the fundamental by behavioural or "irrationally exuberant" traders.

When the cumulative selling pressure exceeds, which is the absorption capacity of behavioural traders, the price drops by a fraction and reach it's post crash price. From this point onwards it grows at r. If the selling pressure never exceeds the bubble is assumed to burst due to exogenous reasons when it reaches it's maximum size $\bar{\ }$. Because of this, along with being strictly increasing, there must exist a nal date $t_0 + \bar{\ }$. This is illustrated in gure 4.3.

Figure 4.3: Price paths, Abreu-Brunnermeier Model

Another component of the model is that rational arbitrageurs become sequentially aware that the stock price growth has exceeded the fundamental value. Or, each instant t from t_0 until $t_0 +$ a new cohort of rational arbitrageurs of mass 1 become aware of the price deviation. As t_0 is a random variable, the arbitrageur does not know how many other arbitrageurs have realised this before or after him. An agent who becomes aware of the bubble at time t_i has a posterior distribution for t_0 with support on $[t_i \;;\; t_i]$ We will here onwards refer to the arbitrageur who becomes aware of the mispricing at time t_i as arbitrageur t_i.

Figure 4.4: Sequential awareness

Figure 4.4 displays the distributions of t_0 for the arbitrageurs t_i, t_j and t_k where $t_k = t_i +$. This synchronization problem creates an incentive for rational traders to time the market which leads to a persistence of the bubble. From the perspective of t_i the distribution or belief of t_0 is

$$(t_0|t_i) = \frac{e \quad e^{(t_i \quad t_0)}}{e \quad 1} \tag{4.5}$$

Another assumption is the following, (Abreu & Brunnermeier, 2003)

$$\frac{}{1 \quad e} < \frac{g \quad r}{1 \quad)} \tag{4.6}$$

This ensures that arbitrageurs don't sell before they become aware of the mispricing. Without the constraint, when is high or $(g \quad r)$ is very small an arbitrageur would sell out immediately. We want to identify the best strategy of a rational arbitrageur t_i. Each arbitrageur is limited in the number of shares he can go long or short. We can normalize the position in the continuum between $[0, 1]$ with 0 being the maximum long position and 1 being the maximum short position, we do this for convenience for the selling pressure analysis.

If we let $(t; t_i)$ denote the selling pressure of arbitrageur t_i at t. The strategy of a rational arbitrageur who became aware of the bubble at time t_i is given by the following mapping $(\;; t_i) : [t_0; t_i + \bar{}] \to [0; 1]$ Which means that $[1 \quad (t; t_i)]$ is the stock holding of trader t_i at time t. The aggregate selling pressure of all rational arbitrageurs at time t is t_0 is

$$s(t; t_0) = \int_{t_0}^{\min(t; t_0 + \;)} (t; t_i)\, dt_i \tag{4.7}$$

And we denote the bursting time of the bubble for a given realization of t_0 as

$$T_0 = \inf\{t \,|\, s(t; t_0) \quad \text{or } t = t_0 + \bar{}\,\} \tag{4.8}$$

So the bubble bursts either when the aggregate selling pressure exceeds the maximum size of the bubble at $t_0 + \bar{}$, whichever point in time comes first. As $(\;|t_i)$ denotes t_i's beliefs about t_0, thus t_i's beliefs about the bursting date are

$$(t|t_i) = \int_{T(t_0)<t} d\;(t_0|t_i) \tag{4.9}$$

The buy or sell execution price of the stock are either the pre crash price $p(t)$ or the post crash price $(1-\beta(t, t_0))p(t)$ less some transaction cost. So the expected execution price is

$$(1-\kappa)p(t) + \kappa(1-\beta(t,t_0))p(t) \quad (4.10)$$

Equation 4.10 is a convex combination of both prices, with $\kappa > 0$ if the selling pressure is larger than $\bar{\kappa}$, and $\kappa = 0$ if the selling pressure is less than or equal to $\bar{\kappa}$ at the time of the bubble bursting.

The point in time t_0, where the price starts to deviate from fundamentals is assumed to follow an exponential distribution with a probability density function $\lambda e^{-\lambda t}$ and a cumulative distribution function $1 - e^{-\lambda t}$. Assume that the bursting time follows the same distribution with a probability density function f_T, which has a rate parameter λ_c and the cumulative distribution function called F_T. Then the hazard rate of the bubble is,

$$h(t|t_i) = \frac{f_T(t|t_i)}{1-F_T(t|t_i)} \quad (4.11)$$

which is the probability that the bubble will burst in the time interval $(t, t+dt)$ given that it has survived up to time t and that the investor becomes aware of the bubble at time t_i. This can be written as,

$$h(t|t_i) = \frac{\lambda_c e^{-\lambda_c(t-t_i)}}{1-(1-e^{-\lambda_c(t-t_i)})} = \lambda_c \quad (4.12)$$

Thus, the exponential distribution of crashes has a hazard rate which is independent of time.

Estimation of λ_c can be achieved by using the following with historical stock or index data. Denote the bursting time as T_0, assume that $T_0 \sim \exp(\lambda_c)$, the expectation value is

$$E(T) = \frac{1}{\lambda_c} \quad (4.13)$$

Denote x_i as the time passed between crashes, a crash would have to be defined as returns being σ standard deviations below the mean for a given time interval. Let n be the number of event measurements. We can write $E(T)$ as the following,

$$\frac{1}{n}\sum_{i=1}^{N} x_i = \frac{1}{\lambda_c} \quad (4.14)$$

isolate λ_c,

$$\lambda_c = \frac{n}{\sum_{i=1}^{N} x_i} \quad (4.15)$$

Equation 4.15 gives an estimate for λ_c or the rate parameter for crashes given that crashes follow an exponential distribution.

Chapter 5.1 in Abreu and Brunnermeier (2003) describes how arbitrageurs never burst the bubble given that, which is interpreted as the dispersion of opinion among arbitrageurs and , which is the absorption capacity of behavioural traders are su ciently large.

The core of the analysis is that dispersion of opinion makes it di cult for arbitrageurs to synchronize selling out of the in ated asset and consequently reducing it's price to the fundamental. This creates an opportunity for rational arbitrageurs to "ride the bubble" or pro t from an asset they know is overpriced as long as they sell out before the bust. Synchronizing events such as news can damage this dispersion of opinions without containing any intrinsic information about the value of the asset. They do however facilitate synchronization and can have an impact out of proportion to it's fundamental content. Synchronizing events arrive at a Poisson arrival rate. Let t_e be the date of such an event and only traders who became aware of the bubble more than $_e$ 0 periods ago observe or look out for this synchronizing event, the idea being that investors become more wary the longer they have known of the bubble. The model suggests a unique equilibrium in the way traders response to a synchronizing event. Each arbitrageur t_i always sells out at the instances of synchronizing events t_e t_i + $_e$ and stays out of the market for all t t_i + . Where represents the time that an arbitrageur stays in the market after realising that there is a bubble given that there is not a synchronizing event. If the synchronized attack fails, in which case he re-enters the market for the time interval t 2 ¹t_e; t_e+ $_e$º unless a new synchronizing event occurs in the interim.

This model argues that bubbles can exist even though the rational traders in the market know that the price is too high and jointly, they have the ability to correct the price. Even though the bubble will eventually burst, there is a long lasting deviation from fundamentals, one where there are pro t opportunities. A crucial assumption of the model is the dispersion of opinion on the timing of the bubble, this assumption models reality as it serves as a metaphor for di erences of opinion and beliefs among rational traders and di erences in information. This dispersion of opinion on the timing of the bubble is what keeps the rational investors from synchronizing their attack on the bubble and reducing the value to its fundamental.

Chapter 5

Empirical Tests

5.1 Introduction

Testing for and identifying asset price bubbles, both rational and irrational is a challenging task. The reason being that in order to identify a bubble, we need to establish the assets fundamental value, which is di cult to measure. Another complication is to distinguish a bubble from non linear dynamics of fundamental values is di cult and perhaps impossible. However, several attempts have been made empirically test for bubbles.

Phillips, Shi, and Yu (2015) tested for bubbles by tting to time series data an autoregressive model. They applied this to a long time series data on the monthly S&P500 stock price index - dividend ratio over the period from January 1871 to December 2010. As a consequence, several known bubble episodes were identi ed including the dot-com bubble and the 2008 subprime mortgage crisis.

Diba and Grossman (1988a) tested for bubbles by analysing whether stock prices are more explosive than their dividend process. If the dividend process follows a linear unit-root process e.g. a random walk, then the price process also follow a unit root process. Under the no bubbles hypothesis they put forward, the change in price (p_t) and the spread between the price and the discounted expected dividend stream ($p_t - d_t \cdot r$) are stationary. Which means p_t and d_t are co-integrated. They test their hypothesis using several unit root tests, autocorrelation patterns and co-integration tests. Their conclusion is that the no-bubble hypothesis cannot be rejected. Campbell and Shiller (1987) did a similar experiment where they tested for cointegration between price and dividends and concluded that deviations from the present value model can be explained as rational bubbles.

This chapter will analyse several indicators of whether an asset's market price is overvalued along with applying them to market indices. These indicators are the PE ratio, CAPE ratio, dividend yield and a variance bound test.

5.2 Price - Earnings Ratio

The price-earnings or P/E ratio is a simple measurement which compares a companies market value per share to its earnings per share or

$$P \bullet E = \frac{\text{Market Value Per Share}}{\text{Earnings Per Share}} \tag{5.1}$$

The ratio is often used when valuing companies as it measures units paid for units earned and thus helpful for evaluating investment options. As discussed earlier, some companies retain dividends for certain time periods because of various reasons e.g. mergers, while others don't pay any. That is why P/E is an important measurement as it is independent of dividend payments. The P/E ratio can however give an insight into whether a companies share price might be overvalued.

Figure 5.1 shows the OMX Iceland All-Share Price Index and its earnings, in ation adjusted to present using the Consumer Price Index. The data is from 1995 through 2019. The in ation adjustment for the price and earnings is calculated using the following,

$$P_{t;T} = P_t \frac{CPI_T}{CPI_t} \tag{5.2}$$

$$E_{t;T} = E_t \frac{CPI_T}{CPI_t} \tag{5.3}$$

$P_{t;T}$ is the index value at t calculated forward to T.

Figure 5.1: OMX Iceland All-Share Price Index and its Earnings

Figure 5.1 does give some insight into whether prices are deviating from earnings. However a clearer depiction would be to look at the ratio between the two, often called the Price Earnings ratio and is plotted in gure 5.2.

An alternate form of P/E is the cyclically adjusted price-to-earnings ratio which is commonly known as CAPE or Shiller P/E is the real price as a ratio of a ten year moving average of earnings adjusted for inflation. Mathematically,

$$CAPE = \frac{Price_t}{\left(\sum_{i=1}^{120} Earnings_{t-i}\right) \cdot 120} \tag{5.4}$$

The idea behind this being that a company's profitability can be highly influenced by the economic cycle depending on the sector said company is in. Some sectors, such as pharmaceuticals and utilities can maintain steady profits during recessions. While others such as car manufacturers are highly dependent on the economic cycle.

Figure 5.2: OMX Iceland All-Share Price Index PE-Ratio and CAPE Ratio

Notable price level deviations from earnings are the Dot Com bubble around the millenium and the subprime mortgage crisis of 2008. The same points in time are interesting for the P/E, CAPE graph, with P/E increasing significantly for the dot com bubble and decreasing because of the subprime mortage crisis. The first calculation of the CAPE ratio is in 2005 as our data only goes back to 1995 and we are using a 10 year average. Data obtained from B. O. Olafsson (2019).

The same methodology can be applied to the U.S. S&P 500 index, in this case we have significantly more data, both in the number of companies and how far back in time the data reaches. Figures 5.3 and 5.4 show the S&P 500 Price, Earnings, PE ratio and CAPE ratio.

Figure 5.3: S&P500 Price and Earnings 1870 - 2019

Figure 5.4: S&P500 PE and CAPE ratio 1870 - 2019

Here we see well known run ups such as the 1920s peak followed by a sharp decline, the Dot Com bubble and the subprime mortgage crisis. Today's prices of the S&P500 deviate extensively from its earnings, whether this is rational or not is di cult to tell. This analysis is based on data and methodology from Robert R. Shiller (U.S. Stock Market 1871-Present , 2019).

5.3 Dividend Yield

The dividend yield represents the dividend-only return of investing in a particular stock and can be expressed as

$$\text{Div}_{\text{Yield}} = \frac{\text{Dividend per share}}{\text{Market price per share}} \tag{5.5}$$

The ratio increases as the market price decreases or the dividends increase. Similarly, it decreases as the market price increases or dividends decrease. The ratio is often used when evaluating investment choices. Intuitively, the dividend yield of the S&P500 could be one indicator of whether there is a bubble in stock prices.

Figure 5.5: S&P500 Dividend Yield 1870 - 2019

Figure 5.5 displays data for the dividend yield of the S&P 500 from 1870 to 2019. When stock prices rise, the dividend yield goes down and thus historical bubbles are noticeable. Such as the 1920's when stock prices rose signi cantly and in the years leading up to 2000 when dividend yields reached an all time low. Currently, dividend yields are low when compared to historical values.

5.4 Variance Bound Tests

Shiller (1981) criticises the Efficient Market Hypothesis and argues that the stock market is inefficient and stock prices fluctuate too much. According to economic theory, the stock price should equal the present value of expected dividends. Dividends are however very stable and fluctuate very little. Consequently stock prices should be stable. In reality though, stock prices fluctuate wildly. Thus, a variance test was developed to examine whether stock prices are too volatile to be justified by subsequent changes in dividends. Let p_t denote the stock price at time t and d_t denote the dividend during period t. The price, p_t should equal the present value of expected dividends or

$$p_t = \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} E_t(d_{t+i}) \tag{5.6}$$

Next, we define p_t^* as the ex post rational price as the present value of actual dividends

$$p_t^* = \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} d_{t+i} \tag{5.7}$$

Now, suppose that p_t is a forecast of the perfect foresight price p_t^* or

$$p_t^* = p_t + e_t \tag{5.8}$$

Where e_t is an error term which we assume to be unrelated to price levels or $E(p_t^* | p_t) = p_t$ and $E(e_t | p_t) = 0$. Thus, p_t and e_t should be uncorrelated. Next, we take the variance of all variables, which yields

$$V(p_t^*) = V(p_t) + V(e_t) \tag{5.9}$$

As variances must be positive, equation 5.9 implies an inequality

$$V(p_t) \leq V(p_t^*) \tag{5.10}$$

which is a variance bound. This states that actual prices should vary less than the foresight price. A forecast should be less volatile than the variable being forecasted. The variance of the ex post rational price is an upper bound on the variance of price. Rewrite equation 5.7 as

$$p_t^* = d_t + \frac{p_{t+1}^*}{1+r} \tag{5.11}$$

We apply this to the S&P500 index data to examine whether a bubble $p_t - p_t^*$ exists. However, this requires backwards induction and the price, p_T^*, which the price path depends heavily on has to be estimated. We assume that the ex-post rational price equals the real price at 01.01.19.

Figure 5.6: S&P500 Real Price v. ex post Rational Price 1870 - 2019

Figure 5.7: S&P500, p-p*, 1870 - 2019

Using this method, we can clearly see historical bubbles and crashes such as in 1929, 2000 and 2008. The estimation of p_T does however create an obvious flaw as the entire price series depends on that value. Shiller applied this on the S&P Composite from 1870 to 1979 and concluded that the variance of the price is much larger than the variance of the ex post rational price and hence, the stock market is inefficient.

Chapter 6

Conclusions

6.1 Summary

The introductory chapters describe bubbles in general and looked into historical examples. Chapter 3.2 describes mathematically the theoretical emergence of price bubbles in detail under the rationality assumption. Chapter 3.3 reaches a similar conclusion through utility maximization arguments. There, the framework is expanded by imposing theoretical restrictions on asset price bubbles such as that once a bubble bursts, it cannot restart and that the existence of a rational bubble would imply it that the asset has been overvalued from the rst day it started trading. Chapter 3.4 describes how risk shifting can lead to bubbles in asset prices due to the limited liability of borrowers. The relationship between the amount of credit provided by the banking system and asset prices was also reviewed along with an analysis of necessary credit expansion by the central bank in order to avoid a nancial crisis. Chapter 3.8 discusses no-arbitrage pricing and how derivatives are priced using this method. The result being that even though the underlying asset can form bubbles, instruments priced using no-arbitrage do not form bubbles in their prices.

Chapter 4 discusses irrational bubbles and how psychological factors and irrational human behaviour can in uence asset prices. In chapter 4.3, a behavioural nance bubble model is introduced which attempts to model a market with bubbles, manias and crashes, with human behaviour as the underlying factor in uencing prices. Chapter 4.4 introduces a model which has both rational and irrational investors. The model argues that bubbles can exist even though the rational traders in the market know that the price is too high and jointly, they have the ability to correct the price. Even though the bubble will eventually burst, there is a long lasting deviation from fundamentals, one where there are pro t opportunities.

Chapter 5 discusses attempts to empirically test for asset price bubbles and looks into metrics which give some insight into whether prices are too high. These metrics are the P/E ratio, CAPE ratio and dividend yield, these ratios are applied on data for the S&P500 and the Icelandic market. We see that presently S&P500 prices deviate signi cantly from earnings and the dividend yield is low.

6.2 Conclusions

As the analysis of this thesis shows, several attempts have been made to model asset price bubbles under the assumptions of rationality and irrationality of investors. As of yet, no model which fully explains the behaviour or emergence of price bubbles exists. These attempts do however give insight into the emergence and behaviour of asset price bubbles.

We can summarize factors which may contribute to the emergence and growth of asset price bubbles. The rst being risk shifting due to shareholders receiving pro ts but their liability of losses being limited, this leads to an incentive for managers to make risky investments and by doing so increasing the risk for debtholders, this can drive up asset prices above their fundamental value. Increasing the amount of credit or credit expansion plays a role in the fueling of bubbles along with interest rates. If credit is expanded, investors have more access to capital which can fuel bubble growth. If interest rates are lowered, capital becomes cheaper which leads to investors choosing risky investments over xed income securities.

Psychological factors such as herd behaviour and trends can have a signi cant e ect on asset prices and create a bubble. The analysis implies that bubbles can exist given that the market consists of both rational and irrational participants. Here, rational traders hold the asset knowing that it's price is too high in order to attempt to pro t from this overvaluation by timing the sell out correctly. Indicators of asset price bubbles are for example high P/E ratios, high CAPE ratios and low dividend yields. Another indicator is to examine whether real prices deviate from the ex-post rational prices through a variance bound test, such a deviation implies that a bubble exists.

Bibliography

Abreu, D. & Brunnermeier, M. K. (2003). Bubbles and Crashes. *Econometrica*, *71*(1), 173 204.

Allen, F. & Gale, D. (2000). Bubbles and Crises. *Royal Economic Society*, *110*.

Amihud, Y. & Mendelson, H. (1986). Asset pricing and the bid-ask spread. *Journal of Financial Economics*, *17*(2), 223 249.

Areal, F. J., Balcombe, K. G., & Rapsomanikis, G. (2014). Testing for bubbles in agricultural commodity markets. *Agricultural Development Economics Division, Food and Agriculture Organization of the United Nations*.

Barber, B. M. & Odean, T. (2001). Boys Will Be Boys: Gender, Overcon dence and Common Stock Investment. *Quarterly Journal of Economics*, *116*(1), 261 292.

Benartzi, S. & Thaler, R. H. (2001). Naive Diversi cation Strategies in De ned Contribution Saving Plans. *American Economic Review*, *91*(1), 79 98.

Blanchard, O. J. & Watson, M. W. (1982). *Bubbles, Rational Expectations and Financial Markets*.

Brunnermeier, M. K. (2008). Bubbles. In L. Blume & S. Durlauf (Eds.), *New Palgrave Dictionary of Economics, Second Edition*.

Brunnermeier, M. K. & Oehmke, M. (2012). *Bubbles, Financial Crises, and Systemic Risk*.

Camerer, C. (1989). Bubbles And Fads In Asset Prices. *Journal of Economic Surveys*, *3*.

Camerer, C. & Weigelt, K. (1988). Experimental Tests of a Sequential Equilibrium Reputation Model. *Econometrica*, *56*(1), 1 36.

Campbell, J. Y. & Shiller, R. J. (1987). Cointegration Tests of Present Value Models. *Journal of Political Economy*, *95*(5), 1062 1088.

Chiang, T. C. & Zheng, D. (2010). An empirical analysis of herd behavior in global stock markets. *Journal of Banking and Finance*, *34*, 1911 1921.

Cochrane, J. H. (2000). *Asset Pricing*.

Diba, B. T. & Grossman, H. I. (1988a). Explosive Rational Bubbles in Stock Prices? *The American Economic Review*, *78*(3), 520 530.

Diba, B. T. & Grossman, H. I. (1988b). The theory of rational bubbles in stock prices. *The Economic Journal*, *98*.

Financial Crisis Inquiry Commission. (2010). *The nancial crisis inquiry report: Final report of the National Commission on the Causes of the Financial and Economic Crisis in the United States*. Retrieved from https://www.gpo.gov/fdsys/pkg/GPO-FCIC/pdf/GPO-FCIC.pdf

French, D. (2007). Anne Goldgar, Tulipmania: money, honor, and knowledge in the Dutch golden age, Chicago, The University of Chicago Press. *History of Economic Ideas*, *15*(3).

Goldberg, D. (2005). Famous Myths of Fiat Money. *Journal of Money, Credit and Banking*, *37*.

Gordon, M. J. & Shapiro, E. (1956). Capital Equipment Analysis: The Required Rate of Pro t. *Management Science*, *3*(1), 102 110.

Hellwig, M. F. (1980). On the aggregation of information in competitive markets. *Journal of Economic Theory*, *22*(3), 477 498.

Huberman, G. & Schwert, G. (1985). Information Aggregation, In ation, and the Pricing of Indexed Bonds. *Journal of Political Economy*, *93*(1), 92 114.

J. Bradford DeLong. (2009). The Simplest Possible Behavioral Finance Bubble Model. *UC Berkeley and NBER*.

Lucas, R. E. (1986). Adaptive Behaviour and Economc Theory. *The Journal of Business*, *59*(4), 401 426.

Malkiel, B. G. (2003). The E cient Market Hypothesis and Its Critics. *Journal of Economic Perspectives*, *17*.

Muth, J. F. (1961). Rational Expectations and the Theory of Price Movements. *Econometrica*, *29*.

National Bureau of Economic Research. (2018). Dow-Jones Industrial Stock Price Index for United States. Retrieved from https://fred.stlouisfed.org/series/M1109BUSM293NNBR

Olafsson, B. O. (2019). Data for the Icelandic Stock Market [1995, .) Retrieved February 12, 2019, from https://notendur.hi.is/boo4/

Olafsson, S. (2018). Lecture notes Fixed income analysis and interest rate models. Reykjavik University.

Phillips, P. C. B., Shi, S., & Yu, J. (2015). Testing for multiple bubbles: Historical episodes of exuberance and collapse in the S&P 500. *International Economic Review*, *56*(4), 1043 1078.

Ritter, J. R. (2003). Behavioral nance. *Paci c-Basin Finance Journal*, *11*(4), 429 437.

Samuelson, P. A. & Nordhaus, W. D. (2010). *Economics: 19th. Edition*. McGraw-Hill/Irwin.

Shiller, R. J. (1981). Do stock prices move too much to be justi ed by changes in subsequent dividends? *American Economic Review*, *71*, 421 436.

Tirole, J. (1982). On the Possibility of Speculation under Rational Expectations. *The Econometric Society*, *50*(5), 1163 1181.

Tirole, J. (1985). Asset Bubbles and Overlapping Generations. *The Econometric Society*, *53*(6), 1499 1528.

U.S. Stock Market 1871-Present. (2019). Retrieved from http://www.econ.yale.edu/~shiller/data/ie_data.xls

Warren E. Bu ett. (2008). Berkshire Hathaway Chairman Letter.

Yahoo, nance. (2018). Nasdaq Composite index IXIC. Retrieved from https://finance.yahoo.com/quote/%5EIXIC/

Appendix A

Code

```

% Numerical values in simulation
T = 100;
P_t = 0.3;
lambda = 1.25;
r = 0.05;
pi = 0.5;
dividend = 0.05;
P_tMinusOne = 0.3;

% Dividend paid
    for i = 1:T
        div = rand;
        if div > pi
            d = 1;
        else
            d = 0;
        end

% Stock price at t+1
        P_tPlusOne = P_t + lambda*(P_t*(1-P_t))*((P_t - P_tMinusOne + d* dividend)/P_tMinusOne-r)

        series(i) = P_t;

        P_tMinusOne = P_t;
        P_t = P_tPlusOne;

    end

% Plot
    plot(series);
    xlabel('Time');
    ylabel('Price');
    set(gcf,'color','w');
    ylim([0.25 0.75])
```