Self-Consistent Rotation Curves for Self-Interacting Dark Matter Halos With Embedded Disks

Arna María Ormsdóttir

Faculty of Physical Sciences
University of Iceland
2020
SELF-CONSISTENT ROTATION CURVES FOR SELF-INTERACTING DARK MATTER HALOS WITH EMBEDDED DISKS

Arna María Ormsdóttir

16 ECTS thesis submitted in partial fulfillment of a Baccalaureus Scientiarum degree in Physics

Advisor
Dr. Jesús Zavala Franco

Faculty of Physical Sciences
School of Engineering and Natural Sciences
University of Iceland
Reykjavík, June 2020
Self-Consistent Rotation Curves for Self-Interacting Dark Matter Halos With Embedded Disks
16 ECTS thesis submitted in partial fulfillment of a Baccalaureus Scientiarum degree in Physics

Copyright © 2020 Arna María Ormsdóttir
All rights reserved

Faculty of Physical Sciences
School of Engineering and Natural Sciences
University of Iceland
Dunhagi 5
107, Reykjavik, Reykjavik

Telephone: 525 4000

Bibliographic information:
Arna María Ormsdóttir, 2020, Self-Consistent Rotation Curves for Self-Interacting Dark Matter Halos With Embedded Disks, B.Sc. degree, Faculty of Physical Sciences, University of Iceland.

Printing: Háskólaprent, Fálkagata 2, 107 Reykjavik
Reykjavik, June 2020
Abstract

The inconsistency that is associated with modelling some types of dark matter halos with cold dark matter is avoided in this thesis by implementing self-interacting dark matter. The halo is split into two regions; an inner region that takes into account the interactions between dark matter particles and an outer region where the interactions are negligible. The Poisson equation of the total potential is solved for the inner region which includes the density of an infinitely thin exponential disk and an isothermal dark matter halo. By combining the two mass components, an axisymmetric equilibrium model is constructed. When the Poisson equation has been solved for each component, the modelled rotation curve is reproduced and fit parameters are altered until a good fit is achieved. The outer region is modelled by a Navarro-Frenk-White profile (often used to model dark matter halos) and the two regions are matched at a radius where the average number of interactions of a dark matter particle is one in the lifetime of the halo.
## Contents

<table>
<thead>
<tr>
<th>List of Figures</th>
<th>ix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 A short overview of dark matter</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Galaxy rotation curves</td>
<td>1</td>
</tr>
<tr>
<td>1.3 Dark matter models</td>
<td>2</td>
</tr>
<tr>
<td>1.4 Rotation curve example dataset</td>
<td>4</td>
</tr>
<tr>
<td>2 Objective of this project</td>
<td>5</td>
</tr>
<tr>
<td>3 Building the model: An isothermal halo hosting a zero-thickness disk</td>
<td>9</td>
</tr>
<tr>
<td>3.1 The baryonic disk</td>
<td>10</td>
</tr>
<tr>
<td>3.2 The isothermal halo</td>
<td>11</td>
</tr>
<tr>
<td>3.2.1 Asymptotic dark matter potential</td>
<td>14</td>
</tr>
<tr>
<td>3.2.2 Inner halo potential</td>
<td>15</td>
</tr>
<tr>
<td>4 Results and Discussion</td>
<td>19</td>
</tr>
<tr>
<td>5 Conclusion</td>
<td>27</td>
</tr>
<tr>
<td>Bibliography</td>
<td>29</td>
</tr>
</tbody>
</table>
List of Figures

1.1 The rotation curve of NGC3198, dimensionless velocity as a function of dimensionless distance from the center. ......................... 4

2.1 A sketch of the DM density profile in two cases. The blue dashed line describes the NFW profile (CDM), it goes as $r^{-1}$ in the center, as $r^{-2}$ in the middle and like $r^{-3}$ in the outskirts. The red solid line is the SIDM profile. It is clear that the SIDM inner region reaches a constant density for $r < r_1$ and the outer region can be modelled as a NFW profile. Figure from Ref. [10]. .............................. 6

4.1 Rotation curve corresponding to the stellar disk and the observed rotation curve .................................................. 20

4.2 The constructed “observed” dimensionless potential corresponding to Eqs. (3.23) and (3.24) ........................................ 21

4.3 Density plot to show the different contributions in Eq. (3.26) and how it compares to the expected density behaviour of the halo $\rho \sim 1/\eta^2$. ................................................................. 21

4.4 The asymptotic DM potential as well as the observed potential to show the correct behaviour of $\psi_{asy}$. The shape of the asymptotic potential near the center where the monopole dominates results in a negative gradient in Eq. 3.2. ................................. 22

4.5 Rotation curve with the disk, asymptotic and combined potential. ................................................................. 22

4.6 Rotation curve with the disk, asymptotic and iterative potential. ................................................................. 23
LIST OF FIGURES

4.7 Rotation curve showing the monopole and the asymptotic potential without the monopole contribution. The monopole rotation curve has higher velocities for $\eta < 2$ ........................................ 24

4.8 Rotation curve with the disk, asymptotic and iterative potential, when the monopole is suppressed ........................................ 25
1 Introduction

1.1 A short overview of dark matter

Dark matter (DM) is rightly called so because it does not seem to interact with electromagnetic waves and is thus undetectable by the astronomical instruments that exist today. It is believed to account for 84% of the total mass in the universe in the form of a new unknown particle that is not in the standard model of particle physics [1]. The pioneer in the field of DM is most often said to be the Swiss-American astronomer Fritz Zwicky, who in the year of 1933 studied the redshifts of different galaxies in the Coma cluster and noticed the very large scatter in their line-of-sight velocities. By using Hubble’s estimate of the average total mass per galaxy he derived, using the virial theorem, the expected velocity dispersion of the cluster to be 80 km/s. The observed velocity dispersion of the cluster was recorded to be approximately 1000 km/s. This enormous difference in velocity dispersion sparked the interest of many scientists to find the explanation of this deviation [2]. The expected mass in the cluster could not balance the observed velocity dispersion and galaxies would have drifted away from the galaxy cluster, unless there is some form of invisible “dark matter” that accounts for the missing mass. Many within the scientific community had its doubts about DM but agreed that more information was needed to solve this puzzle. It wasn’t until the 1970’s with the assistance of radio telescopes, that could measure the 21 cm line (HI) of neutral hydrogen atoms in galaxies, that the flat rotation curves established the existence of a large amounts of missing mass, especially in the outskirts of spiral galaxies [2].

1.2 Galaxy rotation curves

The discovery of the rotation of certain types of galaxies can be traced back to the year 1914 when Slipher and Wolf noticed the inclined absorption lines, in the
nuclear spectra of a few galaxies, when the slit of the spectrogram was lined up with the galaxy’s major axis and the lines were straight when the slit was aligned with the minor axis. They concluded that this was because the observed galaxies were rotating. However, the modern optical observations of rotation curves dates back to the 1950’s when researcher’s implemented the new red sensitivity of photographic plates that ensured the observation of Hα emission lines [3]. With the observation of Hα or HI emission lines the relative radial velocity at that distance from the galaxy center could be derived using the differential linewidth created by the Doppler effect. Thus by taking multiple observations as a function of distance from the center a rotation curve can be constructed. Rotation curves can be used as tools for many purposes, one of which is the study of the distribution of DM in galaxies [3]. They are a good tracer of the total mass within a galaxy because the distribution of matter within a given radius can be obtained from the rotation curve. The total mass derived from rotation curves can then be compared to the expected mass using the luminosity profile of the galaxy. A big difference between the masses indicates the presence of a dominant DM halo in the galaxy. The rotation curves of spiral galaxies can gauge the influence of the DM because it is found that for many spiral galaxies, the rotation curve tends to a constant towards the outer radii that extends much further than the stellar disk. The flat part of the rotation curve is caused by the DM halo that is surrounding the stellar disk and is thought to have the structure that is similar to an isothermal sphere. Thus, the density distribution, $\rho$, approximately declines with the distance from the center as $1/r^2$ near the outer regions of the disk. A better model for the DM halo is that of the spherical Navarro-Frenk-White (NFW) profile [4], that goes as $1/r$ in the center, as $1/r^2$ (like the isothermal halo) in the intermediate region, and as $1/r^3$ in the outer regions [5].

1.3 Dark matter models

Due to the vast evidence that has been accumulated regarding the existence of DM it should be taken as a crucial ingredient in a theory of structure formation. When this is done, the gravitational effect of DM increases the primordial density perturbations during the early Universe, particularly around the time when the cosmic microwave background radiation was emitted, 400,000 years after the Big Bang, [1]. The standard theory of structure formation assumes that DM consists of classical, non-relativistic, collisionless particles that had insignificant thermal velocities early on and is called cold dark matter (CDM). This matter is assumed to behave like a collisionless fluid throughout the age of the Universe with the
exception shortly after the Big Bang (that depends on the formation of DM). DM is believed to be a thermal relic from the Big Bang, just as the rest of the ordinary matter. Shortly after the Big Bang, it is assumed that DM was in thermal equilibrium with the photon-baryon plasma and interacted with particles that are a part of the standard model. When the Universe cooled down, the DM disassociated with the standard model particles. If the interaction strength is assumed to be on the same scale as the weak nuclear force then a new type of yet unidentified particles, known as weakly interacting massive particles (WIMPs) are shown theoretically to have a similar thermal abundance as the observed quantity of DM. WIMPs are perfect CDM particles because they satisfy the conditions mentioned above. The CDM model has been successful in explaining the structure of the Universe on a large scale and it is the model that is most often used to describe DM halos. However, there are other equally successful alternatives to model the DM, which break each of the CDM hypothesis, warm DM (relativistic DM in the early Universe), fuzzy DM (quantum effects are important at galactic scales), and self-interacting DM (SIDM; where DM is collisional), which is relevant for this thesis [1].

The particle physics of DM is still unknown and the existence of DM can only be inferred through its gravitational effects. There are ongoing experiments where the non-gravitational interactions of DM are being studied. The CDM model has proven successful for understanding the large scale structure of the Universe as mentioned above but it seems to have severe challenges in explaining the behaviour of smaller structures like dwarf galaxies. For example, the DM density profile of a dwarf galaxy often has a core instead of the cusp that is predicted by the CDM model, see Fig. 2.1. This suggests that there are other physical mechanisms that come into play other than gravitational effects; one of them could be SIDM [6]. The main differences between CDM and SIDM are that the first is collisionless and the latter takes into account the collision between the DM particles. Thus, in the latter model, heat can be conducted from the warmer outer region to the cooler inner region and the density profile of the halo is therefore flattened. Other differences are that the particle corresponding to the SIDM model cannot be WIMP-like because the cross section needed to flatten the density profile near the center is much larger than the cross section connected to WIMPs. Instead it has an amplitude similar to the strong force for nuclear-nuclear collisions; see Ref. [6].

By using the SIDM model for rotation curves of spiral galaxies we present a self-consistent model that can fit galaxies with similar halo sizes and stellar masses even though they show a variety of behaviour near the center, which lacked in the
1 Introduction

CDM model and MOND\textsuperscript{1} [8].

1.4 Rotation curve example dataset

The rotation curve data we use in this thesis is obtained from VizieR database\textsuperscript{2} [9]. The velocity measurements are derived using the Dopple shifts of spectral lines. The relevant measurements are the galactocentric radius [kpc], observed circular velocity including the error [km/s], the scale length of the disk $h$ [kpc] and the asymptotic velocity along the flat part of the rotation curve $V_{\infty}$ [km/s]. The rotation curve for the spiral galaxy NGC3198 can be seen in Fig. 1.1 and will be used as an example of the modelling procedure described in the following Sections.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{rotation_curve.png}
\caption{The rotation curve of NGC3198, dimensionless velocity as a function of dimensionless distance from the center.}
\end{figure}

\textsuperscript{1}Modified Newtonian dynamics is an incomplete theory of modified gravity that alters Newton’s laws so that the dynamics of galaxies can be explained without the presence of DM. Many rotation curves of spiral galaxies can be understood with this theory [7].

\textsuperscript{2}This research has made use of the VizieR catalogue access tool, CDS, Strasbourg, France (DOI: 10.26093/cds/vizier). The original description of the VizieR service was published in A&AS 143, 23.
2 Objective of this project

The goal of this work was to reproduce the fit of rotation curves for spiral galaxies as was done in a recently published paper [8]. To model the DM halo, the paper assumes that the DM is self-interacting (SIDM), which is a relevant DM model that can explain some of the properties of galaxies in an alternative way to other DM models, more specifically, the standard CDM model.

The DM halo is split into two regions; the inner isothermal halo (produced by self-interactions) and the outer Navarro-Frenk-White (NFW) profile (which follows the behaviour of CDM halos; in the outer regions, self-interactions are irrelevant). The density profile of the isothermal halo in cylindrical coordinates is

\[ \rho_{\text{iso}}(R, z) = \rho_0^{DM} \exp\left(\frac{\Phi_{\text{tot}}(R, z)}{\sigma_v^2}\right) \] (2.1)

where \( \rho_0^{DM} \) is the DM central density, \( \Phi_{\text{tot}} \) is the total gravitational potential and \( \sigma_v \) is the constant DM velocity dispersion. The NFW profile has the density profile in spherical coordinates:

\[ \rho_{\text{NFW}}(r) = \frac{\rho_s r_s^3}{r(r + r_s)^2} \] (2.2)

where \( r_s \) is the scale radius of the halo and \( \rho_s \) is the characteristic density of the halo. The inner and outer regions are matched at a radius \( r_1 \), defined as the radius where a DM particle has scattered once on average in the lifetime of the halo, see Fig. 2.1. By using the continuity of the density and mass at \( r_1 \), the parameters for the inner part directly map to the parameters that describe the outer part. So if the density can be found for one region then automatically the other region can be reproduced because of this continuity. The isothermal parameters \( (\rho_0^{DM}, \sigma_v) \) connect to \( (r_{\text{max}}, V_{\text{max}}) \) or \( (r_s, \rho_s) \), the parameters used for the NFW profile. The radius \( r_{\text{max}} \) is where the DM circular velocity profile (essentially the enclosed mass divided by the radius) reaches the maximum velocity \( V_{\text{max}} \). The reason for the splitting of the halo is that outside \( r_1 \), the halo is not greatly affected by the self-interactions of the DM particles so it can be approximated by the NFW profile.
2 Objective of this project

Figure 2.1: A sketch of the DM density profile in two cases. The blue dashed line describes the NFW profile (CDM), it goes as $r^{-1}$ in the center, as $r^{-2}$ in the middle and like $r^{-3}$ in the outskirts. The red solid line is the SIDM profile. It is clear that the SIDM inner region reaches a constant density for $r < r_1$ and the outer region can be modelled as a NFW profile. Figure from Ref. [10].

The first part is to solve the Poisson equation in the inner region to get the central density and velocity dispersion of the DM. The DM is thermalized due to the self-interactions and is assumed to be in equilibrium with the baryonic matter\(^1\). The Poisson equation becomes:

$$\nabla^2 \Phi_{\text{tot}}(R, z) = 4\pi G \left[ \rho_b(R, z) + \rho_{\text{iso}}(R, z) \right]$$

(2.3)

where $\rho_b$ is the density of baryonic matter and $G$ is the gravitational constant.

When the inner density region has been modelled then the outer region is simply described by a NFW density profile Eq. (2.2). The halos are combined at $r_1$ which is determined by the condition described earlier (one collision on average per particle in the age of the halo):

$$\langle \sigma v_{\text{rel}} \rangle \rho_{\text{NFW}}(r_1) t_{\text{age}}/m = N_{\text{sc}}$$

(2.4)

where $\sigma$ is the self scattering cross section (assumed to be a constant), $v_{\text{rel}}$ is the relative velocity of the DM particle in the halo, $m$ is the mass of the DM particles, $t_{\text{age}} = 10$ Gyr is the constant age of the halo\(^2\) and $N_{\text{sc}}$ is the average number of collisions per particle.

---

\(^1\)Baryonic matter is an umbrella term for all ordinary matter in the physics of galaxies. The reason for the name is that baryons are much heavier than leptons and thus dominate the gravitational interactions of the ordinary matter.

\(^2\)10 Gyr is close to the age of the Universe; in fact, each halo is expected to have a different time of formation, but it has been shown that the precise age in Eq. (2.4) is not relevant.
scatterings per DM particle and is fixed to 1. The halo is assumed to follow a Maxwellian velocity distribution and thus in Eq. (2.4) the velocity is averaged over the Maxwellian velocity distribution (represented by brackets). Thus, the NFW density at $r_1$ is:

$$\rho_{\text{NFW}}(r_1) = \left( \frac{4}{\sqrt{\pi}} \frac{\sigma}{m \sigma_{\text{v}0} t_{\text{age}}} \right)^{-1}$$

(2.5)

where the cross section per unit mass is fixed to $3\text{cm}^2/\text{g}$ and $\sigma_{\text{v}0}$ is the central velocity dispersion of the DM particles. The inner and outer regions can be joined since the density of NFW is fitted for at $r_1$ (given by the parameter $\rho_{0\text{DM}}$ in the inner SIDM core) and $\sigma_{\text{v}0}$ is also fitted for in inner region. A method to solve Eq. (2.3) using an infinitely thin-disk profile of the disk is derived in chapter 3. The model of the disk (in cylindrical coordinates) is:

$$\rho_b(R, z) = \Sigma_0 \exp(-R/h)\delta(z)$$

(2.6)

where $\Sigma_0 = \Sigma(0)$ is the central surface density, $h$ is the scale length of the stellar disk and $\delta(z)$ represents the Dirac delta function. Ref [5] solves Eq. (2.3) and creates numerical templates for the isothermal density profile on the grid of $\alpha$ and $\beta$, which represent the relevant dimensionless parameters of the DM halo and the disk. If the constant variables that describe the disk are known, that is the central surface density $\Sigma_0$ and the scale length of the disk $h$, then the parameters $\alpha$ and $\beta$ give the density of the central DM and the velocity dispersion $\sigma_{\text{v}0}$ of the isothermal halo. The templates of $\alpha$ and $\beta$ can then be interpolated to make a rotation curve for any set of $(\rho_{0\text{DM}}, \sigma_{\text{v}0}, \Sigma_0, h)$. 


3 Building the model: An isothermal halo hosting a zero-thickness disk

This chapter is based on and uses the methods of [5].

There are a few conditions that have to be satisfied for a successful fit to the rotation curve within this modelling:

- The rotation curve $V(R)$ has to be known. The measurements of radial velocity and radius will be called $v_{obs}$ and $r_{ad}$ respectively.

- The rotation curve has to be flat all the way to the outskirts (effectively infinity; $V_\infty$) to have a proper boundary condition.

- The rotation curve has to be regular, that is at zero radius the velocity is zero.

The model is split into two parts corresponding to the two mass components, visible matter (zero-thickness disk) and DM (nonspherical isothermal halo). By combining these masses and constructing an axisymmetric equilibrium model, a rotation curve can be reproduced that fits the observed rotation curve.

The main goal for constructing the model is to solve the Poisson equation for the system, described by

$$
\nabla^2 \Phi_T(R, z) = 4\pi G [\rho_D(R, z) + \rho_{DM}(R, z)]
$$

where $\Phi_T = \Phi_{DM} + \Phi_D$ is the total gravitational potential from the disk and DM, $\rho_D$ is the mass density of the disk, $\rho_{DM}$ is the DM density, renaming the variables $\rho_b$ and $\rho_{iso}$ respectively in Eq. (2.3). The coordinates $(R, z)$ are the standard cylindrical coordinates. When the total potential is known, one can obtain the
3 Building the model: An isothermal halo hosting a zero-thickness disk

rotation curve with the following equation and compare to the measured data.

\[ V_{mod}(R) = \sqrt{R(\partial \Phi_T / \partial R)|_{z=0}} \]  

(3.2)

The gradient of the potential is evaluated at \( z = 0 \), which corresponds to \( \theta = 90^\circ \) in spherical coordinates so in the final result the potential will be taken along that angle.

3.1 The baryonic disk

The disk is the only visible matter that is taken into account in this approach. The bulge that might be present in the galaxy is ignored. The preferred units in the beginning are cylindrical coordinates \((R, z, \theta)\) so the spherical radius is \( r = \sqrt{R^2 + z^2} \). We work with dimensionless quantities using the scale length of the stellar disk, \( h \), and the square of the velocity dispersion, \( \sigma_0^2 = V_\infty^2 / 2 \), for specific energy. The value of \( V_\infty \) for a specific rotation curve can be found in databases such as [9] or it can be used as a free parameter in the fit. The cylindrical coordinates and spherical radius in dimensionless form are

\[ \xi = R/h \quad \zeta = z/h \quad \eta = r/h \]  

(3.3)

and the dimensionless total potential is then

\[ \psi_T = \psi_D + \psi_{DM} = \Phi_T / (V_\infty^2 / 2) \]  

(3.4)

By separating (3.1) into the disk and DM, a solution to the dimensionless equation can be reproduced if the density of the disk is known. This model uses a zero-thickness disk to model the stellar density and the density is said to be proportional to the observed surface brightness profile \( \mu(R) \):

\[ \rho_D(R, z) = \left( \frac{M}{L} \right) \mu(R) \delta(z) \]  

(3.5)

Where \( \delta(z) \) is the Dirac delta-function which matches the zero-thickness description and \( M/L \) is the mass-to-light ratio of the disk. Since the surface brightness profile is observed, the only free parameter in the density is the mass-to-light ratio, as mentioned in Section 2, which is taken to be a constant for a given surface brightness (photometric) profile. The model of the disk is a purely exponential disk so the surface density is described by \( \tilde{\Sigma} = \exp(-\xi) \). The Poisson equation for the disk in cylindrical dimensionless coordinates becomes:

\[ \left( \frac{1}{\xi} \frac{\partial}{\partial \xi} \xi \frac{\partial}{\partial \xi} + \frac{\partial^2}{\partial \zeta^2} \right) \psi_D = -\frac{\beta}{2} \tilde{\Sigma}(\xi) \delta(\zeta) \]  

(3.6)
where the dimensionless parameter $\beta$ is introduced. It represents the weight of the disk in a dimensionless version measured by the mass-to-light ratio $M/L$.

$$\beta = \frac{8\pi G \Sigma(0) h}{V_{\infty}^2} = \frac{M}{L} \frac{8\pi G \mu(0) h}{V_{\infty}^2}$$

(3.7)

The constants are: $\Sigma(0)$, the surface density in the center and $\mu(0)$ is the observed surface brightness in the center. Thus, if the fit to the disk is successful the fit parameter $\beta$ gives the mass-to-light ratio of the disk. Since the disk is purely exponential and the potential has to satisfy the initial condition $\psi_D(0, 0) = 0$ there exists an analytical solution to Eq. (3.6). The solution is

$$\psi_D(\xi, \zeta) = \frac{\beta}{2} \left\{ \int_0^\infty d(hk) \frac{\exp(-hk|\zeta|) J_0(hk \xi)}{[1 + (hk)^2]^{3/2}} - 1 \right\}$$

(3.8)

where $J_0$ is the Bessel functions of the first kind $J_n$, $n = 0$ in this case. It is clear from Eq. (3.8) that the mass-to-light ratio is the only free parameter that is needed to construct the rotation curve for the disk. Eq. (3.8) is adapted from [5]. The solution in [5] had a typo in the way that it was the negative part of Eq. (3.8) which can likely be traced back to the conversion from dimensional potential to a dimensionless form. Another typo was in the denominator where $hk$ was $hk \xi$ in [5]. The deduction of the potential of a zero thickness disk can be found in [11].

Later calculations in this report are done in spherical coordinates so it is convenient to work as well Eq. (3.8) in those coordinates. The spherical coordinates are $(r, \phi, \theta)$ and the corresponding dimensionless parameters are $(\eta, \phi, \theta)$. In Section 3.2, the Poisson equation is expanded into Legendre polynomials of $\cos \theta$ so the disk is also kept as a function of $\cos \theta$. Thus, Eq. (3.8) in spherical coordinates is

$$\psi_D(\eta, \cos \theta) = \frac{\beta}{2} \left\{ \int_0^\infty d(hk) \frac{\exp(-hk|\eta \cos \theta|) J_0(hk \sqrt{1 - \cos^2 \theta})}{[1 + (hk)^2]^{3/2}} - 1 \right\}$$

(3.9)

### 3.2 The isothermal halo

The potential from the halo is not as easily solved as the disk potential. The approach in Ref. [5] is to choose a Maxwellian isothermal distribution to describe the halo. The DM halo is therefore assumed to have the same temperature throughout and therefore has the same velocity distribution. This is an excellent description for the inner part of the halo since we are assuming SIDM, where collisions naturally tend to thermalize the DM distribution. This can be adapted to other
distribution functions if that is desired. Another reason for this choice is that it guarantees that the rotation curve is flat at large radii, which is exactly what is needed since the disk does not contribute much to the flat part of the rotation curve. The Maxwellian distribution function for the DM halo is isotropic which is to say that it is identical in all directions:

\[
f_{DM}(E) = \frac{\rho_{DM}^0}{(2\pi \sigma_{v0}^2)^{3/2}} \exp\left(-\frac{E}{\sigma_{v0}^2}\right)
\]

(3.10)

where the specific energy is \( E = \nu^2/2 + \Phi_T \), \( \sigma_{v0} \) is the velocity dispersion and \( \rho_{DM}^0 \) is the central DM density. The first term in the specific energy corresponds to the kinetic energy and \( \Phi_T \) is the total gravitational potential from the disk and DM. To get the DM density it is only needed to integrate the distribution function with respect to the velocities, \( \nu \):

\[
\rho_{DM}(R,z) = \int d^3\nu f_{DM} = \rho_{DM}^0 \exp\left[-\Phi_T(R,z)/\sigma_{v0}^2\right]
\]

(3.11)

This density is not isotropic and thus not spherically symmetric because of the presence of the zero-thickness disk. The disk breaks the symmetry in the total potential. Eq. (3.11) is a transcendental (or implicit) equation, that is the DM density depends on the total gravitational potential which in turn depends on the density of the DM. Even though this distribution is not spherically symmetric, the disk will dominate at small distances from the center and when the radius is extended further from the center, the influence from the disk becomes negligible. Therefore, the potential reaches spherical symmetry and thus follows the spherical solution for the isothermal halo, which is fitted to match the observed flat rotation curve. In other words, at large radii we expect:

\[
\Phi_T(R,z) \sim \Phi_T(r) \sim V_\infty^2 \ln\left(\frac{r}{r_0}\right)
\]

(3.12)

where the parameter \( r_0 \) is the scale length of the spherical isothermal density profile. By imposing the boundary condition of Eq. (3.12) and inserting it in the Poisson equation we get the following conditions on \( r_0 \) and \( \sigma_{v0} \):

\[
\begin{align*}
V_\infty^2 &= 2\sigma_{v0}^2 \\
V_\infty^2 &= 4\pi G \rho_{DM}^0 r_0^2
\end{align*}
\]

(3.13) (3.14)

The only free parameters for the density in Eq. (3.11) are the DM density in the center \( \rho_{DM}^0 \), which in fact is the total density in the center because of the condition that \( \Phi_T(0,0) = 0 \), and the velocity dispersion \( \sigma_{v0} \). The velocity dispersion is fixed by the value of \( V_\infty^2 = 2\sigma_{v0}^2 \), so \( \sigma_{v0} \) is a constant. The scale length of the DM is also
a fixed parameter because of the conditions implied in Eq. (3.14). Thus, if \( V_\infty \) is known, and given by the asymptotic velocity of the observed rotation curve, then the only free parameter is \( \rho^0_{DM} \) for the DM part. Therefore, it is evident that if the constant mass-to-light ratio for the disk, the observed surface brightness profile \( \mu(R) \) and the central DM density are known, then Eq. (3.1) can be solved.

Given Eq. (3.11), the Poisson equation can be written for the DM in dimensionless cylindrical coordinates:

\[
\left( \frac{1}{\xi} \frac{\partial}{\partial \xi} \xi \frac{\partial}{\partial \xi} + \frac{\partial^2}{\partial \zeta^2} \right) \psi_{DM} = -\alpha \exp[\psi_{DM}(\xi, \zeta) + \psi_D(\xi, \zeta)]
\]

(3.15)

where the dimensionless parameter \( \alpha \) is

\[
\alpha = \frac{8\pi G h^2 \rho^0_{DM}}{V_\infty^2} = \frac{2 h^2}{r_0^2}
\]

(3.16)

and can be interpreted as the dimensionless measure of the DM central density. This equation does not have an analytical solution like the disk so it has to be solved numerically. Before describing that method, let’s introduce the relevant boundary conditions:

\[
\begin{cases}
\psi_{DM}(0, 0) = 0 \\
\psi_{DM}(\eta) \sim -2 \ln \left( \eta \sqrt{\frac{\alpha}{2}} \right) - \psi_D^\infty \quad \eta \gg 1
\end{cases}
\]

(3.17)

(3.18)

where \( \psi_D^\infty \) defines the constant value that the dimensionless potential of the disk reaches at large radii.

The method that is used to solve Eq. (3.15) is based on work done by Prendergast & Tomer [12]. Their article was about non-spherical models for rotating elliptical galaxies and that method can be used if the potential converges to some finite value at infinity. This report is investigating spiral galaxies where the model for the potential diverges at large radii causing a problem. The solution to this problem is to split the isothermal halo into two parts; an asymptotic part and a regular part that converges to some finite value at large radii. The latter potential can thus use the methods of [12] and the asymptotic potential follows Eq. (3.18) at large radii and can thus be constructed. The DM dimensionless potential can then be rewritten as

\[
\psi_{DM} = \psi_{asy} + \psi
\]

(3.19)

The potential \( \psi \) has to be solved by an iterative multipole expansion into Legendre polynomials. First, we describe how the asymptotic potential is obtained.
3 Building the model: An isothermal halo hosting a zero-thickness disk

3.2.1 Asymptotic dark matter potential

The potential that is defined to be $\psi_{asy}$ satisfies the following Poisson equation:

$$\nabla^2 \psi_{asy} = \hat{\rho}_{asy} (3.20)$$

where $\hat{\rho}_{asy}$ is the dimensionless asymptotic density of the DM halo. There are infinitely many solutions of $(\hat{\rho}_{asy}, \psi_{asy})$ that satisfy Eqs. (3.20) and (3.18) but one solution will be shown here. The pair $(\hat{\rho}_{asy}, \psi_{asy})$ are chosen to be spherically symmetric in this case since that is an excellent approximation for the asymptotic part. Justified by the fact that $\psi_{asy}$ is spherically symmetric because the contribution from the disk is negligible and also because the spherical Poisson equation has a simple solution.

First, an “observed” potential is constructed from the observed rotation curve, so it uses the measurement $v_{obs}$ and $rad$ mentioned above:

$$\Phi_{obs}(r) = \int_0^r \frac{V^2(s)}{s} ds = \int_0^r \frac{v_{obs}^2}{rad} drad (3.21)$$

So we make a function using the measurements of the velocity squared and divide by the radial measurements. Then the function is numerically integrated as in Eq. (3.21) using the trapezoidal rule, with the radial measurements as the integration sub-interval. This observed potential has the same asymptotic behaviour as Eq. (3.12) but we use another scaling factor to fit to the observed potential, $r_0^{obs}$, and thus the asymptotic condition at large radii becomes:

$$\Phi_{obs}(r) \sim V_\infty^2 \ln \left( \frac{r}{r_0^{obs}} \right) (3.22)$$

The observed scale length of the DM, $r_0^{obs}$, can be calculated directly from Eq. (3.22). It uses the observed potential and evaluates $r_0^{obs}$ at the maximum radial measurement of the observed rotation curve. The observed potential can be described using the following equations:

$$\begin{cases} 
\Phi_{obs}(r) = \int_0^r \frac{V^2(s)}{s} ds & r \leq \text{max}(rad) \\
\Phi_{obs}(r) = V_\infty^2 \ln \left( \frac{r}{r_0^{obs}} \right) & r > \text{max}(rad) 
\end{cases} (3.23)$$

We convert $\Phi_{obs}(r)$ into dimensionless form and get that the Poisson equation in spherical coordinates for this potential has the asymptotic behaviour:

$$\left( \frac{1}{\eta^2} \frac{d}{d\eta} \eta^2 \frac{d}{d\eta} \right) \psi_{obs}(\eta) \sim -\alpha_{obs} \exp[\psi_{obs}(\eta)]; \eta \gg 1 (3.25)$$
3.2 The isothermal halo

where \( \alpha_{\text{obs}} \) can be directly calculated from Eq. (3.16) using the value of \( r_{\text{obs}}^0 \) found above and \( h \), the scale length of the disk. The form of the Poisson equation is known so the density can be defined from the observed potential and then the (spherical) disk potential is subtracted to prevent having the potential from the disk in the DM density.

\[
\hat{\rho}_{\text{asy}}(\eta) = -\alpha_{\text{obs}} \exp(\psi_{\text{obs}}(\sqrt{\eta^2 + \eta_c^2})) - \hat{\rho}_D^{(0)}(\eta)
\]  

(3.26)

Here a regular core structure with a characteristic size \( \eta_c \) has been introduced\(^1\). The second term in \( \hat{\rho}_{\text{asy}} \) is the monopole term corresponding to the disk. The monopole term gives the spherical contribution to the total disk potential and thus the enclosed mass of the disk is treated as a point mass. It is defined to be negative because it corresponds to the negative dimensionless potential \( \psi_{\text{asy}} \):

\[
\hat{\rho}_D^{(0)}(\eta) = -\frac{\beta}{2} \int_{-1}^{1} \tilde{\Sigma}(\eta) \delta(\zeta) P_0(\cos \theta) d\cos \theta = -\frac{\beta \tilde{\Sigma}(\eta)}{2 \eta}
\]  

(3.27)

The monopole term caused a numerical issue in the center according to our analysis and to circumvent that problem, a core was introduced in the monopole instead of the first term in Eq. (3.26) to suppress the unphysical behaviour of the unaltered Eq. 3.26 in the center; we will come back to this point in Chapter 4. The solution to the spherical Poisson equation is then given by:

\[
\psi_{\text{asy}}(\eta) = \psi_{\text{asy}}(0) + \int_0^\eta \hat{\rho}_{\text{asy}}(\eta') d\eta' - \frac{1}{\eta} \int_0^\eta \eta'^2 \hat{\rho}_{\text{asy}}(\eta') d\eta'
\]  

(3.28)

where the constant \( \psi_{\text{asy}}(0) \) is first taken as a free parameter and later determined by matching the potential \( \psi_{\text{asy}} \) without the constant to the asymptotic behaviour in Eq. (3.18). So the constant is simply \( \psi_{\text{asy}} \) without the constant evaluated at a large radius subtracted from Eq. (3.18) evaluated at the same large radius.

3.2.2 Inner halo potential

The inner halo is constructed via an iterative method using a standard multipole expansion in Legendre polynomials. This can be done because the inner potential \( \psi \) converges to zero at infinity. The objective is to solve the two-dimensional Poisson equation:

\[
\nabla^2 \psi(\eta, \cos \theta) = \hat{\rho}[\eta, \cos \theta; \psi(\eta, \cos \theta)]
\]  

(3.29)

\(^1\)By the end of this thesis, the purpose of this core structure remains unclear.
by using an iterative method, so to obtain the \((n + 1)\) potential \(\psi^{(n+1)}\) it uses the potential from the previous step \(\psi^{(n)}\). That can be described by the following equation:

\[
\nabla^2 \psi^{(n+1)}(\eta, \cos \theta) = \hat{\rho}[\eta, \cos \theta; \psi^{(n)}(\eta, \cos \theta)]
\]

(3.30)

The Poisson equation in this case is Eq. (3.15) when the DM has been split into two parts and the solution to \(\psi\)asy in Eq. (3.20) is added:

\[
\left(\frac{1}{\xi} \frac{\partial}{\partial \xi} \xi \frac{\partial}{\partial \xi} + \frac{\partial^2}{\partial \xi^2}\right) \psi(\eta, \cos \theta) = -\alpha \exp[\psi\)asy\( (\eta) + \psi_D(\eta, \cos \theta) + \psi(\eta, \cos \theta)] - \hat{\rho}_{\text{asy}}(\eta)
\]

(3.31)

The dependence on \(\eta\) and \(\cos \theta\) has been made explicit to show which part is two-dimensional and which is one-dimensional. Ref. [5] writes the functions so they depend on \((\eta, \theta)\) but since we will use the Legendre polynomials of \(\cos \theta\) it is convenient to write the functions depending on \(\cos \theta\). The solution to the inner halo potential part uses the potential from the disk and the asymptotic part.

To make the expansion in Legendre polynomials \(P_k(\cos \theta)\) the coefficients of \(\hat{\rho}\) in Eq. (3.29), which corresponds to the right hand side of Eq. (3.31), have to be determined. The expansion can be written as:

\[
\hat{\rho}^{(n)}(\eta, \cos \theta) = \sum_{k=0}^{\infty} \hat{\rho}_k^{(n)}(\eta) P_k(\cos \theta)
\]

(3.32)

\[
\psi^{(n+1)}(\eta, \cos \theta) = \sum_{k=0}^{\infty} \psi_k^{(n)}(\eta) P_k(\cos \theta)
\]

(3.33)

where \(\hat{\rho}_k^{(n)}(\eta)\) are the Legendre coefficients of \(\hat{\rho}^{(n)}(\eta, \cos \theta)\). The Legendre coefficients can be estimated using the orthogonality of Legendre polynomials. By using Fourier’s trick we get from [13]:

\[
\int_{-1}^{1} P_k(\cos \theta) P_{k'}(\cos \theta) d\cos \theta = \begin{cases} 0, & \text{if } k \neq k' \\ \frac{2}{2k + 1}, & \text{if } k = k' \end{cases}
\]

(3.34)

Thus if Eq. (3.32) is multiplied by \(P_{k'}(\cos \theta)\) and integrated with respect to \(\cos \theta\) we get the expression for the coefficients of the Legendre polynomials:

\[
\hat{\rho}_k^{(n)}(\eta) = \frac{2k + 1}{2} \int_{-1}^{1} \hat{\rho}^{(n)}(\eta, \cos \theta) P_k(\cos \theta) d\cos \theta
\]

(3.35)
When the expansion is successful the general solution of the n-th iterative step is:

$$
\psi^{(n+1)}(\eta, \cos \theta) = \Psi + \left[ \int_0^{\eta} \eta' \rho_0^{(n)}(\eta') d\eta' - \frac{1}{\eta} \int_0^{\eta} \eta' \rho_0^{(n)}(\eta') d\eta' \right]
- \sum_{k=2}^{\infty} \frac{P_k(\cos \theta)}{2k+1} \left[ \eta^k \int_{\eta}^{\infty} \eta^{1-k} \rho_k^{(n)}(\eta') d\eta' + \frac{1}{\eta^{k+1}} \int_0^{\eta} \eta^{k+2} \rho_k^{(n)}(\eta') d\eta' \right]
$$

(3.36)

where $\Psi = \psi(0, \cos \theta) = \psi(0, 0)$. The value of this constant is determined by $\psi_T(0, 0) = \psi(0, 0) + \psi_{asy}(0) + \psi_D(0, 0) = 0$. Since $\psi_D(0, 0) = 0$ then $\Psi = -\psi_{asy}(0)$. The numerical integration used was again the trapezoidal rule. To start the iteration process, an initial guess is needed for $\psi$, which is chosen to be a zero array. The only parameters needed to solve this equation are the Legendre coefficients of Eq. (3.31) and the Legendre polynomial of $\cos \theta$.

An error has to be introduced to estimate whether the potential has converged to the final value. The desired accuracy can be evaluated with the following equation, where $D$ is the applicable domain:

$$
\max_{(\xi, \zeta) \in D} \left| \frac{\psi^{(n+1)} - \psi^{(n)}}{\psi^{(n+1)} + \psi^{(n)}} \right| < \epsilon
$$

(3.37)

It determines the relative error of the last iteration step and compares to the new solution computed in the newest iteration. Thus, the iteration stops if this condition is satisfied. Since, the disk and DM potentials are known, the rotation curve can be reproduced with Eq. (3.2).
4 Results and Discussion

This thesis originally intended to present the full modelling of a SIDM halo with a central core and an NFW profile, in the outskirts. Because of time constraints, the latter part was not included. In this section we present the result of the modelling based on Chapter 3 for the spiral galaxy NGC3198 described in the Introduction. The values $\alpha = 2.6$ and $\beta = 5.9$ were given in [5] as well as the scale length of the disk $h = 2.68\text{kpc}$ and the velocity $V_\infty = 160.1\text{km/s}$. These values were used in the results to test if the model works so the complete fit procedure for $\alpha$ and $\beta$ have not been added into our modelling yet. The velocity and radial measurements for NGC3198 were adapted from Table 2 in [14] and not from the SPARC database as mentioned above. The two-dimensional potential from the zero-thickness disk was calculated using Eq. (3.9). The rotation curve that corresponds to the disk potential can be seen in Fig. 4.1. It uses Eq. (3.2) with $\Phi_T = \Phi_D$ so Eq. (3.9) is first converted to a dimensional form using Eq. (3.4). It is clear from Fig. 4.1 that the stellar disk dominates in the central region of the rotation curve and decreases with radius as expected for an exponential disk.

Next, the DM potential is solved using Eq. (3.15). The halo is split into two regions as mentioned above. It is first needed to construct the “observed” potential as in Eq. (3.21), see Fig. 4.2. The asymptotic behaviour of Eq. (3.22) is matched to Eq. (3.21) at the maximum radial measurement. Thus the derived value of the observed scale length of the DM is $r_0^{obs} = 2.43\text{kpc}$. The value of the observed $\alpha$ is then $\alpha_{obs} = 2h^2/(r_0^{obs})^2 = 2.44$. The monopole term is computed next using Eq. (3.27) and with it, the asymptotic density, where the value of $\eta_c$ in Eq. (3.26) is fixed to zero. The density of the monopole dominates the center of the asymptotic density as is evident from Fig. 4.3. Other values of $\eta_c$ were tried to see if that would change the overall dominance of the monopole, but that was not the case. The first term in Eq. (3.26) does not diverge at zero and thus the regularizing core does not affect it other than shifting the rotation curve of the asymptotic potential to start at larger $\eta$, see Fig. 4.5. The one-dimensional asymptotic potential is then constructed using the asymptotic density, see Fig. 4.4. The figure includes the observed potential to show that the asymptotic potential follows that same asymptotic behaviour at large radii.
4 Results and Discussion

The density $\hat{\rho}_{asy}$ goes roughly like $1/\eta^2$ so when imposed in Eq. (3.28) the potential should be logarithmic as seen in Fig. 4.4. The rotation curve with the asymptotic contribution can be seen in Fig. 4.5. The sum of the velocity contributions from the disk and asymptotic potential are also given in Fig. 4.5 and almost fits the outer region of the rotation curve. The reason that the solid black asymptotic line stops right before $r/h = 2$ is that the shape of the asymptotic potential in the center (see Fig. 4.4) gives a negative gradient which gives unphysical values in Eq. (3.2). The asymptotic part affects the rotation curve the most in the outer region and thus we determined that it did not matter how it looked like in the center because it is unphysical. The inner halo potential was then expected to give a greater contribution in the center to prevent this unphysical behaviour in the final rotation curve.

For the inner halo potential, the Legendre coefficients of the right hand side of Eq. (3.31) were derived and inserted into the general solution to the n-th iterative step of the two-dimensional potential in Eq. (3.36). In this report the accuracy was chosen to be $\epsilon = 10^{-2}$. The iterative process converged relatively quickly to a solution and the difference in accuracy was barely noticeable when using $\epsilon = 10^{-2}$ or $\epsilon = 10^{-4}$. So the first $\epsilon$ was chosen because it is faster. The final rotation curve with the DM and disk is in Fig. 4.6.

Figure 4.1: Rotation curve corresponding to the stellar disk and the observed rotation curve
Figure 4.2: The constructed “observed” dimensionless potential corresponding to Eqs. (3.23) and (3.24).

Figure 4.3: Density plot to show the different contributions in Eq. (3.26) and how it compares to the expected density behaviour of the halo $\rho \sim 1/\eta^2$. 
4 Results and Discussion

Figure 4.4: The asymptotic DM potential as well as the observed potential to show the correct behaviour of $\psi_{\text{asy}}$. The shape of the asymptotic potential near the center where the monopole dominates results in a negative gradient in Eq. 3.2.

Figure 4.5: Rotation curve with the disk, asymptotic and combined potential.
Figure 4.6: Rotation curve with the disk, asymptotic and iterative potential.

It is clear that this total rotation curve does not align with the observed rotation curve since the inner halo potential does not give the right contribution. The problem appears to be in the monopole term that originates in Eq. (3.26) and contributes in an unphysical way to the asymptotic part of the halo near the center and in the intermediate regions. If this is the case, then the zeroth Legendre coefficient is also dominated by the monopole term because of its presence in Eq. (3.26). For $k \geq 2$ in Eq. (3.31) the values of the coefficients were negligible compared to the zeroth coefficient which reflects the monopole (in other words, the inner halo distribution becomes essentially spherical and dominated by the monopole term). Because of the divergence of the monopole, the right hand side of Eq. (3.31), which represents $\hat{\rho}_{DM} - \hat{\rho}_{asy}$, has the monopole behaviour when it should have the isothermal behaviour of the DM halo. The reason that for $\eta > 2$ the inner halo potential rotation curve goes to zero is because the gradient of the potential is negative which results in unphysical values in the rotation curve. These values were exchanged to zero to compute the total velocity which has this strange dip at $\eta \approx 2$. This shape of the inner halo potential is peculiar because it should be almost spherically symmetric and converge to zero at large radii, that is to be regular. To test the iterative method we tried calculating in this way the asymptotic potential to see if it matched the actual asymptotic potential obtained from Eq. (3.28). We put $\alpha = 0$ and changed the sign of $\hat{\rho}_{asy}$ in Eq. (3.31) and it resulted in the same shaped potential shifted in the velocity direction. We then concluded that the Legendre expansion is correct and that the problem lies in the
monopole term.

The monopole was truncated close to zero to get rid of the divergence by introducing a core in the monopole Eq. (3.27). To find the correct value of $\eta_c$, the radius of a regularizing core, a rotation curve of the asymptotic potential was constructed without the monopole. Then the analytical solution of the monopole rotation curve is also computed, see Fig 4.7. Whenever the monopole is larger in than the asymptotic part it will result in unphysical values for the combined rotation curve. So the value of the radius of the regularizing core of the monopole term was chosen to be $\eta_c = 2$.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{rotation_curve}
\caption{Rotation curve showing the monopole and the asymptotic potential without the monopole contribution. The monopole rotation curve has higher velocities for $\eta < 2$}
\end{figure}

Other values of $\eta_c$ were tried and none of them produced a rotation curve that fits to the observed curve as well as the core structure mentioned above. When the monopole is almost zero then $\rho_{asy}$ in Fig. 4.3 follows the orange line approximately. The asymptotic and disk potential resulted in a rotation curve that was a bit higher in amplitude than the observed curve. The inner halo potential then made the correction needed for the perfect fit. This is however, not the standard way of obtaining the rotation curve. The expected result for the asymptotic potential was to contribute in the outskirts of the rotation curve and that the inner halo potential would present the potential needed in the center and converge to zero.
The final rotation curve computed by suppressing the monopole in the center is given in Fig. 4.8. The inner halo potential barely contributes to the rotation curve when the monopole is suppressed and in fact gives only invalid values because the gradient of the inner halo $\psi$ is negative. It does, however, lower the amplitude of the total fit resulting in a more accurate fit. This is because the inner halo potential has the opposite sign when compared to the disk and asymptotic potentials. Thus, when the potentials are added together it decreases the total potential so that it fits to the observed rotation curve.

Figure 4.8: Rotation curve with the disk, asymptotic and iterative potential, when the monopole is suppressed
5 Conclusion

Given the time constraints and the obstacles to manage the divergence of the monopole, only the first part of the objective is presented in this work. The goal was to reproduce the self-consistent modelling of galaxy rotation curves with an infinitely thin disk and an isothermal halo (motivated by Self-interacting DM). The galaxy was split into two mass components; the baryonic matter and the isothermal DM halo. The Poisson equation was solved individually for these components. The stellar disk had an analytical solution but the DM halo had to be solved numerically. The latter was done by splitting the DM into an asymptotic potential and a potential that converges to zero at large radii. We encountered issues when dealing with the monopole term of the stellar disk whose role was to subtract the “spherical” contribution of the disk from the total density in the spherically symmetric total asymptotic density constructed directly from the rotation curve. Since the asymptotic density inferred was smaller than the monopole term in the center, this led to negative values in the rotation curve for the inner halo potential and the asymptotic potential, which are not physical, see Fig. 4.6. A way to fix this divergence is to introduce a regularizing core in the monopole to suppress this behaviour near the center. When that was done the modelled rotation curve fitted to the observed rotation curve, see Fig. 4.8.

The values of $\alpha$ and $\beta$ were known beforehand and used to check if the model works, the fit procedure of $\alpha$ and $\beta$ was not added to the model and is something that could be added in the future developments of our model. In principle, to fit the inner part of the rotation curve the values of $(\rho_{DM}^0, \sigma_v, \Sigma_0, h)$ have to be estimated and with each fit the $\chi^2$ d.o.f. is calculated and the process is iterated manually by adjusting the parameters until a good fit is achieved. When they have been fitted the outer region of the DM halo which has a NFW structure can be modelled. Then the inner and outer regions are matched at $r_1$ fully determined by the SIDM cross section per unit mass as described in Eq. (2.5).

There might have been some inconsistency somewhere in the model that resulted in the dominance of the monopole in the Legendre expansion. However, the model was entirely built on the methods in Ref. [5] and it did not mention any trouble
5 Conclusion

with the monopole but rather they introduced a regularizing core structure in the first term of Eq. (3.26) which did not seem to alter the calculations in this thesis.
Bibliography


Bibliography


