Financial Valuation of Projects with Emphasis on Real Options

by

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Thesis of 30 ECTS credits submitted to the School of Science and Engineering
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Abstract

When it comes to investment decisions, the valuation of investments is an important part of selecting the most profitable or promising projects to be undertaken. Today's most commonly used valuation methods are the Discounted cash flow method and Decision tree analysis, which tend to underestimate the value of risky investment opportunities by failing to capture a certain reality in business today. The market is characterised by change, uncertainty, and competition. As a result, the cash flow will probably be different from what the firms initially expected. When new information is received and uncertainty about market conditions and future cash flows is resolved, firms can have the flexibility to change their original operating strategy. One way to assess the value of flexibility is to use real options based on the same methodology used to price derivatives. With this approach, investment opportunities are valued as an option to invest in real assets. Most investments involve more than one uncertainty. Therefore, it is crucial to know the risks that affect the cash flow in a project. One way to keep uncertainties separate as they are independent and do not affect each other is to make a Quadranomial tree that allows the uncertainties to be resolved simultaneously. This thesis reviews two traditional valuation methods along with a comparison to the real options approach. Various real options can apply to the valuation of tangible assets, and the main ones will be considered. It will also be examined how real solutions are resolved when there is more than one risk.

Verðmat á Verkefnum með Áherslu á Raunvilnanir

Guðrún Helga Grétarsdóttir

október 2021

Útdráttur

Þegar kemur að fjárfestingarákvörðunum er verðmat fjárfestinga mikilvægur þáttur í ferlinu við val á arðbærustu eða efnilegustu verkefnunum sem ráðist er í. Algengustu verðmatsaðferðirnar í dag eru aðferðin með afslátt af sjóðstreymi (e. Discounted cash flow method) og ákvörðunartrésgreining (e. Decision tree analysis), sem hafa tilhneigingu til að vanmeta verðmæti áhættusamra fjárfestingartækifæra með því að ná ekki tilteknum veruleika í viðskiptum í dag. Markaðurinn einkennist af breytingum, óvissu, og samkeppni. Þar af leiðandi mun sjóðstreymið líklega vera annað en fyrirtækin bjuggust við í upphafi. Þegar nýjar upplýsingar berast og óvissa um markaðsaðstæður og framtíðar sjóðstreymi leysast, þá geta fyrirtækin haft sveigjanleika til þess að breyta sinni upphaflegu rekstrarstefnu. Ein leið til þess að meta virði sveigjanleika er að nota Raunvilnanir (e. Real options) sem byggð er á sömu aðferðafræði og er notuð til að verðleggja afleiður. Með þessari nálgun eru fjárfestingartækifæri verðmetin sem valkostur til að fjárfesta í efnislegri eign. Flestar fjárfestingar innihalda fleiri en eina óvissu. Þess vegna er mikilvægt að þekkja þær áhættur sem hafa áhrif á virði fjárstreymis í verkefninu. Aðferð til að halda óvissum aðskildum þar sem þær eru sjálfstæðar og hafa ekki áhrif á hvor aðra er að gera Fjórhyrnings tré (e. Quadranomial tree) sem leyfir óvissunum að leysast samtímis. Í þessari ritgerð er farið yfir tvær hefðbundnar verðmatsaðferðir ásamt raunvilnunum og þær allar bornar saman. Ýmsar raunvilnanir geta átt við þegar það kemur að verðmati efnislegra eigna og verða þær helstu teknar fyrir. Einnig verður skoðað hvernig raunvilnanir leysa geta nýst við að leysa verkefni ef fleiri en ein áhætta er til staðar.

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I dedicate this to my husband, two lovely children and my two furry dogs.

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List of Abbreviations

BSM	Black-Scholes-Merton
CAPM	Capital Asset Pricing Model
DCF	Discounted Cash Flow
DTA	Decision Tree Analysis
EV	Expected Value
FDA	Federal Drug Administration
m	million
NPV	Net Present Value
PV	Present Value
RO	Real Options
WACC	The Weighted Average Cost of Capital

Chapter 1

Introduction

When it comes to investment decisions, financial valuation is essential. Investment decisions identify investment opportunities, exciting projects and determine how much to invest in each project. In order to make a decision, a valuation must be made to assess the uncertainty of the future cash flow that the investment project creates. Therefore, investment valuation is an important part of the process of selecting the most profitable or promising projects to undertake. The valuation process is necessary because it gives the project value. There are three crucial factors in valuation; the cash flow that flows through the entire project period, the discount rate used to discount future flows to account for its uncertainty and the flexibility that exists to change the process of the project. Capital budgeting deals with the allocation of resources among investment projects on a long-term basis. Thus, capital budgeting focuses on sacrificing current consumption to achieve consumption in the coming periods. The difference between today's consumption and future consumption is at the core of the investment choices firms have to make directly or indirectly every day.

Most valuation methods are based on present value calculations, where future cash flows are discounted to the present day with the discount rate. Traditional methods, which include some or all of the three factors mentioned above, project cash flow, the discount factor and the flexibility, into their project valuation calculations are the discounted cash flow and decision tree analysis. Discounted Cash Flow (including net present value) is a well-known method that has been used successfully for several decades in the valuation of projects. Discounted Cash Flow is effective in many cases in project investment decisions that firms need to make. However, the method fails to capture a certain reality in business today. The marketplace is characterised by change, uncertainty and competition. As a result, the cash flow will probably be different from what the firms initially expected. However, when new information is received, uncertainty about market conditions and future cash flows gradually resolves. As a result, firms could have the valuable flexibility to alter their initial operating strategy to take advantage of future opportunities or react to reduce losses. Flexibility includes deferring, expanding, contract, abandoning, or otherwise altering a project at various stages of its operating life. Decision tree analysis is another method used to account for uncertainty and capture the value of flexibility. Decision tree analysis helps firms structure the decision problem by mapping out all feasible alternative managerial actions contingent on the possible chance events. Thus, the method allows management to defer the decision to invest until the end.

Many financial valuation models are used in practice, from simple ones mentioned earlier to more complex methods such as real options. Real options are based on option pricing theory originally from financial options. In 1973, there was a revolution in options pricing when Fischer Black and Myron Scholes published their paper and developed a model for pricing options. In 1979, John Cox, Stephen Ross and Mark Rubinstein came up with the binomial method of pricing options. In recent years, the real options method has gained increasing recognition in assessing uncertain investment opportunities. It has some common features with the other methods but different assumptions. The main advantage of real options is that there is no need to estimate the discount factor. Instead, real options use assessment methods that firms use to estimate the opportunity cost of flexibility in the project and make decisions based on it. The flexibility is that there is no need to invest today. However, they can wait for new information to arrive to take action with real options to expand, change or reduce projects based on changing economic, technological or market conditions. Thus, flexibility is only valuable if firms control it and use new information to change their decisions.

This thesis is structured as follows: Chapter 2 will discuss investments and present value calculations. The discounted cash flow method, net present value, and Decision Tree Analysis will be examined in Chapter 3. Most investment opportunities are exposed to some level of risk discussed in Chapter 4. The characteristics of financial options and the models used to price financial options, the binomial tree and the Black-Scholes-Merton model will be outlined in Chapter 5. Chapter 6 will discuss real options, their features, and the various types of real options and introduce examples. A comparison of Real options with the Discounted cash flow method and the Decision tree analysis and projects with more than one risk will also be considered. Chapter 7 will solve an investment project in the pharmaceutical industry with multiple uncertainties with the three different approaches, discounted cash flow method, Decision Tree Analysis, and real options, to compare their characteristics. Finally, in Chapter 8, we will discuss the results and conclusions.

Chapter 2

Investments

Investment is buying something today for an initial cost to achieve profit in the future. Firms' investments can vary, such as installing types of equipment, buy-in stock, constructing plants, or closing a plant that is losing money [1][2]. Firms create value by investing in projects that have greater value than their required investment. The difference between the investment cost and cash inflow from that investment is the amount of created value. Due to the risk posed by the future cash flow and the time value of money, there is a need to discount the cash flows to ensure the amount of money tomorrow is worth less than the amount of money today [3].

2.1 Project Cash Flows

The project's cash flow represents both the benefits and costs of the project. The first cash flow stream typically is comprised of development phase costs and production phase capital costs. The second stream, commonly referred to as project payoff, is a net cash flow, the difference between the revenues and the costs associated with those revenues in the project's production phase. The main point of project valuation lies in estimating these two cash flow streams over the entire project life cycle and discounting them back to today's value using the appropriate discount rate [4].

2.2 Discount Factor

The discount factor is the rate used to discount the future value of the project cash flows to today's value. The higher the risk is in the project, the higher the discount rate. It adjusts the cash flow for the risk perceived to be associated with the project. Thus, the uncertainty of the cash dictates the project risk flows [4].

$$DF = \frac{1}{(1+r)^t}$$

where:

- DF is the discount factor.
- r is the discount rate per time period.
- t is the number of the time period.

2.3 Flexibility

Most projects involve flexibility, where investors can change the course of the project during its life to maximise its expected return or to minimise its losses. Examples of flexibility in projects are to defer the investment for some time, expand, contract or abandon a project because of change in market conditions or maintain the project's status. A great strategic value is embedded in these decisions, which can be taken advantage of only if investors recognise it and are willing to exercise the decisions. The value of such decisions must be quantified and captured in the project assessment phase to capture the project's actual value. Otherwise, a project of great future strategic value may be rejected because it cannot compete with other projects in a portfolio based on its short-term value only. The additional value of these flexibilities can only be determined using option pricing or Decision Tree Analysis (DTA) [3][4][5][6].

2.4 Present Value

Present value (PV) is today's value of future cash flow. Any valuation starts with the estimation of project cash flow over the project life. Because of the time value of money, each cash flow from the future is converted into today's dollars [4], using the formula:

$$PV = \frac{CF_t}{(1+r)^t} \tag{2.1}$$

where:

- CF_t is the cash flow in period t.
- PV is the present value.
- r is the discount rate per time period.
- t is the number of the time period.

Example
If the annual cash flow on a project is expected to be \$1m two years from now, the annual discount rate of 10%, the PV will be:

$$PV = \frac{1}{(1+0.1)^2} = 826,446$$

This principle of PV is used in every project valuation method. Thus, it is fundamental to any valuation tool, including traditional tools such as the Discount Cash Flow (DCF) method, DTA, and more advanced Real Options (RO) techniques [4].

Chapter 3

Project Valuation Methods

3.1 Discounted Cash Flow Valuation

DCF method aims to calculate the net present value (NPV) of a project over its entire life cycle. It discounts the cash flows into PV using a discount rate that reflects the riskiness of the investment to find the NPV, and it assumes the risk to be unchanging during the life of the project [2][4][7]. Most projects are valued using the DCF method based on the firm's weighted average cost of capital (WACC). Determining the WACC involves using the capital asset pricing model (CAPM) to estimate the rate of return required by equity investors from market information regarding stock prices. This firm-specific information is typically applied to individual investment projects [6]. More about the WACC and the CAPM will be discussed in Chapter 4.

The NPV of the investment is the difference between the PV of the future income discounted at the rate reflecting the riskiness of the estimated cash flow, minus the investment costs that are inherent in an investment opportunity:

$$NPV = \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t} - I_t \tag{3.1}$$

where:

- CF_t is the cash inflow in period t.
- I_t is the investment outlay in period t.
- r is the discount rate reflecting the riskiness of the estimated cash flow.
- T is the life of the project.

If a series of investment outlays are required, the PV of cash outflow is subtracted from the PV of cash inflow in Equation (3.1) to arrive at the NPV.

$$I \equiv \sum_{t=0}^{T} \frac{O_t}{(1+r)^t} \tag{3.2}$$

where:

- O_t is the cash outflow in year t.

DCF method has its foundation in the NPV rule, where the value of any asset is the PV of expected future cash flows [8]. The determination rule is that if the NPV is positive, invest in the project, but reject the project if the NPV is negative [7].

$$NPV\ rule = MAX(at\ t = 0)[0, E_0 V_T - X] \tag{3.3}$$

where:

- $E_0 V_T$ is the expected future value at time 0.
- X is the investment outlay.

It can be seen in the Equation (3.1) that future uncertainty of cash flows is not explicitly modelled in the DCF method; it only discounts expected cash flows. In reality, there are many paths of possible free cash flows that might be realised between the start of the project and its finish. None of them is mapped out with the use of DCF method. That is because the DCF method is constrained to pre-committing today to a go or no go decision. It uses only information that is available today. It is well known that the NPV rule cannot properly capture management's flexibility to adapt and revise later decisions in response to unexpected market developments [2].

The problem with the DCF method is that it systematically undervalues every investment opportunity because it is based on expected future cash flows. Furthermore, the method assumes that once the firm commits to a project, the project's outcome will be unaffected by the future decision of the firm, thereby ignoring any managerial flexibility the project may have and therefore cannot consider the value of flexibility [5][6][7].

3.2 Decision Tree Analysis

DTA is a method that attempts to capture the value of flexibility in discrete time and allows the decision-maker to defer until the end of the period. The basic structure of the decision setting is that the decision-maker is faced with a decision (or a sequence of decisions) of choosing among different alternatives. The consequence of each alternative depends on some uncertain future event or state of nature. Therefore, the decision-maker is assumed to select a strategy consistent with preferences to maximise the value of the project as uncertainties are resolved over the project's life [2][5].

In the DTA, the expected value (EV) approach calculates the project's NPV, and a decision tree is used to map the decision problem visually. In addition, DTA considers possible decisions over the life of the investment project by assigning probabilities to different scenarios.

It captures their outcomes (failure/success), costs, and the results from payoff calculations at each node in the tree. In a decision tree, scenarios can be modelled as the worst, most likely, and best-case scenario. Decision trees provide an effective way to understand decisions and their outcomes by displaying a series of steps. They also help identify all available options and weigh each action against the risks and rewards that each option may provide. In a decision tree, the decision node is represented with squares. Thus, from each decision node comes out a line that speaks for a potential option. A circle at the end of a line denotes potential risk. It expands until every line reaches an endpoint [2][4][7]. See Figure 3.1 in the example down below.

The EV is simply the probability of the event multiplied by the event's cash flow see Equation (3.4). In the DTA, the calculations start on the right side of the tree with the PV of cash flow. The payoffs are then calculated by working from right to left in the tree. Finally, the project's NPV is found after taking all the contingent decisions and the probabilities of their outcome into account [2][4][7]. See the example from the book "Project Valuation Using Real Options: A Practitioner's Gude" [4] below:

$$E(V_t) = puV_t + (1-p)dV_t \tag{3.4}$$

where:

- $E(V_t)$ is the expected value at time t.
- V_t is the payoff at time t.
- p is the probability of the event.
- u is an up movement factor.
- d is a down movement factor.

Example

To illustrate the DTA, let us consider a firm involved in developing technologies. The technical effectiveness of the product first has to be proved through development effort, which is expected to cost \$1m and take one year. Successful development will be followed by the commercialisation of the technology, which is estimated to take an additional year and cost \$2m at that time. DCF method shows a project's payoff of \$15m over the project horizon. Although this payoff is attractive compared to the investment costs, the firm is unsure about the project's technical and commercial success because the respective success probabilities are estimated to be 50% and 70%. Therefore, the firm uses DTA to facilitate the go/no-go decision for the two phases of the investment.

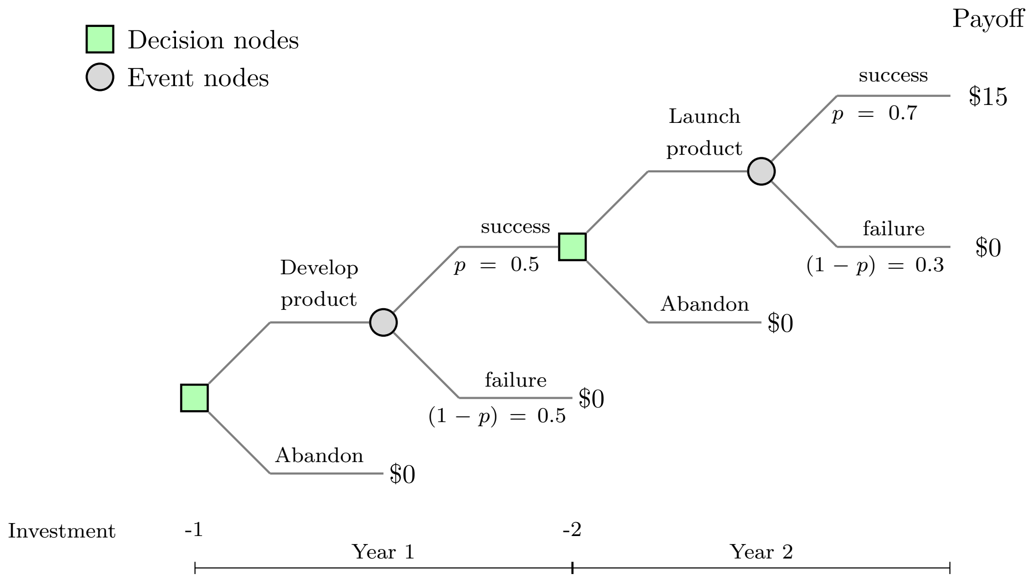


Figure 3.1: Decision Tree

Figure 3.1 shows the decision tree with different decision paths, outcomes, and cost and payoffs associated with different paths. Assuming a discount rate of 10% and starting at the far right of the decision tree calculating the EV of the payoff at year two considering the mutually exclusive outcomes related to the product launch:

$$EV_{2,success} = 0.7(\$15m) = \$10.5m$$
$$EV_{2,fail} = 0.3(\$0) = \$0$$

Add the EV of the two outcomes:

$$EV_2 = \$10.5m + \$0 = \$10.5m$$

Calculate the PV of the payoff at year one by discounting the EV_2 using a discount rate of 10%:

$$PV_1 = \frac{\$10.5m}{(1+0.1)^1} = \$9.55m$$

Calculate the NPV of the project at product launch at year one by substracting the launch cost from the PV_1:

$$NPV_1 = \$9.55m - \$2m = \$7.55m$$

Calculate the EV of the payoff at year one of considering the mutually exclusive outcomes related to the product development:

$$EV_{1,success} = 0.5(\$7.55m) = \$3.77m$$
$$EV_{1,fail} = 0.5(\$0) = \$0$$

Add the EV of the two outcomes:

$$EV_1 = \$3.77m + \$0 = \$3.77m$$

Calculate the PV of the project payoff at year 0 by discounting the EV_1 using a discount rate of 10%:

$$PV_0 = \frac{\$3.77m}{(1+0.1)^1} = \$3.43m$$

Calculate the NPV of the project at year 0 by substrating the development cost from the PV_1:

$$NPV_0 = \$3.43m - \$1m = \$2.43m$$

Although DTA is an efficient valuation tool, there are recognised challenges under specific circumstances. The biggest dilemma is what appropriate discount rate to discount the cash flow when using the decision tree [5]. This will be discussed in Section 6.1.

Chapter 4

Risk

Definition of risk in investment analysis refers to the probability of how far the return received on investment is from the expected return on that investment. For example, investors who buy assets expect to earn a return over the time they hold the asset. However, their actual returns over this holding period may be different from the expected return. This difference between actual and expected returns is a source risk.

The characteristics of any investment can be measured with two variables, the expected return, which represents the opportunity in the investment, and variance, which represents the risk. The expected return and variance are commonly estimated using past returns rather than future returns. The assumption made when using historical variances are the past return distribution, and they are good indicators of the future return distribution. When violating this assumption, the historical estimates may not be a good measure of risk, as is when the asset's characteristics have changed significantly over time [9].

Return (R_i) is the net gain or loss of an investment defined as a percentage change in the market variable between the end of the day $i-1$ and the end of the day i see equation below:

$$R_i = \frac{S_i - S_{i-1}}{S_{i-1}} x100 \tag{4.1}$$

The expected return is the profit or loss that an investor anticipates on investment with known historical returns. Expected return calculations are a crucial piece of business operations and financial theory, included in the well-known models of modern portfolio theory or the Black-Scholes-Merton (BSM) options pricing model [10].

The returns from an investment in an asset are presented by a multi-component random variable **R** and the associated probabilities **P**:

$$\mathbf{R} = (R_1, R_2, \ldots, R_N)$$
$$\mathbf{P} = (p_1, p_2, \ldots, p_N)$$

The following random variables present possible return values, and the expected return is calculated as the EV by multiplying potential outcomes by the chances of them occurring, as illustrated by the following formula:

$$E(\mathbf{R}) = \sum_{i=1}^{N} p_i R_i \tag{4.2}$$

Investment risk depends on the dispersion of possible outcomes, the difference between the actual and expected returns from the investment [9]. The standard measure for dispersion is the variance. Variance is the average value of squared deviation from the mean, which is measured by volatility. When estimating the dispersion of possible outcomes from investing in the stock market, it is assumed that dispersion of returns in the past is a reasonable indicator of what could happen in the future [11]. The greater this variance, the higher the risk is perceived to be [4].

Given a random variable R with EV $E(R)$, the quantity $R - E(R)$ is random, but the EV is zero. The quantity $(R - E(R))^2$ is always non-negative and is significant when R deviates significantly from $E(R)$ and small when it is near $E(R)$. The EV of this squared variable $(R - E(R))^2$ is a valuable measure of how much R tends to vary from its EV [10].

The variance of returns is defined as the equation:

$$\sigma_{\mathbf{R}}^2 = E(\mathbf{R}^2) - (E(\mathbf{R}))^2 \tag{4.3}$$

Covariance is a statistical tool used to determine the relationship between the movement of two asset prices. It measures the directional relationship between the returns on two assets. A positive covariance means that asset returns move together, while a negative covariance means they move inversely. Covariance is calculated by analysing standard deviations from the expected return or multiplying the correlation between the two variables by the standard deviation of each variable. When considering two or more random variables, their mutual dependence can be summarised conveniently by their covariance. That is useful in computations. Covariance is a crucial tool in modern portfolio theory to ascertain what securities to put in a portfolio [10], discussed in more detail in Section 4.2.

Let R_1 and R_2 be two random return variables associated with returns from investments in two assets. Then, the covariance between the two returns is given by:

$$cov(R_1, R_2) = E((R_1 - E(R_1))(R_2 - E(R_2))) \tag{4.4}$$

A correlation coefficient is a statistical measure of the strength of the relationship between the relative movements of two variables. The values range between -1 and 1. For example, suppose two random variables R_1 and R_2, have a correlation $\rho = -1$, a perfect negative correlation, while $\rho = 1$ shows a perfect positive correlation. A correlation of $\rho = 0$ indicates that the two variables are uncorrelated. That is the situation where knowledge of the value of one variable gives no information about the other. If two variables are independent, they are uncorrelated [10]. Correlation statistics can be used in finance and investing. The correlation between two variables is beneficial when investing in financial markets. For example, a correlation can help determine how well a mutual fund performs relative to its benchmark index, another fund, or an asset class. The investor gains diversification benefits by adding a low or negatively correlated mutual fund to an existing portfolio. In other words, investors can use negatively correlated assets or securities to hedge their portfolios and reduce market risk due to volatility or wild price fluctuations. Many investors hedge the price risk, which effectively reduces capital gains or losses because they want the dividend income or yield from the stock or security. The correlation coefficient ($\rho_{1,2}$) between two random variables is defined as the covariance divided by the square root of the variance see equation:

$$\rho_{1,2} = \frac{cov(R_1, R_2)}{\sqrt{\sigma_1^2 \sigma_2^2}} \tag{4.5}$$

where:

- $\rho_{1,2}$ is the correlation coefficient.
- $cov(R_1, R_2)$ is the covariance of variables R_1 and R_2.
- σ_1^2 is the variance of R_1.
- σ_2^2 is the variance of R_2.

4.1 Types of Risk

The reason actual returns may differ from expected returns can be grouped into specific and market. The risk arising from specific affects one or a few investments, while the risk arising from market reasons affects many or all investments. This distinction is critical to the way we assess risk in finance [9].

Specific risk events from the specific group are related to the efficiency of an organisation completing the project and the technology's effectiveness. The risk arising from the market side is called market risk due to the volatility of its expected future payoff in investment driven by market forces, such as market demand and competition. In project valuation, it is essential to recognise and differentiate specific risk from market risk. That is because investors are willing to pay a risk premium for market-driven risks that can not be diversified. However, specific risks can be diversified, so investors do not require a risk premium for that source of risk. That means that the rate used to discount the cash flows is dictated by the type of risk associated with a given cash flow [4].

Market risk arises from the correlation between the asset returns and the market return driven by economy-wide forces affecting all securities in the market, which cannot be diversified away. Specific risk is driven by factors other than the market. This risk results from variability in factors unique to a particular firm or its industry (e.g., the risk that the firm's chief executive officer may quit or die). It is known as a diversifiable risk since the effect of these specific factors beyond the market can be diversified away by holding a portfolio of 15 or more securities [2].

4.2 Diversification

Diversification is a strategy designed to reduce risk by spreading the portfolio across many investments, reducing variability. The reason why portfolio diversification works is that the prices of different stocks do not shift exactly together. Diversification works best when the returns are negative when one business does well, but the other does badly [2][11]. For example, consider a portfolio of investments which at time t is valued at:

$$V(t) = \sum_{i=1}^{N} x_i S_i(t) \tag{4.6}$$

With a fixed asset allocation in a portfolio $\mathbf{w} = (w_1, w_2, \ldots, w_N)$, and a known covariance matrix, the expected return and variance of the portfolio can be calculated as follows:

$$\mu_{portfolio} = \sum_{i=1}^{N} w_i \mu_i \tag{4.7}$$

$$\sigma^2_{portfolio} = \sum_{i,j=1}^{N} w_i C_{i,j} w_j \tag{4.8}$$

4.3 Beta

The contribution of a security to the risk of a diversified portfolio depends on its market risk. Securities are not all equally affected by fluctuations in the market. The risk depends on exposure to macroeconomic events and can be measured as the sensitivity of a stock's return to fluctuations in returns on the market portfolio. This sensitivity is called the stock's beta (β). Therefore, beta is the sensitivity of an investment to the market's movement. The beta for the market portfolio is 1, and for risky investments, the beta is $\beta > 1$. That means that the investment is explicitly sensitive to the market movements and investors require a higher return than the market. For less risky investments, the beta is $\beta < 1$, which means that the investment is not as sensitive to the market's movements, and investors require lower returns than the market [11]. Betas are not available for projects, but they are available for publicly traded firms. However, the beta for single projects can be found by referencing the beta values of similar projects in publicly traded firms. Then the risk is used as a proxy in the CAPM to calculate the expected return from the project. This expected return is then used as a discount rate to discount the cash flow [4].

Beta in the CAPM is defined as:

$$\beta = \frac{COV(R_i, R_m)}{VAR(R_m)} \tag{4.9}$$

4.4 Risk Premium

A risk premium is an extra return that investors require for taking a risk. The market risk premium is the risk premium on the market portfolio. For example, the difference between the return on the market and the interest rate on government bonds is termed the market risk premium [11]:

$$\text{Market risk premium} = r_m - r_f \tag{4.10}$$

where:

- r_m is the return on the market portfolio.
- r_f is the return on risk-free government bonds.

Beta measures risk relative to the market, and therefore the expected risk premium equals beta times the market risk premium [11]:

$$\text{Risk premium} = r - r_f = \beta(r_m - r_f) \tag{4.11}$$

where:

- r_m is the market risk premium.
- r_f is the risk-free interest rate.

4.5 Estimating the Cost of Capital

When valuing investments, a discount rate is needed that reflects the riskiness of the cash flow. The cost of capital is the minimum return the investors demand in return for their risk with the investment. It is the return they can get if they invest somewhere else with the same risk. Therefore, when calculating the NPV of an investment, it uses the cost of capital as a discount rate [3][9].

Two important factors determine the discount rate for a given cash flow stream, the magnitude of risk and the type of risk. In determining the discount rate to be used on a given cash flow stream, the first consideration should be whether uncertainty is associated with the cash flow. Irrespective of whether the cash flow is influenced by specific risk or market risk, no uncertainty means no risk; therefore, an investor will not require a risk premium for the investment. That means that if there is no uncertainty concerning the cash flow stream, the appropriate discount rate is the risk-free rate. If there is uncertainty associated with a particular cash flow stream, the next consideration is whether that stream is influenced by specific risk or market risk. Suppose a specific risk influences it. In that case, the investor will not require a risk premium for the ineptness of the organisation in completing the project or the ineffectiveness of the technology involved. On the other hand, if a cash flow is subject to market risk, one would account for it in some fashion, most commonly by adjusting the discount rate [4].

4.5.1 The Capital Asset Pricing Model

The CAPM is the most widely used method to estimate the cost of capital. The CAPM is a market-based method that addresses the market risk associated with the project net revenues to maximise the shareholders' wealth. It links the expected return of an investment to its proper risk [2][4].

The CAPM states that an investment's expected risk premium should be proportional to both the market risk premium and its beta:

$$r_j = r_f + \beta_j(r_m - r_f) \tag{4.12}$$

where:

- r_j is the expected required return from asset$_j$.
- r_f is the risk-free interest rate.
- r_m is the expected return from the market portfolio.
- β_j is the asset's volatility relative to the market.

The CAPM gives the return that any asset or portfolio in equilibrium must earn to compensate for the market risk, thus offering a cutoff criterion or a hurdle rate for the project's acceptance. The basic message of the CAPM is that the relevant risk premium that investors would pay for a risky capital asset in equilibrium is directly related to that asset's market risk as given by its beta or the covariance of its return with the market. Since the expected rate of return of the project given by the CAPM is the minimum return required by investors from securities having the same market risk. The CAPM specifies the hurdle rate or the risk-adjusted discount rate to determine the project's NPV [2].

Many firms estimate the rate of return required by investors in their securities and use the firm's cost of capital to discount the cash flows on all new projects. However, it must take care of when the firm has issued securities other than equity. Most companies issue a mix of debt and equity [11].

4.5.2 The Weighted Average Cost of Capital

A mixture of debt and equity finances most investments, so investors look for different risks and returns. Therefore, the cost of capital has to be a weighted average of the returns required from debt and equity investors [11]:

$$WACC = k_d(1 - T_m)\frac{D}{D+E} + k_e\frac{E}{D+E} \tag{4.13}$$

Debt (D) and equity (E) are found using market values. The marginal tax rate (T_m) reduces the cost of debt because the DCF method values the tax shield by reducing the WACC. The reason is that the interest tax shield has value and must be included in the valuation [3].

When using the WACC to calculate the cost of capital, there is a need to find comparables to estimate rates of return from debt and equity holders. Thus, for rates of return from debt holders (k_d), there are found other debts with the same risk, and it is assumed that its yield to maturity is the same. However, for rates of return from equity holders (k_e), the CAPM is used to find other equities that are believed to have the exact beta and use them to estimate the cost of equity [7].

Chapter 5

Financial Options

Options are financial securities that give their holder the right, not an obligation, to buy or sell an underlying asset. The underlying asset's value at time t is $S(t)$, the option contract's strike is K, and maturity is T. The option price is paid to acquire the option. The strike price is the price at which the option owner can buy or sell the underlying asset. Since it is a right, not an obligation, the holder can choose not to exercise the right and allow the option to expire. That limits the risk of holding the option to the option price [2][9][12].

Six variables relating to the underlying asset and financial markets are affecting the value of an option:

1. The current value of the underlying asset, $S(t)$
2. The strike price of the option, K
3. Time to the expiration of the option, T
4. Variance in value of the underlying asset, σ
5. The risk-free interest rate corresponding to the life of the option, r
6. The dividends that are expected to be paid on the underlying asset.

5.1 Types of Options

There are two types of options, a call option and a put option. Options can be American or European. The main difference between those options is the time to exercise. The holder can only exercise a European option at maturity of the option, but the holder can exercise an American option whenever in the option's life. The possibility of early exercise makes the American option more valuable than the European option [12].

A call option gives the holder the option to buy an underlying asset (S) on a maturity date (T) for a strike price (K). The price of an asset today is $S(t)$; the holder buys the right to buy this asset at T for K:

- If $S(T) > K$, then the holder exercises his right and profit is $(S(T) - K) > 0$
- If $S(T) < K$, then the holder does not exercise his right

The value of the call option at time T:

$$c(T) = max(S(T) - K, 0) \begin{cases} S(T) - K & \text{if } S(T) > K \\ 0 & \text{if } S(T) < K \end{cases}$$

The value of the call option at time T is discounted today, so we know the call option price today:

$$c(t) = E_t max(S(T) - K, 0)e^{(-r(T-t))} \tag{5.1}$$

Call option becomes more valuable as the stock price increases and less valuable as the strike price exceeds the stock price [12].

A put option gives the holder the option to sell an underlying asset (S) on a maturity date (T) for a strike price (K). The price of an asset today is $S(t)$; the holder buys the right to sell this asset at T for K:

- If $K > S(T)$, then the holder exercises his right and profit is $(K - S(T)) > 0$
- If $S(T) > K$, then the holder does not exercise his right

The value of the put option at time T:

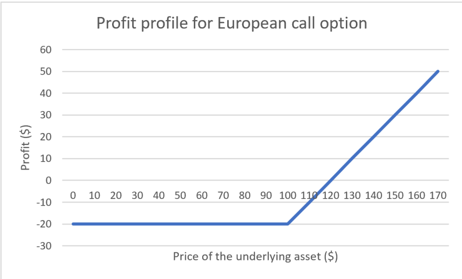
$$p(T) = max(K - S(T), 0) \begin{cases} K - S(T) & \text{if } S(T) < K \\ 0 & \text{if } S(T) > K \end{cases}$$

The value of the put option at time T is discounted today, so we know the put option price today:

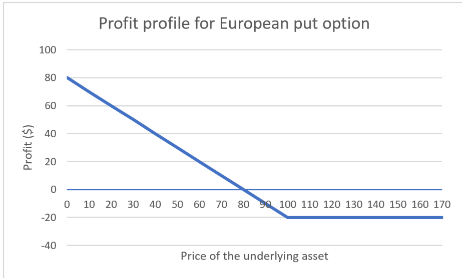
$$p(t) = E_t max(K - S(T), 0)exp^{-r(T-t)} \tag{5.2}$$

Put options become less valuable as the stock prices increases and more valuable as the strike price increases [12].

A profit diagram explains the cash payoff on an option at maturity. For example, consider a European call option that is \$20, called a premium, and the strike price is \$100. The holder will exercise the call option if the underlying asset exceeds the strike price. As seen in Figure 5.1a for the European call option, the payoff is negative if the underlying asset's value is less than the strike price. If the underlying asset's price exceeds the strike price, the gross payoff is the difference between the value of the underlying and the strike price. Net profit is the difference between the gross payoff and the premium of the call. The holder of a call option hopes that the stock price will increase. The holder of a put option is hoping that it will decrease. For example, consider a European put option where the premium is \$20, and the strike price is \$100. The holder will exercise the put option if the underlying asset is below the strike price. As seen in Figure 5.1b for the European put option, the payoff is negative if the underlying asset's value is higher than the strike price. If the underlying asset price goes below the strike price, the gross payoff is the difference between the strike price and the underlying value. Net profit is the difference between the gross payoff and the premium of the put [12].



(a) Profit profile for European call option



(b) Profit profile for European put option

Figure 5.1: Net profit profile for European call and put option

Options are referred to as in the money, at the money, or out of the money. For example, a call option is in the money when $S > K$, at the money when $S = K$, and out of the money when $S < K$. A put option is in the money when $S < K$, at the money when $S = K$, and out of the money when $S > K$. If not previously exercised, the option will be exercised only when it is in the money and on the expiration date [12]. There are two sides to every option contract, the investor who has taken a long position (has bought the option) and the investor who has taken a short position (has sold the option). Investors that take the long position are not obligated to buy or sell; they have the choice to exercise their rights. That means a long position limits the risk for the investor to only the price of the option. On the other hand, investors that take the short position are obligated to buy or sell if the option expires in the money. That means a short position is required to buy or sell and are exposed to more risks and can lose much more than the price of the options [9][12].

5.1.1 Put-Call Parity

For there not to be an arbitrage opportunity, specific relationships between the price of a European call option $c(t)$ and a European put option $p(t)$ with the same strike price K and time to maturity T must hold [12].

At any time t before maturity, the put-call parity states:

$$c(t) + Ke^{-r(T-t)} = p(t) + S(t) \tag{5.3}$$

where:

- $c(t)$ is the price of the call option.
- $Ke^{-r(T-t)}$ is the present value of the strike price
- $p(t)$ is the price of the put option
- $S(t)$ is the value of the underlying asset

Put-call parity is valid at all times:

$$C(t) + K = P(t) + S(t) \tag{5.4}$$

$$LHS = max(S(T) - K, 0) + K \begin{cases} S(T) - K + K = S(T) & \text{if } S(T) > K \\ K & \text{if } S(T) < K \end{cases}$$

$$RHS = max(K - S(T), 0) + S(T) \begin{cases} K - S(T) + S(T) = K & \text{if } S(T) < K \\ S(T) & \text{if } S(T) > K \end{cases}$$

This relationship is known as put-call parity. It shows that the value of a European call with a specific exercise price and exercise date can be deduced from the value of a European put with the same exercise price and exercise date, and vice versa. Put-call parity holds only for European options [12].

5.2 Option Valuation

The options approach to valuation is very different from the traditional tools. The option's value is inferred from the value of a portfolio of traded securities that has the same payoff as the option and mimics its fluctuations in value over time. If the option and portfolio value are not equal, an opportunity to profit from trading would exist. Financial markets move quickly, and arbitrage opportunities are fleeting, bringing discipline and precision to option valuation [13].

When the Black-Scholes-Merton (BSM) model was introduced, Black and Scholes used a replicating portfolio composed of the underlying asset and the risk-free asset with the same cash flows as the option being valued [9]. However, Merton published a paper with a more generalised approach to option valuation and added the final piece: arbitrage [13].

Arbitrage is the process that enforces the Law of One Price, buying an asset at one price and simultaneously selling it at a higher price. Arbitrage opportunities are rare and fleeting because professional investors move their money around to close any pricing gaps quickly. Merton recognised that the value of the option produced by the model must be free of arbitrage opportunities [13].

An option can be priced by assuming there is no arbitrage opportunity for the investor, which can be applied when the underlying asset's price takes a specific form [10][14]. To hold an arbitrage opportunity, the investor needs to construct a riskless portfolio of underlying assets S and an option on these underlying assets whose price is f. The portfolio weights are constructed so that the portfolio's value is independent of the underlying asset's price path; the portfolio is temporarily riskless for a short period of Δt. Then it can be discounted backwards at the risk-free rate to find its value today. Finally, knowing the portfolio value allows us to price the option [12][14].

Standard option valuation typically relies on the following assumptions:

1. Friction-less markets for stock, bonds and options. That means:
   a) there are no transactions costs or taxes
   b) there are no restrictions on short sales, and full use of proceeds is allowed
   c) all shares of all securities are infinitely divisible
   d) borrowing and lending at the same rate are unrestricted.

   These assumptions allow continuous trading.

2. The short-term risk-free interest rate is known or constant over the life of the option.

3. The underlying asset pays no dividends over the life of the option.

4. Stock prices follow a stochastic diffusion Wiener process of the form:

$$\frac{dS}{S} = \alpha dt + \sigma dz \tag{5.5}$$

   where α is the instantaneous (total) expected return on the stock, σ is the instantaneous standard deviation of stock returns (assumed constant), and dz is the differential of a standard Wiener process (with mean 0 and variance dt). This diffusion process is replaced by a multiplicative binomial process or random walk in the discrete-time case discussed below. As the trading interval gets smaller and smaller, the binomial process's distribution becomes equivalent to the log-normal distribution underlying the process in Equation (5.5).

Option pricing models are used to value options, and several models are available to value options. Frequently used models are BSM and Binomial trees, and they have both their advantages and disadvantages in solving the value of options. It depends mostly on estimation issues, transparency and the type of options being considered. That will be explained in the sections below [4]. Two equivalent but seemingly different approaches to valuing simple options are replicating portfolio approach and the risk-neutral probability approach. The replicating portfolio approach may be considered discounting expected cash flow at a risk-adjusted rate and a risk-neutral probability approach, which is equivalent to discounting certainty-equivalent cash flow risk-free rate [7]. These methods will be discussed in more detail in Section 5.2.1.1. The integral premise behind option pricing models is that DCF models tend to understate the value of assets that provide payoffs contingent on the occurrence of an event [9].

5.2.1 Binomial Tree Valuation

The general multiplicative binomial option-pricing approach was popularised by Cox, Ross, and Rubinstein [15]. Binomial tree valuation is a practical technique for pricing an option. It involves constructing a binomial tree. A binomial tree is a graphical representation of possible paths the price of an underlying asset can take during the lifetime of an option. Each node in the tree is associated with a specific value of the underlying asset and an option to buy it or sell it. The underlying assumption is that the price of the underlying asset follows a geometric random walk. Each time step has a certain probability of moving up by a certain percentage amount and a certain probability of moving down by a certain percentage amount. When the time step becomes smaller, this model is the same as the BSM model [12][14].

The binomial tree valuation approach involves dividing the option's life into a large number of small-time intervals of length Δt. It assumes that in each time interval, the price of the underlying asset increase from its initial value of S by a multiplicative factor u with the probability p to Su or decrease by factor d with the probability of $(1 - p)$ to Sd at the end of the period see in Figure 5.2. The up and down factors, u and d, are a function of the volatility of the underlying asset where σ is the volatility represented by the standard deviation of the continuously compounded or logarithmic rate of return if the stock moves up or down [2][4][12] and can be described as follows:

$$u = e^{\sigma \Delta t} \quad (5.6) \qquad d = \frac{1}{u} \quad (5.7)$$

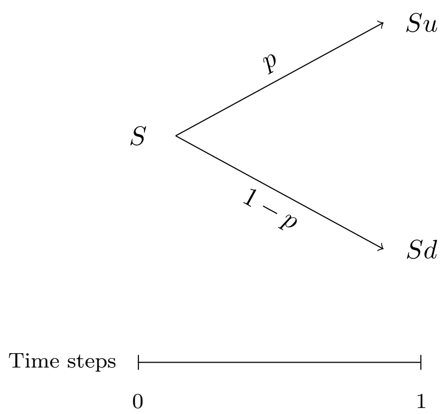


Figure 5.2: Asset price movements in time Δt under the binomial model

5.2.1.1 One-Step Binomial Model

How much should an investor pay for an option to buy or sell an asset at the strike price X in future time T? Start making assumptions about the future value of the underlying asset. For example, assume the value goes up to Su or down to Sd. At time $t = 0$ price of the underlying value is S, and the call option is f. In the future time, the prices of these two assets can be either (Su, f_u) or (Sd, f_d).

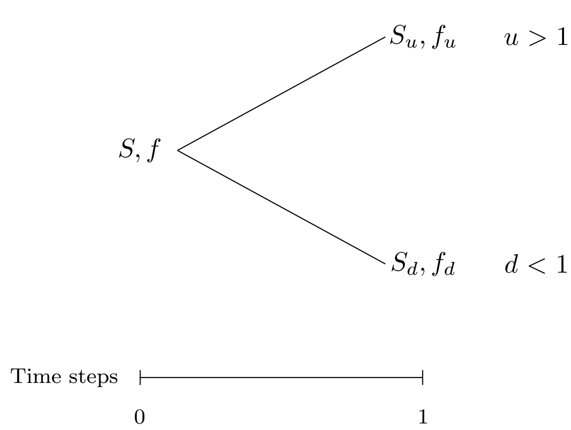


Figure 5.3: General one step tree with stock and option prices

It turns out that a simple argument can be used to price the option in this example. The only assumption needed is that arbitrage opportunities do not exist. Next, we set up a portfolio with the replicating method.

Replicating Portfolio

Replicating theory is the foundation of the BSM equation and the binomial method. The basic idea is enabling the exact pricing of options is that one can construct a portfolio. For example, a portfolio consisting of buying a particular number of Δ in shares of the underlying asset and borrowing an appropriate amount of $\$B$ at the risk-free rate would exactly replicate the future returns of the option in any state of nature. Since the option and this equivalent portfolio would provide the same future returns, they must sell for the same current price to avoid risk-free arbitrage profit opportunities. Thus, we can value the option by determining the cost of constructing its replicating portfolio. That is the cost of the synthetic option equivalent [2].

Example

To illustrate the principle better, let us assume that the current stock price is $S_0 = \$100$. Also, assume that stock price can either go up to $S_u = \$120$ with subjective probability q or go down to $S_d = \$80$ with probability $(1 - q)$.

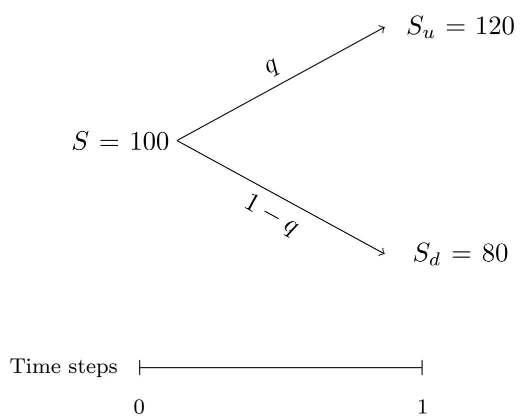


Figure 5.4: Stock price movements for example

Let us consider a call option written on this underlying stock with a strike price of $K = \$110$. Since the stock price can take one of two values, the call option value f can also be one of these two values:

$$f_u = MAX(S_u - K, 0) \text{ with probability } q$$

$$f_d = MAX(S_d - K, 0) \text{ with probability } (1 - q)$$

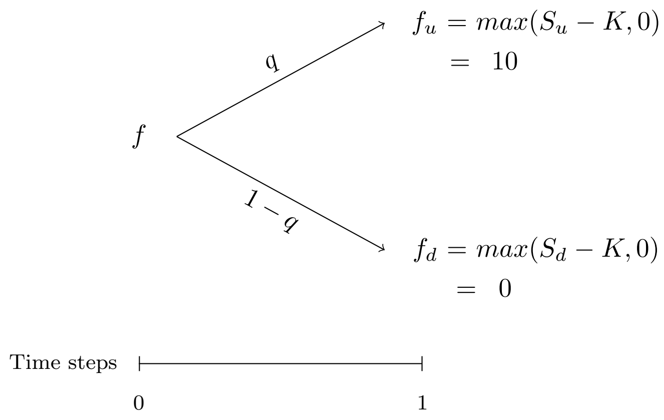


Figure 5.5: Option prices at the final nodes for example

Suppose now we construct a portfolio consisting of buying Δ shares of the underlying stock at its current price, $\$S_t$, financed in part by borrowing an amount of $\$B_t$ at the risk-free interest rate, for a net out-of-pocket of $\Delta S_t - B_t$. That is:

$$\text{Call option} \approx (\text{Buy } \Delta \text{ shares at } \$S_t \ \& \text{ Borrow } \$B_t \text{ at } r_f)$$

or

$$f_t \approx (\Delta S_t - B_t) \tag{5.8}$$

After one period, we would need to repay the principal amount borrowed at the beginning (B_t) with interest, or $(1+r)^t B_t$, for sure. If the portfolio offers the same return as the option at the end of every period in each state, then:

$$f_u = \Delta S_u - (1+r)^T B_t$$

and

$$f_d = \Delta S_d - (1+r)^T B_t$$

Solving these two equations for the two unknowns, Δ and B_t, gives:

$$\Delta = \frac{f_u - f_d}{S_u - S_d} = \frac{10-0}{120-80} = 0.25\text{shares} \tag{5.9}$$

and

$$B_t = \frac{S_d f_u - S_u f_d}{(S_u - S_d)(1+r)} = \frac{\Delta S_d - f_d}{1+r} = \frac{0.25(80-0)}{1.08} = \$18.52 \tag{5.10}$$

The number of shares of the underlying asset that we need to buy to replicate on option over the next period, Δ, is known as the option's delta or hedge ratio and is obtained in the discrete case as the difference of option prices divided by the spread of stock prices.

When substituted back into Equation (5.8), $f_t = \Delta S_t - B_t$, Equations (5.9) and (5.10) we find the value of constructing the portfolio. Since the portfolio correlates perfectly with the value of the option, the current price of the call option is calculated as the current value of the replicating portfolio:

$$f = 0.25(100) - 18.52 = \$6.48$$

Risk-Neutral Valuation

The risk-neutral approach to valuation, introduced by Cox, Ross, and Rubinstein in 1976, is based on the same arguments that underlie the replicating portfolio approach. They identify that the hedge position, combination of the option, and portfolio earn a risk-free return rate. It would have the same value under any preferences for risk. As the underlying asset value fluctuates, dynamic tracking updates the composition of the tracking portfolio to maintain a risk-free hedge position. The return to the hedge position and the composition of the tracking portfolio is the same for all sorts of investors, risk-averse, risk-loving or risk-neutral. The value of the tracking portfolio and the option are independent of risk preferences. Hence, for valuation purposes, Cox, Ross and Rubinstein could assume that everyone is risk-neutral, eliminating the need to estimate any risk premium. The value of an option is the same in the "risk-neutral world" and our everyday world [12][13].

A risk-neutral world has two features that simplify the pricing of derivatives. First, the expected return on the investment is the risk-free rate, and the discount rate used for the expected payoff on an option is the risk-free rate.

Rearranging Equation (5.8) into $\Delta S_t - f_t = B_t$, to construct a portfolio of the underlying asset and the option. For every short position in the option, hold Δ long positions in the underlying asset. So that value of the portfolio is independent of the stock value going up or down [2][12]. Then, the value of the portfolio at time t:

$$V_t = \Delta S_t - f_t \tag{5.11}$$

and at time T:

$$V_{u,T} = \Delta \cdot Su_T - f_{u,T}$$
$$V_{d,T} = \Delta \cdot Sd_T - f_{d,T}$$

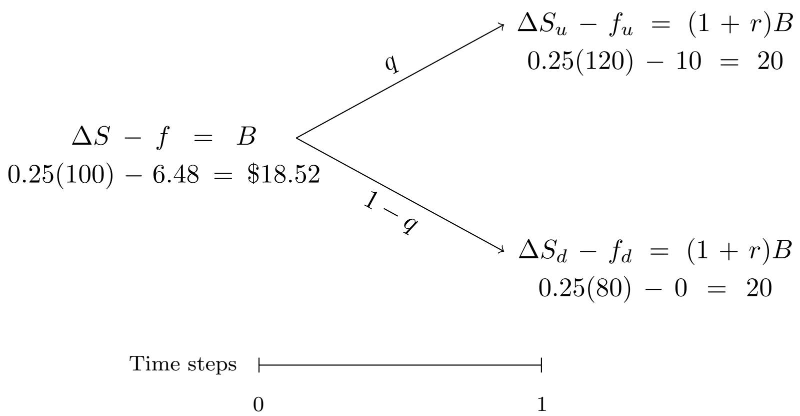


Figure 5.6: Value of the portfolio in example

To proceed with the call option pricing and create a risk-free position, it is demand that $V_{u,T} = V_{d,T}$ fix Δ. In this case, the portfolio is riskless and, for there to be no arbitrage opportunities, it must earn the risk-free rate (r_f). As $V_u(T) = \Delta Su - f_u$ is the end of the period value, the present value of the portfolio must be:

$$V_t = (\Delta Su - f_u)(1 + r_f)^{(T-t)} \tag{5.12}$$

Inserting Δ into Equation (5.12) and solve for f:

$$f = (p_u f_u + p_d f_d)(1 + r_f)^{(T-t)} \tag{5.13}$$

with

$$p_u = \frac{(1 - r_f)^{(T-t)} - d}{u - d} \qquad p_d = \frac{u - (1 - r_f)^{(T-t)}}{u - d} \tag{5.14}$$

If we assume that p_u and p_d is the probability of an up and down movement in the price of the underlying asset, then the expected stock price at T is:

$$E\{S(T)\} = p_u S_u + p_d S_d$$

inserting p_u and p_d from Equation (5.14), we get that:

$$E\{S(T)\} = S(1 - r_f)^{(T-t)}$$

That shows that the stock price grows, on average, at the risk-free rate when p is the probability of an up and down movement. The present value of the call is equal to the expected payouts multiplied by probabilities that adjust them for their risk. In this way [12].

Let us return to the example in Section 5.2.1.1 and illustrate that the risk-neutral valuation gives the same answer as replicating portfolio method.

Example

We define p as the probability of an upward movement in the stock price in a risk-neutral world. We can calculate p from Equation (5.14). Alternative, we can argue that the expected return on the stock in a risk-neutral world must be the risk-free rate of $r_f = 8\%$. That means that p must satisfy:

$$\begin{aligned} 120p + 80(1 - p) &= 100(1 + 0.08) \\ 40p &= 28 \\ p &= 0.7 \end{aligned}$$

At the end of one year, the call option has a 0.7 probability of worth \$10 and a 0.3 probability of being worth zero. Its expected value is, therefore:

$$0.7 \times \$10 + 0.3 \times \$0 = \$7$$

In a risk-neutral world, this should be discounted at a risk-free rate. The value of the option today is, therefore:

$$\$7(1 - 0.08) = \$6.44$$

The replicating method in Section 5.2.1.1 found the same value (with minor round differences).

5.2.1.2 Multi-Step Binomial Method

We can extend the analysis to two and higher step binomial trees. However, first, let us look at a case of a two-step binomial tree.

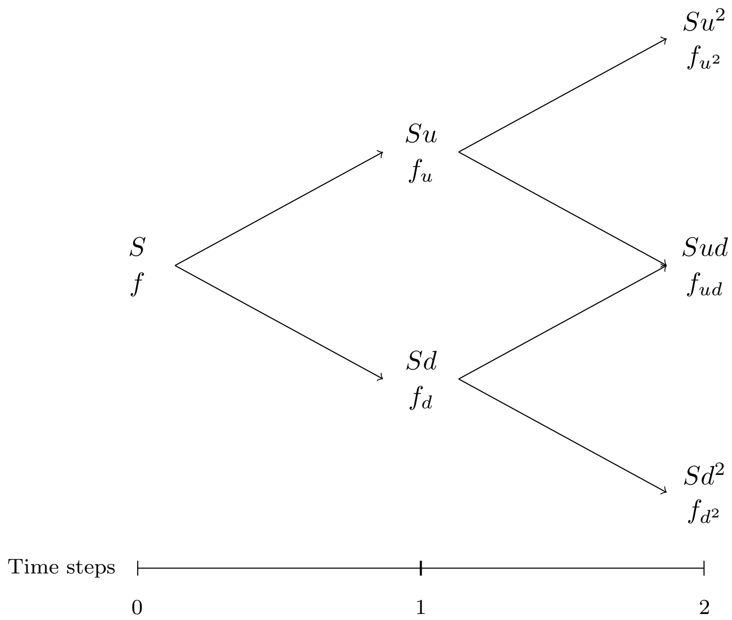


Figure 5.7: General two step tree with stock and option prices

The stock price is initially S_0. Then, it either moves up to u times its initial values or moves down to d times its initial value during each time step. We suppose that the risk-free interest rate is r and the time step length is Δt years.

Because the length of a time step is now Δt rather than T, Equations (5.13) and (5.14) become:

$$f = (p_u f_u + p_d f_d)(1 + r_f)^{(\Delta t)} \tag{5.15}$$

with

$$p_u = \frac{(1 - r_f)^{(\Delta t)} - d}{u - d} \qquad p_d = \frac{u - (1 - r_f)^{(\Delta t)}}{u - d} \tag{5.16}$$

Repeated application of Equation (5.15) gives:

$$f_u = (p_u f_{uu} + p_d f_{ud})(1 + r_f)^{(\Delta t)} \tag{5.17}$$

$$f_d = (p_u f_{ud} + p_d f_{dd})(1 + r_f)^{(\Delta t)} \tag{5.18}$$

$$f = (p_u f_u + p_d f_d)(1 + r_f)^{(\Delta t)} \tag{5.19}$$

Substituting from Equations (5.17) and (5.18) into (5.19), we get:

$$f = (p_u^2 f_{uu} + 2 p_u p_d f_{ud} + p_d^2 f_{dd})(1 + r_f)^{(2\Delta t)} \tag{5.20}$$

That is consistent with the principle of risk-neutral valuation mention earlier. The variables $p_u^2, 2p_u p_d$, and p_d^2 are the probabilities that the upper, middle, and lower final nodes will be reached. The option price is equal to its expected payoff in a risk-neutral world discounted at the risk-free interest rate. As we add more steps to the binomial tree, the risk-neutral valuation principle continues to hold. The option price is always equal to its expected payoff in a risk-neutral world discounted at the risk-free rate [12].

5.2.2 Black-Scholes-Merton Model

The BSM [16] breakthrough recognised the opportunity for replication and based the *pde* on the Law of One Price. By obtaining the *pde* by setting up a risk-less hedge position consisting of the tracking portfolio and the option and equating the hedge return rate to the risk-free interest rate.

$$\frac{\delta F}{\delta t} + rS\frac{\delta F}{\delta S} + \frac{1}{2}\sigma^2 S^2 \frac{\delta^2 F}{\delta S^2} = rF \tag{5.21}$$

Equation (5.21) is the BSM differential equation. It may have many different solutions depending on the derivative defined with S as the underlying asset.

Each particular derivative is found from specific boundary conditions. These specify values of the derivative at the boundaries of possible values of S and t. For example, in the case of a European call option, the boundary condition is:

$$f = max(S - X, 0) \quad \text{When } t = T$$

In the case of a European put option, it is:

$$f = max(X - S, 0) \quad \text{When } t = T$$

The solutions to the BSM equation with these boundary conditions are for the prices of a European call option:

$$c(t) = S(t)N(d_1) - Xe^{-r_f(T-t)}N(d_2) \tag{5.22}$$

and for a put option:

$$p(t) = Xe^{-r_f(T-t})N(-d_2) - S(t)N(-d_1) \tag{5.23}$$

where

$$d_1 = \frac{\ln(\frac{S_t}{X}) + (r_f + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \tag{5.24}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{5.25}$$

The terms $N(d_1)$ and $N(d_2)$ are cumulative probabilities for a unit normal variable z where:

$$N(x) = \int_{-\infty}^{d_1} f(z)dz$$

Where z is distributed normally with mean zero and standard deviation one, and $f(z)$ is the normal density [12][17].
where:

- c is the European call price.
- p is the European put price.
- S_t is the price of the underlying asset.
- $N(d_1)$ is the cumulative normal probability of unit normal variable d_1.
- $N(d_2)$ is the cumulative normal probability of unit normal variable d_2.
- X is the exercise price.
- T is the time to maturity.
- r_f is the risk-free rate.
- σ is the stock price volatility.
- e is the base of natural logarithms, constant = 2, 1728...

Chapter 6

Real Options

Like financial options, RO give their holder the right but not the obligation to take any flexible action in a project at a predetermined cost called the exercise price, at the option's maturity date. Therefore, RO and financial options are valued relative to the underlying asset's price. For example, the underlying asset for a financial option is a security, such as a share in common stock or a bond. In contrast, the underlying asset in RO is a tangible asset, such as a business unit or a project [4][7].

Like financial options, the value of RO depends on the following six variables:

1. The value of the underlying risky asset.
2. The exercise price.
3. The time to expiration of the option.
4. The standard deviation of the value of the underlying risky asset.
5. The risk-free rate of interest over the life of the option.
6. The dividends that are paid of the underlying asset.

RO analysis is valuable because when a project faces high market uncertainty, the option value rises. In that case, the project may obtain more information to study, resolve uncertainty, and facilitate future contingent decisions. Therefore, instead of committing to investing today, investors can make a market survey to resolve uncertainty first. If the results are favourable, investors will likely move forward with the project or even scale it up, thus taking advantage of the project's upside potential. On the other hand, if the results are unfavourable, investors may contract or abandon the project altogether, limiting their losses to the small investment made to resolve uncertainty at the outset [4].

6.1 Comparing Discounted Cash Flow Valuation, Decision Tree Analysis and Real Options

DCF method is an all or nothing strategy that accounts for a project's downside using a risk-adjusted discount rate and expresses risk as a discounted premium. DCF method does not consider the uncertainty with future project outcomes. Some of the limitations of the DCF method can be overcome by using DTA. With DTA, managerial flexibility is modelled in a discrete time manner. It allows for future decisions and maximises the value of the project conditioned on the information available. However, in RO, uncertainty is a crucial factor that drives the option value. Therefore, RO offer valuable information for go/no-go decisions based on evaluating projects against other competing projects in a portfolio. Both DTA and RO consider all cash flows over a project's life and discount cash flow to the present. Therefore, it would be possible to say that both approaches are DCF approaches or that DCF is a RO approach that assumes no flexibility [4][6][7].

DTA and RO apply when uncertainty about the project results and opportunities for dependent decisions. However, there are two fundamental differences between them. The first is that DTA can account for specific and market risks, but the RO approach only addresses market risk. Thus, RO problems will not be valid for specific risks because the theoretical framework does not apply. The second one is that DTA accounts for the risks through the probability of project outcomes. While it considers only a few mutually exclusive possible outcomes, RO account for a wide range of outcomes. That makes a difference in the discount rate used to discount the cash flows. A discount rate used for RO is the risk-free rate. The drawback with DTA is that probabilities of outcomes have to be estimated, which involves subjectivity [4].

DTA is more appropriate in the valuation of projects that do not include market risk, but RO are better when such risk exists. The RO approach is not a substitute but rather a supplement to decision trees. When both market and specific risks exist and opportunities for flexibility to change the project's future course, RO can provide a better valuation combined with DTA. The RO is not a substitute for either DCF method or DTA. Instead, it supplements and integrates the traditional tools into a more sophisticated valuation technique [4]. Although DTA has been an efficient valuation tool for decades, some challenges have been recognised under specific circumstances. For example, the naive DTA approach does not provide the correct valuation in RO valuation [5][18].

A naive approach to valuing projects with RO would be to include decision nodes corresponding to project options into a decision tree model of the project's uncertainties and solve the problem using the same risk-adjusted discount rate appropriate for the project without options. Unfortunately, this naive approach is incorrect because the optimisation at the decision nodes changes the expected future cash flows, thus, the project's risk characteristics. Consequently, the standard deviation of the project cash flow with flexibility is not the same as that of the project without flexibility. As a result, the risk-adjusted discount rate initially determined for the project without options will not be the same as used in RO. However, RO problems can be solved by DTA using risk-neutral probabilities. That implies that we can discount the project's cash flow at the risk-free rate of return and make any necessary adjustments for risk in the probabilities of each state of nature [6][19]. Copland and Antikarov [7] provided an example that illustrates this concept.

Example

Suppose the risk free rate is 8%, and that there is a two-state project with 50-50 chances of cash flows of \$170 or \$65 one year from now that has a cost of capital of 17.5% that will cost \$115 next year. Thus, the PV of the cash flow today is:

$$PV = [0.5(\$170) + 0.5(\$65]]/1.175 = 100\$$$

If an investment decision is required immediately, the project will be rejected. The project's NPV equals the PV of the cash flow minus the investment cost of \$115 next year. Since the investment is certain, it should be discounted at the risk-free rate of 8% and the NPV is:

$$NPV = \$100 - \frac{\$115}{(1 + 0.08)} = \$100 - 106.48 = -6.48$$

The DTA allows the decision-maker to defer until the end of the period and choose whether to spend \$115 based on knowledge at that time. The PV of the decisions is estimated by discounting the expected cash flow, given the right to defer, at the cost of capital as follows:

$$NPV = \frac{0.5(\$55) + 0.5(\$0)}{(1 + 0.175)} = \frac{\$27.5}{1.175} = \$23.40$$

The project's NPV has increased from -\$6.48 given the inflexible precommitment alternative to \$23.40 with the ability to defer. Consequently, the value of the deferral option, using the DTA method, is \$23.4 - (-\$6.48) = \$29.88.

Table 6.1: Cash payoffs for precommitment and for deferral.

	Precommit	Investment	Net precommit	Defer
Up state	\$170	\$115	\$55	MAX[\$55,0]
Down state	\$65	\$115	-\$50	MAX[-\$50,0]

At first glance, this seems to be a good approach. However, on close reflection into the DTA method, the approach is wrong. That is because the DTA method violates the law of one price. The cost of capital of 17.5% is appropriate for 50-50 chances of either \$170 or \$65 and any pattern of perfectly correlated cash flows. However, the cash flows of the deferral option are very different. Look at Table 6.1; the cash flows in the defer column (\$55 or 0) are not perfectly correlated with the project's net cash flow (\$55,-\$50). The answer changes if the decision-maker can defer the investment decision for one year, allowing him to decide after observing next year's prices. The cost of capital, 17.5%, cannot be used because the risk of the project has changed due to the option to defer investment decisions. On the other hand, a set of risk-neutral probabilities for the original project can be determined and used to value the project with a deferral option since the expected cash flow for both problems is the same(\$170 and \$65).

While the correct risk-adjusted discount rate of a project with options is difficult to determine due to the effect on the project risk, the risk-free rate of return can be readily observed in the market. Furthermore, switching from objective probabilities to risk-neutral probabilities allows the NPV with options to estimate even without knowing the correct risk-adjusted discount rate. Therefore, we first use a RO valuation to value this flexibility and then repeat the valuation with the DTA.

In this example, this can be done by defining p as the probability of an upward movement in the price in a risk-neutral world. The expected PV of the project is determined with the objective probabilities and the risk-adjusted discount rate. It can also be calculated from the unknown risk-neutral probabilities and the risk-free discount rate. By equating the two equations for the expected PV of the project, we can solve for the risk-neutral probability p, as follows:

$$\$100 = \frac{p(\$170) + (1 - p)(\$65)}{1 + 0.08}$$

and solve for $p = 0.41$

The project with the option to defer has a net payoff of \$170 - \$115 = \$55 in the upstate and zero in the downstate. Therefore, the NPV of the project with the option to defer is:

$$NPV_{flex} = [0.41(\$55) + 0.59(0)]/1.08 = \$20.86$$

The value of the flexibility to defer is equal to the difference between the project's value with no flexibility -\$6.48, and the value with flexibility to defer \$20.86. Therefore, the value of the option to defer is \$27.34.

The DTA would have provided the same answer if we had used the correct discount rate, calculated as follows:

$$PV = \$20.87 = \frac{0.5(\$55) + 0.5(\$0)}{1 + k}$$

$$k = 31.9\%$$

That confirms that when the DTA used the appropriate discount rate for the project assuming inflexible precommitment (17.5%), it was using the wrong rate for the project's cash flows with flexibility as provided by the deferral option. Thus, in general, the DTA approach will give the wrong answer because it assumes a constant discount rate throughout a decision tree when the riskiness of the cash flow outcomes changes based on where we are located in the tree [3][6][7][18].

Copland [18] and others noted that this shortcoming of DTA is an artefact of their naive analysis, which overlooks market opportunities to borrow and trade considered in the options analysis and confounds time and risk preferences by using a single risk-adjusted discount rate.

6.2 Financial Options v.s. Real Options

As mentioned, financial options are written on traded securities which makes it easier to estimate their parameters. In addition, prices of traded securities are monitored and collected, and from historical data, it is possible to estimate the variance of its returns. With RO, the underlying risky asset, on the other hand, is not traded. Therefore, there is a need to estimate the PV of the underlying asset without flexibility using the traditional NPV. In RO, the underlying asset is a project, investment, or acquisition. If the underlying asset's value goes up, the value of a call option will rise as well. One of the crucial differences between financial and RO is that the owner of a financial option cannot affect the underlying asset's value. However, the management that operates a tangible asset can raise its value and thereby raise all RO that depend on it [4][7][9].

Another significant difference between financial and RO is that firms do not issue contingent shares. However, the agent that issues an option has no influence over the firm's actions and no control over the firm's share price. RO are different because the options are written on the firm's underlying tangible assets controlled by its management. For example, a firm may want to defer a project if its PV is low. However, suppose the firm comes up with a new idea that raises the PV of the underlying project. In that case, the value of the right to defer may fall, and the firm may decide not to defer. Usually, enhancing the value of the underlying real asset also enhances the value of the option [7].

Finally, risk, the underlying uncertainty, is assumed to be exogenous with financial and RO. That is a reasonable assumption for financial options. The uncertainty about the return on a stock is beyond the control or influence of individuals who trade options. However, the actions of a firm that owns a RO may affect the actions of competitors, and consequently, the nature of uncertainty that the firm faces [7].

6.3 Parameters used in Real Option

The biggest challenge in calculating the value of a RO is to estimate the input parameters. The following section defines the input parameters and presents best practices and limitations in their estimation [4].

6.3.1 Underlying Asset Value

Traditional option pricing methods require that markets be complete as discussed in Section 5.2, i.e. marketed security or a portfolio of securities whose payoffs replicate the payoffs of the project in all states and periods. Although this may be a reasonable assumption for financial assets, no such replicating portfolio of securities exists for most tangible assets or projects and markets are said to be incomplete. Copland and Antikarov [7] suggest an alternative discrete-time method based on the assumption that the PV of the project without options is the best-unbiased estimator of the project's market value (the Market Asset Disclaimer (MAD) assumption). With this assumption, the project becomes the underlying asset of the replicating portfolio, thus making the markets complete for the project options. As a result, these options can now be valued with traditional option pricing methods. Another assumption they make, is that the variations in the value of the project follows a random walk. While these assumptions are also subject to several reservations, we will adopt this point of view for this thesis. Therefore, the underlying asset value is calculated as the PV of the expected cash flows, based on the DCF calculations as discussed in Section 3.1.

6.3.2 The Volatility of the Underlying Asset

Volatility is a vital input variable that can significantly impact the option value and is probably the most challenging variable to estimate in RO problems. Volatility is a measure of the variability of the total value of the underlying asset over its lifetime. It signifies the uncertainty associated with the cash flows that comprise the underlying asset value. The volatility factor (σ) used in option valuation is the volatility of the returns, measured as the standard deviation of the natural logarithm of cash flow returns, not the actual cash flows. The return for a given period is the current period cash flow ratio to the preceding one [4].

Volatility is a measure of the variability of the total value of the underlying asset over its lifetime. It signifies the uncertainty associated with the cash flow that comprises the underlying asset value. The volatility factor (σ) used in the options models is the volatility of the rates of return, which is measured as the standard deviation of the natural logarithm of cash flow return, not the actual cash flow. The return for a given period is the current period cash flow ratio to the preceding one. In any option model, the volatility factor used should be consistent with the time step used in corresponding equations. For instance, if the time steps are annual, the volatility factor should be annualised [4].

6.3.3 Strike Price

In RO, the strike price is the investment costs needed to undertake an investment project. The strike price directly impacts the option value. It may change during the option life, so the option valuation equations must be adjusted accordingly. In RO valuation, exercising an option typically involves an action that takes a long time, for instance, developing a product or constructing a new facility [4].

6.3.4 Option Life

Often, the time to maturity of RO is not known at the beginning of the valuation. However, there is a need to define the time variable in the model. Sometimes managers do not know how long they will have an opportunity to invest or exercise an option. Usually, there is no deadline by which the decision must be made. For example, when developing a product, the development time is unknown at the outset and whether or not the final product will be launched is always optional. Therefore, the options life has to be long enough to resolve uncertainty over the option's life. However, it should not be too long because the option value could become worthless as time goes by due to external competition or other unexpected events [4].

6.3.5 Risk-Free Interest Rate

The risk-free annual interest rate used in RO valuation is usually determined based on the return on a government bond with a maturity equivalent to the option's time to maturity [4].

6.4 Valuation of Real Options

Option pricing methods were first developed to value financial options. However, several works made the transition from the valuation of financial options to the valuation of options on tangible assets. Dixit and Pindyck [1], and Trigeorgis [2] were among the first authors to synthesise several of these ideas [6].

Various approaches have been applied in order to value RO. However, the BSM model and binomial methods are the most common models to solve RO problems. Their difference lies in how easily they can be adjusted to account for changes in the underlying variables and how effectively they can explain the results [4].

6.4.1 The Black-Scholes-Merton Equation

The BSM model may seem to be the correct method for valuing RO. The reason is that it is widely employed in financial option valuation. However, its application in RO is limited for many reasons. It is mathematically complex and promotes a "black box" approach, where the intuition behind the application is lost. Therefore, making it challenging to represent the solution to investors. The BSM model was developed for European financial options. A European option can only be exercised at the maturity of the option. However, in RO, the option can be exercised at any time during their life [4].

The BSM equation assumes a log-normal distribution of the underlying asset value, which may not be valid for the cash flows of tangible assets. It also assumes that the increase in the underlying asset value is continuous as dictated by its volatility and does not account for any drastic ups and downs. Furthermore, it allows only one strike price for the option, which can change for RO during its life [4].

These limitations can be overcome by adjusting the BSM model, but the model is already complex, and these adjustments only make it more complex [4].

6.4.2 Binomial Method

The binomial method offers more flexibility than the BSM model. Input parameters such as the strike price and volatility can be changed over the life of the option. The binomial method's essential advantage is transparency in its underlying framework, making the results easy to explain. While the BSM equation gives the most accurate option value, the binomial method is close to it. The reason for that is smaller the time steps used in the binomial method, the closer the method will get to the BSM value [4]. The binomial model is the method of choice for solving the options presented in the rest of this thesis. That is because it is the most effective for illustrative purposes and because the options used in the case study presented in Chapter 7 can not be solved with the BSM model.

6.5 Option Types

RO are classified primarily by the type of flexibility that they offer. An example of simple options is to defer, expand, contract, or abandon. The deferral option and the option to expand are both American call options, where the option can be exercised on or at any time before the expiration date. The investor acquires the right to invest in the project. The option to contract and the option to abandon are both American put options. The investor can exercise the option on or before its expiration date by selling their project assets [4][7]. These types of options are going to be discussed in more detail later in this section. We also explore combinations of these simple options like the option to choose.

An example of more advanced options is compound, switching, rainbow, and compounded rainbow. The compound options are options on options. The value of a compound option depends on the value of another option rather than the underlying asset value. Compound options are common in many multi-phase projects, such as product and drug development. In the Switching option, an investor has the right to switch between two modes of operation; an example would be switching between natural gas and fuel oil for boilers at a manufacturing facility. Switching options are a portfolio of American calls and puts. The rainbow options may be either simple or compound. Options that are driven by multiple sources of uncertainty are called rainbow options. Compound rainbow options are perhaps the most realistic and complex real options to value. However, they cover a broad and important class of decisions [4][7].

6.5.1 Option to Defer

As mentioned, the option to defer is an American call option on the PV of the expected cash flow S_t, with a strike price as the investment outlay I. The option to defer is found in most projects where one has the right to delay the timing of an investment in a project [2][7].

The firm invests I only if prices rise enough and will not commit to the project if prices decline. Just before the expiration, the value of the option to defer is:

$$f_t = max(S_t - I, 0) \tag{6.1}$$

To illustrate the pricing of an option to defer, let us pick some parameters to describe the stochastic process for the value of an underlying risky asset. Consider a research firm keen to undertake a risky research project. Assume that the PV without flexibility is S_0 = \$180m, the annual standard deviation is σ = 40%, and the investment required is I = \$190m. The continuous risk-free rate over the next two years is estimated r_f = 5%.

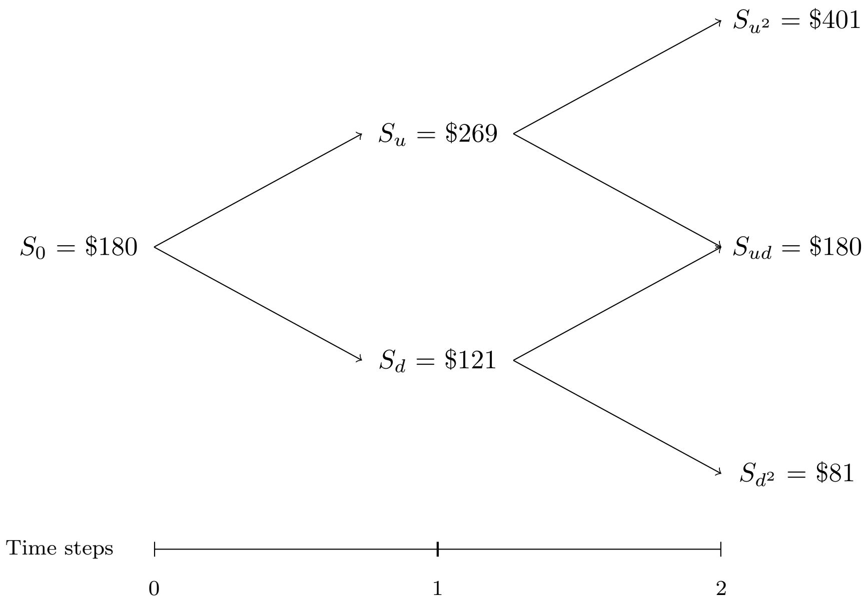


Figure 6.1: Asset price movements in time Δt under the binomial model

An example of a binomial tree can be seen in Figure 6.1. The option's life was defined as two years, and the time interval was one year (Δt = 1). The asset value calculated over the options life where up and down movements (u, d) of the cash flows were calculated as follows:

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.4\times\sqrt{1}} = 1.492 \quad ; \quad d = 1/u = 0.670$$

The risk-neutral probability, q, is calculated as follows:

$$q_u = \frac{e^{(r_f \Delta t)} - d}{u - d} = 0.464$$
$$q_d = (1 - q_u) = 0.536$$

The investment cost of starting the project today is I = \$190m, which gives the firm a negative NPV of -\$10m.

$$NPV_{Noflex} = \$190 - \$180 = -\$10\text{m}$$

According to the NPV rule, the firm should not invest in this project today. Let us assume that the firm can defer its investment until the next period. We start with the terminal nodes representing the tree's last step (see Figure 6.1). Since the firm wants to maximise its return, we calculate the following payoffs:

$$f_{u^2} = max(S_0u^2 - I, 0) = \$211m$$
$$f_{ud} = max(S_0ud - I, 0) = \$0$$
$$f_{d^2} = max(S_0d^2 - I, 0) = \$0$$

At node S_0u^2, the expected asset value is \$401m, compared to the investment cost I of \$190m. At this node, the firm would rather continue than defer the project. When the option value is > 0, the firm would invest in the project and not exercise the option to defer. The option value at this node is \$211m. However, at nodes S_0ud and S_0d^2, the option value is \$0, the firm will exercise the option to defer, and therefore, the decision to not invest will be taken.

On the intermediate nodes, one step away from the last time step, let us start at the top, at node S_0u and calculate the asset value for keeping the option open. That is simply the discounted, at the risk-free rate, a weighted average of potential future option values using the risk-neutral probability as a weight. Then we take the maximum EV of keeping the option open or exercising the option to defer the project.

$$f_u = max\big([q_u f_{u^2} + q_d f_{ud}] \times e^{(-r_f \Delta t)}, S_0u - I, 0\big) = \$93m$$
$$f_d = max\big([q_u f_{ud} + q_d f_{d^2}] \times e^{(-r_f \Delta t)}, S_0d - I, 0\big) = \$0$$

At node S_0u, the option value is > 0, which means the firm will not exercise the option. On the other hand, at node S_0d, the option value is \$0, so the firm will exercise the option to defer.

The value of the option at $t = 0$ is:

$$f_0 = max\big([0.464(\$93m) + 0.536(\$0m)]e^{(-0.05\times1)}, 269 - 190, 0\big)$$
$$f_0 = \$41m$$

Figure 6.2 shows the option values at each node of the binomial tree by backward induction. Thus, each node represents the value maximisation of investing at that point or waiting until the next period.

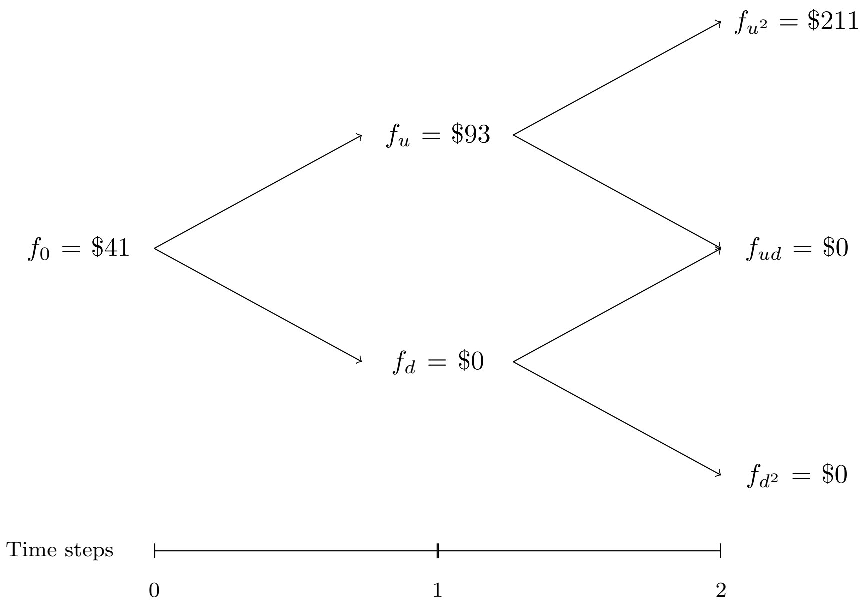


Figure 6.2: Value for the option to defer

From the NPV rule, the value of the project is negative of -$10m. RO, however, results in a NPV of $31m, yielding an additional $21m (-$10m + $31m) due to the option to defer.

$$NPV_{Noflex} = S - I = \$180 - \$190 = -\$10m$$
$$NPV_{Flex} = NPV_{Noflex} + f_0 = -\$10 + \$41 = \$31m$$

6.5.2 Option to Abandon

Abandonment options are American put options where the owner of an asset has the right to sell it for a given price rather than continuing to hold it. The option to abandon is embedded in virtually every project. Abandonment options are essential in research and development, exploration, merger and acquisitions, and development of natural resources. This option is precious where the NPV is marginal, but there is great potential for losses. The losses can be minimised by selling off the assets either on the spot or preferably by prearranged contracts. The flexibility in this option is to abandon the project if the EV falls below the project salvage value, the price of the option [2][4][17].

Abandonment option analysis estimates the value of optimal abandonment and indicates when abandonment should be implemented. In merger and acquisition situations, abandonment is equivalent to being able to bail out of an investment at a floor price which is the estimated exercise price of the abandonment option [7].

The value of the option to abandon is:

$$f_t = max[X - S_t, 0] \tag{6.2}$$

To illustrate the valuation of such an option, we keep the same underlying risky asset as before (see Figure 6.1) but introduce an option to abandon. The NPV with no flexibility is -\$10m, and the firm thinks the project is risky. However, the government wants the project to be undertaken and commits to buying it for X = \$150m if the project fails. Therefore, the firm needs to reconsider and see how this government offer changes the NPV of the project. This offer from the government creates a put option on the projects cash flows, namely the option to abandon.

Let us calculate the value of the option to abandon starting at the terminal nodes as before. Since the firm wants to maximise its return, we find:

$$f_{u^2} = max(X - S_0u^2, 0) = \$0$$
$$f_{ud} = max(X - S_0ud, 0) = \$0$$
$$f_{d^2} = max(X - S_0d^2, 0) = \$69m$$

At node S_0u^2, the expected asset value is \$401m, compared to the \$150m from the government. The option will be exercised if the expected asset value S_t is lower than the exercise price X. If the expected asset value is more significant than X, they will continue rather than abandon the project. The option value at this node is \$0, so the firm will continue with the project. At node S_0d^2, the expected asset value is \$81m, compared to the exercise price of \$150m. The option value is \$69m, and the firm will exercise the option and abandon the project.

On the intermediate nodes, starting at the top, at node S_0u, we calculate the asset value for keeping the option open. That is simply the discounted, at the risk-free rate, a weighted average of potential future option values using the risk-neutral probability as a weight. Then we take the maximum EV of keeping the option open or exercising the option and abandoning the project for \$150m.

$$f_u = max\big([q_u f_{u^2} + q_d f_{ud}] \times e^{(-r_f \Delta t)}, X - S_0u, 0\big) = \$0$$
$$f_d = max\big([q_u f_{ud} + q_d f_{d^2}] \times e^{(-r_f \Delta t)}, X - S_0d, 0\big) = \$35m$$

At node S_0u, the option value is \$0, which means the firm will not exercise the option. However, at node S_0d, the option value is \$35m, so the firm will exercise the option to abandon.

The value of the option to abandon at $t = 0$ is calculated as follows:

$$f_0 = max\big([0.464(\$0m) + 0.536(\$35m)] \times e^{(-0.05\times1)}, X - S_0, 0\big)$$
$$f_0 = \$18m$$

Figure 6.3 shows the option values at each node of the binomial tree by backward induction. Thus, each node represents the value maximisation of investing at that point or waiting until the next period.

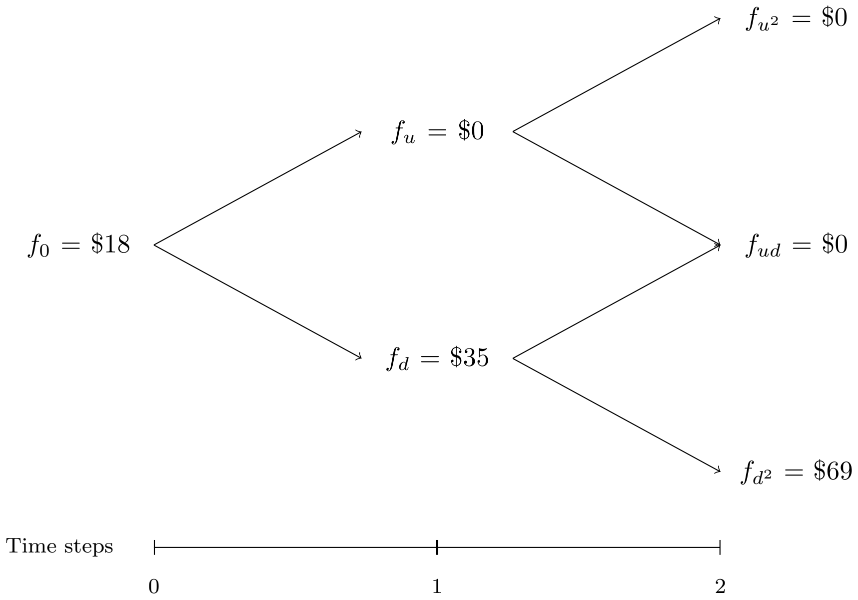


Figure 6.3: Value of the option to abandon

Recall that the NPV with no flexibility was −\$10m, and the firm was not keen to invest in the project. However, the government gave them an offer to buy the project if it failed, so the NPV of the project now with that flexibility is:

$$NPV_{Flex} = -\$10 + \$18 = \$8m$$

Using the option to abandon, RO has added \$18m to the project's NPV. That can make an essential difference in the decision whether to invest in the project or not.

6.5.3 Option to Expand

Expansion options are American call options where a project that turns out better than expected can be expanded by paying an additional cost (I_E) for an increase in value by a factor x_{exp}%. Options to expand are essential in natural-resource industries, consumer goods, and commercial real estate [2][7][20].

The expansion opportunity ($x_{exp} \times S_t - I_E$) needs to be compared with the EV before the expansion (S_t), so we subtract the EV from the expansion opportunity.

The value of the option to expand is:

$$f_t = max\big((x_{exp}S_t - I_E) - S_t, 0\big). \tag{6.3}$$

To illustrate the valuation of such an option, we keep the same underlying risky asset as before (see Figure (6.1) but introduce an option to expand. The firm believes there is great potential for expanding their project and exploring the option to expand. Expand by $x_{exp} = 30\%$ at cost of $I_E = \$60$m.

Let us calculate the value of the option to expand starting at the terminal nodes as before. Since the firm wants to maximise its return, we find:

$$
\begin{aligned}
f_{u^2} &= max\big((S_0u^2 \times x_{exp}) - I_E - S_0u^2, 0\big) = \$60m \\
f_{ud} &= max\big((S_0ud \times x_{exp}) - I_E - S_0ud, 0\big) = \$0 \\
f_{d^2} &= max\big((S_0d^2 \times x_{exp}) - I_E - S_0d^2, 0\big) = \$0
\end{aligned}
$$

At node S_0u^2, the expected asset value is \$401m, compared to the expansion cost I_E = \$60m. At this node the firm would rather exercise the option to expand and invest \$60m and expand the operation by 30%. However, at nodes S_0ud and S_0d^2, the option value is \$0, and the firm will not exercise the option to expand and will keep the project in the same size.

On the intermediate nodes, one step away from the last time step, let us start at the top, at node S_0u and calculate the expected asset value for keeping the option open. That is simply the discounted, at the risk-free rate, a weighted average of potential future option values using the risk-neutral probability as a weight. Then we take the maximum of the EV of keeping the option open to continue with the project in the same size or to exercise the option to expand the project:

$$
\begin{aligned}
f_u &= max\big([q_u f_{uu} + q_d f_{ud}]e^{(-r_f \Delta t)}, (S_0u \times x_{exp}) - I_E - S_0u\big) \\
&= max(27, 21) = \$27m \\
f_d &= max\big([q_u f_{ud} + q_d f_{d^2}]e^{(-r_f \Delta t)}, (S_0d \times x_{exp}) - I_E - S_0d\big) \\
&= max(0, -24) = \$0
\end{aligned}
$$

At the node, S_0u, the expected asset value for keeping the option open is \$27m, which is higher than exercising the option. Therefore, the firm decides to continue with the project of the same size. At node S_0d, the EV for keeping the option open is \$0, but for exercising the option, the value is −\$24m. Maximising \$0 vs −\$24, the firm will keep the option open and continue with the project in the same size.

The value of the option to expand at $t = 0$ is calculated as follows:

$$
\begin{aligned}
f_0 &= max\big([0.464(\$27m) + 0.536(\$0m)]e^{(-0.05\times1)}, (180 \times 1.3) - 60 - 180\big) \\
f_0 &= max(12, -6) = \$12m
\end{aligned}
$$

By taking a better look at nodes S_0u, S_0d, and S_0, the value of keeping the option open is always greater than exercising the option before maturity. If the option is exercised, the value the investors get is the value of the project minus the exercise price. However, by keeping the option open, they get the PV of the project with the option to expand in the future minus discounted PV of the exercise price. Thus, the value of keeping the option open will always be higher than exercising the option. That is a general property of an American call option on a risky asset that pays no dividends [7].

Figure 6.4 shows the option values at each node of the binomial tree by backward induction. Thus, each node represents the value maximisation of expanding the project or continuing the project in the same size.

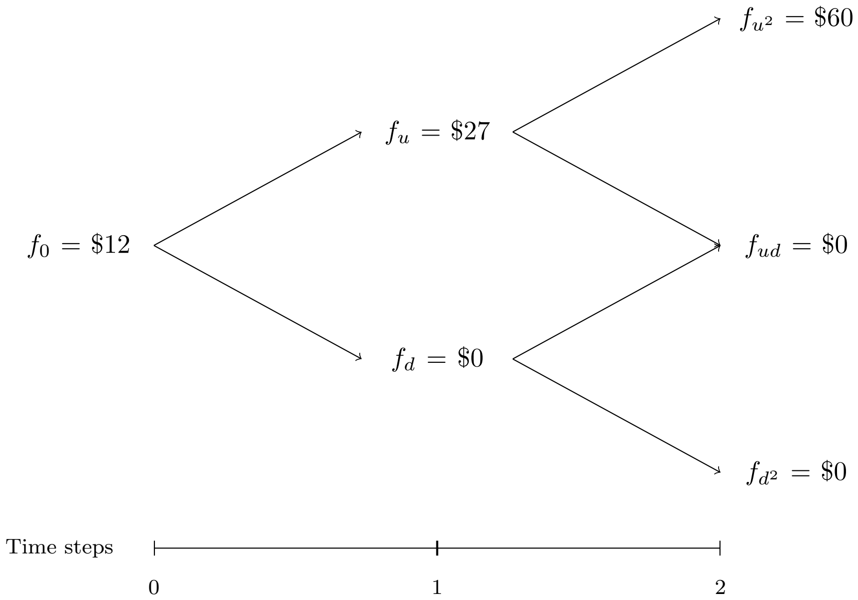


Figure 6.4: Value of the option to expand

Recall that the NPV with no flexibility was $-\$10$m, and the firm was not keen to invest in the project. However, with the option to expand the project over time, the NPV with that flexibility is:

$$NPV_{Flex} = -\$10 + \$12 = \$2m$$

Using the option to expand, RO added to the project NPV a \$12m. That can make the project more appealing for the firm.

6.5.4 Option to Contract

Contraction options are American put options where the owner of the option has the right to contract a project by selling or reduce a fraction $x_{con}\%$ of it for a fixed price I_C thereby shrinking the scale of the project [7][2].

The contraction opportunity $((1 - x_{con}\%) \times S_t + I_C)$ needs to be compared with the EV before the expansion (S_t). Therefore we subtract the EV from the contraction opportunity.

The value of the option to contract is:

$$f_t = max\big(((1 - x_{con})S_t + I_C) - S_t, 0\big) \tag{6.4}$$

To illustrate the valuation of such an option, we keep the same underlying risky asset as before (see Figure 6.1) but introduce an option to contract. The firm believes that the market conditions can turn weaker than initially expected and want to explore the option to contract by $x_{con} = 50\%$ and gain $I_C = \$90$m in savings.

Let us calculate the value of the option to contract, starting at the terminal nodes as before. Since the firm wants to maximise its return, we find:

$$f_{u^2} = max\big(((1 - x_{con}) \times S_0u^2 + I_C) - S_0u^2, 0\big) = \$0$$
$$f_{ud} = max\big(((1 - x_{con}) \times S_0ud + I_C) - S_0ud, 0\big) = \$0$$
$$f_{d^2} = max\big(((1 - x_{con}) \times S_0d^2 + I_C) - S_0d^2, 0\big) = \$50m$$

At node S_0u^2, the expected asset value is \$401, compare to the contraction gain $I_C =$ \$90m. The option value is \$0, therefore will the firm keep the option open and not contract the project. At node S_0d^2, the option value is \$50m and therefore > 0, then the firm will exercise the option to contract the project by 50% and save \$90m.

On the intermediate nodes, starting at the top, at node S_0u, we calculate the expected asset value for keeping the option open. That is simply the discounted, at the risk-free rate, a weighted average of potential future option values using the risk-neutral probability as a weight. Then we take the maximum of the EV of keeping the option open to continue with the project in the same size or to exercise the option to contract the project:

$$\begin{aligned} f_u &= max\big([q_u f_{uu} + q_d f_{ud}]e^{(-r_f \Delta t)}, ((1 - x_{con}) \times S_0u + I_C) - S_0u\big) \\ &= max(0, -44) = \$0 \\ f_d &= max\big([q_u f_{ud} + q_d f_{d^2}]e^{(-r_f \Delta t)}, ((1 - x_{con}) \times S_0u + I_C) - S_0u\big) \\ &= max(25, 30) = \$30m \end{aligned}$$

At node S_0u, the EV for keeping the option open is \$0, but for exercising the option, the value is $-$\$44m. Therefore, maximising \$0 vs $-$\$44, the firm will keep the option open and continue with the same size project. However, at the node, S_0d, the expected asset value for keeping the option open is \$25m, which is lower than exercising the option. Therefore, the decision is to contract the project.

The value of the option to contract at $t = 0$ is calculated as follows:

$$\begin{aligned} f_0 &= max\big([0.464(\$0m) + 0.536(\$30m)]e^{(-0.05\times1)}, ((1 - 0.5) \times 180 - 90) - 180, 0\big) \\ f_0 &= \$15m \end{aligned}$$

Figure 6.5 shows the option values at each node of the binomial tree by backward induction. Thus, each node represents the value maximisation of contracting the project or continuing the project in the same size.

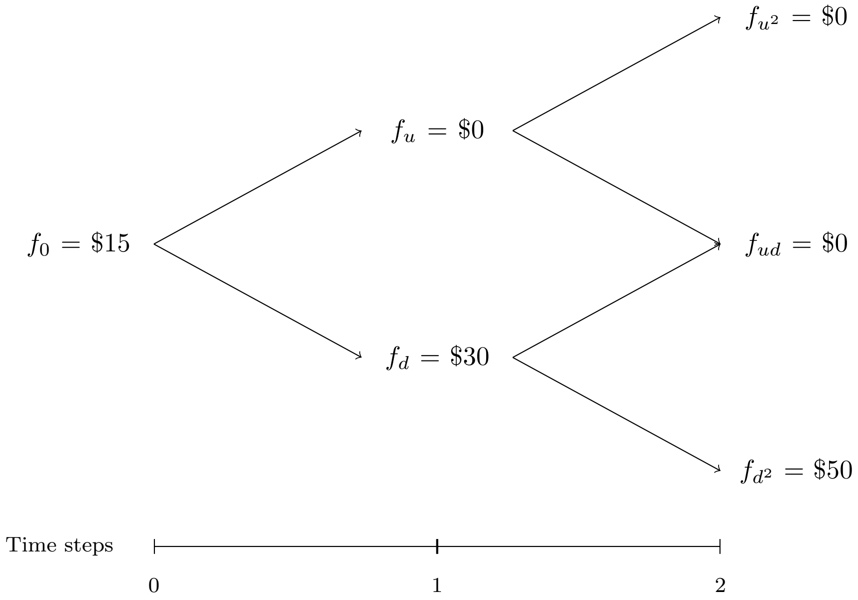


Figure 6.5: Value of the option to contract

Recall that the NPV with no flexibility was −\$10m, and the firm believes that the market conditions could turn out weaker than expected. With an option to contract the project over time, the NPV of the project now with that flexibility is:

$$NPV_{Flex} = -\$10 + \$15 = \$5m$$

Using an option to contract, RO added to the project NPV a \$15m. That can make an essential difference in the decision whether to invest in the project or not.

6.5.5 Option to Choose

The option to choose consists of multiple options combined as a single option. The multiple options are abandonment, expansion, and contraction. It is called a chooser option because the option holder can keep the option open and continue with the project or exercise any options to expand, contract, or abandon. The main advantage of this option is the choice. The option to choose are portfolios of American call and put options that allow their owner to take optimal decisions to choose which action to take at each node [7][4].

Asset values are calculated for each option in every node and compared with the alternative to continue. Either the alternative to continue provides the maximum return, or the option to choose is exercised with the alternative (abandon, expansion or contract) that provides the maximum return [4]. To illustrate the valuation of such an option, we keep the same underlying risky asset as before (see Figure 6.1) but introduce an option to choose. A firm has the flexibility to expand a project by $x_{exp} = 30\%$ at cost of $X_{cost} = \$60$m, contract a project by $x_{con} = 50\%$ and gain $I_C = \$90$m in savings, or abandon a project for the value of $X = \$150$m.

The value of the option to choose is:

$$f_t = max\Big(((1 - x_{con})S_t + I_C) - S_t, (x_{exp}S_t - I_E) - S_t, X - S_t, 0\Big) \tag{6.5}$$

It starts at the terminal nodes as before. Since the firm wants to maximise its return, we find that at node S_0u^2, the expected asset value is \$401m. Let us calculate the value of the assets for exercising each of the available options:

- Abandon: \$150m
- Expand: (1.3 × \$401) − \$60 = \$461m
- Contract: (0.5 × \$401) + \$90 = \$290m

Maximisation shows that the option to expand would be exercised at the node, S_0u^2 so the asset value on that node becomes \$461m. At node S_0ud, the option to contract would be exercised. Finally, at the node, S_0d^2 the option to abandon would be exercised because those actions provide the maximum value, which now becomes the option value on that node.

On the intermediate nodes, starting at the top, at node S_0u calculate the asset value for keeping the option open. That is the discounted weighted average of potential future option values using the risk-neutral probability as weight. So the value at that node is:

$$\begin{aligned} &= [q_u(f_{u^2} + q_d(f_{ud})] \times e^{(-r_f \Delta t)} \\ &= [0.464(461) + 0.536(180)] \times e^{(-0.05(1))} \\ &= \$295m \end{aligned}$$

Now calculate the asset value for exercising each of the available options:

- Abandon: \$150m
- Expand: (1.3 × \$269) − \$60 = \$289m
- Contract: (0.5 × \$269) + \$90 = \$224m

Maximisation shows that the firm would keep the option open at node S_0u. Therefore, the asset value at this node becomes \$295m. For the node, S_0d the firm will also keep the option open, and the asset value becomes \$156m.

The value of the project with the option to choose at $t = 0$ is calculated as follows:

$$\begin{aligned} &= max\big([0.464(\$295m) + 0.536(\$156m)] \times e^{(-0.05\times1)}, 150, 174, 180, 0\big) \\ &= \$210m \end{aligned}$$

The PV of the project with flexibility is \$210m, and we would choose to leave all of our options open at node S_0. The value of the option to choose at $t = 0$ would be:

$$f_0 = \$210 - \$190 = \$20m$$

Figure 6.6 shows the assets values at each node of the binomial tree by backward induction. Thus, each node represents the value maximisation of exercising the chooser option or continuing the project.

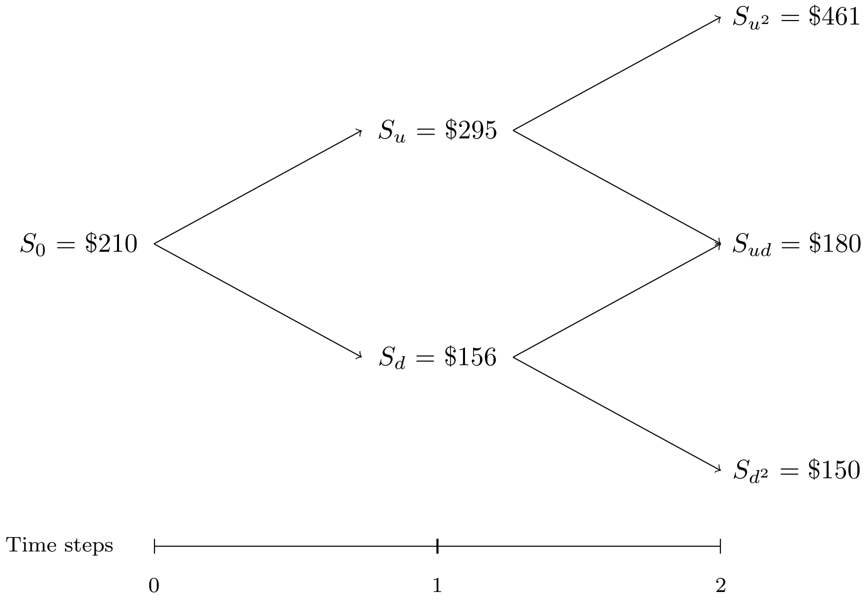


Figure 6.6: Maximisation of assets values for option to choose

Recall that the NPV with no flexibility was −\$10m, and the firm was not keen to invest in the project. Therefore, an option to choose the NPV with that flexibility is:

$$NPV_{Flex} = -\$10 + \$20 = \$10m$$

If the firm considers the individual options separately, rather than a combined chooser option, we can look to the results from the examples here above were:

- Abandonment: \$18m
- Expansion: \$12m
- Contraction: \$15m

As expected, the option to choose (\$20m) has more value than any one of the individual options. However, the chooser option value will always be less than or equal to the summation of the individual options result. That is because the individual options are mutually exclusive and independent of each other. Thus, for example, it is not possible to expand and abandon a project simultaneously [4][7].

6.5.6 Compound Options

Compound options are options on options, and phased investments fit into this category. When a firm sets out to build a factory or start an R&D project, it can choose to do so in the design, engineering, and construction phases. They have the option to stop or defer the project at the end of each phase. Thus, each phase is an option that is contingent on the earlier exercise of other options [7] [4].

A compound option derives its value from another option and not from the underlying asset. The first investment creates the right but not the obligation to make a second investment, which may give the option to make another investment and so on. The compound options give flexibility to the project at any time during its life. The firm has an option contingent on earlier exercise of other options at the end of each phase with compound options. Two or more phases may co-occur (parallel options) or sequence (stage or sequential option). These options are primarily American call or put options [4][7].

A compound option can either be sequential or parallel. Exercising an option to create another option is considered a sequential option. For example, the firm has to finish one stage to enter the next one. In a parallel option, both options are available at the same time. The life of the independent option is longer than or equal to the dependent option. For both sequential and parallel options, valuation is essentially the same except for minor differences. An example of a parallel option is presented in this section. However, an example of a sequential option is presented in the case study in chapter (7) [4][7].

Example

To illustrate a parallel compound option pricing, let us look at an example from the book "Project Valuation Using Real Options A Practitioner's Guide" [4]. A firm is considering offering a service to its customers that involve an investment of \$500m. The license for the service is estimated to cost \$100m. The NPV with no flexibility of the expected cash flows is \$600m, and the annual volatility factor for this payoff is calculated to be 35%. The firm estimates three years to make a go/no-go decision on this project based on the competition. The continuous annual risk-free interest rate over this period is 5%. The firm can start the infrastructure upgrade for this service at any time, but the licenses must be obtained before the upgrade can be tested and the service launched. That creates a parallel option, which the firm can take advantage of in the valuation of the project to make a better investment decision. Since both options, the option to buy the license (independent option) and invest in the upgrade (dependent option), are alive during the same period. Buying the license must be exercised before the upgrade, which constitutes a parallel compound option. What is the value of the option?

An example of a binomial tree with asset price movements can be seen in Figure 6.7. The option's life was defined as three years, and the time intervals were set to one year ($\Delta t = 1 year$). The asset values calculated over the options life where up and down movements (u, d) of the cash flows were calculated as follows:

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.35\times\sqrt{1}} = 1.419 \quad ; \quad d = 1/u = 0.705$$

The risk-neutral probability, q, is calculated as follows:

$$q_u = \frac{e^{(r_f \Delta t)} - d}{u - d}$$
$$q_u = 0.485$$
$$q_d = (1 - q_u) = 0.515$$

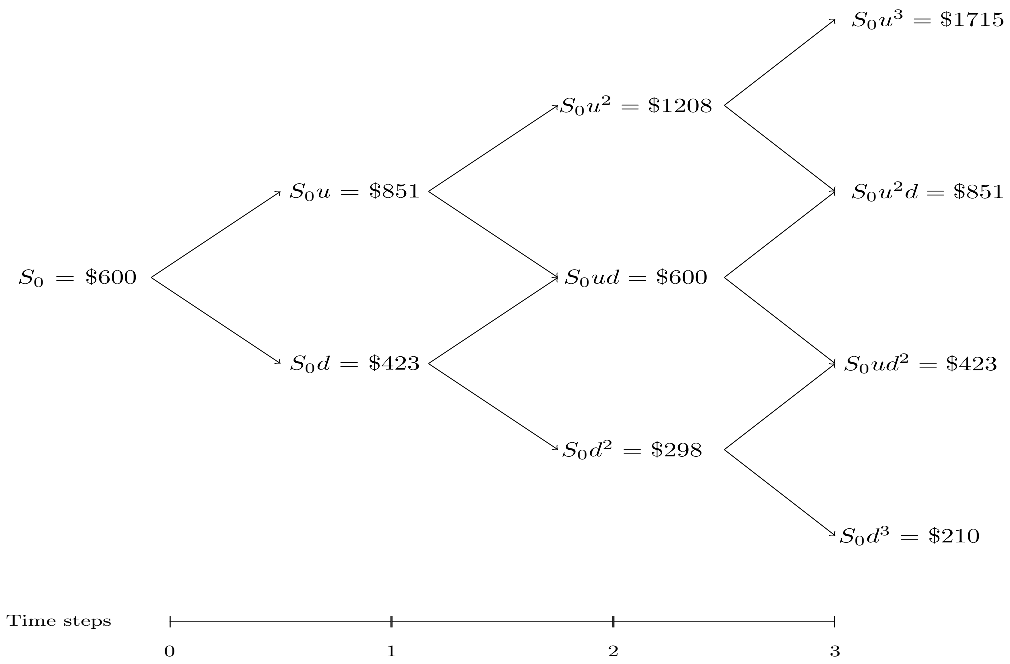


Figure 6.7: Asset price movements in time Δt under the binomial model

Let us start calculating the value of the dependent option value. At terminal nodes, the firm would invest if the payoff is greater than the investment of \$500m; otherwise, the firm would let the option expire. Since the firm wants to maximise its return, we find:

$$
\begin{aligned}
f_{u^3} &= max(S_0u^3 - I, 0) = \$1215m \\
f_{u^2d} &= max(S_0u^2d - I, 0) = \$351m \\
f_{ud^2} &= max(S_0ud^2 - I, 0) = \$0 \\
f_{d^3} &= max(S_0d^3 - I, 0) = \$0
\end{aligned}
$$

At node S_0u^3, the expected asset value is \$1715m, compared to the investment cost I of \$500m. The firm would invest in the project at this node rather than defer, making the net payoff \$1215m. At node S_0d^3, the expected asset value is \$210m, compared to the investment cost of \$500m. The option value is \$0, and the firm will exercise the option, so the decision is not to invest, thereby making the net payoff of \$0.

The firm can either invest \$100m in the project at each intermediate node or keep the option open. Starting at the top, at node S_0u^2 calculate the expected asset value for keeping the option open. That is the discounted, at the risk-free rate, a weighted average of potential future option values using the risk-neutral probability as weight. Thus, the value at that node is:

$$
\begin{aligned}
f_{u^2} &= max\big([q_u f_{u^3} + q_d f_{u^2d}] \times e^{(-r_f \Delta t)}, S_0u^2 - I, 0\big) = \$733m \\
f_{ud} &= max\big([q_u f_{u^2d} + q_d f_{ud^2}] \times e^{(-r_f \Delta t)}, S_0ud - I, 0\big) = \$162m \\
f_{d^2} &= max\big([q_u f_{ud^2} + q_d f_{d^3}] \times e^{(-r_f \Delta t)}, S_0d^2 - I, 0\big) = \$0
\end{aligned}
$$

At node S_0u^2, the EV is \$1208m. Exercising the option at this node will give an option value of \$708m (\$1208 - \$500). Since the value of keeping the option open is \$733m and is larger, the firm would keep the option open and continue; therefore, the option value at S_0u^2 is \$733m. At the node, S_0d^2, the expected asset value for keeping the option open is \$0. If, on the other hand, the firm will exercise the option to invest, the expected asset value would be \$298 - \$600 = -\$302m, so the decision is to keep the option open and make the net payoff of \$0.

Completing the option valuation to time = 0 using the approach outlined above:

$$f_u = max\big([q_u f_{u^2} + q_d f_{ud}] \times e^{(-r_f \;\; t)}, S_0 u - I, 0\big) = \$418m$$
$$f_d = max\big([q_u f_{ud} + q_d f_{d^2}] \times e^{(-r_f \Delta t)}, S_0 d - I, 0\big) = \$75m$$

The value of the option at $t = 0$:

$$f_0 = max\big([0.485(\$418m) + 0.515(\$75m)] \times e^{(-0.05\times 1)}, 600 - 500, 0\big),$$
$$f_0 = \$229m$$

Figure 6.8 shows the option values at each node of the binomial tree by backward induction. Thus, each node represents the value maximisation of investing at that point or waiting until the next period.

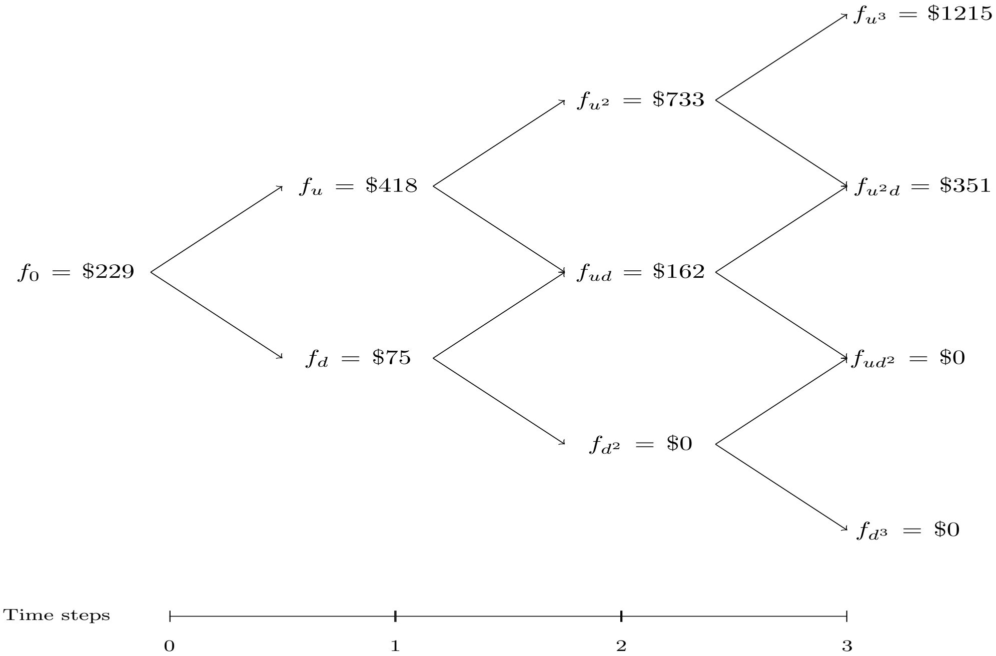


Figure 6.8: Option values for the dependent option of parallel compound option, which are the same as the asset values for the independent option

The option value for the independent option, buying the license using the dependent option, the upgrade, as the underlying asset's values. Figure 6.8 show also the underlying asset values, which are the same as the option values from the upgrade option.

Start by calculating the option value for the independent option at each tree node by backward induction. Each node represents the value maximisation of investing versus continuation, where the firm can invest \$100m in the project or keep the option open. At terminal nodes, the firm would invest if the payoff is greater than the investment of \$100m; otherwise, they would let the option expire.

Let us start with the terminal nodes. Since the firm wants to maximise its return, we find:

$$
\begin{aligned}
f_{u^3} &= max(S_0u^3 - I_2, 0) = \$1115m \\
f_{u^2d} &= max(S_0u^2d - I_2, 0) = \$351m \\
f_{ud^2} &= max(S_0ud^2 - I_2, 0) = \$0 \\
f_{d^3} &= max(S_0d^3 - I_2, 0) = \$0
\end{aligned}
$$

At node S_0u^3, the expected asset value is \$1215m, compared to the investment cost I_2 of \$100m they would continue rather than defer the project. When the option value is > 0, the firm would invest in the project and realise a net payoff of \$1115m, making the option value at this node \$1115m. At node S_0d^3, the expected asset value is \$0; therefore, no investment would be made, resulting in \$0 at that node.

The firm can either invest \$100m in the project at each intermediate node or keep the option open. Starting at the top, at node S_0u^2 calculate the expected asset value for keeping the option open. That is the discounted, at the risk-free rate, a weighted average of potential future option values using the risk-neutral probability as weight. Thus, the value at that node is:

$$
\begin{aligned}
f_{u^2} &= max\big([q_uf_{u^3} + q_df_{u^2d}] \times e^{(-r_f\Delta t)}, S_0u^2 - I, 0\big) = \$638m \\
f_{ud} &= max\big([q_uf_{u^2d} + q_df_{ud^2}] \times e^{(-r_f\Delta t)}, S_0ud - I, 0\big) = \$116m \\
f_{d^2} &= max\big([q_uf_{ud^2} + q_df_{d^3}] \times e^{(-r_f\Delta t)}, S_0d^2 - I, 0\big) = \$0
\end{aligned}
$$

At node S_0u^2, the EV is \$733m. Exercising the option at this node will give an option value of \$633m (\$733 - \$100). Since the value of keeping the option open is \$638m and is larger, the firm would keep the option open and continue; therefore, the option value at S_0u^2 is \$638m. At nodes S_0ud and S_0d^2, the decision is to keep the options open and make the net payoffs of \$116 and \$0.

Completing the option valuation to time = 0 using the approach outlined above:

$$
\begin{aligned}
f_u &= max\big([q_uf_{u^2} + q_df_{ud}] \times e^{(-r_f\ t)}, S_0u - I, 0\big) = \$315m \\
f_d &= max\big([q_uf_{ud} + q_df_d] \times e^{(-r_f\Delta t)}, S_0d - I, 0\big) = \$54m
\end{aligned}
$$

The value of the option at $t = 0$:

$$
\begin{aligned}
&f_0 = max\big([0.485(\$315m) + 0.515(\$54m)] \times e^{(-0.05\times1)}, 229 - 100, 0\big), \\
&f_0 = \$188m
\end{aligned}
$$

Figure 6.9 shows the option values at each node of the binomial tree by backward induction. Thus, each node represents the value maximisation of investing at that point or continue.

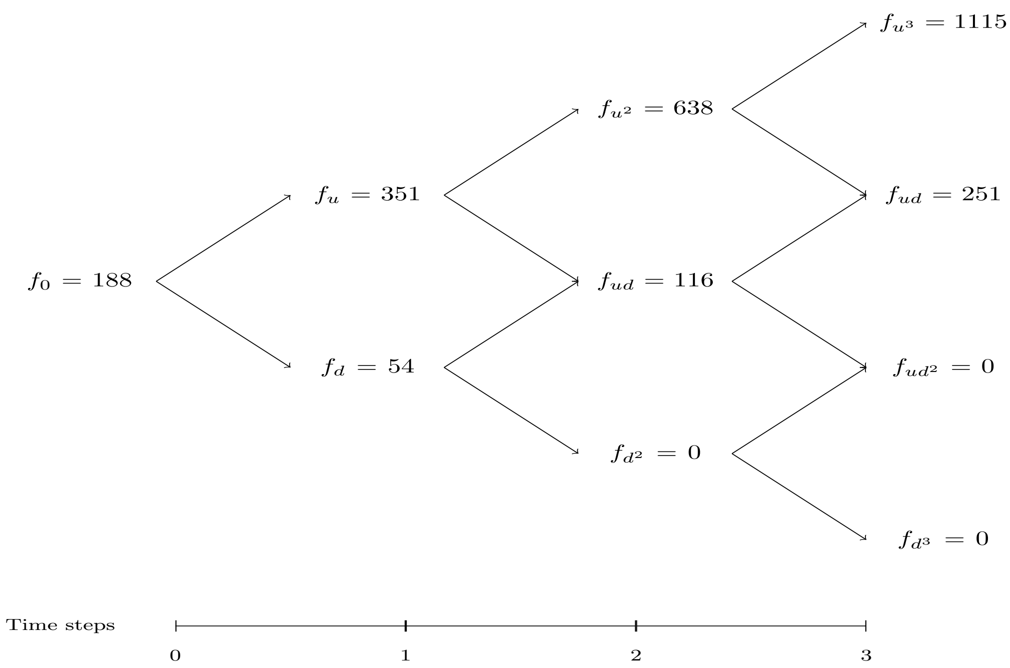


Figure 6.9: Option values for the independent option of parallel compound option

The NPV with no flexibility for this investment is calculated to be \$0, assuming the total investment is \$600m (\$500 + \$100). Therefore, this project would not be considered for investment because of this NPV. However, the RO shows an NPV of \$188m due to the parallel compound option. Therefore, this flexibility embedded in this project makes it a more attractive investment.

The minor differences between the parallel and sequential options are that both options are simultaneously alive in the parallel option. One must be exercised before the exercise of the other is complete, thereby creating a parallel compound option. The infrastructure construction can be started at any time, but the license must be obtained before the upgrade can be tested and the services launched. That creates a parallel option. The option solution method is essentially the same for sequential and parallel options, yielding the same RO value. The difference lies in framing the option[4].

6.5.7 Rainbow Option

A *rainbow option* is an option that is driven by multiple sources of uncertainty regarding the price of a unit of output, the quantity that might be sold, and by uncertain interest rates that affect the PV of the project. The uncertainty may be related to the input parameters used in RO valuation or the individual components that make up an input parameter. In addition, there may be changes in the uncertainty itself over the option lifetime. The options solutions method is the same as for a single volatility factor, except it involves a quadrinomial tree instead of a binomial. Thus, the asset can take one of four values moving from one node to the nodes of the next step in the tree. The BSM model cannot accommodate the multiple sources of uncertainty and is not helpful for rainbow options [7][4]. The quadrinomial tree approach will be discussed in more detail in Section 6.7.1. A phased investment decision often has economic and technological uncertainty, and then we have a compound rainbow option. Those are the most realistic RO and perhaps the most complex to show how to value. Technological uncertainty is significant at the start of the project but decreases as the investment is made. Economic uncertainty grows more diffuse through time and increases as the investment is made [7]. An example of a rainbow option will be covered in Chapter 7.

6.6 The Four-Step Process

Copland and Antikarov [7] use this method almost all the time for their client application. This method breaks down the calculations for RO valuation into four steps [7]. Thus, it keeps track of calculations and helps to break down the calculations into smaller units, not forgetting to consider anything in the calculations. Figure 6.10 shows the four-step process.

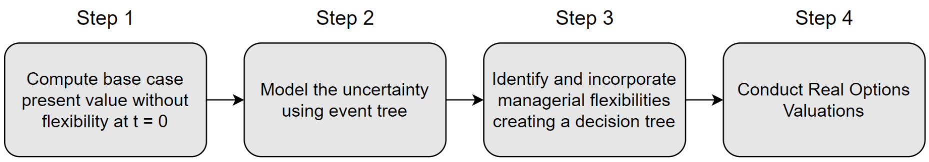


Figure 6.10: A four-step process

Step 1: The objective is to determine a base case with standard DCF analysis using traditional techniques where cash flow is forecasted over the project's lifetime. Later on, it is checked to see that the option-pricing solution assumes no flexibility equals the DCF results.
Step 2: The objective is to understand how the PV develops over time. It is to build an event tree based on the set of combined uncertainties that drive the volatility of the project. An event tree does not have any decisions built into it. Instead, it should model the uncertainty that drives the value of the underlying risky asset through time. Event trees explicitly map out the cash flow and use objective probabilities and the cost of capital to calculate the project value without flexibility. Thus, this value should equal the value from Step 1. Estimate volatility will be discussed in more detail in Section (6.7).
Step 3: The objective is to analyse the event tree to identify and incorporate management flexibility to respond to new information. By putting decisions that the management may make, into the nodes of the event tree, we can turn it into a decision tree. The event tree models the set of values that the underlying risky asset may take through time. The decision tree shows the payout of optimal decisions, depending on the state of nature. Therefore, its payoffs are those that result from the option or options we are trying to evaluate.
Step 4: The objective is to value the total project. The valuation of the payoffs in the decision tree uses either replicating portfolios or risk-neutral probabilities. The RO includes the NPV base case and the option value.

6.7 Estimating Volatility

Although a single volatility factor is used in most option models, most real projects have more than one uncertainty. Therefore, it is crucial to recognise that uncertainties are related to several variables, such as unit price and quantity expected to be sold, controlling the project cash flow. Furthermore, the volatility factor can be different for the revenue versus the cost component of the PV representing the underlying asset value [4].

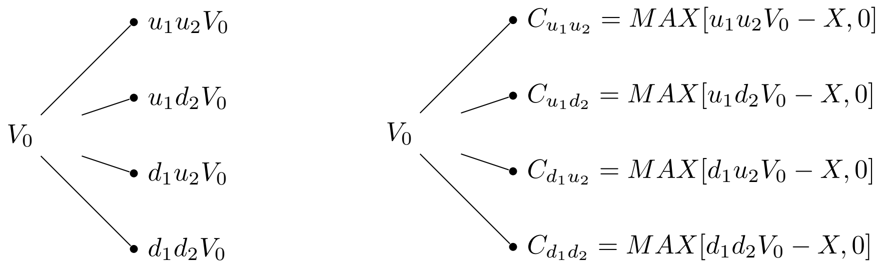
Two choices in addressing different sources of volatility are the consolidated approach and the separated approach. The consolidated approach combines uncertainties (e.g. the price, quantity, and variable cost) into one estimated volatility output. The data we obtain on the variability of drivers of project uncertainty, comes from historical data and assumes that the future is like the past. Subjective data that is estimated by the management can also be used. After estimating the multiple uncertainties that drive the value of a project with a consolidated approach, they are combined via a Monte Carlo analysis into a single uncertainty. That single estimate of volatility is all that is needed to build a binomial tree. The details of the consolidated approach for estimating the volatility of the value of the project and Monte Carlo simulation are described in the book "Real Options, a practitioner's guide"[7].

In some cases, however, it will not be helpful to combine uncertainties. For example, when decisions must be tied to a particular uncertainty, combining them into the value of the project will not help. Instead, the separated approach can be used, where two or more sources of uncertainty must be estimated separately. For many projects, the most significant uncertainties are technology, regulation changes, and competitors' moves. Most of those uncertainties are independent of each other, evolve differently over time and impact the asset value in different directions. Therefore, do not get resolved smoothly over time, as in the consolidating approach. Instead, they are resolved when the information becomes available [7][4]. An example is the uncertainty of an Federal Drug Administration (FDA) approval of a new drug. When the result is announced, the value of the project moves dramatically up if the drug was approved or to zero if approval was denied. As a result, we cannot simply estimate the volatility of the project and use it in a standard binomial model to generate the event tree. Instead, we need to build an event tree that reflects the actual uncertainty over the time of the project to get optimal execution of the RO and correct valuation. It is almost impossible to draw an event tree with more than two uncertainties. Not much has been written about this, but Tom Copland and Vladimir Antikarov go into it in their book "Real Options a practitioner's guide" [7]. They point out that the volatility of a project is not the same as the volatility of any input variables (e.g. the price or the quantity of the product), nor is it equal to the volatility of the company's equity [7][4]. The way to keep the significant uncertainties separate is to use an options framework known as rainbow options (defined in the Section 6.5.7) because they explicitly allow for project valuation where multiple sources of uncertainty exist. An example of the rainbow option is calculated in the Chapter 7.The way to do this is to keep the significant uncertainties separate and explicitly model their interaction and effect on the project's value by performing a quadranomial tree. Quadranomial approach can be done if the uncertainties are uncorrelated or correlated. In this thesis we introduce quadranomial approach with uncorrelated uncertainties (see Section 6.7.1). However Tom Copland and Vladimir Antikarov go into an approach with correlated uncertainties in their book "Real Options a practitioner's guide" [7].

6.7.1 Quadranomial Approach without Correlated Uncertainties

The quadranomial approach is a two-variable binomial tree. It allows the uncertainties to be resolved simultaneously. For example, consider a rainbow option with payoffs based on the value of an asset driven by two sources of uncertainty. Each uncertainty is assumed to follow a Gauss-Wiener process. The quadranomial event tree has four branches at every node and generalises the binomial event tree with two branches. To develop the tree, we need to have estimates of annual standard deviations of the changes in the asset value when driven by each uncertainty, σ_1 and σ_2. This information is equivalent to knowing the joint distribution of up and down movements generated by the two uncertainties. We also need to know the exercise prices. These might be expenditures on phases of experimentation or market development [7].

Figure 6.11 illustrates the four outcomes that are possible at the end of one period, assuming that the starting value for the risky asset is V_0 and that it is multiplicative. The up and down movements are u_1 and d_1 when driven by the first source of uncertainty, and u_2 and d_2 when driven by the second source of uncertainty [7].



(a) Value of the asset is affected by two sources of uncertainty

(b) The value of flexibility, C_0

Figure 6.11: Quadranomial values of the underlying and a call option after one period

We cannot use the replicating portfolio approach to solve the quadranomial. Because we end up with the following four equations, but only two unknowns, m and B.

$$mu_1u_2V_0 + (1 + r_f)B = C_{u_1u_2}$$
$$mu_1d_2V_0 + (1 + r_f)B = C_{u_1d_2}$$
$$md_1u_2V_0 + (1 + r_f)B = C_{d_1u_2}$$
$$md_1d_2V_0 + (1 + r_f)B = C_{d_1d_2}$$

Therefore, we proceed by solving for the risk-neutral probabilities for each branch of the quadranomial and use them in the following valuation formula:

$$C_0 = \frac{p_{u_1u_2}C_{u_1u_2} + p_{u_1d_2}C_{u_1d_2} + p_{d_1u_2}C_{d_1u_2} + p_{d_1d_2}C_{d_1d_2}}{(1 + r_f)} \tag{6.6}$$

If the two uncertainties are independent of each other, then the risk-neutral probability of each branch, the quadranomial, is equal to the product of the risk-neutral probabilities for that branch, based on each separate source of uncertainty.

That is because of the subject of unconditional versus conditional probabilities. There are two uncertainties X and Y. Next period with probabilities, p_{uX} and (1-p_{uX}) respectively, X can only take one of two values:

$$X_t \varepsilon [X_u, X_d]$$

Similarly, with probabilities p_{uY} and (1-p_{uY}) respectively, Y can only take one of two values:

$$Y_t \varepsilon [Y_u, Y_d]$$

If the two uncertainties are independent, the probability of Y going up does not change if we know X has moved up. In this case, the conditional probabilities for X and Y are equal to their respective unconditional probabilities:

$$p(Y_u \mid X_u) = p_{uY}$$
$$p(X_u \mid Y_u) = p_{uX}$$

Given independence, we can find the probabilities for each of the four possible combinations $[X_u, X_d]$ and $[Y_u, Y_d]$ by simply multiplying the probabilities.

This fact produces four equations:

$$\begin{aligned} p_{u_1u_2} &= p_{u_1}p_{u_2} \\ p_{u_1d_2} &= p_{u_1}p_{d_2} \\ p_{d_1u_2} &= p_{d_1}p_{u_2} \\ p_{d_1d_2} &= p_{d_1}p_{d_2} \end{aligned} \tag{6.7}$$

The quadranomial tree is appropriate when the sources of uncertainty are not related to each other. In real-world situations, the different sources of risk, especially market risk, may not be completely independent. However, as long as the correlation is insignificant, the quadrinomial method is expected to provide a good approximation of the actual option value. Specific risk related to the technical effectiveness of a project, on the other hand, is independent of the market risk and can be accounted for in the options valuation. Therefore, the DTA addresses the specific risk. A binomial or quadrinomial approach is used to account for the market risk, depending on whether it is a simple or a rainbow option [4].

Chapter 7

Case Study

In this thesis the purpose of the case study is to illustrate how the already mentioned quadranomial approach can be used to solve a RO problem. These problems can be found in many industries but in this case we will assume the pharmaceutical industry. It should be mentioned that the figures that are used in the calculations are hypothetical, however, in real life these need to be estimated with great accuracy.

7.1 Pharmaceutical Industry

The success of pharmaceutical firms is highly dependent on research and development (R&D). Firms currently perform a DCF method or the more advanced DTA to identify value-increasing projects. These methods, however, cannot properly capture the value of the flexibility to readjust plans of biotechnology projects. For example, many potential drugs are identified as failures during the R&D stages. Stopping further developments might save an essential part of R&D investment expenditures while adding a significant value to a project's NPV. Moreover, traditional valuation tools give no insight into how these future decision contingencies affect the risk of a project during its lifetime. The RO approach may solve these problems and provide a richer investment decision framework [21].

7.2 Assumptions

PharmIce is putting a new drug on the market, but it has to go through testing. The project will have three testing phases, and all phases need approval by the FDA. If the drug has received the required approvals it either goes straight to the market or it will take an additional year to perform an indications test phase, a comparative test of the new drug versus other emerging drugs. If the indications test is successful, it will increase the sale of the new drug.

Assumptions about the probability of success is estimated 60% for phase 1, and the investment cost is \$10m. In Phase 2, the probability of success is 50%, and the investment cost is \$15m. In Phase 3, the probability of success is 70%, and the investment cost is \$35m. The investment cost to go straight to the market is assumed to be a precommitment and is \$98m. For the indications testing, the probability for success is 50%, and the investment cost is \$13m (see Table 7.1).

Dimasi [22] shows that more than half of clinical research failures occur in phase 2. Therefore, in this thesis the probability of success was assumed to be the lowest in that particular phase. Borissiouk and Pell [21] indicate that one of five drugs that entered phase 1 would reach the market with a cumulative probability for the technical phases of 20%. Hence, it was assumed that cumulative probability of the technical phases in this thesis would be the same.

Table 7.1: Assumptions for PharmaIce

	Prob. of Success	Invest. cost (millions)
Phase 1	60%	10
Phase 2	50%	15
Phase 3	70%	35
Straight to market		98
Indication test phase		13
Success	50%	
Fail	50%	

In the test phases, there is a technological uncertainty, and in addition to that, PharmIce faces product/marketing uncertainty. The firm estimates that the drug could sell for $30 per unit today with a shift of 22% a year; also, the price is expected to grow at an annual rate of 11%. Although the quantity of sales is also uncertain, the estimated annual sales growth is contingent on whether they go straight to the market or conduct further indications testing. If they go straight to the market, the expected sale is 2m units per year. If the drug goes through indications testing with success,it is estimated that 2.4m units will be sold per year. However, only 1.6m units per year will be sold if the drug fails the indications testing (see Table 7.2). Furthermore, it was estimated that once the drug goes to the market, its terminal value will be four times its cash flow.

Table 7.2: Assumptions for Product/Market uncertainty

Product/Market uncertainty		
Estimate price for drug	30	per unit
Standard deviation	22%	annually
Growth	11%	annually
Quantity Straight to market	2	million unit annually
Conduct indication testing		
success	2.4	million unit per year
fail	1.6	million unit per year

The estimation of free cash flow was defined as the total revenue minus total cost, where the variable cost was estimated $4 per unit, minus the fixed cost of $2m (see Equation (7.1)). Furthermore, we are assuming a WACC of 10% and a risk-free rate (r_f) of 8% (see Table 7.3).

$$CF = [(P - VC)Q - F] \tag{7.1}$$

where:

- CF is the cash inflow.
- P is the investment outlay.

- VC is the variable cost.
- Q_i is the quantity of unit per year.
- F is the fixed cost.

Table 7.3: Inputs for Cash Flow

Free cash flow	
Variable cost per unit	4
Fixed cost (in millions)	2
WACC	10%
Risk free rate	8%

7.3 Calculations

This project is calculated using the quadranomial approach because we want to keep the uncertainties separate where they do not resolve smoothly over time. Instead, they are resolved when the information becomes available. We use the four-step process shown in Figure 7.1, where we start modelling the uncertainties but do not compute the base case like mentioned in Section 6.6. To keep the uncertainties separate, we need to build an event tree that reflects the substantial uncertainty over time, as mentioned in Section 6.7. Then calculate the NPV with no flexibility by using risk-neutral probabilities and the risk-free rate to discount the end of branch cash flow backwards in time. The third step adds flexibility by adding decisions to the tree, creating a decision tree, and valuing the investment by inserting the option theory using the RO valuation.

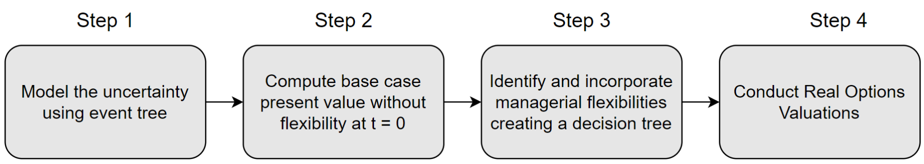


Figure 7.1: A Four-step process

7.3.1 Modelling the Uncertainties

We start by modelling the uncertainties by using event trees. Technological and product/market uncertainties evolve simultaneously through time. Technological uncertainty is assumed to be independent of the economy and is illustrated in Figure 7.2.

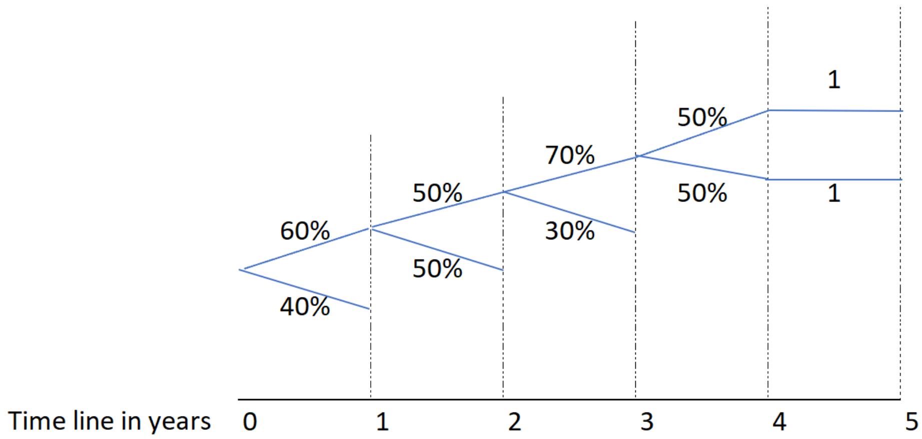


Figure 7.2: Technological uncertainty

The methodology that is used to model the price uncertainty is a random walk. It starts with price today of P_0 = \$30 per unit and up and down movements are u = 1.246, d = 0.803 that are calculated with Equations (5.6) and (5.7) with the volatility of σ = 22%. It takes four time periods to go straight to the market but five time periods to do the indications testing. Figure 7.3 shows the evolution of price uncertainty through time. It is assumed that the project pays no dividends.

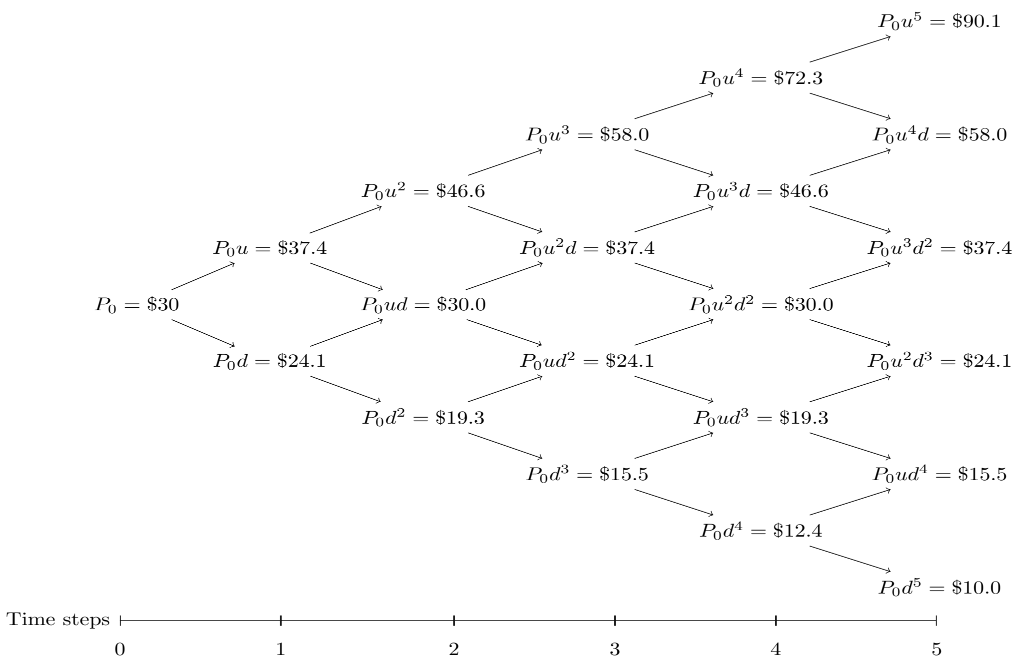


Figure 7.3: Evolution of price

The third source of uncertainty (see Figure 7.4) is the number of drug doses that might be sold each year. The unconditional expectation is 2m units if the decision is to go straight to the market, after phase 3. Alternatively, the decision could be to spend an additional \$10m for indications testing. In that case we would forego any sales for an additional one year to learn whether the market for the product will be 2.4m units or 1.6m units.

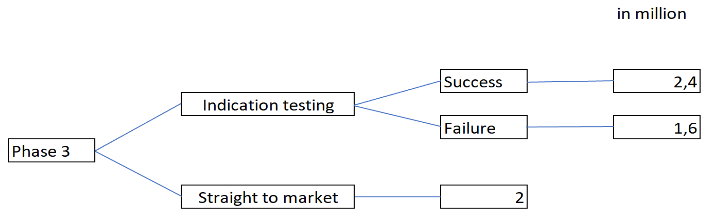


Figure 7.4: Number of doses uncertainty

The final task in modelling the uncertainties is to use a quadranomial tree to combine price and quantity uncertainties into a single product/market uncertainty. Figure 7.5 shows the quadranomial event tree of cash flow. The cash flow is calculated with Equation (7.1) for each node to combine price and product uncertainties. The drug price in period four and five are the prices we work with to find the cash flow. This is because we are interested in what the cash flow will be when we go straight to the market or into indications testing.

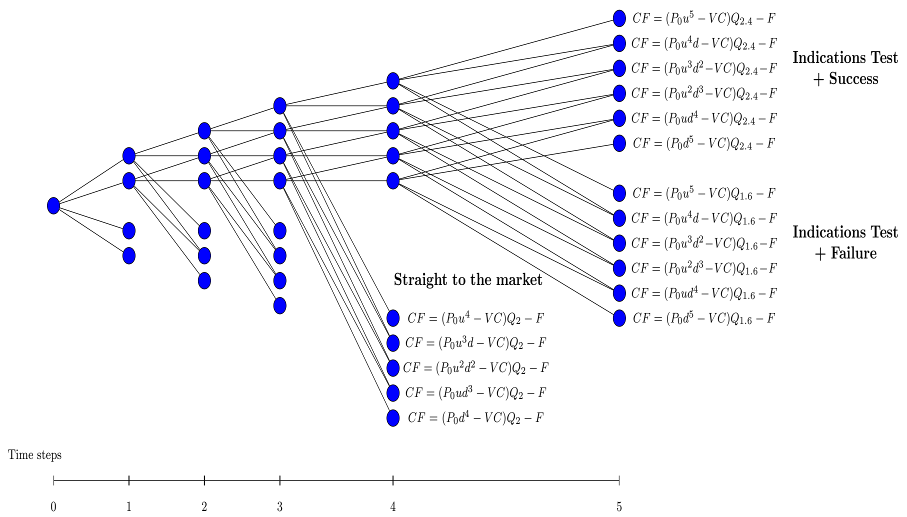


Figure 7.5: Quadranomial event tree for cash flow

Let us calculate one node for the cash flow to go straight to the market with Equation (7.1). The price (P) of the drug to go straight to the market can be seen in Figure 7.3. The price for period four is P_0u^4 = \$72.3. As can be seen in Figure 7.4 the quantity (Q) of units per year that is needed to go straight to the market is 2m units. As already mentioned, the variable cost (VC) is \$4 per unit, and the fixed cost (F) is \$2m (see Table 7.3). When we put all these information together, we get:

$$CF = [(72.3 - 4)2 - 2]$$
$$CF = \$134.7\text{m}$$

The same calculations are used to calculate other nodes in the case of going straight to the market. For the nodes to do the indications testing, the same calculations are used, except we use the price for period five (see Figure 7.3).

Figure 7.6 shows the event tree of terminal value, conditional on the market price per dose and the number of doses in each state of nature. The terminal value is found by multiplying the cash flow from Figure 7.5 by four at each node.

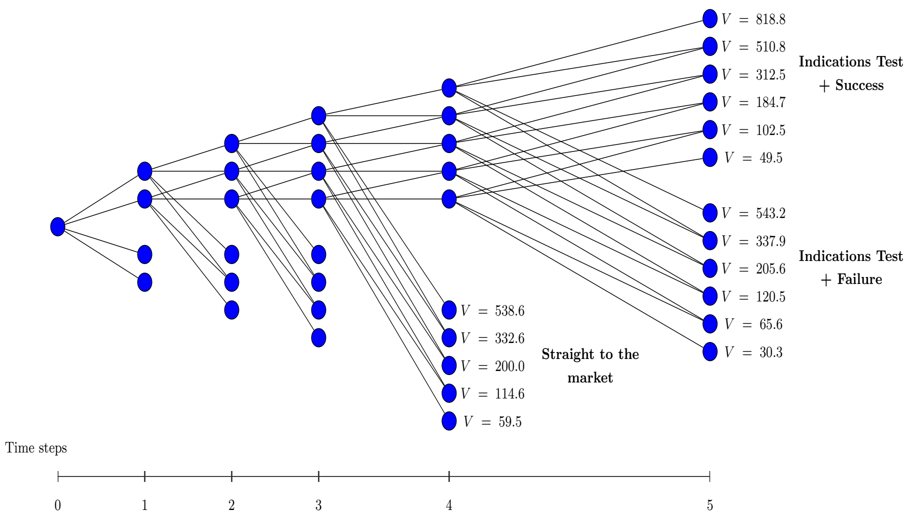


Figure 7.6: Quadranomial event tree for terminal value

7.3.2 Compute Base Case

The DCF method is used to find the expected cash flow of the investment over the time of the project by discounting cash flow to today with the information known today. Since the DCF method is constrained to pre-committing today to take a decision, it treats whether to go straight to the market or conduct indications testing as mutually exclusive alternatives at the beginning of the project and chooses the one with a higher NPV.

7.3.2.1 Indications Testing

Start with an indications testing alternative by calculating the PV of the expected outlay. The outlay is discounted at the risk-free rate because the outlay is the exercise price of a call option whose value depends on technological uncertainty independent of the market economy (see Table 7.4). It is also assumed that the expenditure of $98m to go to the market is precommitment.

Table 7.4: PV calculations for investment outlay for indication testing

Indication testing

Year	0	1	2	3	4
Investments	10	15	35	13	98
Discount Factor	1,00	0,93	0,86	0,79	0,74
PV	10.0	13.9	30.0	10.3	72.0
PV(Investment)	136.25				

The PV of the expected cash flow for the indications testing alternative is obtained by multiplying the certainty equivalent cash flow by the risk-neutral probabilities at each node, and discounting to the present. The risk-free rate is used to discount the cash flow since the technological risk is independent of the market risk. That makes the beta in the CAPM zero, and the discount factor equals the risk-free rate.

To calculate the risk-neutral probability, we need to calculate the objective probabilities of price uncertainty. Only price uncertainty is market-related. In Figure 7.3, we can see how the prices evolve. We know from the previous calculations that $u = 1.246$ and $d = 0.803$. The price growth is expected to be 11% annually. Therefore, the objective probability of an upward movement is determined from today's price multiplied with the growth rate which must equal the expected price. The corresponding equation is:

$$\begin{aligned} P_0(1+g) &= puP_0 + (1-p)dP_0 \\ (1+g) &= pu + d - pd \\ p &= \frac{1+g-d}{u-d} = \frac{1.11-0.803}{1.246-0.803} \\ p &= 0.69 \end{aligned}$$

Next we calculate the risk-neutral probabilities; the price will grow at the risk-free rate so that we have:

$$p' = \frac{e^{r_f} - d}{u-d} = \frac{e^{0.08} - 0.803}{1.246 - 0.803}$$
$$p' = 0.63$$

Risk-neutral probabilities are used to calculate the value at each node, working backwards in the tree (see Figure 7.7). For example, at node **M** we have:

$$V_M = \frac{p'818.8 + (1-p')510.8}{1+r_f}$$

$$V_M = 653.5$$

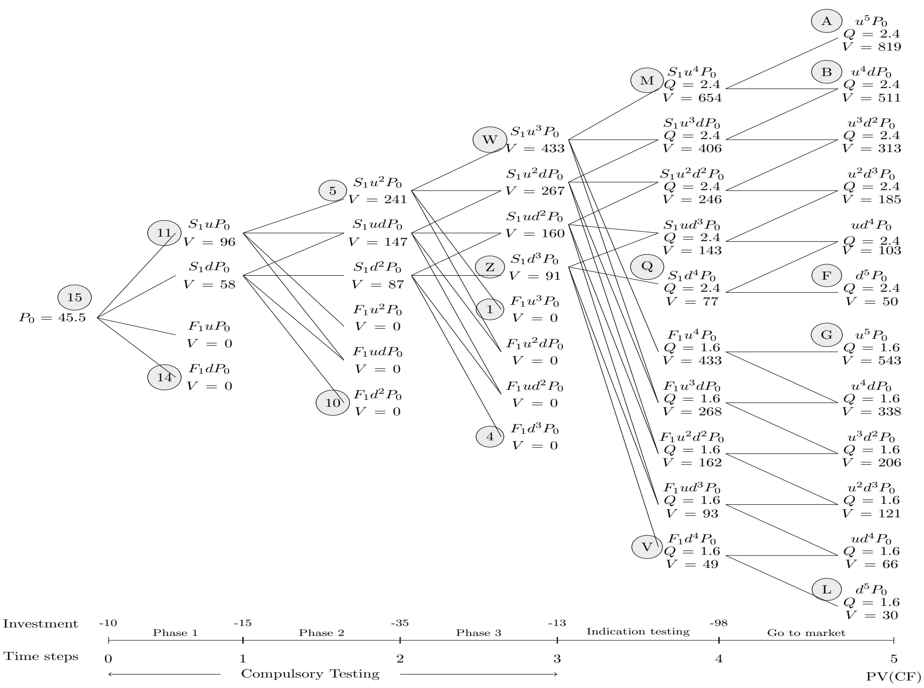


Figure 7.7: PV calculations with indications testing

Note that year five and beyond have only product/market uncertainty because all technological uncertainty was resolved by the end of indications testing in year four. Therefore, there were only two branches from each node in period four.

When calculating the value at node **W**, there are two sources of uncertainty independent of each other and four branches, a quadranomial tree. Risk-neutral probabilities need to be estimated to determine the value at node **W**, one for each branch. Price and quantity uncertainty are independent; we can use Equations (6.7) to calculate the risk-neutral probabilities, as shown in Table 7.5.

Table 7.5: Quadranomial risk-neutral probabilities for Indication testing

			Price	
			prob up 63,3%	prob down 36,7%
Quantity	Prob up	50%	31,6%	18,4%
	prob down	50%	31,6%	18,4%

The value at node **W** is calculated by multiplying the payoffs by the risk-neutral probabilities and discounting the result at the risk-free rate:

$$V_W = \frac{0.316(653.5) + 0.184(405.6) + 0.316(433.2) + 0.184(267.9)}{1.08} = \frac{432.7}{1.08}$$

$$V_W = \$432.9$$

Similar calculations are used to calculate values back through the tree to the root node, where the PV of this project is $45.5m (see Figure 7.7. At each testing phase the risk-neutral probabilities are different from the objective probabilities. In Table 7.6, we can see the independent uncertainties calculated for each step in the tree.

Table 7.6: Risk-neutral probabilities for each step in the quadranomial tree

Objective probabilities			Price		
			Prob. Up 63,3%	Prob. Down 36,7%	
Quantity	Prob. Up	50,0%	31,6%	18,4%	Indication testing
	Prob. Down	50,0%	31,6%	18,4%	
Technology	Prob. Up	70,0%	44,3%	25,7%	Phase 3
	Prob. Down	30,0%	19,0%	11,0%	
Technology	Prob. Up	50,0%	31,6%	18,4%	Phase 2
	Prob. Down	50,0%	31,6%	18,4%	
Technology	Prob. Up	60,0%	38,0%	22,0%	Phase 1
	Prob. Down	40,0%	25,3%	14,7%	

From the above calculations we can now calculate the NPV of the indications testing alternative as follows:

$$NPV_{(IT)} = 45.5 - 136.3 = -\$90.8$$

7.3.2.2 Straight To the Market

The other mutually exclusive alternative is an event tree without indications testing where we pre-commit to going straight to the market. The process for calculating the project NPV is the same as before. In Table 7.7, we can see the PV calculations of the expected outlay to go straight to the market. In Figure 7.8, we can see the PV of going straight to market.

Table 7.7: PV calculations for investment outlay to go straight to market

Straight to the market

Year	0	1	2	3
Investments	10	15	35	98
DF	1,00	0,93	0,86	0,79
PV	10.0	13.9	30.0	77.8
PV(Investment)	131.69			

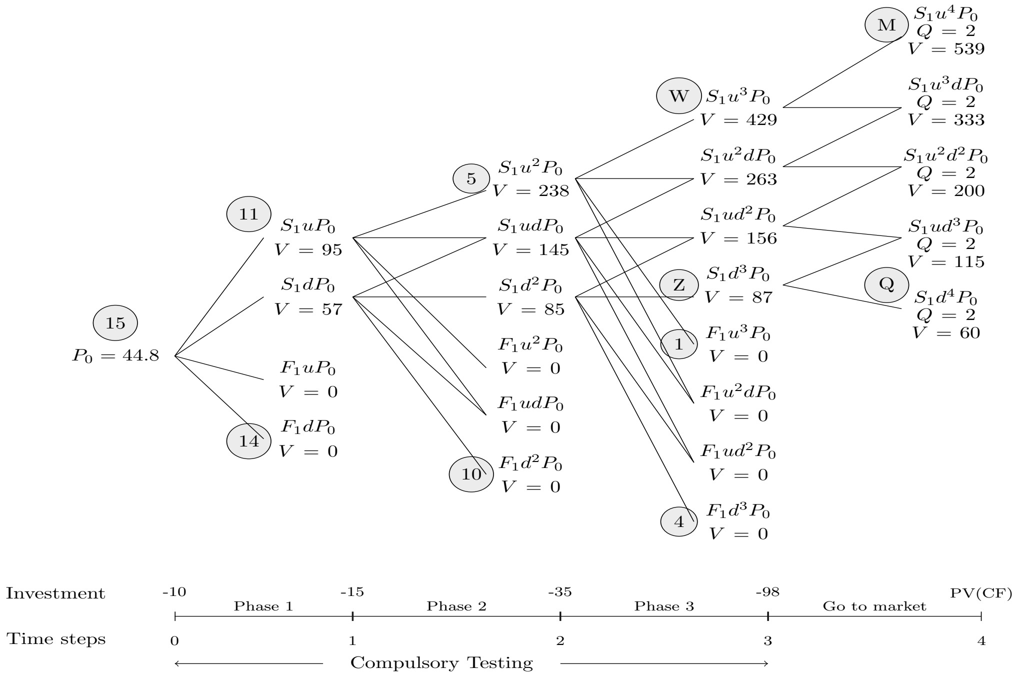


Figure 7.8: PV calculations with no indications testing

From the above calculations we can now calculate the NPV of the alternative of going straight to the market as follows:

$$NPV_{(SM)} = 44.8 - 131.7 = -\$86.9$$

By comparing the two NPV's we can see that the loss in wealth is slightly less of going straight to the market. Therefore, going straight to the market would be a better mutually exclusive alternative. However, because the NPV's are both negative we would reject both alternatives in this case.

7.3.3 Creating a Decision Tree

The third step is to analyse the event tree to identify and incorporate managerial flexibility. Incorporating flexibility transforms event trees into decision trees. In Figure 7.9, we can see the decision tree that captures the relevant decisions up to the product launch. The objective probabilities of success or failure of each phase are given in the figure.

At the end of the first two test phases, the decision is either to go ahead by making the required investment or to abandon. At the end of the third phase, however, we have three decisions to consider. Either we spend \$13m to do indications testing, spend \$98m to go straight to the market, or abandon.

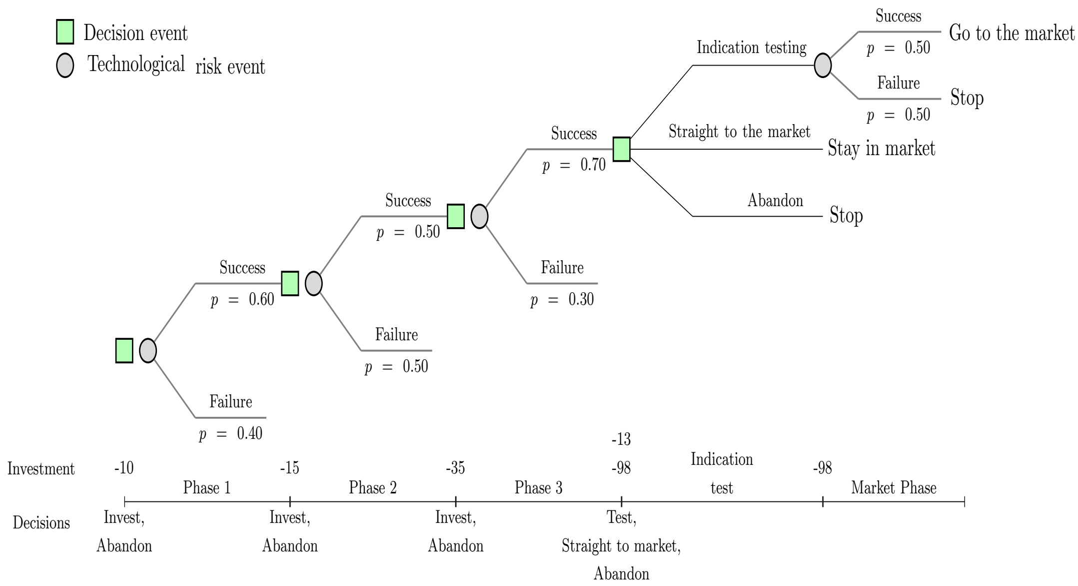


Figure 7.9: Decision Tree for PharmaX

7.3.4 Conduct Real Option Valuation

Next, we turn to RO to estimate the value of the project with flexibility. In this case, we use the option to abandon and the compound option to invest. Figure 7.10 shows the decision tree with two sources of uncertainty, which would require a quadranomial solution, for the RO calculations and the optimal decisions. At the end of each node in the first compulsory test phase, the decision is either to exercise the option to make the required investment or to abandon. At the end of the third phase, however, we have three options to consider. Either we spend \$13m to do indications testing, spend \$98m to go straight to the market, or abandon.

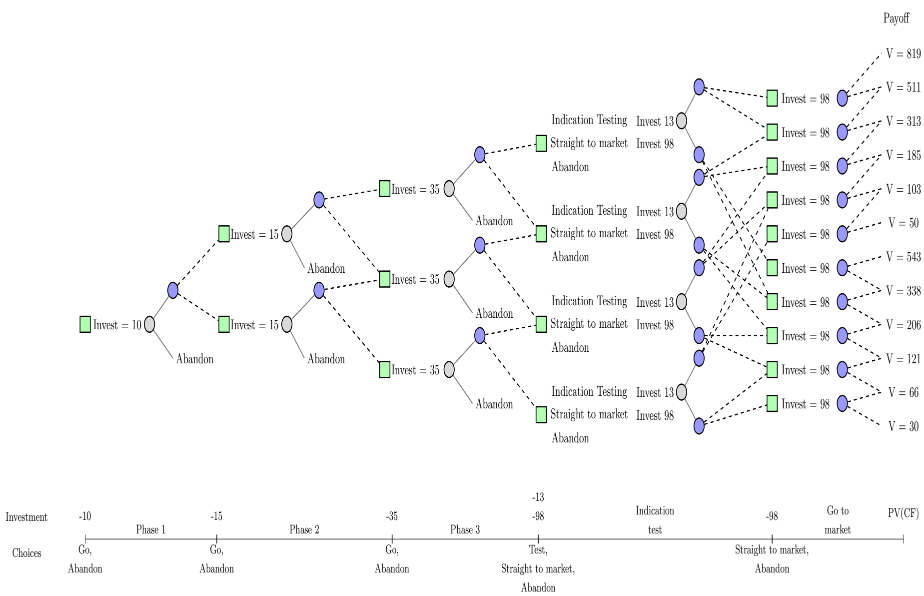


Figure 7.10: Decision tree for PharmaIce with technological and market uncertainty

In year 5, we have terminal values representing the value maximisation of exercising the option by investing in the project versus letting the option expire. Lets assume that we decide to invest in the project at this point in time and that the indications testing will be successful. In that case the price will either go up or down as shown in Figure 7.11. It can be seen that if the price goes up the payoff will be \$818.8m (see node **A**) but if the price goes down the payoff will be \$510.8m (see node **B**).

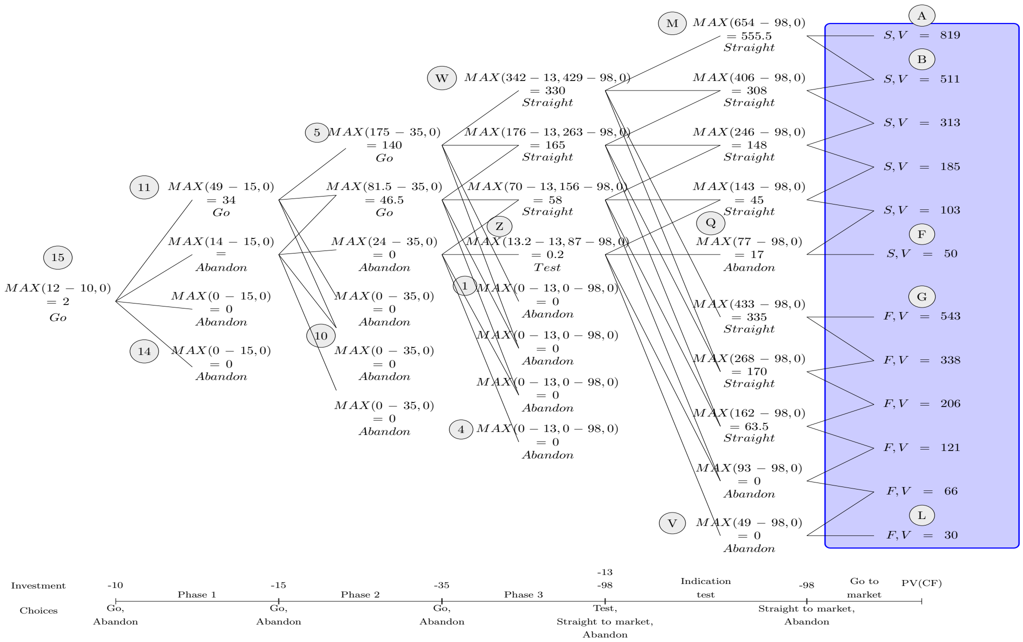


Figure 7.11: Terminal nodes

In year 4, see Figure 7.12, only the market uncertainty exists because the technical uncertainty was already cleared in year 3. Therefore, only the risk-neutral probability is used in calculating the EV of keeping the option open and accounting for the optimal downstream decision. In year 4, node **M**, the expected asset value for keeping the option open, considering the optimal downstream decision, is \$653.5m. The decisions are to go straight to the market or abandon. For example, if the option is exercised by investing \$98m, the expected asset value would be:

$$V_M = MAX(653.5 - 98.0) = \$555.5$$

Therefore, the decision is to go straight to market.

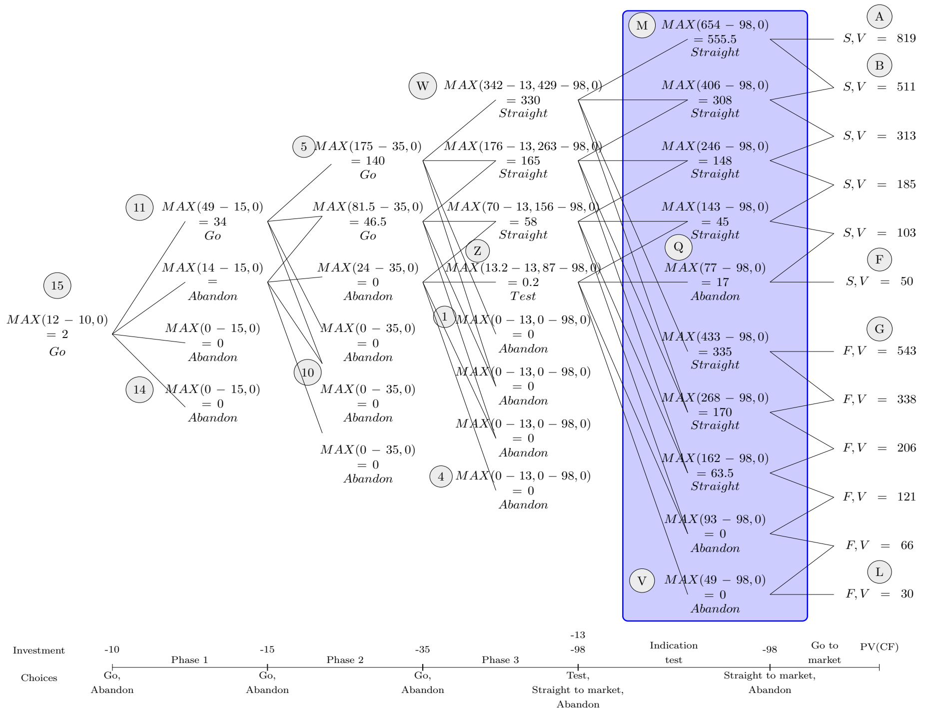


Figure 7.12: Step 4

At the end of the third phase (see Figure 7.13), we have three options to consider. Either we spend \$13m to do indications testing, spend \$98m to go straight to the market, or abandon.

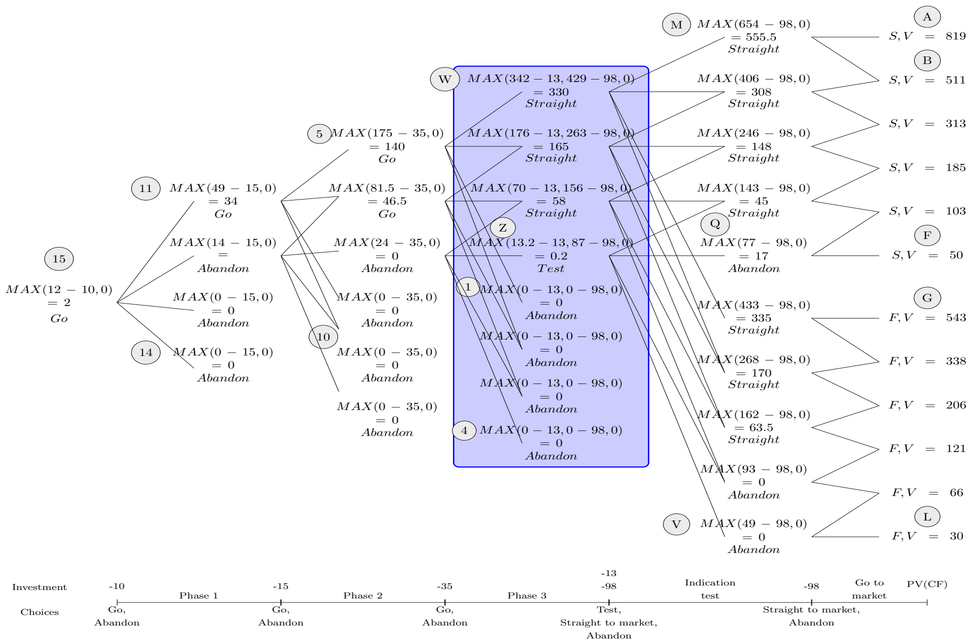


Figure 7.13: Step 3

To illustrate, let us go through the calculations at node **W**. If we decide to do indications testing, the result could be successful or a failure with either a high or a low price.

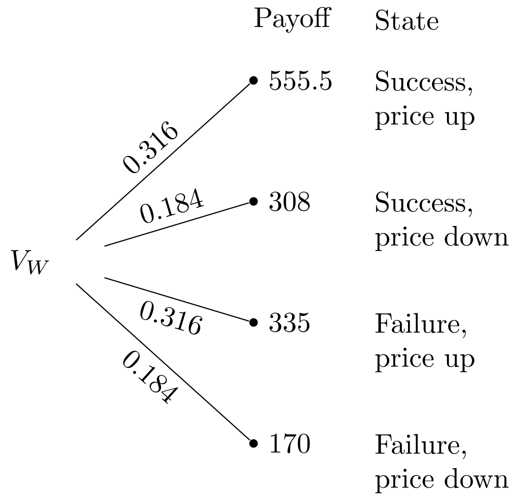


Figure 7.14: Value of testing

Figure 7.14 shows the quadranomial risk-neutral probabilities and the payoff for each state of nature. Then we calculate the value:

$$Test\ V_W = \frac{0.316(555.5) + 0.184(308) + 0.316(335) + 0.184(170)}{1.05} = \frac{369.35}{1.08} = \$342$$

The other investment opportunity at node **W** is to go straight to the market, which costs \$98m, and has the following PV:

$$Straight\ to\ market\ V_W = \frac{0.63(539) + 0.367(333)}{1.08} = \$429$$

Since the NPV of going straight to the market is greater than the NPV of indications testing, we have:

$$MAX[428.7 - 98] > MAX[342.1 - 13]$$
$$330.7 > 329.1$$

The decision is to go straight to market at node **W**. Similar calculations are made at the other nodes.

At the end of the first two test phases, the decision is to exercise the option to make the required investment or abandon the project. Because we have two uncertainties, we use quadranomial risk-neutral probabilities in Table 7.6 to calculate the EV of keeping the option open and accounting for the optimal downstream decision.

Calculations for the next two steps are similar; let us illustrate node **5** where the value is calculated as follows:

$$V_5 = \frac{0.443(330.7) + 0.257(164.9) + 0.19(0) + 0.11(0)}{1.08} = \$174.9$$

$$MAX[174.9 - 35, 0]$$

Since the NPV of investing is greater than the NPV of abandoning, the decision is to invest.

By working back to the root node we find that RO of the project is positive ($11.9m), and the decision is made to invest $10m to begin Phase 1 of compulsory testing. Then, at the end of the third phase of compulsory testing, we go straight to the market in those states of nature where prices are high enough but we do indications testing when the prices are lower. As can be seen, technology uncertainty is more important here than price/product uncertainty because if technology uncertainty fails, then price and quantity do not matter.

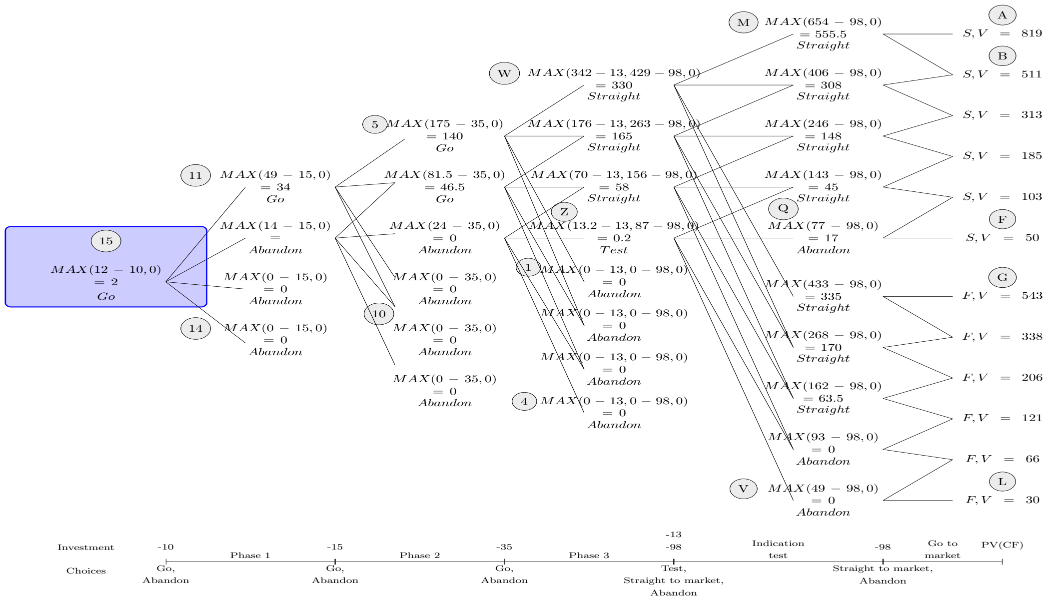


Figure 7.15: Real option calculation and optimal decision

Chapter 8

Discussion and Conclusion

Firms invest in projects to generate profits. Most projects involve flexibility where firms can decide and change the project's process in order to maximise the expected return, or minimise its losses. Much value is built into these decisions, but it can only be captured if the management of the firm is aware and executes. The value of these decisions is found and measured in the project's evaluation phase, to find the real value of the project. If the decisions are not made in time there is a risk that the project will be rejected. The value of this flexibility can only be found by using the option pricing method or DTA.

In this thesis the aim was to use RO valuation methods to solve various tasks with the assumptions of MAD and the random walk. RO were compared with the more traditional valuation methods, DCF and DTA. We also looked at how the RO approach resolved in a phased investment project that was driven by multiple uncertainties, affecting the project's PV, called compound rainbow options.

When the valuation methods discussed in this thesis were compared, we could see that the DCF method does not capture the value of flexibility, as it focuses on the decision being made today, and no changes can be made after that decision. The DTA method can better handle the value of flexibility. However, the main drawback of the DTA is that the probabilities of outcomes have to be estimated and an appropriate discount rate has to be chosen, but both involve subjectivity. RO valuations on the other hand, are based on the risk-free world which offers much more flexibility in the valuation of highly risky projects with uncertain future cash flow. In comparison the DTA is more appropriate in the valuation of projects that are affected by specific risks. RO works better when market risk is present, but when both market and specific risk exist and opportunities for flexibility to change the project's future course are present, RO can provide a better valuation combined with DTA.

The main result of this thesis is that it is not possible to ignore the fact that RO valuations has many advantages over other traditional methods of investment valuations for risky projects. For example, using it gives a better idea of the correct price of the project. Furthermore, the example where both types of risks exist, which was solved by mixing the RO and DTA, showed that their cooperation can solve very complex situations. To date not much has been written about using the quadranomial method in solving compounded rainbow options. However, we could see that this method is useful when there are multiple sources of uncertainties under consideration.

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