MS Thesis
in Economics

Precautionary Saving
and the Timing of Transfers

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Does the timing of transfer payments matter? If the transfers have a regressive payment schedule, with regard to income, then I find that it does. Paying the transfers in the period they accrue rather than with a one period time lag provides households with better insurance against negative income dynamics, especially for liquidity constrained households. Forward looking households are risk averse and hence accumulate, for consumption smoothing motives, precautionary savings. Immediate payment of transfers aids households in smoothing their consumption and as a result they marginally reduce their precautionary savings.

Here households are heterogeneous and optimize facing uninsurable household-specific idiosyncratic risks and economy-wide technology shocks. Uncertainties regarding future income greatly effects household behavior. Socializing further insurance from negative income dynamics by changing the timing of transfer payments lowers perceived risks and hence private insurance measures of households. As a result of lower precautionary savings needed the aggregate savings rate marginally changes and the general equilibrium model developed moves to a new steady-state over fifty periods. The accumulative effect on aggregate capital is that it is 0.65% lower in the new steady-state. The change in savings is very different between households and so is their substitution of leisure and consumption for those savings. Some households increase their consumption while others reduce it but in the aggregate consumption is 0.25% lower in the new steady-state.

The most unifying response of households is that they generally find it optimal to reduce their labour supply. In the new steady-state hours worked and labour supply are reduced by 0.7% and 0.4% in the aggregate, respectively. Static analysis gives that, for any given capital holding and labour productivity status, the change in household behavior results in greater welfare for
overwhelming majority of households. Importantly though, in a dynamic setting under uncertainties the said change in individual household’s behavior results in different optimal consumption, labour supply and capital holding paths for the households and the economy going forward. With welfare being monotonically increasing in capital welfare is 0.25% lower in the aggregate in the new steady-state. Furthermore, with greater response from low wealth households there is a marginal increase in inequality raising the coefficients of both wealth and labour income Ginis.
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Chapter 1

Introduction

Does the timing of transfer payments matter? That is the single question asked in this thesis. To elaborate further, what is the impact of reducing the effective tax wedge by changing from a tax system that is designed in such a way that it collects gross tax in the period revenues are earned but pays transfers one period after they accrue, to a system of net taxation were transfers are deducted as they accrue from that period’s gross taxes?

In general it is the case that transfer payment systems that have a regressive payment schedule with regard to income pay out transfers with a time lag because of within period income dynamics. That is to say each period’s transfer payments can not be determined within that period and paid out on a cash basis since that period’s total income is not known until at the very end of the period. This is always the case if the tax and the transfer payment schedules are determined independendly and as a result do not complement each other to form in combination a single flat rate schedule. Therefore in practice, it is a fact for tax and transfer systems that are independendly determined and that have transfer payments somehow paid out on a cash basis will need to have year-end corrections made except only for taxpayers on fixed income. If, however, the tax and transfer schedules are designed so that they complement each other in such a way that tax rates and transfer deduction rates sum to a single flat rate then net taxation within the tax year on a monthly basis is possible. A flat rate lends itself perfectly to cash settlement. The single flat rate is applied to the sum of labour income and the maximum transfer payment amount to determine both taxes and transfer deductions payable in
It is the total amount payable that is the key to the simultaneous settlement of both systems. If someone happens to pay too much taxes at one time then that would be offset simultaneously by a too little deduction of transfers, leaving only very minimal if any year-end corrections to be made.

A few years back when working for a labour union I proposed a new design of the tax and transfer system in Iceland that would make such net taxation possible. I have always wanted to know how the advantages and disadvantages of adopting net taxation weigh against each other. Especially since gross taxation is the norm in most countries and net taxation is not. To answer this quantitatively I develop a dynamic general equilibrium model based on an extended version of the stochastic neoclassical growth model augmented by a government sector which has its main function income redistribution through transfers to households financed with progressive taxation. The model is calibrated to reflect the characteristics of the Icelandic economy mainly using data from 1992/93 and 2005/06.

In a standard complete market, representative agent macroeconomic model framework the question posed would not yield an interesting answer. Introducing uninsurable idiosyncratic risk changes that as precautionary motives for saving come into play (Leland 1968; Sandmo 1970; Dreze and Modigliani 1972). Precautionary wealth is defined as the difference between the wealth that consumers would hold in the absence of uncertainty and the amount they hold when uncertainty is present (Kimball 1990). To aid households the Government socializes, to some extent, insurance against said risks by paying out transfers. Government policies in that respect, in the presence of incomplete markets, can therefore have an effect on household behavior, saving in particular. Many recent studies have noted the economic importance of precautionary saving. Caballero (1990) and Normandin (1994) have pointed out that precautionary saving may be able to explain excess sensitivity of aggregate consumption to movements in income. Carroll (1992) and Carroll and Dunn (1997) have argued that precautionary behavior is important factor in consumption-led business cycles. Furthermore, simulations in Hubbard, Skinner, and Zeldes (1994) suggest that precautionary saving is non-trivial and could account for nearly half of the aggregate capital stock. As a result deviation from the permanent income hypothesis is therefore often observed, e.g. Challe and Ragot (2009) that analyze evidence from the latest financial crisis in 2007-09.

An important class of models that deviate from the complete markets assumption is that of heterogeneous household models, where uninsured idiosyncratic labour income risk coupled with borrowing constraint generates heterogeneity in individuals income and asset holdings (e.g., Bewley, 1983; Aiyagari, 1994; Krussel and Smith, 1998). Because of the technical and computational diffi-
For the first generation of heterogeneous household models focused on stationary environments with aggregate certainties in which idiosyncratic labour income risk was assumed to be time-invariant (e.g., Huggett, 1997; Aiyagari, 1994). Krussel and Smith (1997, 1998) were the first to introduce aggregate shocks and time-varying labour income risk into this framework. The model developed here is an extension of that latest model framework in that it has greater labour income heterogeneity and endogeneous labour supply introduced.

Changing the timing of transfer payments further socializes the insurance against negative labour income dynamics in that it smoothes take home pay over time. On accrual basis the transfer payments ex ante are equivalent independent of timing and with balanced budget fiscal policy the Government debt position is unaffected. Furthermore, it is assumed that households are infinitely lived so that there is no redistribution across generations so a failure of the Ricardian equivalence in the model developed would be fully attributable to the insurance effect of the transfer system. Viewing the progressivity of individual’s personal tax liability as a form of insurance Kimball and Mankiw (1989) find that the timing of taxes matters when uninsurable risk and heterogeneity are introduced. They derive there result through analytical analysis, unlike the numerical optimization model framework developed here. Although very different in many respects, e.g. their taxes are not distortionary, their research question and findings are the closest to the question asked and the findings derived here that I have found. The failure of Ricardian equivalence that they find, was discussed by Barro (1974, p. 1115) and Tobin (1980, pp. 59-60) and first analyzed using a two-period model by Chan (1983). Shortly thereafter Barsky, Mankiw, and Zeldes (1986) extended that analytical analysis further and argued then that the insurance effect is likely to be quantitatively important. The model simulation results that I get support that argument.

Earlier Arthur Okun (1975) argued that there is a equality efficiency tradeoff and used the metaphor of the leaky bucket to describe that efforts to socially insure households, to get a more equitable outcome, are likely to suffer from efficiency losses. The leakage would result from transaction costs and distortion of incentives. To answer the question of whether further socializing against negative income dynamics by changing the timing of transfers has that leaky bucket effect is the aim of this thesis.

To briefly introduce the model developed then the main features are as follows. Households are heterogenous with regard to wealth and labour income. Necessarily so, since without differences in labour income and asset holdings there of course is no need for transfers. Furthermore, to incorporate variability in the accrueement of transfer payments there has to be some labour income dynamics
introduced. To that end the households are first allocated different labour productivity statuses and are then subjected to household-specific idiosyncratic productivity shocks between periods that can change their productivity statuses. Households can not insure against this risk and can therefore be hit with an unemployment spell or they can lose or advance their skills. Furthermore, households are subjected to economy-wide technology shocks in each period that results in fluctuating factor prices that are endogeneously determined. There are therefore two stochastic processes acting on the households. So, as households meet their fortunes and misfortunes they non-trivially decide on their saving decision resulting in aggregate capital dynamics that affects the interest rate. Importantly, intertemporally optimizing households must be able to forecast factor prices correctly. To accurately forecast factor prices households would need to be able to forecast the distribution dynamics of aggregate capital and labour with complete foresight. Such complete forward looking rationality is a strong assumption and is complicated to model. Therefore in the model the households are assumed to be boundedly rational in that they only use the first moment of the distribution dynamics of aggregate capital and labour to forecast factor prices.

With factor prices and individual household’s productivity status given the households adjust their labour supply along the intensive margin, i.e. their hours worked. Adjustment along the extensive margin is not allowed since the unemployment rate is exogeneously given. Modelling e.g. search unemployment, although interesting, is complicated and is left for further research. The model so specified manages quite succesfully to match observed labour income mobility and labour income distribution. The wealth distribution in the model on the other hand is not as observed since competitive markets for both factors are assumed so there are no profits to be had that can skew and concentrate the wealth distribution. The resulting wealth distribution thus reflects only the differences in productivity histories of households and their savings decisions. That does not hinder the analysis though since it is the labour market dynamics that matters the most and that the model simulates adequately.

The organisation of the thesis is as follows. Chapter 2 introduces the model in detail. In chapter 3, the model is calibrated with regard to characteristics of the Icelandic economy. In chapter 4, the results are presented. Chapter 5 concludes. Appendix A describes in detail the algorithm developed to compute the solution to the model.
The model economy analyzed in this thesis is a modified version of the stochastic neoclassical growth model. This model can also be interpreted as an extension of Heer and Trede (2000) and of Castaneda et al. (1998) in that it merges the features of their two different model economies into one model economy and then adds a system of income redistributive transfer payments. The key features of the model economy analyzed are: (i) it includes a large number of infinitely lived heterogeneous households; (ii) it is augmented by a government sector which has as its main function an income redistribution through taxation and transfer of benefits to households; (iii) households face uninsurable household-specific idiosyncratic productivity shocks and economy-wide aggregate technology shocks; (iv) the households can not borrow and can therefore be liquidity constrained; (v) the households accumulate assets both for precautionary reasons as a substitute of insurance against said shocks to smooth consumption and to take advantage of higher expected future rates of returns; and (vi) the households are boundedly rational when forecasting future factor prices in that they only use the first moment of the distribution dynamics of aggregate capital and labour.

In the following description of the model three different sectors are depicted: households, firms and the Government. Households maximize discounted lifetime utility with regard to their intertemporal consumption and labour supply. Firms maximize their profits and produce with constant returns to scale using labour and capital as inputs. The Government taxes consumption, labour and capital income and spends the revenues on Government consumption, unem-
ployment compensation and transfer payments.

2.1 Households

2.1.1 Population

Households are of measure one and infinitely lived. It is assumed that at each point in time, the economy is inhabited by a continuum of households of different types, $j \in J \equiv \{1, \ldots, J\}$. Households are heterogeneous with regard to their labour productivity factor $\epsilon^j$ and in the transition probabilities of their idiosyncratic productivity processes, $\pi^j$, which is described below. It is also assumed that the labour productivity factor takes a value from the finite set $\delta = \{\epsilon^1, \epsilon^2, \ldots, \epsilon^{nc}\}$, where $\epsilon^1 = 0$ describes the state of unemployment. As a result of the stochastic labour income heterogeneity of households they are necessarily also heterogeneous in their wealth $k_i$, $i \in [0, 1]$.

2.1.2 Preferences

Household $j$, which is characterized by productivity $c^j_t$ and wealth $k^j_t$ in period $t$, maximizes its intertemporal utility with regard to consumption $c^j_t$ and labour supply $n^j_t$:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c^j_t, 1 - n^j_t)$$

(2.1)

where $\beta < 1$ is a discount factor and expectations are conditioned on the information set of the household at time 0. The choice of the functional form for utility follows Castaneda et al. (1998). Instantaneous utility $u(c_t, 1-n_t)$ is assumed to be additively separable in the utility from consumption and the utility from leisure as given by:

$$u(c_t, 1-n_t) = \frac{c_t^{1-\eta}}{1-\eta} + \gamma_0 \frac{(1-n_t)^{1-\gamma_1}}{1-\gamma_1}$$

(2.2)

Where $\eta$ is the intertemporal elasticity of substitution and $\{\gamma_0, \gamma_1\}$ are parameters of disutility from working. Most quantitative studies of general equilibrium models specify a Cobb-Douglas functional form of utility. In that case, however, the elasticity of individual labour supply with regard to wealth is larger than for the utility function (2.2) and, consequently, the distribution of

1As we consider only one type of asset, we will refer to $k$ as capital, wealth and asset interchangeably.
working hours varies more and is less in accordance with empirical observations than for this choice of the utility function [16].

2.1.3 The stochastic processes

2.1.3.1 The household-specific productivity shock

It is assumed that each household type faces an idiosyncratic random shock that can change their efficiency type. There have been many empirical studies on the time-series behavior of earnings and wages through the years that find substantial persistence in the shocks to log-earnings. More recently, some computable general equilibrium models with income heterogeneity and exogenous labour supply have used a regression to the mean process for log-labour earnings, i.e. \( \ln y_t - \ln \bar{y} = \rho (\ln y_{t-1} - \ln \bar{y}) + \epsilon_t \) where \( \epsilon_t \sim N(0, \sigma^2_\eta) \). Atkinson et al. (1992) report that estimates of the regression towards the mean parameter \( \rho \) varies from 0.65 to 0.95 in annual data. I will therefore specify the log-earnings process as an AR(1) process. The process needs to be discretized for computational purposes. The process can easily be approximated with a first-order finite-state Markov chain with conditional transition probabilities given by

\[
\pi_i(\epsilon'|\epsilon) = Pr \{ \epsilon_{t+1} = \epsilon'|\epsilon_t = \epsilon \} = \begin{pmatrix}
\epsilon^1_{11} & \epsilon^1_{12} & \cdots \\
\epsilon^2_{21} & \epsilon^2_{22} & \cdots \\
\vdots & \vdots & \ddots 
\end{pmatrix}
\]

where \( \epsilon, \epsilon' \in \mathcal{E} \) and \( i=\{1, 2\} \).

2.1.3.2 The economy-wide technology shock

It is assumed that there is an exogenous stochastic economy-wide technology process \( \{ Z_t \} \). In this model, a restriction of the very simple case of two possible states is applied to keep the state space to a reasonable size to reduce the computational burden. Hence the economy only experiences good or bad times with technology levels \( Z_g \) and \( Z_b \), respectively, with \( Z_g > Z_b \). The process between states follows a stationary finite-state Markov chain with transition probabilities given by

\[
\Pi(Z'|Z) = Pr \{ Z_{t+1} = Z'|Z_t = Z \} = \begin{pmatrix}
Z_{gg} & Z_{gb} \\
Z_{bg} & Z_{bb}
\end{pmatrix}
\]

where \( Z, Z' \in \mathcal{Z} \).
2.1.3.3 The joint processes

The household-specific productivities, of course, depend on the aggregate productivity $Z_t$. In good times agents have higher probabilities of being a high efficiency type than in bad times. In particular, the probability of being in the lowest efficiency type $\epsilon^1=0$, i.e. being unemployed, is greater in bad times than in good times. Conditional on the realizations of $Z_t$ and $Z_{t+1}$ the idiosyncratic household-specific shocks are assumed to be independently distributed across households and identically distributed within each household type. Consequently, the joint process of the two shocks, $Z_t$ and $\epsilon_t$, can be written as a Markov chain with $n=n_\epsilon \times n_Z$ states. Their transition probabilities are given by

$$
\Gamma_i(Z', \epsilon'|Z, \epsilon) = \Pr\{Z_{t+1} = Z', \epsilon_{t+1} = \epsilon'|Z_t = Z, \epsilon_t = \epsilon\} \quad (2.5)
$$

Households know the laws of motion of both $\{\epsilon_t\}$ and $\{Z_t\}$, and they observe the realizations of both stochastic processes at the beginning of each period.

2.1.4 The households decision problem

The households decision problem is a dynamic programming problem. A recursive representation of the problem is given by the following Bellman equation as follows

$$
V(\epsilon, k, Z, m, N) = \max_{c, n, k'} \left[ u(c, 1-n) + \beta E\{V(\epsilon', k', Z', m', N')\}\right] \quad (2.6)
$$

Where $V(\epsilon, k, Z, m, N)$ is the value of a five dimensional objective function of a household characterized by productivity $\epsilon$, wealth $k$, hours worked $n$ and with economy-wide aggregates characterized by technology level $Z$, aggregate capital $m$ and aggregate employment $N$. The households optimal policy rules are also a function of five variables, i.e. $c(\epsilon, k, Z, m, N)$, $n(\epsilon, k, Z, m, N)$ and $k'(\epsilon, k, Z, m, N)$ for consumption, hours worked and next-period capital respectively. The solution to (2.6) is subject to the households budget constraint, the government policy, the stochastic processes as given by (2.5), a vector of aggregates and as a function thereof the relative factor prices $\{w, r\}$ both current and next-period. Each of these constraints to the solution of (2.6) will be discussed in turn.

The households are not allowed to borrow

$$
k^j \geq 0 \quad (2.7)
$$

In addition, the household faces a budget constraint. The household receives income from labour $n_t$ and capital $k_t$. It also receives unemployment compensation if unemployed ($\epsilon = \epsilon^1$) and transfer payments, which the government
2.1 Households

pays out for income redistribution purposes. The household then spends its revenue on consumption $c_t$ and next-period wealth $k_{t+1}$. The household $j$’s budget constraint is therefore given by

$$k_{t+1}^j = (1 + r_t(1 - \tau_r))k_t^j + (w_t n_t^j \epsilon_t^j (1 - \tau_w) + ta_t) + \mathbb{1}_{\epsilon = \epsilon_1} \times b_t + (tp_t - w_{t-1} n_{t-1}^j \epsilon_{t-1}^j \zeta_{tp}) - (1 + \tau_c)c_t^j$$  \hspace{1cm} (2.8)

where $\{r_t, w_t, \tau_c, \tau_r, \tau_w, ta_t, \zeta_{tp}\}$ denote the interest rate, the wage rate, the sales tax rate, capital gains tax rate, the tax rate on labour income, the personal tax allowance and the transfer payments deduction rate, respectively. $\mathbb{1}_{\epsilon = \epsilon_1}$ is an indicator function which takes the value one if the household is unemployed ($\epsilon = \epsilon_1$) and zero otherwise. $\{b_t, tp_t\}$ denote the unemployment compensation and the transfer payment, respectively. The transfer payment is a function of previous-period income from labour in the benchmark model economy, but is changed to current-period income from labour when I change the Government policy rule in the alternative model economy. In fact, it is the only thing that changes.

The Government policy rules consist of setting exogenously the parameter values for $\{\tau_r, \tau_w\}$ but the sales tax rate $\{\tau_c\}$ is determined endogenously in the model so that the Government budget is balanced over the business cycle. The triplets $\{b_t, tp_t, ta_t\}$ are also partly determined endogenously since they are a fixed ratio of the average labour income of efficiency type 2 ($\epsilon = \epsilon_2$) but the ratio is determined exogenously.

To solve (2.6) the agents need to know next-period factor prices. But since there is aggregate uncertainty in the model economy due to (2.4) the next-period aggregate capital stock and employment is not known for certain. The household’s income and savings decisions depend on the aggregate productivity level and, for that reason, the distribution of capital and employment changes over time. Rational expectations of next-period factor prices for capital and labour $\{w, r\}$ that clear the market can therefore not be formed with absolute certainty because of the stochastic nature of the model economy. It is therefore assumed that the households are only boundedly rational when they form their expectations for the law of motion of the aggregate capital stock and employment following Krussel and Smith (1998). In particular, the households are assumed to use only the first $I$ moments $m$ to predict the law of motion for the distribution of aggregate capital, with the first moment being $m_1 = K$, and that they perceive the law of motion $m$ as follows

$$m' = H_I(m, Z)$$  \hspace{1cm} (2.9)

A simple parameterized functional form for $H_I(m, Z)$ is chosen, again following Krussel and Smith (1998), with $m = m_1 = K$:

$$\ln K' = \begin{cases} \gamma_{0g} + \gamma_{1g} \ln K & \text{if } Z = Z_g, \\ \gamma_{0b} + \gamma_{1b} \ln K & \text{if } Z = Z_b \end{cases}$$  \hspace{1cm} (2.10)
Given the boundedly rational expectation of next-period capital stock the household forms an expectation of the next-period employment level based on past realizations of employment levels for various aggregate capital stock levels. The next-period factor prices \( \{ w, r \} \) can then be derived by the households.

### 2.2 Production

Firms are owned by the households and they maximize profits with respect to their labour and capital demand. It is assumed that aggregate output, \( Y_t \), depends on aggregate capital, \( K_t \), on the aggregate labour input, \( N_t \), and on the economy-wide technology shock, \( Z_t \), through a constant returns to scale Cobb-Douglas aggregate production function:

\[
Y_t = f(K_t, N_t, Z_t) \equiv Z_t K_t^\alpha N_t^{1-\alpha} \tag{2.11}
\]

Competitive factor and product markets are assumed implying that in a market equilibrium, factors and products are compensated according to their marginal products and profits are zero:

\[
w_t = Z_t (1 - \alpha) \left( \frac{K_t}{N_t} \right)^\alpha \tag{2.12}
\]

\[
r_t = Z_t \alpha \left( \frac{N_t}{K_t} \right)^{1-\alpha} - \delta \tag{2.13}
\]

where \( \delta \) denotes the constant depreciation rate of the capital stock.

### 2.3 Government

Government expenditures consist of Government consumption \( G_t \), unemployment compensation \( B_t \) and transfer payments \( TP_t \). The Government consumption does not have an effect on either utility nor production in the model economy. The Government expenditures are financed by a labour income tax, capital gains tax and a sales tax. The sales tax is a simple proportional tax on consumption where the tax rate is endogenously determined so that the Government budget is balanced over the business cycle. The sales tax varies only between two states, i.e. sales tax during good times and sales tax during bad times. The capital gains tax is a flat rate tax on income from capital. The labour income tax is a progressive tax. The personal tax allowance makes the tax progressive although the tax rate is a single flat rate. The transfer
payments and the unemployment compensation are partly endogenously determined as a fixed exogenously determined ratio of the average labour income of efficiency type $2$ ($\epsilon = \epsilon^2$). Taxation, compensation and transfer payments are discussed in more detail in the next chapter on the calibration of the model.

The Government budget is set to balance over the business cycle - the simulated time period - so that all Government expenditures are fully financed by tax revenues $T_t$:

$$\sum_t G_t + \sum_t B_t + \sum_t TP_t = \sum_t T_t$$

(2.14)

### 2.4 The definition of equilibrium

The concept of equilibrium used in this thesis uses a recursive representation of the household’s following Stokey et al (1989). Since the household’s decision problem is a finite-state, discounted dynamic program, an optimal stationary Markov solution to this problem always exists. A recursive competitive equilibrium for a given set of Government policy parameters is ($i$) a value function $V(\epsilon, k, Z, m, N)$, ($ii$) individual policy rules $c(\epsilon, k, Z, m, N)$, $n(\epsilon, k, Z, m, N)$ and $k'(\epsilon, k, Z, m, N)$ for consumption, hours worked and next-period capital, respectively. ($iii$) The pricing processes $r(Z, m, N)$ and $w(Z, m, N)$. ($iv$) The distribution of the individual states $(\epsilon, k)$ for given aggregate state variables $(Z, m, N)$ in period $t$ denoted by $f(\epsilon, k; Z, m, N)$. The dynamics of the distribution of the individual states are described by

$$f'(\epsilon', k'; Z', m', N') = \sum_{\epsilon} \Gamma(Z', \epsilon' | Z, \epsilon) f(\epsilon, k; Z, m, N)$$

(2.15)

and finally ($v$) a vector of aggregates $K, N, C, T, B$ and $TP$ such that:

1. Factor inputs, consumption, tax revenues, unemployment compensation and transfer payments are obtained aggregating over households:

$$K = \sum_{\epsilon} \int_k k f(\epsilon, k; Z, m, N) \; d\psi(\Omega)$$

(2.16)

$$N = \sum_{\epsilon} \int_k n f(\epsilon, k; Z, m, N) \; d\psi(\Omega)$$

(2.17)

$$C = \sum_{\epsilon} \int_k c(\epsilon, k, Z, m, N) f(\epsilon, k; Z, m, N) \; d\psi(\Omega)$$

(2.18)
\[
T = \left( \tau_w w N - \sum_{\epsilon=\epsilon_2}^{\epsilon_1} \int_k \tau a f(\epsilon, k; Z, m, N) \, d\psi(\Omega) \right) \\
+ \tau_r r K + \left( 1 - \frac{1}{1 + \tau_c} \right) C
\]

\[ (2.19) \]

\[ B = \int_k 1_{\epsilon = \epsilon_1} b \, d\psi(\Omega) \quad (2.20) \]

\[ TP = \sum_{\epsilon} \int_k t p f(\epsilon, k; Z, m, N) \, d\psi(\Omega) \quad (2.21) \]

where \( \Omega \) denotes the Government policy. The measure \( \psi \) is a function of \( \Omega \) and describes the distribution of both household-specific state variables \( k \) and \( \epsilon \).

2. \( c(\epsilon, k, Z, m, N) \), \( n(\epsilon, k, Z, m, N) \) and \( k'(\epsilon, k, Z, m, N) \) are optimal decision rules and solve the household decision problem described in (2.6).

3. Factor prices (2.12) and (2.13) are equal to their marginal productivities.

4. The goods market clears:

\[ f(Z, K, N) + (1 - \delta)K = C + K' + G = C + K + G \quad (2.22) \]

5. The Government budget (2.14) is balanced over the business cycle.
The aim is to compute the quantitative effects of a different fiscal policy regime on output, welfare, employment, capital holdings, wealth and income distributions. To that end the model economy is calibrated to the characteristics of the Icelandic economy. In calibrating special attention is given to households employment decision characteristics, the lorenz curve of income distribution, income mobility, the tax and transfer systems. Effort is also made to match key characteristics and ratios from the national income and product accounts as closely as possible.

It has to be taken into consideration that the model economy is a Walrasian type economy based on the neoclassical growth model were there are no product market imperfections. Hence there is less concentration of wealth in the model than what is usually seen in real world economies. This can not be calibrated away. The wealth distribution seen in the model economy is the result of the heterogeneous-household modelling of the labour market and is the result of fortunes and misfortunes of households when they are allocated their productivity series. There are no market activity based profits that can skew the wealth distribution in the direction of what is seen in real world economies. Although market imperfections are not modelled it is nevertheless possible to study redistributive problems in a way that is not at all possible in the representative-agent model. The representative agent model can not answer the question of how different redistributive fiscal policies affect welfare and the distribution of income and wealth. In fact, it does not provide an answer to the question of how the dispersion of income and wealth arises in
the first place.

It is however possible to calibrate the labour market characteristics adequately in the model. Which is the objective at hand so that the quantitative effect of a different redistributive fiscal policy on heterogenous labour supplying households can be measured.

The model periods correspond to years. The calibration of parameter values is summerized in Table 3.1, but the model parameters will now be discussed in turn.

3.1 Utility

For the utility function parameters, the discount rate of \( \beta = 0.955 \) is chosen following an estimate, from Icelandic data, by T. Einarsson (2001). I set \( \eta = 2 \), but empirical estimates of the intertemporal elasticity of substitution \( 1/\eta \) vary considerably. Heer and Trede (2001) performed a sensitivity analysis for a heterogeneous-household model economy with the same utility function as (2.2) and found that the results are robust with regard to the choice \( \eta \in [1, 4] \). The parameters \( \gamma_0 \) and \( \gamma_1 \) are chosen in order to imply (i) an average working time of approximately \( 2/5 \) and (ii) a coefficient of variation of workers’ labour supply equal to the empirical value estimate taken from Heer and Trede (2001). For the model \( \gamma_0 = 0.13 \) and \( \gamma_1 = 10 \) are used and that results in that, in the benchmark case, the average hours worked by workers is equal to 0.282 and the coefficient of variation of working hours is 0.286.

3.2 Productivity and technology

The productivities \( \epsilon \in E = \{ \epsilon^1, \ldots, \epsilon^n \} \) are chosen so that they can replicate the distribution of hourly wage rates by discretization. Wage rates are proportional to productivity. The number of productivities is set to \( n = 5 \) which is sufficient to capture most of the wage distribution except for the by far highest wage rates. \( \epsilon^1 \) as mentioned before represents the state of unemployment and is therefore set to zero. The productivities \( \{ \epsilon^2, \epsilon^3, \epsilon^4, \epsilon^5 \} \) are estimated from

---

1 Long working hours in Iceland is a well-known fact. According to surveys undertaken by The Statistical Bureau of Iceland the average working week for those engaged in full-time job is approximately 50 hours [30].

2 I do not go higher than \( n = 5 \) to keep the state space to a reasonable size for reasons discussed in Appendix A.
Icelandic tax returns filed in 2006\cite{28}. The four productivities correspond to the average wages of earners in each of the quartiles, respectively. Normalizing the average of the four productivities to unity, from the 50\textsuperscript{th} percentile, the calibration arrives at
\[
\{\epsilon^2, \epsilon^3, \epsilon^4, \epsilon^5\} = \{0.2734, 0.7770, 1.3390, 2.7086\}
\] (3.1)

The economy-wide technology shock is assumed to take only two values \(Z \in \mathcal{Z} = \{Z_g, Z_b\} = \{1.0480, 0.9728\}\). They are found from statistics observed in the solow residual\cite{31}, assuming a constant returns to scale Cobb-Douglas production technology, and some trial and error to match Icelandic economy aggregate statistics and characteristics.

### 3.3 The Markov transition matrices

#### 3.3.1 Household-specific productivity transitions

There are four transition matrices needed to capture the possible transition probabilities between household-specific productivities for all the possible aggregate technology state dynamics. There is one transition matrix needed for productivity mobility when going from good times to good times between periods during expansions and one for going from bad times to bad times during recessions. There are also two matrices needed to give the transition probabilities when going from good times to bad times and vice versa.

The transition probability into and out of unemployment, \(\pi_i (\epsilon' > 0|\epsilon = 0)\) and \(\pi_i (\epsilon' = 0|\epsilon > 0)\) where \(\epsilon'\) represents next period’s productivity, are calibrated so that the average unemployment rate is 2.0% in good times and 5.2% in bad times. The average duration of an unemployment spell is 1.30 periods in good times and 1.45 periods in bad times. The probability of becoming unemployed is positive for all household types. During unemployment spells the worker’s human capital depreciates and accordingly his probability of becoming as high a productivity type as before goes considerably down, but still remains positive.

For the transition from good times to bad times it is assumed that all unemployed household remain unemployed. Secondly, it is also assumed that the employment level shifts to reflect the employment level during bad times, that is to say I let an unemployment spell hit the required number of employed households so that the unemployment rate rises to bad times levels. Aside from this increased risk of unemployment all employed households get to keep

\footnote{Using Icelandic annual data from the sample period 1990-2006.\cite{31}}
their productivity type. The same principle applies for the transition from bad times to good times. No employed household becomes unemployed during the transition, all employed households get to keep their productivity type and the employment level shifts to reflect the employment level during good times. That is, the required number of unemployed households are lifted out of the unemployment spell by elevating them to a higher productivity type so that the unemployment rate goes back down to levels seen in good times.

The remaining 16 transition probabilities for the good times to good times and bad times to bad times matrices, respectively, are calibrated such that (i) the model economy matches the observed quartile to quartile transition probabilities of household wages as reported in tax returns filed for wages earned in 2005 and 2006 on the one hand and 1992 and 1993 on the other for good times and bad times, respectively. (ii) each row in the transition matrix sums to one.

So, the four Markov transition matrices that result are

\[
\pi_{Zgg}(\epsilon' | \epsilon) = \begin{pmatrix}
0.3084 & 0.1194 & 0.1650 & 0.2350 & 0.1722 \\
0.0355 & 0.7348 & 0.1507 & 0.0534 & 0.0256 \\
0.0073 & 0.1913 & 0.6478 & 0.1346 & 0.0189 \\
0.0080 & 0.0437 & 0.1649 & 0.6558 & 0.1277 \\
0.0064 & 0.0217 & 0.0178 & 0.1321 & 0.8220
\end{pmatrix}
\] (3.2)

\[
\pi_{Zbb}(\epsilon' | \epsilon) = \begin{pmatrix}
0.3084 & 0.1194 & 0.1650 & 0.2350 & 0.1722 \\
0.0355 & 0.7348 & 0.1507 & 0.0534 & 0.0256 \\
0.0073 & 0.1913 & 0.6478 & 0.1346 & 0.0189 \\
0.0080 & 0.0437 & 0.1649 & 0.6558 & 0.1277 \\
0.0064 & 0.0217 & 0.0178 & 0.1321 & 0.8220
\end{pmatrix}
\] (3.3)

\[
\pi_{Zgb}(\epsilon' | \epsilon) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0.0106 & 0.9645 & 0 & 0 & 0 \\
0.0072 & 0 & 0.9927 & 0 & 0 \\
0.0073 & 0 & 0 & 0.9920 & 0 \\
0.0070 & 0 & 0 & 0 & 0.9936
\end{pmatrix}
\] (3.4)

\[
\pi_{Zbg}(\epsilon' | \epsilon) = \begin{pmatrix}
0.3911 & 0.1051 & 0.1453 & 0.2069 & 0.1516 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\] (3.5)
3.3.2 Economy-wide technology transitions

It is very difficult to discern the length or persistence of business cycles. Here it is chosen to have the volatility of output in the model economy match as closely as possible the value of about 4 per cent observed in yearly Icelandic data. This is a somewhat difficult target since the volatility of output in Iceland is larger than seen in most OECD economies which have this statistic in the 2 - 3 per cent range, see Gudmundsson and Zoega (2000), and this model may be too abstract to capture that.

So, the Markov transition matrix that I arrived at is simply

\[
\Pi(Z'|Z) = \begin{pmatrix}
Z_{gg} & Z_{gb} \\
Z_{bg} & Z_{bb}
\end{pmatrix} = \begin{pmatrix}
0.80 & 0.20 \\
0.20 & 0.80
\end{pmatrix}
\]  \hspace{1cm} (3.6)

Following Castaneda et al. (1998) it is assumed, as is customarily made in most quantitative studies of business cycles, that the expected durations of expansions and recessions are equal in length or, equivalently, that they are equally persistent.

3.4 Government expenditures

3.4.1 Government consumption

The Government consumption $G$ is calibrated such that the average sales tax needed to balance the Government budget is approximately equal to the Icelandic average sales tax rate of 18.6 per cent. The Government consumption share in output is set equal to 30 per cent which results in a sales tax of 17.2 per cent in the model economy at average. The aim of calibrating the required sales tax is to have the three main taxes that make up the tax wedge, i.e. the labour income tax, the capital gains tax and the sales tax, as close to their respective Icelandic rates as possible. These taxes are the ones that affect the consumption, saving and employment decisions the most and have to suffice since the model is purposefully too abstract to be able reflect all the details of Government revenues and expenditures.
3.4.2 Unemployment compensation

Unemployment compensation $b$ is set equal to 60 per cent of the average wage of the lowest productivity workers, $\epsilon^2\bar{n}^2w$, net of labour income taxes. This replacement ratio is computed using (3.1) and information about actual unemployment compensations relative to earned wages reported in tax returns filed in 2006. The unemployment compensation is therefore partly endogenously determined in the model and is a function of the average labour supply of productivity type 2 ($\epsilon = \epsilon^2$) and the wage rate in each period.

3.4.3 Transfer payments

The transfer payment system in Iceland is relatively complicated and so much so that it can not be accurately modeled. I therefore resort to a stylized version that (i) reflects the relative importance or value of transfer payments to respective household types and (ii) approximates the somewhat non-linear function that determines the deduction of transfer payments to a household with a simple linear function.

The transfer payments are calibrated such that the highest possible transfer payment to a household is 40 per cent of the unemployment compensation at each time. Transfer payments are therefore also partly endogenously determined. The highest possible transfer payment goes to households of type $\epsilon = \epsilon^1$ and are without deductions. Households that earn labour income have their transfer payments deducted by 3 per cent of wages earned.

3.5 Taxation

3.5.1 The labour income and capital gains tax

The capital gains tax is easily calibrated to the Icelandic one as a flat rate of 10 per cent. The labour income tax is also calibrated at a flat rate of 37.4 per cent of wages earned, but there is also a personal tax allowance that can be deducted from labour taxes owed and that feature makes the labour income tax progressive. The personal tax allowances is calibrated such that it is 37.4 per cent of the unemployment compensation in each period thus making the unemployment compensation the limit below which no labour income taxes need to be paid of wages earned. The personal tax allowance is therefore endogenously determined as a function of the wage rate in each period.
3.5.2 The sales tax

The sales tax rate is endogenously determined in the model and is not calibrated directly.

3.6 Production

The production elasticity of capital, $\alpha = 0.337$, and the annual rate of constant capital depreciation, $\delta = 0.0683$, are taken from T. Einarsson (2001) and are based on Icelandic annual data from the second half of the 1990’s.

<table>
<thead>
<tr>
<th>Description</th>
<th>Function</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility function</td>
<td>$c_1^{\frac{\eta}{1-\eta}} + \gamma_0 (1-n_1)^{1-\gamma_1}$</td>
<td>$\eta = 2, \gamma_0 = 0.13, \gamma_1 = 10$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>$\beta = 0.955$</td>
</tr>
<tr>
<td>Production function</td>
<td>$f(Z, K, N) = ZK^\alpha N^{1-\alpha}$</td>
<td>$\alpha = 0.337$</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$\delta$</td>
<td>$\delta = 0.0683$</td>
</tr>
<tr>
<td>Government consumption</td>
<td>$\bar{G} = \gamma^g f(Z, \bar{K}, N)$</td>
<td>$\gamma^g = 30%$</td>
</tr>
<tr>
<td>Unemployment compensation</td>
<td>$b$</td>
<td>$b = 0.60e^2n^2w$</td>
</tr>
<tr>
<td>Transfer payments</td>
<td>$tp - (\epsilon^2n^2w \times \zeta_{tp})$</td>
<td>$tp = 0.40b, \zeta_{tp} = 0.03$</td>
</tr>
<tr>
<td>Personal tax allowance</td>
<td>$ta$</td>
<td>$ta = 0.374b$</td>
</tr>
</tbody>
</table>

Table 3.1: Calibration of parameter values
In this chapter the quantitative results of the model simulations are reported. The effects of changing the timing of payments on employment, consumption/savings, welfare and the distribution of wealth and income are analyzed. The properties of the neoclassical growth model are well understood, it being the most widely used framework for macroeconomic analysis, so here the emphasis will be on analyzing the properties special to the extensions developed in the model. The discussion begins by analyzing in detail the equilibrium properties of a benchmark model characterized by the parameterization presented in Table 3.1 were the payment of transfers lags the accrualment of them. It then proceeds to analyze the effects of changing the timing of transfers by first looking at the transition period that follows and then the properties of the new equilibrium that results.

4.1 Equilibrium properties of the benchmark model

4.1.1 Welfare and policy functions properties

The policy functions solve the respective heterogeneous household’s problem and I find that the optimal policy rules for labour supply, consumption and next-period capital behave as expected ex ante. The properties of the welfare function and the three policy functions are depicted in the four panels of Figure 4.1 by plotting up the simulation results. The properties are:
labour supply: It results from the simulation that labour supply is an increasing function of productivity, as the substitution effect is stronger than the income effect, but a decreasing function of wealth as the marginal utility of income declines with higher wealth.

Figure 4.1: Simulation results for policy and welfare functions
Panel a shows how well-behaved the simulated value function is over the capital and productivity space. Panel b shows the decreasing differences between current and next-period capital holdings a prerequisite for a steady-state aggregate capital stock. Panels c and d show actual simulated values for consumption and hours worked for each household type over households capital space.

consumption: Consumption is an increasing function of both productivity and wealth as expected lifetime income increases with productivity and the marginal utility of intertemporal deferral of consumption decreases with wealth.
4.1 Equilibrium properties of the benchmark model

**next-period capital**: Next-period capital is an increasing function of productivity, as consumption increases at a slower rate than labour income. Importantly, next-period capital is only increasing in wealth up to a point where it is no longer feasible to defer consumption and becomes decreasing in wealth after that. That property is very important and gives rise to the existence of the all important steady-state of aggregate capital which stabilizes the economy.

**welfare**: The effects of these optimal policy rules are summed up in the welfare function which turns out to be monotonically increasing in both productivity and wealth. With welfare dropping sharply as households get closer and closer to being liquidity constrained, see panel a in Figure 4.1.

**A closer look at labour supply**

Panel d in Figure 4.1 reveals that the wage rate differential, on account of productivity heterogeneity, clearly results in that a high-productivity household always supplies more labour for a given wealth than a low-productivity household does. The difference in labour supply is minimal for the lowest-wealth households were the labour supply differential is a mere 2.70% with all households working as much as possible. That difference in supply widens as wealth increases since a low-productivity household decreases its supply more sharply than a high-productivity household as leisure is more expensive to high-productivity households. The difference in supply goes up to 59.1% for the highest-wealth households leaving the low-productivity household supplying only 40% of the labour that highest-productivity household supplies.

<table>
<thead>
<tr>
<th>percentiles</th>
<th>Unemployed</th>
<th>$\epsilon^2$</th>
<th>$\epsilon^3$</th>
<th>$\epsilon^4$</th>
<th>$\epsilon^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% lowest</td>
<td>-75.4 (-77.9)</td>
<td>-22.9 (-49.6)</td>
<td>29.11 (217.7)</td>
<td>43.4 (533.6)</td>
<td>60.8 (1497.4)</td>
</tr>
<tr>
<td>1 - 2.5%</td>
<td>-212.2 (-58.6)</td>
<td>-55.3 (-31.8)</td>
<td>23.0 (40.3)</td>
<td>40.5 (116.5)</td>
<td>59.9 (345.8)</td>
</tr>
<tr>
<td>2.5 - 5.0%</td>
<td>-337.4 (-40.2)</td>
<td>-89.8 (-22.2)</td>
<td>14.9 (11.3)</td>
<td>37.0 (45.4)</td>
<td>59.2 (148.7)</td>
</tr>
<tr>
<td>5.0 - 10%</td>
<td>-407.8 (-26.0)</td>
<td>-129.6 (-15.8)</td>
<td>3.1 (1.43)</td>
<td>31.3 (18.8)</td>
<td>56.6 (68.8)</td>
</tr>
<tr>
<td>10 - 25%</td>
<td>-364.4 (-14.1)</td>
<td>-169.5 (-10.0)</td>
<td>-16.91 (-2.3)</td>
<td>20.2 (5.5)</td>
<td>52.1 (27.8)</td>
</tr>
<tr>
<td>25 - 50%</td>
<td>-257.3 (-7.6)</td>
<td>-170.3 (-6.1)</td>
<td>-38.0 (-2.8)</td>
<td>5.6 (0.8)</td>
<td>44.3 (10.7)</td>
</tr>
<tr>
<td>50 - 75%</td>
<td>-184.3 (-4.9)</td>
<td>-146.9 (-4.3)</td>
<td>-49.5 (-2.5)</td>
<td>-6.4 (-0.5)</td>
<td>36.1 (5.1)</td>
</tr>
<tr>
<td>whole space</td>
<td>-234.5 (-10.1)</td>
<td>-146.2 (-7.2)</td>
<td>-37.0 (0.9)</td>
<td>2.4 (9.7)</td>
<td>40.0 (35.7)</td>
</tr>
</tbody>
</table>

Table 4.1: Savings ratios as a percentage of income (wealth)

**Savings ratios**

To offer a more detailed look at the next-period capital decision the savings ratios of different percentiles are given in Table 4.1. First of all, it is evident that unemployed households that are in the 1% lowest wealth percentile con-
sume both the transfers they receive and 78% of their assets also. Further, households in that same wealth percentile but enjoy the fortune of having the highest-productivity status choose to save 60.8% of their income. Despite such a high savings ratio the consumption of the highest-productivity household is 5.5 times that of the unemployed one resulting in a great difference in welfare as can be seen in panels a and c in Figure 4.1. But high-productivity households eventually get to a certain wealth position were they start to lower their savings ratio and increase consumption. This is in line with what the policy rules stipulate and it is a prerequisite for the existence of a steady-state.

**Behavior in-line with expectations**
The substitution and income effects are thus in line with what one could expect ex ante. The policy rules give rise to a well behaved welfare function and a steady-state of the economy as aggregate capital stabilizes.

### 4.1.2 Labour market properties

![Lorenz curves for the simulated and empirical distributions.](image)

The Lorenz curve of the simulated labour income distribution (broken line) is plotted in the left-panel along with the empirical distribution (solid line). The right-panel plots the Lorenz curves for simulated wage rates (broken line) and labour income (solid line).

Labour productivity heterogeneity, idiosyncratic productivity shocks and the economy-wide technology shock together create labour market dynamics that result in an endogenous distribution of labour income within the model. To see how well the labour market dynamics fits the data the simulated labour income distribution can be compared with the empirical one. The simulated Gini coefficient of gross labour income is 0.440. That compares favorably with
4.1 Equilibrium properties of the benchmark model

the empirical Gini coefficient of 0.447 calculated from the 2006 tax returns.[28]

A more difficult and very important property to emulate is to have the simulated Lorenz curve adequately fit the empirical one. The Lorenz curve of the simulated labour income distribution (broken line) is graphed in Figure 4.2 panel a along with the empirical distribution (solid line). As can be seen from the graph the two distributions are very similar except for the very highest income. This could be fixed by increasing the number of available productivities in the model. I deemed the computational burden of expanding the state-space not worth the benefit since the focus is mostly on low to middle income households. The difference after all is quite small as can be seen from the comparison of the Gini coefficients of the simulated and empirical distributions.

It is interesting to look at the Gini coefficient of the distribution of wage rates. It is only 0.383. There is therefore slightly more equality when it comes to wage rates. The wage rate differential can therefore only partially explain the inequality in labour income distribution. The rest must be explained with differences in the labour supplied. Ex ante it is not obvious how the labour supply factor affects the income distribution because there are two opposite effects in play. On the one hand high-productivity households supply more labour for given wealth. But on the other hand high-productivity households are also richer than the low-productivity households on average, which tends to reduce the supply of average working hours of high-productivity households. The result of the model simulation is that in general the substitution effect is stronger than the income effect as can be seen in Table 4.2 were differences in working hours is shown.

<table>
<thead>
<tr>
<th>Gini</th>
<th>w</th>
<th>wxn</th>
<th>k</th>
<th>( \eta_{n,w} )</th>
<th>( \sigma_{n}/\bar{n} )</th>
<th>( \sigma_{\epsilon_n}/N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark case</td>
<td>0.383</td>
<td>0.440</td>
<td>0.391</td>
<td>0.22</td>
<td>0.286</td>
<td>0.791</td>
</tr>
<tr>
<td>empirical value</td>
<td>-</td>
<td>0.447</td>
<td>0.5-0.89</td>
<td>0.20</td>
<td>0.324</td>
<td>0.689</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>mean</th>
<th>( \epsilon^2 )</th>
<th>( \epsilon^3 )</th>
<th>( \epsilon^4 )</th>
<th>( \epsilon^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>working hours</td>
<td>0.282</td>
<td>0.234</td>
<td>0.299</td>
<td>0.291</td>
</tr>
</tbody>
</table>

Table 4.2: Key labour market statistics of the benchmark case

Regarding other labour market properties then it results that aggregate effective labour supply amounts to \( N = 0.377 \) with average working hours equal to \( n=0.282 \). This fits well with empirical data that shows for the period 2003-2008 average working hours to be 0.279.[30]
Additionally, in the model households work 3.99% more in good times than in bad with average working hours being 0.287 in good times and 0.276 in bad as a result of wages being 8.34% higher in good times than in bad times. Hours worked range from 0.234 for the second lowest productivity to 0.303 for the highest productivity households. Further, working hours vary less than effective labour reflecting the optimization behavior of the working agents who work longer if they are more productive. The variational coefficient of working hours (effective labour) is equal to $\sigma_n/\bar{n}=0.286$ ($\sigma_{en}/N=0.791$) (see the last two columns of Table 4.2). This corresponds closely to empirical estimates taken from Heer and Trede (2001). The standard deviation of working hours (effective labour) supply is $\sigma_{n}=0.66\%$ ($\sigma_{en}=8.86\%$). Also, it results that labour supply elasticity with regard to the wage rate, $\eta_{n,w}$, is moderate, amounting to 0.213 for the average worker. Again, this compares favorably with the data. Bianchi et al. (2000) estimates that elasticities for Icelandic male labour supply have an upper bound of 0.40 and are likely to be well below that.

4.1.3 Factor prices and the aggregates over the business cycle

In the simulation the mean aggregate capital stock amounts to $K = 4.91$ which is associated with a capital/output coefficient equal to $K/Y = 3.48$. During 1990-2008, the empirical value of $K/Y$ was equal to 3.21 in Iceland for the whole economy[31]. Further, the standard deviation of the simulated capital stock is 16.1% while the empirical one for the 1990-2008 period is 11.9% at fixed prices. The simulated values of the aggregate capital stock can be seen in Figure 4.3. Despite considerable fluctuations households are able to predict the size of the capital stock with a prediction error of only 1.61%. The household’s estimate of the parameterized function for the law of motion stated in equation (2.10) are as follows

$$\ln K’ = \begin{cases} 
0.0915 + 0.9474 \ln K & \text{if } Z = Z_g, \\
0.0676 + 0.9531 \ln K & \text{if } Z = Z_b
\end{cases}$$

indicating a slightly higher autocorrelation in bad times than in good. At the same time the lower constant or drift parameter in bad times indicates a lower level of growth in bad times than in good. That is in line with the difference found in the simulated capital stock during good times and bad with the average capital stock being 4.94 in good times and 4.90 in bad. This results in higher interest rates in good times, 7.45%, than in bad with the average rate differential being 0.66%. The range in which the capital stock fluctuates is larger than the difference in the average capital stock between good times and bad would indicate as can be seen from Figure 4.3. The minimum capital stock is 4.44 and the maximum is 5.42.
4.1 Equilibrium properties of the benchmark model

Both capital and labour in the aggregate effect the price of both factors as can be seen from equations (2.11) and (2.12). Wages rise (decline) with an increase (decrease) in aggregate capital (labour) while interest rates fall (rise). Both factors greatly effect each other. The elasticity of wages with regard to aggregate capital is $\eta_{w,K} = 0.29$. Additionally, the elasticity of working hours with regard to aggregate capital is small or $\eta_{h,K} = 0.05$ while the elasticity of employment with regard to aggregate capital is $\eta_{e,K} = 0.06$.

<table>
<thead>
<tr>
<th></th>
<th>$r$</th>
<th>$w$</th>
<th>$K$</th>
<th>$N$</th>
<th>$C$</th>
<th>$Y$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>good times</td>
<td>0.074</td>
<td>1.677</td>
<td>4.815</td>
<td>0.376</td>
<td>0.481</td>
<td>0.987</td>
<td>-64.918</td>
</tr>
<tr>
<td>bad times</td>
<td>0.069</td>
<td>1.553</td>
<td>4.749</td>
<td>0.373</td>
<td>0.469</td>
<td>0.907</td>
<td>-65.174</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\eta_{w,K}$</th>
<th>$\eta_{h,K}$</th>
<th>$\eta_{e,K}$</th>
<th>$\eta_{C,K}$</th>
<th>$\sigma_K$</th>
<th>$\sigma_C$</th>
<th>$\sigma_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark case</td>
<td>0.29</td>
<td>0.05</td>
<td>0.06</td>
<td>0.08</td>
<td>0.1630</td>
<td>0.0295</td>
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<td>empirical value</td>
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<td>0.0402</td>
<td>0.0412</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3: Key aggregate statistics of the benchmark case

Turning to output then it results from the discussion above that output is 8.87% greater in good times than in bad because of the effect a technology shock has on factor prices and the effect that has on household behavior. The volatility of simulated output compares favorably with the empirical one with standard deviation being 4.20% and 4.12%, respectively.
The volatility of simulated consumption is not as great as the empirical one. That may perhaps be explained with the fact that consumption in Iceland is more volatile than in other economies largely because of the exchange rate and how open the the economy is. Consumption in Iceland is very much affected by the the exchange rate. When the real exchange rate is favorable consumption of imports in general and durables in particular goes up and vice versa if the opposite is the case. This amplifies consumption volatility and since this aspect is not modelled here a lower standard deviation of simulated consumption than what is empirically observed results. The standard deviation of simulated consumption is 2.95% while the empirical one is 4.02%.[31].

Consumption fluctuates over the business cycle in response to changes in factor prices and capital holdings. On average there is 2.50% more consumption in good times than in bad. That is not surprising considering that in good times income from labour and capital is 15.02% higher than in bad times. That increase in income mostly benefits high-productivity and high-wealth households that have a high savings ratio already so most of the increase goes to increase savings and in the process raising the aggregate capital stock by nearly 1%. In Table 4.3 key statistics of the benchmark case are displayed.

4.1.4 The Government sector

Government expenditures and revenues are mostly endogenously determined in the model. The expenditures consist of exogenously given public consumption set at 30% of output and endogenously determined unemployment compensations and transfer payments. Tax revenues consist of labour and capital income taxes based on exogenously given tax rates and on a sales tax that is collected in the amount necessary to balance the budget by endogenously determining the sales tax rate. The sales tax rate required is determined for good times and bad. In good times a sales tax rate of 16.09% balances the budget but in bad times a rate of 17.85% is required. The total unemployment compensation paid out in bad times is 2.1 times that what is paid in good times. This results from more than twice as much unemployment in bad times than in good and from the fact that unemployment compensation per household is a lower amount in bad times because of it being a function of the wage rate and hours worked. Regarding transfers on the other hand I find that in bad times total payments are 6.86% higher in bad times than in good.

1Note: All empirical data are HP-filtered with the smoothing parameter set to 100.
4.2 Effects of changing to the new tax system

Counter-cyclical automatic stabilisers
Tax revenues behave as expected with revenues increasing in good times with more robust economic activity and a progressive labour income tax. The average labour income tax rate goes from being 32.96% in bad times to 33.32% in good times. Hence, it results that tax revenues are 15.1% higher in good times than in bad times at the same time that expenditures are 13.7% lower in good times than in bad and vice versa. The Government should run a surplus in good times and a deficit in bad times. This is typical of fiscal policy since the government purposfully designs the tax and transfer system in such a way that counter-cyclical results through automatic fiscal stabilisation and that is clearly evident in the model simulation.

<table>
<thead>
<tr>
<th></th>
<th>unemploym.</th>
<th>sales tax</th>
<th>ave. transfer payments</th>
<th>labour taxes (%)</th>
<th>capital taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>good times</td>
<td>0.0656</td>
<td>16.32%</td>
<td>0.0134</td>
<td>0.2163(33.32)</td>
<td>0.016</td>
</tr>
<tr>
<td>bad times</td>
<td>0.0607</td>
<td>18.04%</td>
<td>0.0143</td>
<td>0.2155(32.96)</td>
<td>0.013</td>
</tr>
</tbody>
</table>

Table 4.4: Key government statistics of the benchmark case

To sum up
All things considered the model is well behaved and fits the data relatively well, as can be seen in Table 4.2 and 4.3. It is therefore well suited to answer the question posed and that is the subject of next section.

4.2 Effects of changing to the new tax system

The change in the tax system consists of changing only when transfer payments are paid. No parameters are changed. In the new system households get transfer payments based on their current period income instead of their last period income. The change is simulated as follows. First the benchmark model is simulated over 3000 periods to ensure that the endogenous distributions of wealth and productivity depend not on their initial distribution. Then the distribution in period 3000 is used as the distribution in the last period before the change in timing. Then, the model is simulated over yet another 3000 periods. First with the old system continuing to serve as a benchmark and then again with the new system. The differences observed between these two simulations are reported below.
4.2.1 Welfare and policy functions changes

4.2.1.1 Welfare changes

The effect of changing the timing is to some extent summed up in the respective welfare functions. To get a fair comparison the welfare comparison between the two systems it needs to be made in such a way that there are comparable transfer payments in the two systems. So, for each productivity pair a three period productivity sequence is set up that starts and ends at the same productivity but the second period is of the other productivity, e.g. to compare the welfare of an unemployed household that enjoyed the highest-productivity in the previous period before becoming unemployed then the sequence $\epsilon^1 \rightarrow \epsilon^5 \rightarrow \epsilon^1$ ensures comparable transfer payments and income in the last two periods combined between the systems. Only the last two periods are used in the comparison. The differences between households welfare values are plotted in Figure 4.4. The differences are between households with identical wealth and productivity pairs. Those differences only give the static difference in welfare since they do not take into account the dynamic effects that results from the changes in households policy rules. The dynamic effects are analyzed in a subsection below. The static differences plotted in panels a-f in Figure (4.4) are now discussed in turn.

panel a: The welfare differences of unemployed households, over the current and previous period, are plotted for all five possible previous period productivities. What is immediately clear is that low-wealth unemployed households that enjoyed either of the two highest productivities in the previous period are best off in the new system with up to 4% increase in welfare. Greater income smoothing in the new system gives rise to this result. The same goes for those households that had the third highest productivity with welfare increasing up to 2%. The welfare benefits drop quickly, however, as household wealth increases as income smoothing becomes relatively less important. For those households that are unemployed in all periods and have a low-wealth position are worse off by up to 0.5%. In this case there is no difference in the households income streams between the two systems. There is therefore no smoothing advantages today but when the household makes its forward looking expectations it takes into account any possible future income smoothing advantages of the new system. But what outweighs that future advantage for the low-wealth household is the fact that it foresees that in the period the unemployment spell ends it gets one more transfer payment as if it was still unemployed in the old system but not in the new system (transfer payments prior to the previous period are not taken into consideration). The same applies to second lowest productivity household but to a smaller extent. This is true only for low-wealth households since the relative importance of that one
Figure 4.4: Differences in welfare depending on productivities history.

In panel a the welfare differences of unemployed households, over the current and previous period, are plotted for all five possible previous period productivities. The y-axis shows the percentage differences. The same applies to the other panels except for panel f that shows differences when there is no change in productivity.

Extra large transfer payment decreases and as a result welfare gains become marginally positive as wealth increases.

Panel b: The same holds for the welfare differences of the second lowest productivity households as they converge to the same positive values as those of the unemployed households. The convergence, however, is at a slower pace and it starts from a smaller welfare difference.
panels c to e: For the three highest productivity types the welfare difference is always positive regardless of previous period productivity. In all cases they converge so that the greatest welfare gains are to be had in a descending order with respect to previous period productivity from highest productivity to the lowest. This is also the case with respect to current period productivity status. This convergence order for high-wealth households does not hold for low-wealth households and it is in fact almost the opposite. For the highest productivity household the greatest welfare gain goes to the one that was unemployed in the previous period and it amounts to almost 4%. This mirrors the result when going from highest productivity to the lowest as the same logic applies, i.e. there is a benefit from income smoothing. The welfare gain, though, converges quickly with increasing wealth. Again, the same applies to second lowest productivity households but to a smaller extent.

Looking at welfare gains when previous period productivity was among the three highest the results are more staple as these households never get close to being liquidity constrained as some low-wealth low-productivity households. The benefits of income smoothing are therefore not as important since these households are saving in both periods. There are though welfare gains to be had and they lie in the range of 0.2-0.4%. These households derive a benefit from possible future income smoothing benefits if they were, for example, to be hit with an unemployment spell.

panel f: Here the cases were the productivities never change are shown in one graph, i.e. were the sequence is $\epsilon^j \rightarrow \epsilon^j \rightarrow \epsilon^j$, $j=1,\ldots,5$. These five series show the welfare differences in the forward looking expectations since there is no income smoothing benefit in the current period. This graph shows that the timing of transfer payments matters even if there is no change in productivity between periods.

<table>
<thead>
<tr>
<th>$%$</th>
<th>$\epsilon^1 \epsilon^1$</th>
<th>$\epsilon^2 \epsilon^2$</th>
<th>$\epsilon^3 \epsilon^3$</th>
<th>$\epsilon^4 \epsilon^4$</th>
<th>$\epsilon^5 \epsilon^5$</th>
<th>mean</th>
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</thead>
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<tr>
<td>$\epsilon^1 \epsilon^1$</td>
<td>-0.14</td>
<td>-0.11</td>
<td>-0.04</td>
<td>0.05</td>
<td>0.11</td>
<td>-0.03</td>
</tr>
<tr>
<td>$\epsilon^2 \epsilon^2$</td>
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<td>-0.02</td>
<td>0.06</td>
<td>0.12</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\epsilon^3 \epsilon^3$</td>
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<td>-0.02</td>
<td>0.02</td>
<td>0.09</td>
<td>0.14</td>
<td>0.04</td>
</tr>
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<td>0.09</td>
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<td>0.04</td>
<td>0.10</td>
<td>0.15</td>
<td>0.05</td>
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</tbody>
</table>

Table 4.5: Equally weighted averages of differences in welfare

Finally, in Table 4.5 the equally weighted average welfare benefits of households over the capital space are displayed with the current period productivities cat-
4.2 Effects of changing to the new tax system

egorized into rows and previous period productivities into columns. What can be seen from the table is that welfare benefits are increasing in both current and previous period productivity. The greatest welfare benefits go to the most productive current period households that also were the most productive in the previous period while the least benefit goes to the unemployed households that were also unemployed in the previous period. The upper and lower triangles of the table are symmetric since the three period productivity sequence ensures that the productivity sequence between current and previous period does not matter. The average welfare benefit over both the productivity and capital space is 0.05%.

4.2.1.2 Policy function changes

Figure 4.5: Smoothed differences when there is no change in productivity. The y-axis gives the percentage differences (100 grid point moving average) of the respective aggregates over the household’s capital space when there is no change in productivity between periods.

Changes in the policy rules means that households alter their behavior and that leads to the above mentioned changes in welfare. The differences in households policy rules for labour supply, consumption and next-period capital are depicted in Figure 4.5 for the case were the productivities never change, i.e. were again the sequence is $\epsilon^1 \rightarrow \epsilon^j \rightarrow \epsilon^1$, $j=1, \ldots, 5$. Figure 4.5 isolates the effect
that changes in the forward looking expectations of households has on behavior and welfare.

Looking at the panels then first of all it is clear that households with second to fourth productivity all reduce their labour supply by a few percentages. With the reduction increasing with wealth. Secondly, households that have gain the second to fourth productivities reduce their next-period capital by up to 2% with the comparative dissaving decreasing with higher wealth. Third, the differences in consumption are more non-linear in nature. Consumption is a function of the labour supply and savings decisions of the households. The function being optimized by the households is the welfare function which is very well behaved as can be seen from panel d. One also has to take into account that the percentage differences in consumption are from very different levels of consumption between households, e.g. the consumption of the highest productivity households is more than five times that of the unemployed one.

A more detailed discussion of the three policy functions follows below were income smoothing benefits between current and previous period also come into play.

labour supply

When it comes to labour supply the results are clear, nearly all employed households either reduce their labour supply or keep it nearly unchanged regardless of what their previous period productivity was. In Figure B.1 in appendix B it can be seen that even though the degree to which households reduce their labour supply varies the story is very similar in every case, that in response to a change in timing of transfer payments households choose to reduce their labour supply. By doing that households lower their income and that necessarily means that they also must reduce their consumption and/or lower their savings or dissave.

<table>
<thead>
<tr>
<th>%</th>
<th>$\epsilon^1\epsilon^3$</th>
<th>$\epsilon^2\epsilon^3$</th>
<th>$\epsilon^3\epsilon^3$</th>
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<td>-1.42</td>
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<td>-2.57</td>
<td>-1.79</td>
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<td>-2.86</td>
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</tr>
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<td>-1.20</td>
<td>0.02</td>
<td>-0.87</td>
</tr>
<tr>
<td>mean</td>
<td>-0.82</td>
<td>-2.44</td>
<td>-2.25</td>
<td>-2.00</td>
<td>-0.87</td>
<td>-1.68</td>
</tr>
</tbody>
</table>

Table 4.6: Equally weighted averages of differences in hours worked

From the diagonal of Table 4.6 it can be seen that the most productive house-
4.2 Effects of changing to the new tax system

holds do not reduce their labour supply at all while the other employed households all reduce their supply by a similar amount or by 2.4-3.3%. So, if a household was neither the most productive nor unemployed in either the current or previous period - as given by the first and last rows and columns - then labour supply drops by 2.4-3.3% or else by nearly half that. On average then unemployed households and the most productive ones reduce their labour supply by around 0.85% while the other reduce it by 2.0-2.4% resulting in an average equally weighted labour supply reduction of 1.68% over both the productivity and capital space.

next-period capital

Analyzing the effect the change in timing has on next-period capital decision is very important since the savings rate affects the steady-state of the economy. From Table 4.7 it can be seen that the decision of how much to save today is greatly affected by whether or not there is income smoothing effect between current and previous period in the new system since if there is a benefit then households save some of it. The greater the income smoothing, i.e. greater difference in productivity between periods, the more they save as can be seen from the average of the first and fifth row of Table 4.7.

<table>
<thead>
<tr>
<th>%</th>
<th>$\epsilon_1^1\epsilon_1^3$</th>
<th>$\epsilon_2^1\epsilon_3^3$</th>
<th>$\epsilon_3^1\epsilon_3^3$</th>
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<th>$\epsilon_5^1\epsilon_3^3$</th>
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<td>0.34</td>
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<td>0.05</td>
<td>0.35</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Table 4.7: Equally weighted averages of differences in next-period capital

This increase in current period savings weighs against the decision of households to decrease precautionary savings because of foreseeable smoother income profile in the future. On the diagonal of Table 4.7 the productivities do not change and the precautionary effect is therefore isolated. Weighing these two effects together then household saving of current period income smoothing benefit outweighs the drop in precautionary savings with next-period capital being 0.18% higher than before when averaged equally over both the productivity and capital space. But households are not equally distributed over the state space. So whether or not the drop in precautionary savings dominates depends on how often household productivities change over time. If the productivities rarely change then lesser precautionary savings dominates while if the productivities frequently change then savings of current period income smoothing benefits may dominate. The model is calibrated so that the
probabilities of employed households having the same productivity in the next
period lies in the range of 60-80% leaving the lesser precautionary savings to
dominate as will be discussed in a subsection below on the new steady-state
equilibrium.

consumption
The differences in consumption between the two systems are very non-linear
with labour supply and next-period capital switching between the grid points
off the state space. The fluctuation between positive and negative differences
in consumption can be seen from panel b from Figure 4.6 for the $\epsilon^2\epsilon^2$ pair.
Taking averages over the capital space masks how volatile the differences are
but as can be seen from Table 4.8 the differences are more often than not pos-
tive for the top two productivities while they are more often negative for the
three lower productivities. The average percentage increase in consumption
over both the capital and productivity space is 0.01%.

But one has to be careful when interpreting these simple equally weighted aver-
ages over the state spaces because of underlying volatility and non-linearities.

<table>
<thead>
<tr>
<th>%</th>
<th>$\epsilon^1\epsilon^1$</th>
<th>$\epsilon^2\epsilon^1$</th>
<th>$\epsilon^3\epsilon^1$</th>
<th>$\epsilon^4\epsilon^1$</th>
<th>$\epsilon^5\epsilon^1$</th>
<th>mean</th>
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</thead>
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<td>-0.25</td>
<td>0.13</td>
<td>0.17</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 4.8: Equally weighted averages of differences in consumption

To sum up the changes in the three policy functions and to get a better under-
standing of the differences as a function of capital holdings the differences of
all three policy functions are plotted together in Figure 4.6 for the $\epsilon^2\epsilon^2$ pair.
From it the trade off between leisure, consumption and savings can be seen.
The series are of comparable units, i.e. actual differences, were the change in
income, savings, consumption and also transfer payments all sum to zero.
4.2 Effects of changing to the new tax system

4.2.2 The transition period

These changes in policy functions imply that the model economy has a new steady-state equilibrium. The change in the aggregate savings ratio largely determines the new steady-state. To reach the new steady-state the economy goes through a transition period of about 50 periods and after that the differences are almost stationary. In Figure 4.7 the differences between the two simulations are plotted for four aggregates, i.e. labour, consumption, capital and welfare.

From panel b it is evident that households respond to the change by immediately reducing their labour supply and they do so by nearly 0.9% leading to a corresponding drop in labour income. What is interesting is that following that drop in income the households do not immediately reduce their consumption, as can be seen from panel c, and that means that households in the aggregate begin to eat into their savings. Aggregate capital therefore immediately starts the decline towards the new equilibrium, see panel a. As a result of all of this aggregate welfare initially rises marginally by 0.1% and stays positive for 7 periods.

The behavior of households continues to evolve as the capital stock decreases. Labour supply begins to rise again marginally, raising income again. The drop
in labour supply however still remains much lower than before and that leads to a reduction in the capital stock at the same time households begin to reduce their consumption. That slows down the rate that the capital stock is decreasing at. So as the capital stock decreases labour supply keeps rising and consumption keeps falling until the reduction of the capital stock reaches the steady-state and stabilises the economy.

Aggregate welfare drops throughout the transition period. A quick glance at the panels gives us, roughly, that at the end of the transition period welfare has dropped by 0.20%, hours worked by 0.75% and labour supply by 0.40%, consumption by 0.35% and the capital stock by 0.65%. It is evident from these figures that timing of transfer payments does matter. In the next subsection a more detailed discussion of the new equilibrium follows.
4.2 Effects of changing to the new tax system

4.2.3 The new equilibrium properties

After transition over about 50 periods the model economy reaches a new steady-state. It is stochastic on the account of the continual economy wide technology shocks. The changes in the equilibrium levels of the aggregates and the aggregate savings rate, although marginal, are significant.

4.2.3.1 Labour market properties

Aggregate hours worked and labour supplied are 0.7% and 0.4% lower in the new steady-state, respectively. That has a direct effect on the wage rate, income distribution and output. With wage rate being a function of the ratio of capital to labour, see equation (2.12), and with capital dropping by nearly twice as much as labour in the aggregate the wage rate is only 0.12% lower in the new state. The lower wage rate is thus not the primary cause of the reduction in hours worked. Households reduce their hours worked to a different degree because of wage rate heterogeneity. With low wage households reducing their labour supply more than high wage households results in that income inequality increases.

<table>
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<th>$w_kn$</th>
<th>$k$</th>
<th>$\eta_{n,w}$</th>
<th>$\sigma_{n}/\bar{n}$</th>
<th>$\sigma_{cn}/N$</th>
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<td>0.394</td>
<td>0.21</td>
<td>0.289</td>
<td>0.793</td>
</tr>
</tbody>
</table>

Table 4.9: Key labour market statistics of the benchmark case

As can be seen from Table 4.9 average working hours go from 0.282 to 0.280, a 0.7% drop, while working hours of the second lowest productivity households drop by nearly twice that or by 1.2% and highest productivity households only reduce their labour supply by a mere 0.1%, hence the lesser drop in aggregate labour than in hours worked. How much this skews the labour income distribution can be seen in that the Gini coefficient of labour income marginally rises by 0.6%. Further, the variation of working hours increases from before by as much as 1% while the variation of effective labour only increases by 0.6% (see the last two columns of Table 4.9).
It is easily determined how much aggregate output drops because of said drop in aggregate labour. Output is 0.3% lower in the new steady-state because of fewer hours worked.

4.2.3.2 The aggregates and factor prices

The change in aggregate savings ratio during the transition period determines both capital and consumption in the aggregate in the new steady-state. In response to a lesser need for precautionary savings a great majority of households reduce their capital holdings. Because of that the aggregate savings ratio, during the transition period, drops until the new aggregate capital stock is reached. After that it is reached the average aggregate savings ratio over good times and bad becomes the same as before the change but as a percentage of a lower base of income and capital. Importantly, even though the average aggregate savings rate is the same as before households save less in good times and dissave less in bad times than before. The aggregate savings ratio in the first 50 periods as a percentage of income goes from being 2.20% to being 2.08%, on average. But as a percentage of wealth the aggregate savings ratio goes from being 2.37% to being 2.24%, on average, a difference of 0.0125% per period that accumulates over 50 periods to over 0.6%. The change in household savings is very heterogeneous.

<table>
<thead>
<tr>
<th>perctiles</th>
<th>Unemployed</th>
<th>$\epsilon^2$</th>
<th>$\epsilon^3$</th>
<th>$\epsilon^4$</th>
<th>$\epsilon^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% lowest</td>
<td>22.2 (6.6)</td>
<td>6.1 (5.7)</td>
<td>-0.9 (-6.8)</td>
<td>-1.6 (-32.9)</td>
<td>-0.3 (-18.8)</td>
</tr>
<tr>
<td>1 - 2.5%</td>
<td>60.1 (4.6)</td>
<td>10.6 (3.9)</td>
<td>-1.0 (-2.6)</td>
<td>-2.1 (-10.8)</td>
<td>-0.5 (-6.6)</td>
</tr>
<tr>
<td>2.5 - 5.0%</td>
<td>94.4 (2.5)</td>
<td>13.4 (1.6)</td>
<td>-0.9 (-0.8)</td>
<td>-2.0 (-3.5)</td>
<td>-0.4 (-2.4)</td>
</tr>
<tr>
<td>5.0 - 10%</td>
<td>103.7 (1.4)</td>
<td>17.9 (0.9)</td>
<td>-0.8 (-0.4)</td>
<td>-2.0 (-1.7)</td>
<td>-0.5 (-1.3)</td>
</tr>
<tr>
<td>10 - 25%</td>
<td>68.4 (0.6)</td>
<td>18.8 (0.4)</td>
<td>-0.6 (-0.1)</td>
<td>-2.0 (-0.7)</td>
<td>-0.6 (-0.6)</td>
</tr>
<tr>
<td>25 - 50%</td>
<td>30.2 (0.3)</td>
<td>13.4 (0.2)</td>
<td>-0.4 (-0.0)</td>
<td>-2.4 (-0.3)</td>
<td>-0.7 (-0.3)</td>
</tr>
<tr>
<td>50 - 75%</td>
<td>13.8 (0.1)</td>
<td>8.3 (0.1)</td>
<td>-0.2 (0.0)</td>
<td>-2.6 (-0.2)</td>
<td>-0.8 (-0.2)</td>
</tr>
<tr>
<td>population</td>
<td>32.0 (0.5)</td>
<td>11.0 (0.3)</td>
<td>-0.4 (-0.2)</td>
<td>-2.4 (-0.9)</td>
<td>-0.7 (-0.64)</td>
</tr>
</tbody>
</table>

Table 4.10: Differences in savings ratios as a percentage of income (wealth)

In Table 4.10 the change in saving ratios are given as percentage of income (wealth). The two lowest productivity type households increase their savings ratios, that is they dissave less, while households of the top three productivities reduce their savings. The lowest wealth households change their savings the most, especially in the bottom 1% percentile.
4.2 Effects of changing to the new tax system

The new aggregate steady-state of the capital stock is reached when the savings ratio becomes the same as before and that condition is met when the households have in the aggregate 0.6-0.7% lower capital holdings than before the change. When the aggregate savings ratio has stabilised then the aggregate consumption also stabilises along with labour supply. On average aggregate hours worked are 0.7% fewer in the new steady-state but since low wage households are the ones that reduce their working hours disproportionately aggregate labour supply is only reduced by 0.4%, on average. The fact that there is less aggregate labour supplied at the same time that aggregate capital is reduced is an indication of how disproportional the change in household behavior is since labour supply is a decreasing function of capital. Aggregate consumption is 0.25%, on average, lower in the new steady-state on account of less income. The cost of enjoying more leisure, fewer hours worked, is less aggregate consumption when the aggregate savings rate stays the same. As a result aggregate output is 0.5% lower than it was before the change because of both lower capital and labour in the aggregate.

<table>
<thead>
<tr>
<th></th>
<th>( r )</th>
<th>( w )</th>
<th>( K )</th>
<th>( N )</th>
<th>( C )</th>
<th>( Y )</th>
<th>( V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>good times</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>benchmark case</td>
<td>0.074</td>
<td>1.677</td>
<td>4.815</td>
<td>0.376</td>
<td>0.481</td>
<td>0.987</td>
<td>-64.918</td>
</tr>
<tr>
<td>deviation case</td>
<td>0.075</td>
<td>1.675</td>
<td>4.784</td>
<td>0.375</td>
<td>0.478</td>
<td>0.982</td>
<td>-65.07</td>
</tr>
<tr>
<td>bad times</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>benchmark case</td>
<td>0.069</td>
<td>1.553</td>
<td>4.749</td>
<td>0.373</td>
<td>0.469</td>
<td>0.907</td>
<td>-65.174</td>
</tr>
<tr>
<td>deviation case</td>
<td>0.070</td>
<td>1.551</td>
<td>4.719</td>
<td>0.372</td>
<td>0.467</td>
<td>0.090</td>
<td>-65.339</td>
</tr>
</tbody>
</table>

\[
\begin{array}{cccccccc}
\eta_{w,K} & \eta_{h,K} & \eta_{c,K} & \eta_{c,K} & \sigma_K & \sigma_C & \sigma_Y \\
\text{benchmark case} & 0.29 & 0.05 & 0.06 & 0.08 & 0.1630 & 0.0295 & 0.0420 \\
\text{deviation case} & (0.29) & (0.05) & (0.06) & (0.08) & 0.1627 & 0.0295 & 0.0417 \\
\end{array}
\]

Table 4.11: Key aggregate statistics of the benchmark case

Finally, the standard deviations of consumption, capital and output in the aggregate are all marginally lower after the change in timing as can be seen in Table 4.11 where the key aggregate statistics before and after the change are shown.

4.2.3.3 Government sector

The effect on the Government sector is, in general, that the size of it decreases. Public consumption being a fixed percentage of output is lower by the same
amount as output or by 0.5%. For that reason and the fact that transfer payments are lower in the new steady-state the result is that lower tax revenues are needed to balance the budget. The sales tax rate is therefore lower. Before the change the sales tax rates needed to balance the budget were 16.08% and 17.85% in good times and bad, respectively. But in the new steady-state a sales tax rates of 15.96% and 17.83% in good times and bad, respectively, suffice to balance the budget.

<table>
<thead>
<tr>
<th>good times</th>
<th>unempl. comp. ub</th>
<th>sales average</th>
<th>labour transfer</th>
<th>capital taxes (%)</th>
<th>capital taxes</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark case</td>
<td>0.0649</td>
<td>16.08%</td>
<td>0.0132</td>
<td>0.2163(33.32)</td>
<td>0.016</td>
</tr>
<tr>
<td>deviation case</td>
<td>0.0643</td>
<td>15.96%</td>
<td>0.0128</td>
<td>0.2155(33.35)</td>
<td>0.016</td>
</tr>
<tr>
<td>bad times</td>
<td>unempl. comp. ub</td>
<td>sales average</td>
<td>labour transfer</td>
<td>capital taxes (%)</td>
<td>capital taxes</td>
</tr>
<tr>
<td>benchmark case</td>
<td>0.0606</td>
<td>17.85%</td>
<td>0.0139</td>
<td>0.1913(32.96)</td>
<td>0.014</td>
</tr>
<tr>
<td>deviation case</td>
<td>0.0600</td>
<td>17.83%</td>
<td>0.0140</td>
<td>0.1905(33.00)</td>
<td>0.014</td>
</tr>
</tbody>
</table>

Table 4.12: Key government statistics of the benchmark case

Unemployment compensations are 1.2% lower even though the unemployment rate is exogenously determined since the compensations are a direct function of the wage rate and labour supply of the second lowest productivity households which both are lower in the new steady-state. Transfer payments decrease on the one hand by the same percentage as the unemployment compensations do since transfer payment amounts are a fixed percentage of the unemployment compensation, but on the other hand households labour income is now lower in the aggregate which means that minimum transfer payments are reduced by a lesser amount. From Table 4.12 the effect of the change in timing can be seen directly. Transfer payments in good times are lower than before but they are higher than before in bad times even though transfer amounts per households are lower.

Labour tax revenues are lower in the new steady-state by 0.4% since labour supplied is reduced and the aggregate wage rate is lower than before. Capital tax revenues are only 0.3% lower since higher interest rate counter the reduction of the aggregate capital stock. So even though on accrual basis the Government is indifferent of when it pays the transfer payments households are clearly not indifferent as they change their behavior and in the process reduce the size of the Government and the economy.
4.2 Effects of changing to the new tax system

4.2.3.4 Aggregate welfare effect

A static analysis of the change in the timing of transfer payments would give that a gain in welfare would result from the change. But what happens when the dynamics over time are considered is that the behavioral change of households leads them and the economy down a different path than before. Households need to make their decisions under uncertainty and they are not able to predict the shocks they are subjected to. By socializing further insurance from negative income dynamics a great majority of households decrease their private insurance measures today as the risk has been lowered. Private insurance measures consist of capital accumulation. So less private insurance measures in the aggregate leads to a lower capital stock and with welfare being increasing in capital a drop in aggregate welfare results in future periods. Insurance coverage and the uncertainties of future shocks determines thus the steady-state savings rate and capital holdings in the aggregate. All the aggregates are a function of the aggregate steady-state savings rate but the relationship between aggregate capital and aggregate savings rate defines the steady-state of the economy. The aggregate savings rate is a function of the aggregate capital and vice versa.

Socializing insurance when households are not able to predict individual shocks lowers the aggregate precautinary savings and hence also aggregate capital and welfare in future periods even though it raises aggregate welfare initially. From the simulation I find that the aggregate welfare is 0.25% lower on average in the new steady-state. Which makes the change in timing a mixed blessing at best. For a given household capital holding and productivity status at any given time it is better, in overwhelming majority of cases, to be in the new system at that particular time. But for that same household, in not that distant future, it would be better for it to be in the old system from that period on.

<table>
<thead>
<tr>
<th></th>
<th>all</th>
<th>$\epsilon^1$</th>
<th>$\epsilon^2$</th>
<th>$\epsilon^3$</th>
<th>$\epsilon^4$</th>
<th>$\epsilon^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>benchmark case</td>
<td>-64.70</td>
<td>-66.22</td>
<td>-75.33</td>
<td>-69.88</td>
<td>-61.15</td>
<td>-52.26</td>
</tr>
<tr>
<td>deviation case</td>
<td>-64.86</td>
<td>-66.37</td>
<td>-75.58</td>
<td>-70.09</td>
<td>-61.28</td>
<td>-52.31</td>
</tr>
<tr>
<td>%</td>
<td>-0.25%</td>
<td>-0.22%</td>
<td>-0.32%</td>
<td>-0.31%</td>
<td>-0.21%</td>
<td>-0.09%</td>
</tr>
</tbody>
</table>

Table 4.13: Aggregate welfare of each productivity type

In Table 4.13 the differences in welfare are given in the aggregate and for the individual household productivity type in the period 2998. Highest productivity households are the least negatively affected while the second and third productivity types the most.
As a main result I find that the timing does matter. The economy moves to a new steady-state with a smaller aggregate capital stock, less consumption and more leisure. The incentives have thus been distorted with an important aggregate effect. If the welfare values in some sense sum up the effect then there is net loss from the change in timing with aggregate welfare being lower in the new steady-state. So, in the context of the leaky bucket metaphor then it can be said that there is leakage on account of distorted incentives. Private insurance measures are replaced by Government insurance measures with negative welfare effects and marginal increase in inequality as measured by the Gini coefficient. Importantly, the leakage is a long term phenomena in that it takes around 50 periods for the full effect to materialize according to this model specifics. That makes this finding very difficult to empirically validate.

It can be said that at any given time for almost any given capital holding and productivity status that households that are hit with a negative income shock are better off in the new system viewed statically. That effect supports the ex ante believe that advantages outweigh disadvantages. Who would prefer e.g. to have his unemployment compensation paid out with one period time lag? to point to a similiar situation. But that effect does not hold in a dynamic setting with uncertainties since a few periods later those same households are worse off in the new system because of said change in incentives. Households should rather have accumulated private savings to smooth out negative income shocks rather than rely on Government insurance. Advantages do not outweigh disadvantages in the long term although they do in the short term.
The goal here has been to extend the neoclassical growth model framework to be as general and realistic as possible to answer the question posed. Other authors analyzing the interaction between e.g. taxes and precautionary saving have relied on closed form analytical analysis. Resorting to analytical analysis can be limiting when determining the model specifics, especially in the case of dynamic heterogeneous household models since they generally do not have an analytical solution. I hope the findings of this thesis have further developed the understanding of the implications of Government policies.
Solving the model - computation

The model has no analytical solution. With only very few exceptions, dynamic heterogeneous-household general equilibrium models do not allow for the derivation of analytical results. One therefore needs to resort to numerical optimization. Algorithms to solve heterogeneous-household models with an endogenous distribution of agents have only recently been introduced into the economic literature. The algorithms used to solve the model developed are adopted from various studies in this area. Most notably HEER and TREDE (2001), CASTANEDA ET AL. (1998) and KRUSSEL AND SMITH (1998). The adoption takes into account important extensions in the model and mixes together various properties from the different models in the above mentioned studies.

The computation of the solution consists of three main steps, the computation of the policy functions, the simulation of the distribution dynamics and the aggregation of individual state variables and imposing the aggregate consistency conditions. The complete solution algorithm is described in the following steps with a more detailed description of individual steps to follow.
A.1 Main algorithm

Step 1: Compute the ergodic distribution of the efficiency types.

Step 2: Make initial guesses of the aggregate capital stock \(K\), aggregate employment \(N\) and the sales tax rate \(\tau_c\) that balances the government budget.

Step 3: Compute the wage rate \(w\) and interest rate \(r\).

Step 4: Choose the order 1 of moments \(m\).

Step 5: Guess the parameterized functional form for \(H_I\) in \((x,x)\) and choose the initial parameters of \(H_I\).

Step 6: Initialize the value functions.

Step 7: Solve the household’s five dimensional optimization problem and compute \(V(\epsilon,a,Z,N,m)\) and the decision functions.

Step 8: Simulate the dynamics of the distribution of assets for each efficiency type.

Step 9: Compute hours worked, consumption, income, earnings and transfers for each simulated household.

Step 10: Use the time path for the distribution to estimate the law of motion for the moments \(m\).

Step 11: Compute the aggregate capital stock \(K\), aggregate employment \(N\) and taxes \(T\) that solve the aggregate consistency conditions.

Step 12: Compute the sales tax rate \(\tau_c\) that solves the government budget.

Step 13: Check for convergence of the value functions. Update the variables and return to step 7 if necessary.

Step 14: Check the accuracy of the computed optimal policy function by computing the residual function for the intertemporal and intratemporal first-order conditions. If necessary increase the number of grid points.

In step 7 the optimization problem of the household is solved with value function iteration. For that the state space is discretized. The value function and policy functions are five dimensional objects. To keep the state space grid to a reasonable number of grid points there are only two states of technology, \(Z \in \{Z_g,Z_b\}\), five efficiency types \(i=1,\ldots,5\), five aggregate capital states, an equispaced grid \(K\) of 1000 points for individual assets states, on the interval
[0, $k^{\max}$] and finally it is assumed that the household can only choose discrete values from an equispaced grid $N$ of 100 points from the interval [0,1] for its labour supply. The upper bound on individual capital $k^{\max}$=12 is found never to be binding.

In order to find the maximum of the rhs of the Bellman equation (2.6), an iteration over the next-period capital stock $k^1 \in K$ is required and the optimal labour supply $n \in N$ for every $k \in K$, efficiency type $e^i$, aggregate capital stock and technology state. This amounts to $2 \times 5 \times 1000 \times 100 \times 1000 \times 4 + 2 \times 5 \times 1000 \times 1000$ iterations. To reduce the number of iterations the monotonicity conditions are exploited, i.e. that the value function is a monotone increasing function of assets $k$, that consumption is strictly positive and monotone increasing in $k$ and that labour supply is a monotone decreasing function of assets $k$. More specifically given an optimal next-period capital stock $k^1$ and labour supply $n$ for a particular current-period capital stock $k_i$, efficiency type and aggregate state, the iteration over next-period capital stock for $k_{i+1} > k_i$ starts at the optimal next-periods capital stock $k^1$ for $k_i$ and iterate upwards. Similarly, the iteration over the labour supply $n$ starts at the optimal labour supply for $k_i$ and iterate downwards. The iteration stops if $c \leq 0$.

During the first iterations a high accuracy of the value function and the policy functions is not needed so a lot of computational time is saved by iterating only 10 times over the value function for the first 10 iterations of the algorithm. The number of iterations is increased over the value functions to 20 thereafter until convergence. The algorithm converges after 15 iterations and 30 hours of computational time$^1$.

In the model economy there is aggregate uncertainty and therefore a non-stationary capital distribution. Households need to predict next-period factor prices $w^1$ and $r^1$, which are functions of both aggregate capital $K^1$ and aggregate employment $N^1$ as well as exogenous technology level $Z^1$. In order to predict the aggregate capital stock $K^1$, households need to know the dynamics of capital distribution. Households are boundedly rational and only use partial information about the distribution, namely its first $m$ moments. In accordance with CASTANEDA ET AL.(1998), $m = 1$ is chosen. A simple parameterized functional form for $m^1 = H_I(m,Z)$ is also chosen following KRUSSEL and SMITH (1998):

$$\ln K^1 = \begin{cases} \gamma_0 g + \gamma_1 g \ln K & \text{if } Z = Z_g, \\ \gamma_0 b + \gamma_1 b \ln K & \text{if } Z = Z_b \end{cases} \quad (A.1)$$

$^1$On a Intel 2.66Ghz quad-processor computer
As the aggregate productivity is a stochastic variable, the economy needs to be simulated. Again following KRUSSEL and SMITH (1998) and using 5000 households in order to approximate the population. A uniform initial distribution of assets $k$ is chosen equal to the average capital stock of the economy and then the efficiency types are distributed in accordance with their ergodic distribution. The dynamics of the economy are simulated over 3000 periods and the first 100 or so are discarded so that the initial distribution of assets does not have any effect on the regression results. The law of motion for aggregate capital in good and bad times respectively is estimated by separating the simulated observation points into two subsamples and estimate the parameters \( \{ \gamma_0, \gamma_1 \} \) with ols regression.

But to simulate the economy the optimal policy functions computed before with value function iteration are used. The optimal next-period asset level $k'$ is, of course, a function of efficiency type status, current-period assets $k$, the aggregate productivity level $Z$, the aggregate capital stock $K$ and the aggregate employment level $N$: $k' = k'(\epsilon, k, Z, m, N)$. The individual asset level $k$ and the aggregate capital stock $K$ need not be a grid point $k_i$ or $K_j$, respectively, during simulation of the economy. Therefore, one has to use bilinear interpolation in order to compute the optimal next-period asset level $k'$ off grid points in the simulation.

The law of large numbers on the simulation results needs to be imposed. That is, while the behavior of 5000 agents is tracked, the fraction of each efficiency type needs to match the ergodic distribution of the efficiency types in both good times and bad times. A random number generator is used in order to simulate the motion of the individuals efficiency status according to their appropriate conditional probabilities. In each period $t$, the fraction of each efficiency type is checked to see if it is equal to the ergodic distribution fractions. If not, households are chosen at random and their efficiency status is changed accordingly to impose the law of large numbers.

Given the results of the households savings decisions, one has the household consumption, income, earnings, hours worked and transfers for each simulated household for the optimal policy rules. Aggregating the simulated households the time paths for the above mentioned variables are generated. Also, the aggregate capital stock $K$, aggregate employment $N$ and taxes $T$ need to be computed to solve for the aggregate consistency conditions. Given the aggregates one solves for the sales tax that balances the government budget over the business cycle. The variables are updated before iterating again over steps 7 to 12.
Appendix B

Figures
Figure B.1: Differences in labour supply depending on productivities history.

In panel a the welfare differences of unemployed households, over the current and previous period, are plotted for all five possible previous period productivities. The y-axis shows the percentage differences. The same applies to the other panels except for panel f that shows differences when there is no change in productivity. Mind the scales of the x and y-axis and the fact that these are moving average series where more often than not values are switching between being zero or not. Series are smoothed to make them easier to compare. Even though the differences are somewhat non-linear the labour supply profiles are well behaved.
Figure B.2: Differences in consumption depending on productivities history.
In panel a the welfare differences of unemployed households, over the current and previous period, are plotted for all five possible previous period productivities. The y-axis shows the percentage differences. The same applies to the other panels except for panel f that shows differences when there is no change in productivity. Mind the scales of the x and y-axis and the fact that these are moving average series where more often than not values are switching between being zero or not. Series are smoothed for easier comparison. Series are smoothed to make them easier to compare. Even though the differences are very non-linear the consumption profiles are well behaved.


