Concrete walls founded on earthquake areas

Helgi S. Ólafsson

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Abstract

Calculations made by structural engineers of structural models are almost fixed to the ground (cantilever), even though in reality, the structures are founded on bedrock or soil bed.

In the project, 10 walls are modeled in a structural software program, five squat walls and five slender walls. The walls have different foundations, fixed to the ground (cantilever), founded on bedrock and founded on soil bed (gravel) with and without damping. Earthquake load react under all walls, i.e. time history analysis.

The main object of this project is to show how different foundations change the behavior of walls during seismic activity. The analysis of this behavior requires models that take into account not only the structure, but also the bedrock, soil bed and the dynamic interaction forces existing between them.

The wall and support were modeled using finite element method (FEM) and boundary element method (BEM). The calculations were carried out using the software ANSYS. The soil-structure interaction (SSI), model was separated into near-field and far-field using FEM to represent the near-field, and viscous dampeners placed around the FEM model as boundary elements to represent the far-field.

The results show that the effects of rocking the walls changes their behavior profoundly; displacements increased while the stress and shear forces diminished.

A careful estimate would be that walls on bedrock are 25% less reinforced than fixed walls, and the walls on gravel bed are 50% less reinforced than fixed walls.
Preface

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Helgi S. Ólafsson
Concrete walls founded on earthquake areas

Masters thesis in Structural engineering
Helgi S. Ólafsson 5

Table of contents

Abstract ............................................................................................................................. 3
Preface ............................................................................................................................... 4
List of Figures ................................................................................................................... 8
List of Table .................................................................................................................... 10
List of Graphs ................................................................................................................. 10
Nation Index ................................................................................................................... 11

1. Introduction ............................................................................................................. 15
   1.1. The European standard for earthquakes ................................................................. 17
   1.2. Objective .................................................................................................................. 18
   1.3. Overview ................................................................................................................... 19

2. Earthquakes ............................................................................................................. 20
   2.1. Introduction ................................................................................................................ 20
   2.2. Causes of earthquakes ................................................................................................ 20
   2.3. Seismic Waves .......................................................................................................... 23
       2.3.1. Body waves .............................................................................................................. 23
       2.3.2. Surface waves .......................................................................................................... 26
   2.4. Earthquake magnitude ............................................................................................. 27
   2.5. Earthquakes in Iceland ............................................................................................. 28
   2.6. Time history ............................................................................................................... 30
   2.7. Prevent earthquake damages to structures ............................................................... 31

3. Soil-structure interaction theory ........................................................................... 34
   3.1. SSI introduction ........................................................................................................ 34
   3.2. Equation of motion.................................................................................................... 35
   3.3. Illustration of Soil-Structure Interaction .................................................................. 37
   3.4. Formulation of a Soil-Structure Interaction .............................................................. 41
       3.4.1. Near-field, finite element method FEM ................................................................. 42
       3.4.2. Far-field, boundary element method BEM ........................................................... 42

4. The computer program, ANSYS ........................................................................... 45
   4.1. Introduction ............................................................................................................... 45
   4.2. Concrete element ...................................................................................................... 46
Concrete walls founded on earthquake areas

4.3. Soil and rock elements ....................................................................................................... 47
4.4. Contact elements ............................................................................................................. 48
4.5. Damping elements .......................................................................................................... 50

5. Modeling of walls and foundations ............................................................................... 51
   5.1. Introduction .................................................................................................................. 51
   5.2. Squat wall models ........................................................................................................ 52
   5.3. Slender wall models ..................................................................................................... 53
   5.4. Material properties ....................................................................................................... 55

6. Analysis results ................................................................................................................. 58
   6.1. Introduction .................................................................................................................. 58
   6.2. Results for the squat walls ......................................................................................... 59
       6.2.1. Squat wall fixed on ground .................................................................................... 59
       6.2.2. Squat wall on bedrock .......................................................................................... 59
       6.2.3. Squat wall on bedrock with damping ....................................................................... 60
       6.2.4. Squat wall on gravel bed ....................................................................................... 60
       6.2.5. Squat wall on gravel bed with damping ................................................................. 61
   6.3. Results for the slender walls ...................................................................................... 63
       6.3.1. Slender wall fixed on ground ............................................................................... 63
       6.3.2. Slender wall on bedrock ....................................................................................... 64
       6.3.3. Slender wall on bedrock with damping ................................................................. 65
       6.3.4. Slender wall on gravel bed .................................................................................... 66
       6.3.5. Slender wall on gravel bed with damping ............................................................. 67

7. Discussion ......................................................................................................................... 69
   7.1. Further research .......................................................................................................... 71

8. References ....................................................................................................................... 73
9. Appendixes.................................................................................................................. 76

Appendix A - Wall and foundation models in ANSYS ................................................. 76
  Squat Wall .................................................................................................................. 76
  Squat wall cantilever, with time history in x-direction: .............................................. 83
  Squat wall on bedrock, with time history in x-direction: ............................................. 85
  Squat wall on gravel bed, with time history in x-direction: ........................................ 90
  Squat wall on bedrock or gravel bed with damping, with time history in x-direction: ...................................................... 91
  Slender Wall ........................................................................................................... 95
  Slender wall cantilever, with time history in x-direction: ........................................... 97
  Slender wall on bedrock or gravel bed, with time history in x-direction: ....................... 97
  Slender wall on bedrock or gravel bed with damping, with time history in x-direction: ...................................................... 97
  Get a results from ANSYS ....................................................................................... 98

Appendix B - Time history in x-direction, from Hella 2000 ........................................ 100
List of Figures

Figure 1-1. Dual frame-wall structure ................................................................. 16
Figure 1-2. Diagrams for dual frame-wall structure ............................................. 16
Figure 2-1. Global tectonic plate boundaries ....................................................... 20
Figure 2-2. Worldwide earthquake distribution .................................................. 21
Figure 2-3. Definitions of earthquake sources location ......................................... 22
Figure 2-4. Faulting and landform ................................................................... 22
Figure 2-5. Ground motion near the ground surface due to P-waves .................. 23
Figure 2-6. Ground motion near the ground surface due to S-waves .................. 24
Figure 2-7. Ground motion near the ground surface due to Love waves ............. 26
Figure 2-8. Ground motion near the ground surface due to Rayleigh waves ....... 26
Figure 2-9. The South Iceland seismic zone ....................................................... 28
Figure 2-10. Simple rules for plan layouts of aseismic buildings ......................... 32
Figure 2-11. Simple rules for elevation shapes of aseismic buildings .................. 33
Figure 2-12. Simple rules for vertical frames in aseismic buildings ...................... 33
Figure 3-1. Single-degree of freedom systems subjected to base shaking .......... 35
Figure 3-2. Model with three dynamic of freedom ............................................. 37
Figure 3-3. A soil-structure interaction system .................................................. 41
Figure 3-4. Forces acting on unit cube .............................................................. 42
Figure 3-5. Viscous boundary considered in the 3D finite element model .......... 43
Figure 4-1. Solid65 ............................................................................................ 46
Figure 4-2. Solid45 ............................................................................................ 47
Figure 4-3. Conta174 ....................................................................................... 48
Figure 4-4. TARGE170 .................................................................................... 49
Figure 4-5. MATRIX27 .................................................................................... 50
Figure 4-6. MATRIX27 .................................................................................... 50
Figure 5-1. Squat wall fixed on ground (cantilever) ............................................ 52
Figure 5-2. Squat walls on bedrock (Basalt), with and with out damping .......... 52
Figure 5-3. Squat walls on 1.0m thick mean gravel bed, with and with out damping .... 52
Figure 5-4. Slender wall fixed on ground (cantilever) ......................................... 53
Figure 5-5. Slender walls on bedrock (Basalt), with and with out damping ........ 54
Figure 5-6. Slender walls on 1.0m thick mean gravel bed, with and with out damping .... 54
Figure 5-7. Bedrock, Basalt ............................................................................. 55
Figure 5-8. Soil, mean gravel bed ..................................................................... 55
Concrete walls founded on earthquake areas

**Figure 6-1.** Nodes to monitor displacements. .................................................58
**Figure 6-2.** Squat wall fixed on ground, displacement in x-direction. .................59
**Figure 6-3.** Squat wall fixed on ground, displacement in z-direction.
**Figure 6-4.** Squat wall fixed on ground, stress in x-direction.
**Figure 6-5.** Squat wall fixed on ground, stress in z-direction.
**Figure 6-6.** Squat wall on bedrock, displacement in x-direction. .......................59
**Figure 6-7.** Squat wall on bedrock, displacement in z-direction.
**Figure 6-8.** Squat wall on bedrock, stress in x-direction.
**Figure 6-9.** Squat wall on bedrock, stress in z-direction.
**Figure 6-10.** Squat wall on bedrock with damping, displacement in x-direction. ....60
**Figure 6-11.** Squat wall on bedrock with damping, displacement in z-direction.
**Figure 6-12.** Squat wall on bedrock with damping, stress in x-direction.
**Figure 6-13.** Squat wall on bedrock with damping, stress in z-direction.
**Figure 6-14.** Squat wall on gravel bed, displacement in x-direction. ...................60
**Figure 6-15.** Squat wall on gravel bed, displacement in z-direction.
**Figure 6-16.** Squat wall on gravel bed, stress in x-direction.
**Figure 6-17.** Squat wall on gravel bed, stress in z-direction.
**Figure 6-18.** Squat wall on gravel bed with damping, displacement in x-direction. ....61
**Figure 6-19.** Squat wall on gravel bed with damping, displacement in z-direction.
**Figure 6-20.** Squat wall on gravel bed with damping, stress in x-direction.
**Figure 6-21.** Squat wall on gravel bed with damping, stress in z-direction.
**Figure 6-22.** Slender wall fixed on ground, displacement in x-direction. ...............63
**Figure 6-23.** Slender wall fixed on ground, displacement in z-direction.
**Figure 6-24.** Slender wall fixed on ground, stress in x-direction.
**Figure 6-25.** Slender wall fixed on ground, stress in z-direction.
**Figure 6-26.** Slender wall on bedrock, displacement in x-direction. .....................64
**Figure 6-27.** Slender wall on bedrock, displacement in z-direction.
**Figure 6-28.** Slender wall on bedrock, stress in x-direction.
**Figure 6-29.** Slender wall on bedrock, stress in z-direction.
**Figure 6-30.** Slender wall on bedrock with damping, displacement in x-direction. .....65
**Figure 6-31.** Slender wall on bedrock with damping, displacement in z-direction.
**Figure 6-32.** Slender wall on bedrock with damping, stress in x-direction.
**Figure 6-33.** Slender wall on bedrock with damping, stress in z-direction.
**Figure 6-34.** Slender wall on gravel bed, displacement in x-direction. .................66
**Figure 6-35.** Slender wall on gravel bed, displacement in z-direction.
**Figure 6-36.** Slender wall on gravel bed, stress in x-direction.
**Figure 6-37.** Slender wall on gravel bed, stress in z-direction.
Concrete walls founded on earthquake areas

Figure 6-38. Slender wall on gravel bed with damping, displacement in x-direction. .......................... 67
Figure 6-39. Slender wall on gravel bed with damping, displacement in z-direction.
Figure 6-40. Slender wall on gravel bed with damping, stress in x-direction.
Figure 6-41. Slender wall on gravel bed with damping, stress in z-direction.
Figure 7-1. Proposal of concrete house. .................................................................................................. 71
Figure 7-2. Structure, with shear wall, beam and column. ................................................................. 72

List of Tables

Table 2-1. Properties of major magnitude scales. .................................................................................. 27
Table 5-1. Parameters for squat and slender walls and foundation. ..................................................... 56
Table 5-2. Parameters for foundation. ...................................................................................................... 56
Table 6-1. List of maximum displacement for squat walls. ................................................................. 62
Table 6-2. List of maximum stresses for squat walls. .............................................................................. 62
Table 6-3. List of total shear-forces for squat walls. ............................................................................. 62
Table 6-4. List of maximum displacement for slender walls. .............................................................. 68
Table 6-5. List of maximum stresses for slender walls. ......................................................................... 68
Table 6-6. List of total shear-forces for slender walls.......................................................................... 68

List of Graphs

Graph 2-1. Time history from Hella 2000. .............................................................................................. 30
Natation Index

Roman upper and lower case

$A$ = Fault rupture area, (misgengi brotsvæðis).
$A_n, A_{11}, A_{12}$ = Viscous dampers controlling fields, (segju dempunarsvæði).
$A_{bar}$ = Cross section of reinforcement, (þversniðsflatarmál járnbendingar).
$A_{wall}$ = Cross section of wall, (þversniðsflatarmál veggjar).
$a$ = Radius of circular footing, (radius (geisli) hringlaga undirstöðu)
$[C]$ = Damping matrix, (dempunarfylki).
$c$ = Damping coefficient (force per velocity), (dempunarstuðull).
$c_g$ = Cohesion, (samloðun).
$E$ = Modulus of elasticity, (fjaðurstuðull).
$f_c$ = Compressive strength for concrete, (þrýstistyrkur steypu).
$f_t$ = Tensile strength for concrete, (togstyrkur steypu).
$f_y$ = Yield strength for reinforcement, (flotstyrkur járnbendingar).
$G$ = Shear modulus, (skerstuðull).
$g$ = Acceleration of gravity, (þyngdarhröðun), $g = 9.81$ (m/s$^2$).
$h$ = Height, (hæð).
$i$ = Complex number, (tvinntala), $i^2 = -1$.
$[K]$ = Stiffness matrix, (stifleikafylki).
$k$ = Spring coefficient, (dempunarstuðull).
$l$ = Length, (lengd).
$l_t$ = Total length of wall, (heildarlengd veggjar).
$M_0$ = Seismic moment, (skjálftavægi).
$M_L$ = Local or Richter magnitude, (staðbundinn eða Richter styrkur).
$M_S$ = Surface magnitude, (yfirborðsstyrkur).
$M_W$ = Moment magnitude, (vægisstyrkur).
$m$ = Mass, (massi).
$m_b$ = Body wave magnitude, (P- og S- bylgju styrkur).
$P$ = Force (kraftur).
$Q$ = Load, (álag).
$q$ = Distributed load (jafndreiftálag).
$\{R\}$ = Load vector (álagsvigur).
$t$ = Wall thickness, (þykkt veggjar).
$u$ = Displacement, (færsla).
\( \ddot{u} \) = Velocity, (hraði).
\( \dddot{u} \) = Acceleration, (hröðun).
\( u_{ed} \) = Elastic deformation, (formbreyting á fjaðursviði).
\( u_f \) = Displacement of foundation, (færsla undirstöðu).
\( u_g \) = Ground displacement, (færsla jarðar).
\( v_p \) = Wave propagation velocity, (P-bylgju hraði).
\( v_s \) = Shear wave velocity, (S-bylgju hraði).
\( u_t \) = Total displacement, (heildar færsla).
\( \Delta u \) = Average slip between opposite side of fault, (meðal misgengi milli gagnstæðrar brotlínu).

**Greek lower case**

\( \zeta \) = Material damping, (efnis dempun).
\( \theta_f \) = Foundation rotation, (snúningur undirstöðu).
\( \theta_g \) = Ground rotation, (snúningur jarðar).
\( \lambda \) = Lame parameter, (Lame kennistærð).
\( \mu \) = Friction coefficient (núningsstuðull).
\( \nu \) = Poisson’s ratio, (Poisson hlutfall).
\( \rho \) = Mass density, (rúmþyngd).
\( \sigma \) = Normal stress (spenna).
\( \tau \) = Shear stress (skerspenna).
\( \omega \) = Frequency, (tiðni).
\( \chi \) = Reinforcement ratio, (járnahlutfall).

**Units**

\[ 1 \text{kg} \cdot 9,81 \text{m/s}^2 = 9,81 \text{N} \]
10000 N = 10 kN ≈ 1 ton = 1000 kg
Force and load: \( N \), \( N/m \), \( N/m^2 \), \( kN \), \( kN/m \), \( kN/m^2 \)

Unit mass: \( kg/m^3 \)

Unit weight: \( N/m^3 \), \( kN/m^3 \)

Stress and strength: \( N/mm^2 = MN/m^2 = MPa \)
Moment: \( Nm \), \( kNm \)
Glossary

Bedrock, (klöpp).

Boundary Element Method (BEM), (jaðarbúta aðferð): A numerical methods based on boundary integral equations, are very well suited for dynamic soil-structure interaction problems. The basic difference between BEM and FEM; is the fact that BEM only needs to solve the unknowns on the boundaries, whereas FEM solves for a chosen region of space and requires a boundary condition bounding that region.

Damping, (dempun): The force or energy lost in the process of material deformation.

Dashpot: A damper which resists motion via viscous friction.

Earthquake, (jarðskjálfti): The vibration of earth produced by the rapid release of accumulated energy in elastically-strained rocks.

Epicenter, (skjálftamðja): The projection on the surface of the Earth directly above the focus or hypocenter.

Far-field, (fjarsviðsáhrif): Beyond near-field. A zone of the Earth’s crust within which the two sides have moved; faults may be hundreds of kilometers long, from one to over 160 kilometers deep, and not readily apparent on the ground surface.


Focus, (skjalftauptök): The point of origin of an earthquake.

Free-field, (frjálssviðsáhrif): The motion that would occur in rock or soil in the absence of the structure.

Gravel bed, (malar fylling).

Hollow concrete block, (holsteinn).

Hypocenter (same as focus): The location of initial radiation of seismic waves (i.e., the first location of dynamic rupture).

Near-field, (nærsviðsáhrif): Within one source dimension of the epicenter, where source dimension refers to the length or width of faulting, whichever is less.

Normal fault, (siggengi).
Peak ground acceleration (PGA), (hámarks jarðhröðun): Maximum recorded acceleration amplitude.

Peak ground velocity (PGV), (hámarks jarðhraði).

Peak ground displacement (PGD), (hármarks jarðfærsla).

Reverse fault, (samgengi).

Soil-structure interaction (SSI), (samverkun jarðvegs og mannvirkis): The coupling between a structure and its supporting medium (bedrock or soil bed) during an earthquake.

Strike-slip fault, (sniðgengi).

Tectonic, (jarðhnik): Relating to, causing, or resulting from structural deformation of the Earth’s crust.

Time history, (tímaröð): A series of values (describing a strong-motion parameter) as a function of time.
1. Introduction

The response of soil to earthquake excitation is highly complex and depends on a large range of factors, many of which cannot be established with any certainty.

In the field of civil engineering, nearly all projects are built on, or into the ground. Buildings, bridges, highways, tunnels, walls, towers, masts, canals or dams must be founded in, or on the surface of Earth.

It is common in the field of structural engineering to rigidly fix the foundation of a structure to the ground while carrying out design calculations. This is done to make calculations easier and to deliver quick solutions for static load cases and design combinations. For such analysis, fixed approach is usually acceptable. However, during earthquakes, fixed-ground calculations do not depict the actual behavior of the structure.

The purpose of this project is to analyse concrete walls on different foundations. It has been shown in Southern Iceland that earthquakes in the year 2000 of moment magnitude $M_w=6.6$ and year 2008 of $M_w=6.3$, less reinforced structures on bedrock and gravel bed escaped with less damage than would have been expected. The earthquake forces had less impact on these structures because of the translation and rocking the foundation allowed [23, 33]. When the effect of the earthquakes is assessed, and the destruction following in their wake is examined, it can be said the southern part of Iceland came through amazingly well and better than one would have expected, with no residential buildings collapsing [30].
It has been shown that different fixed walls change moments and displacements as figure 1-2 shows.

These diagrams can be considered realistic only if the wall is completely fixed to the ground. Otherwise, the fixed-end moment of the shear wall is further reduced without any significant reduction of the shear forces while the moments of the beams coupling the frame and the wall are increased. The diagrams on figure 1-2 correspond to 100%, 50% and 25% rotational restraint of the fixed end. The fact that shear wall failures during strong earthquakes are almost always due to shear (exhibit X-shaped cracks) and seldom to flexure, should be attributed to the elastic rotation of the foundations [18].

Figure 1-1. Dual frame-wall structure.

Figure 1-2. Moment (kNm) and displacement (mm) diagrams for dual frame-wall structure in figure 1-1 for different fixity. (a) Wall fully fixed on the ground. (b) Wall flexibly supported on the ground with 50% fixity. (c) Wall flexibly supported on the ground with 25% fixity.
1.1. The European standard for earthquakes

In Eurocode 8 part 1, one note about soil-structure interaction, reads: “Special attention should be paid if soils have very low values of $v_S$, low internal damping and an abnormally extended range of linear behaviour and can therefore produce anomalous seismic site amplification and soil-structure interaction effects”.

In Eurocode 8 part 5 are a few notes on when the effects of dynamic soil-structure interaction should be taken into account:

- Structures where P-δ effects play a significant role.
- Structures with massive or deep-seated foundations, such as bridge piers, offshore caissons, and silos.
- Slender tall structures, such as towers and chimneys.
- Structures supported on very soft soils, with average shear wave velocity $v_{s,max}$ less than 100 m/s.

As a result of dynamic soil-structure interaction, the seismic response of a flexibly-supported structure, i.e. a structure founded on deformable ground, will differ in several ways from that of the same structure founded on fixed base and subjected to an identical free-field excitation, for the following reasons:

- The foundation motion of the flexibly-supported structure will differ from the free field motion and may include an important rocking component of the fixed-base structure.
- The fundamental period of vibration of the flexibly-supported structure will be longer than that of the fixed-base structure.
- The natural periods, mode shapes and modal participation factors of the flexibly-supported structure will be different from those of the fixed-base structure.
- The overall damping of the flexibly-supported structure will include both the radiation and the internal damping generated at the soil-foundation interface, in addition to the damping associated with the superstructure.

For the majority of common building structures, the effects of soil-structure interaction (SSI), tend to be beneficial, since they reduce the bending moments and shear forces in the various members of the superstructure.
1.2. Objective

This study focuses on shear walls and was conducted to study the effects of allowing them to rock and translate on foundations. For most of these studies, the focus is on near-field and far-field effects on the seismic input. The most complicated part of these calculations is to accurately depict the motion at the site. The motion also varies with each seismic input with different frequencies.

The studies used the BEM-FEM modeling technique. The finite element method (FEM) was used to represent the near-field and the boundary element method (BEM) to model the continuity of the bedrock and soil bed. However for the BEM method many assumptions must be made, so the results depend on those. Furthermore, it must be taken into consideration that radiation effect from the seismic input is dissipated through the hysteresis forming of the soil while using non-linear behavior, instead of truncating the wave at the boundaries of the model with dashpots or using silent boundaries. This excludes the availability for modal analysis, but in this thesis the focus is on transient response.

The purpose of the project is to answer the following questions:

1. Will there be more displacement on a wall founded on bedrock and gravel bed than cantilever?

2. Is there a difference in stress and shear forces with a wall cantilever or founded on bedrock or gravel bed?

3. Is there a difference between the reinforcement in a wall founded on bedrock and gravel bed versus cantilever?
1.3. Overview

The thesis is divided to seven chapters, with the main chapters as follows:

Second chapter: The causes of earthquakes and different types of seismic waves.

Third chapter: Soil-structure interaction theory.

Fourth chapter: The elements of the computer software used for modelling all the walls and foundations.

Fifth chapter: Modeling of the walls and foundations in ANSYS.

Sixth chapter: Analysis results.

Seventh chapter: Discussion and further work.

The thesis also contains two appendixes.
2. Earthquakes

2.1. Introduction

This chapter provides the causes of earthquakes, why an earthquake occurs and the different types of seismic waves; body waves and surface waves. The propagation velocities of P- and S-waves are involved in the wall models. The time history graph is in this chapter, and will be used in the wall models in chapter 5. Also some items to prevent damages of structures caused by earthquakes.

2.2. Causes of earthquakes

Earthquake motion is caused by the quick release of stored potential energy into the kinetic energy of motion. Earthquake occurrences may be explained by the theory of large-scale tectonic processes, referred to as plate tectonics [1]. These plates are driven by the convective motion of the material in the Earth’s mantle, which in turn is driven by heat generated at the Earth’s core [14]. The Earth’s crust is divided into about 15 large plates. The continental-sized plates are the African, American, Antarctic, Australian-Indian, Eurasian and Pacific. The major plates are shown in Figure 2-1.

![Figure 2-1. Global tectonic plate boundaries. Arrows indicate directions of plate movement.](image_url)
The plates either rubbed together on the side or thrust from each other, so that new crust is formed. The plates can also squeeze each other so that the old crust melts again. On all of these plate junctions, tension that rises is resolved by an earthquake.

![Figure 2-2. Worldwide earthquake distribution.](image)

When an earthquake occurs, seismic waves radiate away from the source and travel rapidly through the earth’s crust. When these waves reach the ground surface, the produce shaking that may last from seconds to minutes.

The point at which rupture begins and the first seismic waves originate is called the focus or hypocenter of the earthquake. From the focus, the rupture spread across the fault [2]. Although fault rupture can extend to the ground surface, the focus is located at some focal depth below the ground surface. The point on the ground surface directly above the focus is called the epicenter. The distance on the ground surface between an observer or site and the epicenter is known as the epicenter distance and the distance between the observer and the focus is called the focal distance [9]. It is convenient to view figure 2-3 in this context.
There are three main types of faults, based on how adjacent blocks of rock move relative to each other. The three basic fault types are shown in figure 2-4. The normal fault; where one block of rock drops down relative to the other. The strike-slip fault; where the fault blocks slide horizontally past each other. And the reverse fault, where one fault block moves upward relative to the other.

Other terminology includes; near-field, within one source dimension of the epicenter, where source dimension refers to the width or length of faulting, whichever is shorter and far-field, beyond near-field [14]. Further discussed of near-field and far-field in section 3.
2.3. Seismic Waves

Earthquake shaking is generated by two types of elastic seismic waves: body and surface waves. The shaking felt is generally a combination of these waves, especially at small distance from the source or near-field [1].

2.3.1. Body waves

Body waves travel through the Earth’s interior layers, two kinds of body waves exist. They include longitudinal or primary waves, also known as “P-waves” and transverse or secondary waves, also called “S-waves”.

P-waves cause alternate push or compression and pull or tension in the rock as shown in figure 2-5. These P-waves, just like sound waves, are able to travel through both solid rock, such as granite mountains, and liquid material such as volcanic magma or the water of the oceans [5]. P-waves are seismic waves with relatively little damage potential. In most earthquakes, the P waves are felt first.

![Figure 2-5. Ground motion near the ground surface due to P-waves. High velocity long waves compression and extension.](image-url)
S-waves propagation, by contrast, causes vertical and horizontal side-to-side motion, which can cause significant damage. Such waves introduce shear stresses in the rock along their paths as displayed in figure 2-6. S-waves are thus also defined as shear waves, they are analogous to electromagnetic waves, show large amplitudes and long periods, and cannot propagate in fluids [1].

![Figure 2-6. Ground motion near the ground surface due to S-waves. High velocity shear waves.](image)

Body waves (P and S) are named after their arrival time as measured by seismographs at observation sites. P-waves travel faster, at speed between 1.5 - 8 kilometers per seconds while S-waves are slower, usually traveling at 50% of the speed of P-waves [1].

The actual speed of P and S seismic waves depends on the density and elastic properties of the rocks and soil through which they pass [5].
Body waves may be described by Navier’s equation for an infinite, homogeneous, isotropic, elastic medium in the absence of body forces [1]. The propagation velocities of P- and S-waves within an isotropic elastic medium denoted as $v_p$ and $v_s$ respectively, are as follows:

$$v_p = \sqrt{\frac{E \cdot (1-v)}{\rho \cdot (1+\nu) \cdot (1-2\cdot\nu)}} \quad (2.1)$$

$$v_s = \sqrt{\frac{E}{2 \cdot \rho \cdot (1+\nu)}} \quad (2.2)$$

Where, $v_p$ = Wave propagation velocity.
$v_s$ = Shear wave velocity.
$E$ = Modulus of elasticity.
$\rho$ = Mass density.
$\nu$ = Poisson’s ratio.

The ratio of P- and S-wave velocities is as follows:

$$\frac{v_s}{v_p} = \sqrt{\frac{1-2\cdot\nu}{2\cdot(1-\nu)}} \quad (2.3)$$

For $\nu$-values, Poisson’s ratio characterising ordinary soil types, i.e. with $\nu$ ranging between 0.30 and 0.50:

$$0 \leq v_s \leq 0.53 \cdot v_p \quad (2.4)$$
2.3.2. Surface waves

Surface waves in earthquakes can be divided into two types. The first is called a Love wave. Its motion is essentially the same as that of S-waves that have no vertical displacement. It moves the ground from side to side in a horizontal plane parallel to Earth’s surface, but at right angles to the direction of propagation, as can be seen from the illustration in figure 2-7.

![Figure 2-7. Ground motion near the ground surface due to Love waves. Shear waves.](image)

The second type of surface wave is known as a Rayleigh wave. Like rolling ocean waves, the pieces of rock disturbed by a Rayleigh wave move both vertically and horizontally in a vertical plane pointed in the direction in which the waves are travelling. As shown by the arrows in figure 2-8. Each piece of rock moves in an ellipse as the wave passes.

![Figure 2-8. Ground motion near the ground surface due to Rayleigh waves. Which resemble ordinary water waves except particle movement is reversed.](image)

Surface waves travel more slowly than body waves, and of the two surface waves, Love waves generally travel faster than Rayleigh waves. Thus, as the waves radiate outwards from the earthquake source into the rocks of the Earth’s crust, the different types of waves separate out from one another in a predictable pattern [5].
2.4. Earthquake magnitude

The earthquake magnitude is defined as the logarithm of the measured amplitude, corrected for distance [29]. Earthquake size is expressed in several ways. Local or Richter magnitude $M_L$, exhibits several limitations, it is a regional or a local scale. While body wave magnitude $m_b$, surface magnitude $M_S$, and moment magnitude $M_W$, are worldwide scales.

The main properties of magnitude scales are summarized in table 2-1.

<table>
<thead>
<tr>
<th>Scale type</th>
<th>Author</th>
<th>Earthquake size</th>
<th>Earthquake depth</th>
<th>Epicentre distance (km)</th>
<th>Reference parameter</th>
<th>Applicability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_L$</td>
<td>Richter (1935)</td>
<td>Small</td>
<td>Shallow</td>
<td>&lt;600</td>
<td>Wave amplitude</td>
<td>Regional (California)</td>
</tr>
<tr>
<td>$m_b$</td>
<td>Guteberg and Richter (1956)</td>
<td>Small to medium</td>
<td>Deep</td>
<td>&gt;1000</td>
<td>Wave amplitude (P-waves)</td>
<td>Worldwide</td>
</tr>
<tr>
<td>$M_W$</td>
<td>Kanamori (1977)</td>
<td>All</td>
<td>All</td>
<td>All</td>
<td>Seismic moment</td>
<td>Worldwide</td>
</tr>
</tbody>
</table>

Table 2-1. Properties of major magnitude scales [1].

Moment magnitude $M_W$, can be used to measure the whole spectrum of ground motions. Moment magnitude is defined as a function of the seismic moment $M_0$. This measures the extent of deformation at the earthquake source and can be evaluated as follows:

$$M_0 = G \cdot A \cdot \Delta u$$  \hspace{1cm} (2.5)

Where, $G =$ shear modulus of the material surrounding the fault.

$A =$ fault rupture area.

$\Delta u =$ average slip between opposite side of the fault.

$$M_W = 0.67 \cdot \log(M_0) - 10.70$$  \hspace{1cm} (2.6)
2.5. Earthquakes in Iceland

Iceland lies on the Mid Atlantic Ridge and is split by the divergent plate boundary between the North American plate and the Eurasian plate. The plates are moving relative to each other at an average rate of approximately 20 mm/year.

Due to volcanic activity, different sea levels through the ages and glacial drift make the Icelandic geology quite complex at many sites [31].

In southern Iceland, there is an effective seismic zone as shown in figure 2-9. Last earthquake there was in the year 2008 of moment magnitude $M_W = 6.3$. Few years earlier, two large earthquakes occurred in the year 2000, both of moment magnitude $M_W = 6.6$ [3]. The highest measured acceleration in these earthquakes was 0.84g.

Figure 2-9. Yellow lines denote the western volcanic zone (WVZ) and the presently more active eastern volcanic zone (EVZ). The South Iceland seismic zone (SISZ) is indicated as well as its prolongation in the Reykjanes peninsula (RP). The direction of the relative plate motion is shown by arrows. The faults of the two large earthquakes that ruptured on June 17 and 21 2000 are indicated by 17 and 21 respectively.
None of these earthquakes caused serious injuries on people. No buildings collapsed, and the buildings most damaged in the earthquakes were older structures particularly. Buildings built on poor foundation should be mentioned, houses constructed using building blocks made of lava aggregate and houses with floating base slabs resting on fill of poor quality, as well as houses with masonry partitions. Also, a few old poorly built unreinforced concrete and hollow concrete block houses were damaged.

However, it can be generally concluded that well-constructed wooden houses and reinforced concrete houses withstood the earthquakes well, sustaining little or no damage. One can also find houses of masonry construction that withstood the earthquakes or sustained little damage despite great excitation. It often appeared somewhat a matter of chance which houses suffered damage [30].
2.6. Time history

A numerical integration method, usually referred to as time history analysis, is required to get more accurate responses of structure [35].

Time histories theoretically contain complete information about the motion at the instrumented location, recording three traces or orthogonal records, two horizontal and one vertical. The maximum amplitude of record acceleration is termed the peak ground acceleration (PGA), peak ground velocity (PGV) and peak ground displacement (PGD), are the maximum respective amplitudes of velocity and displacement [15].

One of the recorded motions of the earthquake in year 2000, the so-called Hella 2000, in June the 17th where the peak acceleration reached 0.47g [28], was used in the wall models.

Graph 2-1. Time history from Hella 2000 in June 17th, horizontal displacement in E-W-direction.

Eurocode 8 part 1, stated: “The two horizontal and the vertical components of the seismic action shall be assumed to act simultaneously”. In the wall models analysis, time history in one direction will be used, as the author thinks it is likely enough because the analysis is a wall analysis not a building analyses.

Eurocode 8 part 1, stated in another paragraph: “If time-history analyses are required, a set of at least three ground motion records should be used”. In this thesis, aim is not to follow Eurocode 8, because of the number of different models author decided to have one time history.
2.7. Prevent earthquake damages to structures

Behavior of a structure during an earthquake depends on two basic parameters:
- The intensity of the earthquake.
- The quality of the structure and the foundations.

The intensity of the earthquake is a parameter with very high uncertainty, whose expected maximum value during the lifetime of the structure can be estimated based on very limited data and on questionable evaluation of any existing historical information.

The quality of the structure is a parameter which exhibits a sufficient level of reliability since it depends on the configuration of the structural system, the design procedure, the detailing of structural elements and careful construction [18].

Damage caused by earthquakes is due to correlation between natural period of vibration of structures and frequency of earthquakes.

Despite of proportion dynamic of structures and earthquakes, many structures cover that hold the field with the escape frequency band where the earthquake is strongest due to extension of oscillation.

Recommendations concerning structural configuration:
- Buildings regular in plan and in elevation, without re-entrant corners and discontinuities.
- Concrete shear walls should span the whole distance between adjacent columns.
- All the structural elements should be interconnected.
- Short columns.
- Flat slab system without any beams should be avoided.
- Large discontinuities in the infill system should be avoided.
- Weak points in the slab endangering its diaphragmatic action should be avoided.
- Structures have to be composed of strong columns and weak beams.
- Foundations:
  - Site be free of risks of soil rupture.
  - Permanent settlements.
  - All footings should rest on the same horizontal level.
  - Only one foundation type should in general be used for the same structure.
Earthquakes repeatedly demonstrate that the simplest structures have the greatest chance of survival. The ability to understand the overall behavior of simple structure is markedly greater than it is for a complex one, e.g. torsional effects are particularly hard to predict on an irregular structure.

Symmetry is desirable for much the same reasons. Symmetry is important in both directions in plan, as shown in figure 2-10, and helps in elevation as well. Lack of symmetry produces torsional effects which are sometimes difficult to assess and can be very destructive.

Buildings of H-, L-, T- and Y-shape in plan have often been severely damaged in earthquakes. Such plan forms should only be adopted if an appropriate three-dimensional earthquake is used in the design [36].

<table>
<thead>
<tr>
<th>Suitable</th>
<th>Unsuitable</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Suitable" /></td>
<td><img src="image2" alt="Unsuitable" /></td>
<td>Ideal for behavior and analysis.</td>
</tr>
<tr>
<td><img src="image3" alt="Suitable" /></td>
<td><img src="image4" alt="Unsuitable" /></td>
<td>Good symmetry, analysis less easy.</td>
</tr>
<tr>
<td><img src="image5" alt="Suitable" /></td>
<td><img src="image6" alt="Unsuitable" /></td>
<td>Beware of differential behavior at opposite ends of long buildings.</td>
</tr>
<tr>
<td><img src="image7" alt="Suitable" /></td>
<td><img src="image8" alt="Unsuitable" /></td>
<td>Bad for asymmetrical effects.</td>
</tr>
<tr>
<td><img src="image9" alt="Suitable" /></td>
<td><img src="image10" alt="Unsuitable" /></td>
<td>Although symmetrical, long wings give behavior prediction problems.</td>
</tr>
<tr>
<td><img src="image11" alt="Suitable" /></td>
<td><img src="image12" alt="Unsuitable" /></td>
<td>Projecting access towers. Problems with analysis and detailing.</td>
</tr>
<tr>
<td><img src="image13" alt="Suitable" /></td>
<td><img src="image14" alt="Unsuitable" /></td>
<td>Asymmetry of members resisting horizontal shear. Analysis and torsion problems.</td>
</tr>
</tbody>
</table>

Figure 2-10. Simple rules for plan layouts of aseismic buildings. For unsuitable plan layouts, dynamic analysis and careful detailing is necessary.
As indicated in figure 2-11, very slender structures and those with sudden changes in width should be avoided in strong earthquake areas. Very slender buildings have high column forces and foundation stability may be difficult to achieve. Also higher mode contributions may add significantly to the seismic response of the superstructure.

**Suitable**  
**Unsuitable**  
**Comments**

Very slender buildings may have excessive horizontal deflections.

Effects of facade setbacks cannot be predicted by normal code equivalent static analyses.

**Figure 2-11.** Simple rules for elevation shapes of aseismic buildings. For unsuitable elevation shapes, dynamic analysis and careful detailing is necessary.

A building will have more possibility to surviving an earthquake if all columns and walls are continuous and without offsets from roof to foundation, as the example in figure 2-12 shows.

**Suitable**  
**Unsuitable**  
**Comments**

Avoid low redundancy of cantilevers: no fail-safe mechanism.

Avoid changes of stiffness with height. Problems with analysis and detailing.

Remarks as above “Soft storey” demonstrably vulnerable.

**Figure 2-12.** Simple rules for vertical frames in aseismic buildings. For unsuitable vertical frames, dynamic analysis and careful detailing is necessary.
3. Soil-structure interaction theory

3.1. SSI introduction

Most of structures involve some type of structural elements which have direct contact with the ground. When external forces, such as earthquakes, act on these systems, neither the structural displacements nor the ground displacements, are independent. The process in which the response of the soil influences the motion of the structure and the motion of the structure influences the response of the soil is termed as soil-structure interaction (SSI) [11].

All structural systems have a dynamic response of soil systems that depends on inertia, stiffness and damping in common.

The most common dynamic analysis method is to determine the free-field ground motion at the site of the structure and then apply the motion at the base of the structure assuming that the base is fixed. This may be true in cases where the structure is founded on bedrock. However, if the structure is founded on soil bed, the earthquake motion at the base of the structure is not likely to be identical to the free-field ground motion. The presence of the structure will modify the free-field motions because the soil and structure will interact and create a dynamic system quite different from the free-field condition. This soil-structure interaction will result in a structural response that may be different from the structural response computed from a fixed base structure subjected to a free-field ground motion.
3.2. Equation of motion

Dynamic loading often results from vibration of the supports of system rather than from dynamic external loads. To evaluate the response of such systems, it is necessary to develop an equation of motion for loading caused by base shaking.

Figure 3-1 explains where \( m \) denotes effective values of mass, \( k \) is the spring coefficient and \( c \) is the damping coefficient. The mass, spring and damping are associated with the fundamental mode of vibration of the structure built in at its base, \( h \) is the distance from the base to the centroid of the inertial forces [8]. For displacement, \( u_t \) denote total displacement, \( u_g \) represent the ground displacement, \( \theta_g \) the ground rotation and \( u_{ed} \) the elastic deformation.

![Figure 3-1. Single-degree of freedom systems subjected to base shaking.](image-url)
For damped free vibrations, the equation of motion can be written as [17]:

\[ m \ddot{u} + c \dot{u} + k u = -m \ddot{g} \]  

(3.1)

The impressions of the spring and damping in equation 3.1. are subject to displacement and velocity to the system, with respect to bottom of the system but effects of the mass are dependent on total acceleration of the system.

The fixed-base frequency of the structure is denoted as \( \omega \):

\[ \omega = \sqrt{\frac{k}{m}} \]  

(3.2)
3.3. Illustration of Soil-Structure Interaction

The effect of soil-structure interaction can be illustrated with the idealized model. The structure is modeled with mass $m$, a lateral stiffness with a spring coefficient $k$, damper with a coefficient $c$ and the height of the structure is $h$. The corresponding coefficient is denoted as $k_h$ and $c_h$ in the horizontal direction, $k_r$ and $c_r$ in the rotational (rocking) direction. All spring and dampers have a length approaching zero.

The system depicted in figure 3-2 possesses three degrees of freedom:

- The horizontal displacement of the mass $m$.
- The horizontal displacement of the foundation $u_f$.
- The rotation $\theta_f$ of the foundation.

Figure 3-2. Model with three dynamic of freedom. The stiffness and damping characteristics of the compliant soil-foundation system can be represented by the translational and rotational springs and dashpots.
The foundation dashpots represent two sources of damping: material damping caused by inelastic behavior of the soil supporting the foundation and radiation damping that occurs as dynamic forces in the structure causes the foundation to deform the soil, producing stress waves that travel away from the foundation.

The amount of material damping will depend on the level of the strain induced in the soil; if the strain are high, material damping can be substantial, but if they are low, the material damping may be negligible. In contrast, radiation damping is a purely geometric effect that exists at low as well as high strain amplitudes. For typical foundation, radiation damping is often much greater than material damping [9].

The total displacement amplitudes $u_t$, can be split into these components, extending from figure 3-2:

$$u_t = u_g + u_f + h \cdot \theta_f + u_{ed}$$ (3.3)

Where,

- $u_g$ = Ground displacement.
- $u_f$ = Horizontal displacement of foundation.
- $h$ = Height of the structure.
- $\theta_f$ = Foundation rotation.
- $u_{ed}$ = Elastic deformation.

Neglecting material damping in the soil $\xi_g = 0$, the horizontal force imposed on the soil by the foundation would be:

$$P_x = k_x \cdot u_f + c_x \cdot \dot{u}_f$$ (3.4)

Where the subscript $x$ denotes the horizontal direction for a purely elastic soil, for $\xi_g = 0$ conditions.

For harmonic excitation at frequency $\omega$, material damping can be introduced by the use of a complex stiffness, so that:

$$P_h = k_x \cdot \left(1 + i \cdot 2 \cdot \xi_x + i \cdot 2 \cdot \xi_g \right) \cdot u_f$$ (3.5)
Since: $P_h = k_h \cdot u_f + c_h \cdot \dot{u}_f$, the horizontal stiffness and damping coefficients are:

$$k_h = k_x = \frac{8 \cdot G \cdot a}{2 - \nu}$$

(3.6)

$$c_h = c_x + \frac{2}{\omega} \cdot \xi g \cdot k_x$$

(3.7)

$$c_x = \frac{4 \cdot 6 \cdot \rho \cdot v_s \cdot a^2}{2 - \nu}$$

(3.8)

Where, $G$ = Shear modulus.
$a$ = Radius of circular footing.
$\nu$ = Poisson’s ratio.
$\rho$ = Mass density.
$v_s$ = Soil shear wave velocity.

The first term on the right side of equation (3.7) corresponds to radiation damping, and the second to material damping. If the structure is rigid $k = \infty$ and the foundation unable to rotate $k_r = \infty$, the natural frequency for translational vibration would be:

$$\omega_h = \frac{k_h}{m}$$

(3.9)

Repeating the same process for the rocking mode of vibration produces:

$$k_r = k_{\theta} = \frac{8 \cdot G \cdot a^3}{3 \cdot (1 - \nu)}$$

(3.10)

$$c_r = c_{\theta} + \frac{2}{\omega} \cdot \xi g \cdot k_{\theta}$$

(3.11)

$$c_{\theta} = \frac{0.4 \cdot \rho \cdot v_s \cdot a^4}{1 - \nu}$$

(3.12)

Where the supscript $\theta$ denotes the absence of material damping.
If the structure is rigid \( k = \infty \) and the foundation unable to translate \( k_h = \infty \), the natural frequency for rocking would be:

\[
\omega_r = \sqrt{\frac{k_r}{m \cdot h^2}}
\]  

(3.13)

Insight into the soil-structure interaction problem can now be gained by developing an equivalent of a single degree of freedom system (SDOF). Using the subscript \( e \) to describe the properties of this equivalent system, the equation of motion can be written as:

\[
\left( -m \cdot \omega^2 + i \cdot \omega \cdot c_e + k_e \right) \cdot u_{ed} = m \cdot \omega^2 \cdot U_g
\]  

(3.14)

Where \( U_g \) is the equivalent seismic input motion.

For the equivalent system:

\[
k_e = m \cdot \omega^2_e
\]  

(3.15)

The foundation translation of the equivalent system can be shown to be:

\[
u_f = \frac{\omega^2_e}{\omega^2_h} \cdot \left( 1 + 2 \cdot \xi_i - 2 \cdot \xi_h \cdot i - 2 \cdot \xi_g \cdot i \right) \cdot u_{ed}
\]  

(3.16)

The foundation rotation:

\[
\theta_f = \frac{1}{h} \cdot \frac{\omega^2_e}{\omega^2_r} \cdot \left( 1 + 2 \cdot \xi_i - 2 \cdot \xi_0 \cdot i - 2 \cdot \xi_g \cdot i \right) \cdot u_{ed}
\]  

(3.17)

Then the motion of the mass relative to the free-field motion is given by the sum of the foundation displacement \( u_f \), the displacement of the top of the structure due to rotation of the base, \( h \theta \) and the displacement due to distortion of the structure \( u_{ed} \)

\[
u_{ed} + u_f + h \cdot \theta = \omega_f^2 \left[ \frac{1}{\omega^2_e} + 2 \cdot \left( \xi - \xi_g \right) \cdot i \cdot \left( \frac{1}{\omega^2_e} - \frac{1}{\omega^2_f} \right) - \frac{2 \cdot \xi_h}{\omega^2_h} - \frac{2 \cdot \xi_0 \cdot i}{\omega^2_0} \right] \cdot u_{ed}
\]  

(3.18)
3.4. Formulation of a Soil-Structure Interaction

The simulation of the infinite medium in the numerical method is a very important topic in the dynamic soil-structure interaction problems. The general approach for treating this problem is to divide the infinite medium into the near-field (truncated layer), which includes the irregularity as well as the non-homogeneity of the foundation; and the far-field, which is simplified as an isotropic homogeneous elastic medium [12].

![Figure 3-3. A soil-structure interaction system.](image)

The near-field is modeled using finite elements, and the far-field is treated by adding special artificial boundaries or special connection elements. The soil bed is in most cases a semi-infinite medium, and this unbounded domain should be enlarged so large to the extent that the simultaneous modeling together with the structure may be impractical. In a dynamic problem, it may be insufficient to prescribe a zero displacement at a large distance from the structure, as it is routinely done in static problems [20].
3.4.1. Near-field, finite element method FEM

The system equations of motion, for typical dynamic and undamped system, can be assembled from the element matrices as:

\[
[M]\{\ddot{u}\} + [K]\{u\} = \{R\}
\]

(3.19)

Where,

- \([M]\) = Mass matrix.
- \(\ddot{u}\) = Acceleration.
- \([K]\) = Stiffness matrix.
- \(u\) = Displacement.
- \(\{R\}\) = Load vector.

3.4.2. Far-field, boundary element method BEM

To calculate the properties of the boundary condition, it is necessary to consider a plane wave propagating in the x-direction. The forces that cause wave propagation are shown acting on a unit cube in figure 3-4.

\[
\rho \frac{d^2 u}{dt^2} + \frac{d\sigma_x}{dx} = 0
\]

(3.20)

Where,

- \(\rho\) = Mass density.
- \(u\) = Displacement.
- \(\sigma_x\) = Stress in x-direction.
The one-dimensional partial differential equation in the classical wave propagation form as:

\[
\frac{d^2 u}{dt^2} - v_p^2 \frac{d^2 u}{dx^2} = 0
\]  
(3.21)

Where, \(v_p\) = Wave propagation velocity.

Viscous boundaries can be used with the finite element mesh as shown in figure 3-5. In the figure \(A_n\), \(A_{11}\) and \(A_{12}\) are the fields that controlled viscous dampers, \(\sigma\) and \(\tau\) are the normal and shear stresses occurred in the boundaries of the medium. The subscripts \(n\) and \(t\) represent normal and tangent direction in the boundary.

![Figure 3-5. Viscous boundary considered in the 3D finite element model.](image)

The speed of pressure and shear waves travelling through a homogenous and isotropic medium has been obtained, the viscous boundaries are determined by the area of the elements for each damper and the \(\rho\) of the material, which is the force acting at the truncated boundaries equal to the velocity times the damping constant.

\[
N_n = A_n \cdot \sigma_n \quad \rightarrow \quad N_n + C_n \cdot \dot{u}_n = 0
\]

\[
N_{11} = A_{11} \cdot \tau_{11} \quad \rightarrow \quad N_{11} + C_{11} \cdot \dot{u}_{11} = 0
\]

\[
N_{12} = A_{12} \cdot \tau_{12} \quad \rightarrow \quad N_{12} + C_{12} \cdot \dot{u}_{12} = 0
\]  
(3.22)
Where the damping matrix can be considered as:

\[
C = \begin{pmatrix}
A_n \cdot \rho_v \cdot v_p & 0 & 0 \\
0 & A_h \cdot \rho_v \cdot v_s & 0 \\
0 & 0 & A_{2v} \cdot \rho_v \cdot v_s
\end{pmatrix}
\]  
(3.23)

When the viscous boundary is taken into consideration, the well-known equation of motion for the system considered in this study can be written for the damping case as:

\[
[M] \{\ddot{u}\} + [C] \{\dot{u}\} + [K] \{u\} = \{R\}
\]  
(3.24)

Where the load vector \( \{R\} \) is determined from the near-field/far-field displacements.
4. The computer program, ANSYS

4.1. Introduction

ANSYS is a general purpose finite element modeling package for numerically solving a wide variety of problems. These problems include, static or dynamic structural analysis, both linear and non-linear, heat transfer and fluid problems, as well as acoustic and electro-magnetic problems.

In general, a finite element solution may be broken into the following three stages. This is a general guideline that can be used for setting up any finite element analysis:

1. **Preprocessing:** *defining the problem*; the major steps in preprocessing are:
   - Define keypoints, lines, areas or volumes.
   - Define element type and material, geometric properties.
   - Mesh lines, areas or volumes as required.

2. **Solution:** *assigning loads, constraints and solving*; here are possibilities to:
   - Specify the loads (point or pressure).
   - Constraints (translational and rotational).
   - Solve the resulting set of equations.

3. **Postprocessing:** *further processing and viewing of the results*; in this stage may wish to see:
   - Lists of nodal displacements.
   - Element forces and moments.
   - Deflection plots.
   - Stress contour diagrams.

The walls and the foundations was modeled using the finite element method (FEM), these calculations was carried out using ANSYS. The soil-structure interaction (SSI) model was separated into near-field and far-field using FEM to represent the near-field and viscous dampeners, boundary element method (BEM) placed around the FEM model to represent the far-field.
In ANSYS, the element SOLID65 was used to represent the lightly reinforced C25 concrete nonlinearity. SOLID45 is used to represent both the bedrock and the gravel bed, with linear-plastic properties used in that element. To model the contact between the wall and the ground, the area between them was made TARGE170 to define the surface for the contact element. CONTA174 is used to represent contact and sliding between elements, the contact element overlays the solid boundaries between the wall and the ground and interacts with them through their surface element when it penetrates one of the target segment elements. MATRIX27 is used to represent a damping matrix, an arbitrary element without a specified geometry but its response can be specified by stiffness, damping or mass coefficients.

### 4.2. Concrete element

SOLID65 is used for the three-dimensional modeling of solids with reinforcing bars. SOLID65 is capable of cracking in tension and crushing in compression. In concrete applications, the solid capability of the element may be used to model the concrete while the rebar capability is available for modeling reinforcement behavior. Other cases for which the element is also applicable would be reinforced composites (such as fiberglass). The element in figure 4-1, is defined by eight nodes and the isotropic material properties, having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The element has one solid material and up to three different rebar specifications may be defined. Use the MAT command to input the concrete material properties. Rebar specifications, which are input as real constants, include the material number MAT, the volume ratio VR, and the orientation angles THETA and PHI.

![Solid65 diagram](image)

**Figure 4-1. Solid65. Geometry, node location, and the coordinate system for this element are shown [10].**
4.3. Soil and rock elements

SOLID45 is used for the three-dimensional modeling of solid structures. The element in figure 4-2, is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions.

The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. These features work well for bedrock and soil bed elements. A reduced integration option with an hourglass control is available. The element is defined by eight nodes and the orthotropic material properties. Orthotropic material directions correspond to the element coordinate directions. Pressures may be input as surface loads on the element faces. Positive pressures act into the element, and temperatures and fluences may be put as the element body loads at the nodes.

![Figure 4-2. Solid45. Geometry, node location, and the coordinate system for this element are shown [10].](image-url)
4.4. Contact elements

CONTA174 Element Description

CONTA174 is used to represent contact and sliding between three-dimensional target surfaces TARGE170 and a deformable surface, defined by this element. The element in figure 4-3, is applicable to three-dimensional structural and coupled field contact analyses. This element is located on the surfaces of three-dimensional solid. It has the same geometric characteristics as the solid element face with which it is connected.

Contact occurs when the element surface penetrates one of the target segment elements on a specified target surface. Coulomb and shear stress friction is allowed. The element is defined by eight nodes (the underlying solid or shell element has midside nodes). It can degenerate to a six node element depending on the shape of the underlying solid or shell elements. The node ordering is consistent with the node ordering for the underlying solid element. The positive normal is given by the right-hand rule going around the nodes of the element and is identical to the external normal direction of the underlying solid element surface.

Figure 4-3. Conta174. Geometry and node location are shown [10].
TARGET170 Element Description

TARGET170 is used to represent various three-dimensional target surfaces for the associated contact elements. The contact elements themselves overlay the solid elements describing the boundary of a deformable body and are potentially in contact with the target surface, defined by TARGET170. This target surface in figure 4-4, is discretized by a set of target segment elements TARGET170 and is paired with its associated contact surface via a shared real constant set. It is possible to impose any translational or rotational displacement, temperature, voltage, and magnetic potential on the target segment element. Also impose forces and moments on target elements.

For rigid target surfaces, these elements can easily model complex target shapes. For flexible targets, these elements will overlay the solid elements describing the boundary of the deformable target body.

Figure 4-4. TARGET170. Geometry and node location are shown [10].
4.5. Damping elements

MATRIX27 represents an arbitrary element whose geometry is undefined but elastic kinematic response can be specified by stiffness, damping, or mass coefficients. The matrix is assumed to relate two nodes, each with six degrees of freedom per node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes. The matrix's translational degrees of freedom are activated for one node while the other node has no function and are fixed, thus negating all forces at the truncated boundaries.

Figure 4-5. MATRIX27. The node locations and the coordinate system for this element are shown [10].

The element is defined by two nodes and the matrix coefficients. The stiffness, damping, or mass matrix constants are input as real constants.

The units of the stiffness constants are: \( \frac{\text{Force}}{\text{Length}} \) or \( \frac{\text{Force} \cdot \text{Length}}{\text{Radian}} \)

The units of the damping constants are: \( \frac{\text{Force} \cdot \text{Time}}{\text{Length}} \) or \( \frac{\text{Force} \cdot \text{Length} \cdot \text{Time}}{\text{Radian}} \)

The mass constants should have units of: \( \frac{\text{Force} \cdot \text{Time}}{\text{Length}} \) or \( \frac{\text{Force} \cdot \text{Time} \cdot \text{Length}}{\text{Radian}} \)

All matrixes generated by this element are 12 by 12. The degrees of freedom are ordered as \( \text{UX, UY, UZ, ROTX, ROTY, ROTZ} \) for node I followed by the same for node J.

Figure 4-6. MATRIX27. The symmetric matrix has this form [10].
5. Modeling of walls and foundations

5.1. Introduction

Five different squat wall models and five different slender wall models have been investigated. All the shear walls have flanges at their ends.

Definition of squat walls is: \( \frac{h}{l} < 2 \) and for slender walls is: \( \frac{h}{l} > 2 \) [4].

Where, \( h \) = Height of the walls.
\( l \) = Length of the walls.

The wall models were set up to show how different foundation support changes their behavior during seismic excitation.
The aim is to look for difference displacements, see if there is a difference in tensile stress and compressive stress, as well as different shear forces.

All necessary elements are available in ANSYS, as mentioned in section 4.

On top of the walls acts distributed load \( q \), given as follows:

| Imaginary 0.2m thick concrete plate on top of walls: \( 5.4m \times 4.2m = 22.7m^2 \) |
| Self weight, concrete plate: \( \frac{24.0kN}{m^3} \times 0.2m = 4.8kN/m^2 \) |
| Self weight, anhydrit and flooring: \( 1.2kN/m^2 \) |
| Live load: \( 2.0kN/m^2 \) |

\[
Q = 8.0kN/m^2 \times 22.7m^2 = 181.6kN \quad l_t = 2.2m + 3.0m + 2.2m = 7.4m
\]

\[
q_0 = \frac{Q}{l_t} = \frac{181.6kN}{7.4m} = 24.5kN/m \quad \Rightarrow \quad 24.5kN/m \times 1000 = 24500.0N/m
\]

\[
q = \frac{q_0}{t} = \frac{24500.0N/m}{0.2m} = 122500.0N/m^2
\]

Where, \( Q \) = Total load.
\( l_t \) = Total length of wall.
\( q \) = Distributed load on top of each wall.
\( t \) = Wall thickness.
5.2. Squat wall models

One of the models examined was where squat wall was rigidly fixed on the ground (cantilever) as in figure 5-1. Second case was where a squat wall was founded on bedrock, with and without damping as in figure 5-2. The third case examined had a squat wall founded on a one meter thick mean gravel foundation, with and without damping as in figure 5-3. When the walls are founded on bedrock and gravel bed with contact element, they are allowed to rock and translate.

**Figure 5-1.** Squat wall fixed on ground (cantilever), (mm).

**Figure 5-2.** Squat walls on bedrock (Basalt), with and without damping, (mm).

**Figure 5-3.** Squat walls on 1.0m thick mean gravel bed, with and without damping, (mm).
5.3. Slender wall models

The slender wall models are three times higher than the squat wall models and the distributed load reacts on every story. Otherwise, the slender models are similar to squat models, i.e. in figure 5-4 the slender wall is rigidly fixed. In figure 5-5 the slender wall is founded on bedrock, with and without damping. In figure 5-6 the slender wall is founded on a one meter thick mean gravel, with and without damping. When the walls are founded on bedrock and gravel bed with contact element, they are allowed to rock and translate.

*Figure 5-4. Slender wall fixed on ground (cantilever), (mm).*
Concrete walls founded on earthquake areas

Masters thesis in Structural engineering
Helgi S. Ólafsson

Figure 5-5. Slender walls on bedrock (Basalt), with and with out damping, (mm).

Figure 5-6. Slender walls on 1.0m thick mean gravel bed, with and with out damping, (mm).


5.4. Material properties

The walls are made of concrete C25, which have compressive strength \( f_c = 25.0 \text{MPa} \) and tensile strength \( f_t = 3.3 \text{MPa} \) [26].

The reinforcement is B500, which have yield strength \( f_y = 500.0 \text{MPa} \).

All the walls are reinforced in single layer in the middle, with K10 c250 in both directions.

Giving a reinforcement ratio: \( \chi = \frac{A_{\text{bar}}}{A_{\text{wall}}} = 0.0015 = 0.15\% \)

Bedrock and soil behaviour under dynamic loading depends upon many factors.

The bedrock where the walls are founded on, is basalt. It is appropriate because the bedrock of Iceland is mostly basalt and about 70% of the total surface of Earth is basalt [6].

Basalt is made of igneous rock, formed by solidification of magma, either at depth forming plutonic rocks, at shallow depth form dike rocks, or at the surface forming volcanic rocks. Basalt consists of four mineral types, i.e. feldspar, olivine, pyroxene and magnetite [29].

The soil where the walls are founded is mean gravel bed, with particle size 20 – 60 mm, cohesion \( c_g = 0.0 \) and angle of internal friction \( \varphi = 40^\circ \).

The coefficient of the friction is \( \mu_g = 0.6 \) for the gravel isolation layer, and for bedrock the friction is \( \mu_b = 0.7 \).

Contact opening stiffness was chosen, as no tension was allowed to form in the contact element.

Some soils increase in strength under rapid cyclic loading, while others such as saturated sands or sensitive clays may lose strength with vibration [36].
More values used for the elements in these calculations are given in Table 5-1 and 5-2.

<table>
<thead>
<tr>
<th>Element</th>
<th>$E$ (N/m$^2$)</th>
<th>$G$ (N/m$^2$)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>$25 \cdot 10^9$</td>
<td>$10.9 \cdot 10^9$</td>
<td>2400</td>
<td>0.15</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>$210 \cdot 10^9$</td>
<td>$80.8 \cdot 10^9$</td>
<td>7800</td>
<td>0.30</td>
</tr>
<tr>
<td>Gravel bed</td>
<td>$70 \cdot 10^6$</td>
<td>$25.9 \cdot 10^6$</td>
<td>2100</td>
<td>0.35</td>
</tr>
<tr>
<td>Bedrock</td>
<td>$50 \cdot 10^9$</td>
<td>$18.5 \cdot 10^9$</td>
<td>2800</td>
<td>0.35</td>
</tr>
</tbody>
</table>

*Table 5-1. Parameters for squat and slender walls and foundation.*

<table>
<thead>
<tr>
<th>Element</th>
<th>$\lambda$ (N/m$^2$)</th>
<th>$v_p$ (m/s)</th>
<th>$v_s$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravel bed</td>
<td>$60.5 \cdot 10^6$</td>
<td>231.3</td>
<td>111.1</td>
</tr>
<tr>
<td>Bedrock</td>
<td>$43.2 \cdot 10^9$</td>
<td>5353.5</td>
<td>2571.7</td>
</tr>
</tbody>
</table>

*Table 5-2. Parameters for foundation.*

Where, $E =$ Modulus of elasticity. $\lambda =$ Lame parameter. $G =$ Shear modulus. $\nu_P =$ P-wave velocity. $\rho =$ Mass density. $\nu_S =$ Shear wave velocity. $\nu =$ Poisson’s ratio.

The following equations are applied where calculating the shear modulus and Lame parameter:

$$G = \frac{E}{2 \cdot (1 + \nu)} \quad (5.1)$$

$$\lambda = \frac{E \cdot \nu}{(1 + \nu) \cdot (1 - 2 \cdot \nu)} \quad (5.2)$$
Also it is feasible to use formula 2.1 and 2.2 in chapter 2 to find $v_P$ and $v_S$.

$$v_P = \sqrt{\frac{\lambda + 2G}{\rho}}$$

(5.3)

$$v_S = \sqrt{\frac{G}{\rho}}$$

(5.4)

Damping matrix can be written as:

$$C = \begin{pmatrix} A \cdot \rho \cdot v_P & 0 & 0 \\ 0 & A \cdot \rho \cdot v_S & 0 \\ 0 & 0 & A \cdot \rho \cdot v_S \end{pmatrix}$$

(5.5)

The values in matrix C, are put in MATRIX27 in ANSYS to represent the damping matrix for far-field conditions. It is an arbitrary element without a specified geometry but in this case its response is specified by damping coefficients. The matrix's translational degrees of freedom are activated for one node, while the other node has no function and are fixed, thus negating all forces at the truncated boundaries according to equations [22, 27].
6. Analysis results

6.1. Introduction

The results are according to time history from Hella year 2000, which was allowed to work under the fixed walls and under the foundations for other walls in the x-direction. Two nodes are chosen, as in figure 6-1, to monitor the displacements over time, one at the top and other at the bottom. Both compressive- and tensile stresses of the walls are maximum stesses over the time history.

![Squat wall and Slender wall](image)

*Figure 6-1. The nodes represent where the displacements and rocking are monitored. The coordinate system for all the walls are the same, i.e. x-direction is horizontal and z-direction is vertical*

Results from the analytical study by using ANSYS finite element program, are collected for squat walls in tables 6-1, 6-2 and 6-3. For the slender walls, the results are collected in tables 6-4, 6-5 and 6-6.

Ten cases are shown, five for squat walls and five for slender walls:
1. Wall perfectly fixed to the ground.
2. Wall founded on bedrock and allowed to rock and translate with near-field effects.
3. Wall founded on bedrock and allowed to rock and translate with near- and far-field effects (damped).
4. Wall founded on gravel bed and allowed to rock and translate with near-field effects.
5. Wall founded on gravel bed and allowed to rock and translate with near- and far-field effects (damped).
6.2. Results for the squat walls

6.2.1. Squat wall fixed on ground

Figure 6-2. Displacement in x-direction: In upper part of wall, 24.246mm. In lower part of wall, 24.280mm.

Figure 6-3. Displacement in z-direction: In upper part of wall, -0.056mm. In lower part of wall, 0.000mm.

Figure 6-4. Stress in x-direction: Compressive stress, -0.252MPa. Tensile stress, 0.177MPa.

Figure 6-5. Stress in z-direction: Compressive stress, -1.727MPa. Tensile stress, 1.289MPa.

6.2.2. Squat wall on bedrock

Figure 6-6. Displacement in x-direction: In upper part of wall, 24.290mm. In lower part of wall, 24.295mm.

Figure 6-7. Displacement in z-direction: In upper part of wall, -0.487mm. In lower part of wall, -0.162mm.

Figure 6-8. Stress in x-direction: Compressive stress, -0.165MPa. Tensile stress, 0.144MPa.

Figure 6-9. Stress in z-direction: Compressive stress, -1.298MPa. Tensile stress, 0.269MPa.
Concrete walls founded on earthquake areas

6.2.3. Squat wall on bedrock with damping

**Figure 6-10.** Displacement in x-direction:
- In upper part of wall, 24.291mm.
- In lower part of wall, 24.295mm.

**Figure 6-11.** Displacement in z-direction:
- In upper part of wall, 0.003mm.
- In lower part of wall, -0.032mm.

**Figure 6-12.** Stress in x-direction:
- Compressive stress, -0.165MPa.
- Tensile stress, 0.132MPa.

**Figure 6-13.** Stress in z-direction:
- Compressive stress, -1.263MPa.
- Tensile stress, 0.276MPa.

6.2.4. Squat wall on gravel bed

**Figure 6-14.** Displacement in x-direction:
- In upper part of wall, 25.090mm.
- In lower part of wall, 25.123mm.

**Figure 6-15.** Displacement in z-direction:
- In upper part of wall, -1.911mm.
- In lower part of wall, -1.899mm.

**Figure 6-16.** Stress in x-direction:
- Compressive stress, -0.062MPa.
- Tensile stress, 0.112MPa.

**Figure 6-17.** Stress in z-direction:
- Compressive stress, -0.556MPa.
- Tensile stress, 0.122MPa.
6.2.5. Squat wall on gravel bed with damping

**Figure 6-18.** Displacement in x-direction:
In upper part of wall, 25.604mm.
In lower part of wall, 25.389mm.

**Figure 6-19.** Displacement in z-direction:
In upper part of wall, -0.882mm.
In lower part of wall, -1.034mm.

**Figure 6-20.** Stress in x-direction:
Compressive stress, -0.045MPa.
Tensile stress, 0.121MPa.

**Figure 6-21.** Stress in z-direction:
Compressive stress, -0.488MPa.
Tensile stress, 0.084MPa.
Results for all the squat walls are collected in tables:

<table>
<thead>
<tr>
<th>Squat wall models</th>
<th>Displacement, x-direction</th>
<th>Displacement, z-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top (mm)</td>
<td>Bottom (mm)</td>
</tr>
<tr>
<td>Fixed</td>
<td>24.246</td>
<td>24.280</td>
</tr>
<tr>
<td>Bedrock</td>
<td>24.290</td>
<td>24.295</td>
</tr>
<tr>
<td>Bedrock damped</td>
<td>24.291</td>
<td>24.295</td>
</tr>
<tr>
<td>Gravel bed</td>
<td>25.090</td>
<td>25.123</td>
</tr>
<tr>
<td>Gravel bed damped</td>
<td>25.604</td>
<td>25.389</td>
</tr>
</tbody>
</table>

*Table 6-1.* List of maximum displacement for squat walls. Negative value in z-direction means down direction and positive value means up direction.

<table>
<thead>
<tr>
<th>Squat wall models</th>
<th>Comp. stress x-direction (MPa)</th>
<th>Tens. stress x-direction (MPa)</th>
<th>Comp. stress z-direction (MPa)</th>
<th>Tens. stress z-direction (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>-0.252</td>
<td>0.177</td>
<td>-1.727</td>
<td>1.289</td>
</tr>
<tr>
<td>Bedrock</td>
<td>-0.165</td>
<td>0.144</td>
<td>-1.298</td>
<td>0.269</td>
</tr>
<tr>
<td>Bedrock damped</td>
<td>-0.165</td>
<td>0.132</td>
<td>-1.263</td>
<td>0.276</td>
</tr>
<tr>
<td>Gravel bed</td>
<td>-0.062</td>
<td>0.112</td>
<td>-0.556</td>
<td>0.122</td>
</tr>
<tr>
<td>Gravel bed damped</td>
<td>-0.045</td>
<td>0.121</td>
<td>-0.488</td>
<td>0.084</td>
</tr>
</tbody>
</table>

*Table 6-2.* List of maximum stresses for squat walls. Negative value is compression stress and positive value is tension stress.

<table>
<thead>
<tr>
<th>Squat wall models</th>
<th>Shear force, x-direction (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>242.09</td>
</tr>
<tr>
<td>Bedrock</td>
<td>185.05</td>
</tr>
<tr>
<td>Bedrock damped</td>
<td>170.89</td>
</tr>
<tr>
<td>Gravel bed</td>
<td>116.69</td>
</tr>
<tr>
<td>Gravel bed damped</td>
<td>102.77</td>
</tr>
</tbody>
</table>

*Table 6-3.* List of total shear-forces for squat walls.
6.3. Results for the slender walls

6.3.1. Slender wall fixed on ground

Figure 6-22. Displacement in x-direction:
In upper part of wall, 23.983mm.
In lower part of wall, 24.280mm.

Figure 6-23. Displacement in z-direction:
In upper part of wall, 0.205mm.
In lower part of wall, 0.000mm.

Figure 6-24. Stress in x-direction:
Compressive stress, -0.550MPa.
Tensile stress, 0.438MPa.

Figure 6-25. Stress in z-direction:
Compressive stress, -3.440MPa.
Tensile stress, 2.790MPa.
6.3.2. Slender wall on bedrock

**Figure 6-26.** Displacement in x-direction:
In upper part of wall, 37.585 mm.
In lower part of wall, 24.333mm.

**Figure 6-27.** Displacement in z-direction:
In upper part of wall, 6.337mm.
In lower part of wall, 6.363mm.

**Figure 6-28.** Stress in x-direction:
Compressive stress, -0.305MPa.
Tensile stress, 0.246MPa.

**Figure 6-29.** Stress in z-direction:
Compressive stress, -2.515MPa.
Tensile stress, 0.707MPa.
6.3.3. Slender wall on bedrock with damping

**Figure 6-30.** Displacement in x-direction:
- In upper part of wall, 57.898mm.
- In lower part of wall, 24.290mm.

**Figure 6-31.** Displacement in z-direction:
- In upper part of wall, 17.716mm.
- In lower part of wall, 17.836mm.

**Figure 6-32.** Stress in x-direction:
- Compressive stress, -0.338MPa.
- Tensile stress, 0.255MPa.

**Figure 6-33.** Stress in z-direction:
- Compressive stress, -1.794MPa.
- Tensile stress, 0.641MPa.
6.3.4. Slender wall on gravel bed

Figure 6-34. Displacement in x-direction: 
In upper part of wall, 103.416mm. 
In lower part of wall, 24.693mm.

Figure 6-35. Displacement in z-direction: 
In upper part of wall, 28.623mm. 
In lower part of wall, 29.144mm.

Figure 6-36. Stress in x-direction: 
Compressive stress, -0.221MPa. 
Tensile stress, 0.065MPa.

Figure 6-37. Stress in z-direction: 
Compressive stress, -0.877MPa. 
Tensile stress, 0.538MPa.
6.3.5. Slender wall on gravel bed with damping

**Figure 6-38.** Displacement in x-direction:  
In upper part of wall, 133.449mm.  
In lower part of wall, 24.507mm.

**Figure 6-39.** Displacement in z-direction:  
In upper part of wall, 31.911mm.  
In lower part of wall, 32.689mm.

**Figure 6-40.** Stress in x-direction:  
Compressive stress, -0.224MPa.  
Tensile stress, 0.048MPa.

**Figure 6-41.** Stress in z-direction:  
Compressive stress, -0.878MPa.  
Tensile stress, 0.542MPa.
Results for all the slender walls are collected in tables:

<table>
<thead>
<tr>
<th>Slender wall models</th>
<th>Displacement, x-direction</th>
<th>Displacement, z-direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top (mm)</td>
<td>Bottom (mm)</td>
</tr>
<tr>
<td>Fixed</td>
<td>23.983</td>
<td>24.280</td>
</tr>
<tr>
<td>Bedrock</td>
<td>37.585</td>
<td>24.333</td>
</tr>
<tr>
<td>Bedrock damped</td>
<td>57.898</td>
<td>24.290</td>
</tr>
<tr>
<td>Gravel bed</td>
<td>103.416</td>
<td>24.693</td>
</tr>
<tr>
<td>Gravel bed damped</td>
<td>133.449</td>
<td>24.507</td>
</tr>
</tbody>
</table>

*Table 6-4. List of maximum displacement for slender walls. Negative value in z-direction means down direction and positive value means up direction.*

<table>
<thead>
<tr>
<th>Slender wall models</th>
<th>Comp. stress x-direction (MPa)</th>
<th>Tens. stress x-direction (MPa)</th>
<th>Comp. stress z-direction (MPa)</th>
<th>Tens. stress z-direction (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>-0.550</td>
<td>0.438</td>
<td>-3.440</td>
<td>2.790</td>
</tr>
<tr>
<td>Bedrock</td>
<td>-0.305</td>
<td>0.246</td>
<td>-2.515</td>
<td>0.707</td>
</tr>
<tr>
<td>Bedrock damped</td>
<td>-0.338</td>
<td>0.255</td>
<td>-1.794</td>
<td>0.641</td>
</tr>
<tr>
<td>Gravel bed</td>
<td>-0.221</td>
<td>0.065</td>
<td>-0.877</td>
<td>0.538</td>
</tr>
<tr>
<td>Gravel bed damped</td>
<td>-0.224</td>
<td>0.048</td>
<td>-0.878</td>
<td>0.542</td>
</tr>
</tbody>
</table>

*Table 6-5. List of maximum stresses for slender walls. Negative value is compression stress and positive value is tension stress.*

<table>
<thead>
<tr>
<th>Squat wall models</th>
<th>Shear force, x-direction (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
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<tr>
<td>Bedrock</td>
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</tr>
<tr>
<td>Bedrock damped</td>
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<tr>
<td>Gravel bed</td>
<td>402.31</td>
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<tr>
<td>Gravel bed damped</td>
<td>393.94</td>
</tr>
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</table>

*Table 6-6. List of total shear-forces for slender walls.*
7. Discussion

These results are in accordance with what has previously been stated, the effects of rocking of the wall changes its behavior profoundly. Displacements increased while the stresses and shear forces diminished.

The purpose of the project was to get answers to the following questions:

1. **Will there be more displacement on a wall founded on bedrock and gravel bed than cantilever?**
   For squat walls, there is little more displacement in walls on bedrock and gravel bed than fixed wall. The difference is not great, nearly the same for walls on bedrock versus fixed wall and only about 1.5mm for walls on gravel bed versus fixed wall.

   For slender walls, there is much more displacement in walls on bedrock and gravel bed than fixed wall. The difference is almost 50% with walls on bedrock versus fixed walls and about 80% more for walls on gravel bed than fixed wall.

2. **Is there a difference in stress and shear forces with a wall cantilever or founded on bedrock or gravel bed?**
   For squat walls, there is about 30% less stress in walls on bedrock than fixed wall. The difference for walls on gravel bed is 80% less stress than with fixed wall. The shear force is approximately 25% less for walls on bedrock than fixed wall, walls on gravel bed are 50% less than fixed wall.

   For slender walls, the stress is around 40% less for walls on bedrock than when fixed wall, and 60% less stress for walls on gravel bed than fixed wall. Due to shear force for wall on bedrock are nearly 20% less than fixed wall, when the shear force for wall on damped bedrock is 25% less than fixed wall. For the walls on gravel bed and damped gravel bed are the difference nearly 50% less than fixed wall.

3. **Is there a difference between the reinforcement in a wall founded on bedrock and gravel bed versus cantilever?**
   According to these results, carefully estimated; 25% less reinforcement for wall on bedrock versus fixed wall and 50% less reinforcement for wall on gravel versus fixed wall.
Most of software used to design structures nowadays, using usually finite element method. Including the computer program ANSYS, which has been used in analysis of this project.

ANSYS is well known for approaching various challenges close to reality. The use of such programs is complex and can be difficult, for example to find out optimal settings each time. However, it is worth remembering that the finite element method is an approach method based on divided structures in a number of elements which have final numbers of degrees of freedom (DOFs). The elements are connected together in node points that each have 6 degrees of freedom, 3 displacements and 3 rotations.
7.1. **Further research**

Further research needs to be done to confirm the results of this project.

More numerical examples should be analysed for different soil types and foundation conditions. For examples; do thicker foundations have a greater impact than thin ones, different friction factors, different modulus of elasticity.

As mentioned in section 2.6. it is desirable to use more than one time history, but it was impossible in this project because of the time it took to model all the squat and slender walls and foundations in ANSYS.

Continuity on this project could include recasting concrete walls in a laboratory and test them on a shaketable. That is one of the reason why the walls look like they did in the project, to have comparison with the reality walls and foundations versus model walls and foundations, to minimize the construction costs.

Suitable would be also, to analyse an entire houses and foundations in computer software and compare it together with reality houses and foundations on a shaketable, as models on figure 7-1 shows. The main disadvantages are the solving time in the computer software would be too long and the construction costs in the reality.

*Figure 7-1. Proposal of concrete house. (a) When the house is fixed to the ground (cantilever) and (b) when the house is allowed to rock and translate on foundations.*
It is also necessary to further research when a beam is connected between column and shear wall, same as figures 7-2 or 1-2 shows. If such model is designed cantilever, but is in the reality with soil-structure interaction effects; that means the shear force and moment increases in the beam and in the column, while it reducing on the shear wall. Then there is a risk that the reinforcement in the beam and the column are under designed and the reinforcement in the shear wall is significantly over designed.

*Figure 7-2. Structure, with shear wall, beam and column. (a) When the structure is fixed to the ground (cantilever) and (b) when the structure is allowed to rock and translate on foundations.*
8. References


Concrete walls founded on earthquake areas


Concrete walls founded on earthquake areas


9. Appendixes

Appendix A - Wall and foundation models in ANSYS

Squat Wall

All units are put in the computer software ANSYS in Newton and meters. ANSYS deliver the outcomes in Newton and or meters.

Go to Preferences and set as Structural

To determine size of grid: WorkPlane -> WP settings
Turn on grid, WorkPlane -> Display working plane
Concrete walls founded on earthquake areas

Draw the base form of the wall on grid, Preprocessor -> Modeling -> Create -> Keypoints -> On working plane or In active CS

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<tr>
<td>16</td>
<td></td>
<td>6.2, 3.1, 0</td>
</tr>
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Do every part as area, Preprocessor -> Modeling -> Create -> Areas -> Arbitrary -> Through KPs.

Connect the area together, Preprocessor -> Modeling -> Operate -> Booleans -> Partition -> Areas.

Do the wall volume by extrude the wall up 3.4 meters in z-direction, Preprocessor -> Operate -> Extrude -> Areas -> By XYZ Offset.

Materials properties for wall, Preprocessor -> Material Props -> Material Models

The concrete tensile strength 3,3MPa and compressive strength 25MPa:
Material properties for the reinforcement, Preprocessor -> Material Props -> Material Models

Put reinforcement in to the wall, Preprocessor -> Real Constant -> Add/Edit/Delete.
When reinforcement turn in x-direction, then THETA = 0 og PHI = 0.
When reinforcement turn in y-direction, then THETA = 90 og PHI = 0.
When reinforcement turn in z-direction, then THETA = 0 og PHI = 90.
The reinforcement in the middle-wall (wall in x-direction), has Real constant Set No. 1:
Reinforcement ratio vertical, z-direction, simple grid 12 pieces K10 (c250):
\[ A_{wall,z} = 3000mm \cdot 200mm = 600000mm^2 \]
\[ A_{bar,z} = 12 \text{ pce} \cdot \frac{\pi \cdot 10^2}{4} = 942mm^2 \]
Reinforcement ratio = \( \frac{A_{bar,z}}{A_{wall,z}} = 0,0015 = 0,15\% \)

Reinforcement ratio horizontal, x-direction, simple grid 14 pieces K10 (c250):
\[ A_{wall,x} = 3500mm \cdot 200mm = 700000mm^2 \]
\[ A_{bar,x} = 14 \text{ pce} \cdot \frac{\pi \cdot 10^2}{4} = 1100mm^2 \]
Reinforcement ratio = \( \frac{A_{bar,x}}{A_{wall,x}} = 0,0015 = 0,15\% \)

The reinforcement in the end-wall (wall in y-direction), has Real constant Set No. 2:
Reinforcement ratio vertical, z-direction, simple grid 9 pieces K10 c250:
\[ A_{wall,z} = 2200mm \cdot 200mm = 440000mm^2 \]
\[ A_{bar,z} = 9 \text{ pce} \cdot \frac{\pi \cdot 10^2}{4} = 706mm^2 \]
Reinforcement ratio = \( \frac{A_{bar,z}}{A_{wall,z}} = 0,0015 = 0,15\% \)

Reinforcement ratio horizontal, y-direction, simple grid 14 pieces K10 c250:
\[ A_{wall,y} = 3400mm \cdot 200mm = 680000mm^2 \]
\[ A_{bar,y} = 14 \text{ pce} \cdot \frac{\pi \cdot 10^2}{4} = 1100mm^2 \]
Reinforcement ratio = \( \frac{A_{bar,y}}{A_{wall,y}} = 0,0015 = 0,15\% \)
Define the wall with reinforcement in appropriate directions, Preprocessor -> Meshing -> Mesh Attributes -> Picked Volumes.

Meshing the wall, Preprocessor -> Meshing -> MeshTool.
Check for correct reinforcement direction, write in ANSYS command window.

/ESHAPE,1
/TYPE,,BASIC
/DEVICE,VECTOR.ON
EPLLOT

Plot -> Element PlotCtrls -> Device Options -> Vector mode ON

Put in distributed load on top of the wall, 122500 N/m², Solution -> Define Loads -> Apply -> Structural -> Pressure -> On Areas

Put in acceleration of gravity, 9,81 m/s², Solution -> Define Loads -> Apply -> Structural -> Inertia -> Gravity -> Global
Squat wall cantilever, with time history in x-direction:

Put fundaments under the wall, Solution -> Define loads -> Apply -> Structural -> Displacement -> On Areas, fundament are cantilever, select All DOF.

Earthquake load, i.e. time history put in, Solution -> Define Loads -> Apply -> Structural -> Displacement -> On Areas

Select displacement in x-direction. And select new table.

Call the table dx, displacement in x-direction.

Use 2500 point. Select read from file.

Browse for file.
Concrete walls founded on earthquake areas

Go to file and select Apply/Quit

Solve the structure as Transient, Solution -> Analysis Type -> New Analysis

Solution -> Sol’n Controls.

Solution -> Solve -> Current LS.
Squat wall on bedrock, with time history in x-direction:

Turn on grid, WorkPlane -> Display working plane
Draw base form of rock on grid, Preprocessor -> Modeling -> Create -> Keypoints -> On working plane or In active CS

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<td>36</td>
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</table>

Take away the fundament under the wall, Solution -> Define Loads -> Delete -> Displacement -> On Area.

Area for bedrock, Preprocessor -> Modeling -> Create -> Areas -> Arbitrary -> Through KPs.

Do the bedrock volume by extrude, down 1 meters in z-direction, Preprocessor -> Operate -> Extrude -> Areas -> By XYZ Offset
Define element for bedrock, Preprocessor -> Element Type -> Add/Edit/Delete
Solid – Brick 8node 45.

Material properties for bedrock, Preprocessor -> Material Props -> Material Models

Define bedrock, Preprocessor -> Meshing -> Mesh Attributes -> Picked Volumes

Meshing the bedrock:
Select vertical end-lines and divide mesh in 1. For horizontal end-lines is 9 for x-direction, but 6 for y-direction, i.e. interval 1 meter.
Preprocessor-> Meshing->SizeCntrs->ManualSize-> Line -> PickedLines

Preprocessor -> Meshing -> Mesh Tool
Create contact element between wall and rock,

Depress on contact wizard,

Depress pick target and then upper edge of the rock:

Select the lower edge of the wall:
Friction factor is estimated 0.7 for bedrock.

Put fundaments under the bedrock, Solution -> Define loads -> Apply -> Structural -> Displacement -> On Areas, fundaments are cantilever, select All DOF.

Earthquake load i.e. time history, Solution -> Define Loads -> Apply -> Structural -> Displacement -> On Areas

Select displacement in x-direction. And select new table.
Concrete walls founded on earthquake areas

Select read from file.

Use 2500 point.

Browse for file.

Go to file and select Apply/Quit.

Solve the structure as Transient.
Squat wall on gravel bed, with time history in x-direction:

Change the material properties from bedrock to gravel bed, Preprocessor -> Material Props -> Material Models

Change the friction factor from 0.7 for bedrock to 0.6 for gravel bed:

Also possible to change the friction factor in Contact manager -> properties:

Earthquake load are the same as for wall on bedrock.

Solve the structure as Transient.
Concrete walls founded on earthquake areas

Squat wall on bedrock or gravel bed with damping, with time history in x-direction:

Define element for the damping matrix, Preprocessor -> Element Type -> Add/Edit/Delete

User Matrix – Damp Matrix27.

Select Options in Element Types.
Preprocessor -> Real Constants -> Add/Edit/Delete

Put in point for the dampers, Preprocessor -> Modeling -> Create -> Keypoints -> In Active CS

<p>| | | | | | |</p>
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</table>

Preprocessor -> Modeling -> Areas -> Arbitrary -> Through KPs
Concrete walls founded on earthquake areas

Extrude the outmost area about 1 meter, Preprocessor -> Modeling -> Operate -> Extrude

-> Areas -> By XYZ Offset

Meshing for outermost area:
Pick lines in x-direction and divide the mesh in 9, for line in y-direction is divided by 6, i.e interval is 1 meter.
Preprocessor-> Meshing->SizeCntrls->ManualSize->Lines -> PickedLines

The outmost parts are Solid 45, Preprocessor -> Meshing -> Mesh Attributes -> Picked Volumes
Preprocessor -> Meshing -> MeshTool
Concrete walls founded on earthquake areas

Connecting the dampers (dampersmatrix MATRIX27) between bedrock or gravel bed and outmost parts: Preprocessor -> Modeling -> Create -> Elements -> Elem Attributes.

Put MATRIX27 between bedrock or gravel bed and outmost parts: Preprocessor -> Modeling -> Create -> Elements -> Auto Numbered -> Thru Nodes

Fix outmost parts: Solution -> Define load -> Apply -> Structural -> Displacement -> On Areas, cantilever, All DOF.

Solve the structure as Transient.
**Slender Wall**

Use the same squat walls model but rise the slender walls model about three story, Before that have to unmesh every thing and then: Preprocessor -> Modeling -> Copy -> Volumes

Pick All

Conect the structure together:
Preprocessor -> Modeling -> Operate -> Booleans -> Glue -> Volumes

Define the wall with reinforcement in appropriate direction, Preprocessor -> Meshing -> Mesh Attributes -> Picked Volumes

Mesh the wall, Preprocessor -> Meshing -> MeshTool
Distributed load between story and on top of the wall, 122500 N/m². Solution -> Define Loads -> Apply -> Structural -> Pressure -> On Areas
Concrete walls founded on earthquake areas

Slender wall cantilever, with time history in x-direction:

Put fundament under the wall:
Solution -> Define loads -> Apply -> Structural -> Displacement -> On Areas, fundaments are cantilever, select All DOF.

To put in earthquake load i.e. time history are the same as for squat walls.

Solve the structure as, Solution -> Solve -> Current LS.

Slender wall on bedrock or gravel bed, with time history in x-direction:

See modeling as for squat walls

Slender wall on bedrock or gravel bed with damping, with time history in x-direction:

See modeling as for squat walls
Get a results from ANSYS

Plot Results -> Contour Plot -> Nodal Solu

Get list and graph for squat walls:

UX or UZ top is nr.300, element 272
UX or UZ middle is nr. 395, element 218
UX or UZ bottom is nr.288, element 164

TimeHist Postpro -> Define Variable

TimeHist Postpro -> Graph Variables
Concrete walls founded on earthquake areas

TimeHist Postpro -> List Variable

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Get list and graph for slender walls:

UX or UZ top is nr.3940, element 2846
UX or UZ middle is nr. 2265, element 1496
UX or UZ bottom is nr.288, element 164

TimeHist Postpro -> Define Variable
TimeHist Postpro -> Graph Variables
TimeHist Postpro -> List Variable
### Appendix B - Time history in x-direction, from Hella 2000.

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![Graph of Time history in x-direction](image.png)