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Theory and practice in mathematics teacher education
Building a learning community

In Iceland education of compulsory school teachers has been at Bachelor of Education level since 1971. Iceland University of Education1 has been the main institution educating teachers for compulsory schools and has been graduating 200-250 compulsory school teachers with a B. Ed. degree yearly for the last 10-15 years. The teacher education has been under constant development and students have been able to specialize in mathematics teaching and learning to a different degree. The most recent reforms open more possibilities for specialization both within educational studies and certain professional areas or subjects. Specialization in mathematics and mathematics education is currently 80 ECTS out of 180 ECTS. The latest reforms also allow students to commence graduate studies directly after finishing undergraduate degrees.

Iceland University of Education places great emphasis on teacher education as a research based professional education. Professional depth and comprehensive overview should characterize the studies and it is also considered of great importance that students are active and responsible for their studies. Connection to research, connection to the field, and focus on creativity and communication are the three threads that should be intertwined in the content of all courses.

Research in mathematics education has been developing the last decades. The main emphasis was on curricular research in the 1960’s, on the students in the 1980’s and in the last decade the focus has shifted to the teachers. There has been a growing interest in teachers as learners; how teachers learn to think and act in particular ways and what contributes to their learning.

In planning mathematics teacher education we have been developing the courses in relation to research findings. The focus has gradually changed from emphasis on teaching methods to children’s learning and the student teachers’ learning. To be competent in mathematics teaching teachers must be capable of interpreting, organizing, and teaching according to the goals of the curriculum as well as being able to understand the learning of their students and develop their own teaching. An important feature in the development of mathematics teaching is being aware of one’s own understanding of mathematics. Through inquiry

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1 On July 1st 2008 Iceland University of Education and the University of Iceland merged and the former is now the School of Education within the University of Iceland.
learners can go beyond the use and application of algorithms and rules, develop understanding of general relationships in mathematics, and deal with problematic aspects of the abstraction and formalism that is central to mathematics. We believe that student teachers build up their knowledge and understanding, through research, communication to the field and creative studies. An important part of our approach is a continuous action for change as we question, refine, and critique our practice.

In this article we will describe and analyze two pre-service courses in mathematics education we have taught at the Iceland University of Education for a few years. The students start their specialisation in mathematics and mathematics education in the third semester and the two courses Mathematics learning - research and development (6 ECTS) and Mathematics teaching and the professional development of the mathematics teachers (6 ECTS) are taught in the fourth and the fifth semester. We will look into the background of the courses, the content and the ways of working. We will also explore the reasons for our choice of structure, content, readings and assignments. We will describe how we build a community of inquiry in which reflective development of own mathematical understanding as well as practice in mathematics teaching results in a learning community. Student teachers are encouraged to look critically at their own mathematical learning and teaching and to modify these through their learning-in-practice. Through case writing of their practices and critical reflection in connection with their studies on research in mathematics education the participants build a learning community and we, the teacher educators, are looking into our practice through constant discussion and reflection on our teaching. We analyze cases from our teaching and draw conclusions from the learning we have observed taking place, both our own learning and the student teachers.

Research on teacher education
In recent years there has been a considerable focus on teachers in mathematics education research (Clarke, Emanuelsson, Jablonka, & Mok, 2006; Grevholm, 2004, , 2006, , 2007; Hiebert, Morris, & Glass, 2003; Hill, Ball, & Schilling, 2007; Kilpatrick, 2004; Sfard, 2005). There also seems to be a shift in focus from what works to why and how teachers do what they do (Sfard, 2005; Sherin, Sherin, & Madanes, 2000).

Wood, one of the editors of The Journal of Mathematics Teacher Education, has in her editorials “argued for the importance of generating and sharing knowledge about the complexity in mathematics teaching” (Wood & Berry, 2003, p. 195). In the 11 years the journal has been published it has played an important role in building up a knowledge base for teacher educators who wish to improve the education of mathematics teachers. The collaboration of teachers, teacher educators and students seems to be a central issue in many of the research studies
reported on in the journal. Team work, learning communities, networks and design research are often suggested as ways to help teachers to tackle and meet the complexity of teaching mathematics (Jaworski, 2005, 2006; Krainer, 2003; Wood, 2002; Wood & Berry, 2003).

Groups of experts in mathematics education who have defined important mathematical competencies or proficiencies for pupils have also defined which competencies or proficiencies teachers need to have to be able to teach pupils so that they achieve these competencies (Kilpatrick, 2004; Niss, 2004). Even though Kilpatrick uses the word proficiency and Niss the word competency it is clear that there are many common threads and similar views on what it means to be a good mathematics teacher. The choice of words is probably rather a reflection of how the same words can have a different meaning in different contexts and cultures than being different views on mathematics teaching. It is clear that both find it important that teachers have a good knowledge of curriculum and understanding of the content to be taught, they also need to be able to plan and carry out teaching and build on their understanding of students learning and to collaborate with other teachers and develop professionally. These views on what it means to be a good mathematics teacher also make it clear that educating mathematics teachers is a complicated task.

Grevholm (2006; Grevholm, 2007) also emphasizes that teaching is a complicated profession and that it is only possible to a certain extent to prepare the students for it in pre service teacher education. She points out that it will never be possible to meet all needs, demands and expectations in pre service teacher education. Therefore teacher students have to build up knowledge on different ways to develop professionally and to seek further education. To get insight into research communities and pedagogical content knowledge is another important factor. This can be done by emphasizing on subject didactics in both pre service education and professional development of teachers. For mathematics teachers mathematics education or didactics of mathematics therefore plays a central role.

If students are to start to develop their professional identity in their pre service teacher education the teacher education needs to be research based and linked to practice. The students also have to learn to share their experiences with others and create learning communities themselves that will support their learning as teachers (Grevholm, 2006; Hiebert, Morris, & Glass, 2003). Student teachers have to realize that in their pre service education they are only starting to create knowledge base and they are entering a profession where lifelong learning is essential.

An important factor in research based mathematics teacher education is the use of research based literature and that students get insight into the work and methods used by researchers. They also have to learn to conduct some small research projects themselves and get the prerequisites for starting researcher educa-
tion. Students have to learn to present ideas in writing in a scientific way and reason and present their ideas in public in a professional way (Grevholm, 2004).

One of the main tasks of teacher education programs is to develop amongst the student’s a knowledge of teaching. If students are to obtain a profound knowledge of teaching and to develop a practice that is different from what they themselves experienced as pupils they need learning opportunities that are more powerful than simply reading and talking about new pedagogical ideas. The best learning environment is when the students are given the opportunity to do, study and reflect in collaboration with others and by looking closely at the pupils and their work and share what they see with others. This requires a close connection to schools and teaching practices. Students need both opportunities to try out their ideas in practice and knowledge of and support to reflect upon and interpret practice (Darling-Hammond, 1998).

We have found it important to model teaching that enhances inclusive education. This is frequently addressed in the book *Preparing mathematics and science teachers for diverse classrooms* (Rodriguez & Kitchen, 2005). The goal of the book is to offer teacher educators stories of creative approaches that work with prospective and practising mathematics and science teachers to prepare them to work with diverse learners in schools. Moore (2005) discusses the transformation from theory to practice. He concludes that if teachers are expected to teach for diversity and understanding, they need opportunity to develop and enhance their pedagogical knowledge. “They need to experience their own mathematics learning in an environment that reflects the environment they are expected to create in their own classroom. Transformative mathematics pedagogy affords them that opportunity and provides concrete examples of how to meet the demands of an increasingly diverse student population within our ever-changing global society” (p. 186).

Hiebert, Morris, & Glass (2003) propose a teacher preparation program that focuses on helping students to acquire the tools they need to learn to teach rather than the finished competencies of effective teaching. The two main goals of the program are, firstly, that the students become mathematically proficient and, secondly, that they learn to teach in a manner that can help their students to become mathematically proficient. They suggest that all lessons the prospective teachers give should be treated as experiments and that they also need to know how to learn from these classroom teaching experiments. They also describe learning environments the prospective teachers must learn to create in order to sustain their own and other teachers learning. Teacher students need to become responsible for their own professional development when they enter into the profession of teaching and also for the development of their own profession. Teacher preparation programs can only be the starting point to this long term
process and they have to make the students responsible for creating and sustain-
ing a knowledge base for mathematics teaching.

Courses on mathematics teaching and learning
Following is a discussion on two courses that students who specialize in mathe-
matics education take at the Iceland University of Education. The focus at the
former course is on learning; both the students own learning and children’s learn-
ing of mathematics. At the latter course teaching and teacher development is the
core subject as well as the preparation of teacher practice.

Mathematics learning - research and development
The main goal of the course is that the student teachers develop awareness of
their own learning of mathematics as well as of how young children and teenag-
ers develop in their learning of mathematics. The students read research on
mathematical learning and discuss in class, they explore with mathematical prob-
lems in class, discuss their thinking about the tasks and write a diary of their ex-
plorations. They also work with young children and teenagers and try ideas based
on their studies. The focus is on the student teacher as active learner that is re-
sponsible for her/his own learning both in class and in practice in schools with
diverse groups of children.

The main course book is: Thinking Mathematically. Integrating Arithmetic &
on many years research on children’s mathematical thinking, their intuitive un-
derstanding of basic number concepts and arithmetic in relation to their devel-
opment in algebraic thinking. The researchers worked with a group of elementary
school teachers for sixteen years to study how children learn mathematics and
mathematical way of thinking. They found that children throughout the elemen-
tary grades are capable of learning powerful unifying ideas of mathematics that
are the foundation of both arithmetic and algebra. The book provides a frame-
work for teachers and students to reflect on their implicit knowledge of arithme-
tic concepts and procedures, to ask questions about why we do things the way we
do and how we know that we can do them that way. Learning mathematics with
understanding is not only for a few mathematically gifted students, it is most
critical for students at risk of failing in mathematics to engage in the kind of ac-
tivities provided in the book. The reader gets profound knowledge of the com-
plex work of teaching needed to bring student’s capacity for mathematical ideas
and thinking into productive contact with the rich territory of mathematics. An
accompanying CD provides rich illustrations of ideas in the book (extended in-
teractions with individual children or classroom episodes) all clearly linked to the
text.
The student teachers are responsible for leading discussion about the book and being prepared to participate in the discussions. In their presentation they use the episodes from classrooms presented at the CD and often refer to their own experience both as learners of mathematics and as teachers. They also read new articles in research journals on mathematics teaching and learning, both articles that we choose about constructive mathematics teaching where children are encouraged to explore and investigate in their mathematics learning and articles they find themselves. We find it important for teachers to read research articles on teaching and learning to be able to develop their teaching. In this course student teachers are guided through reading such articles and how to find research that is valuable for their work.

John Mason’s book *Learning and doing mathematics* (Mason, 1999) is used as support for the students own explorations with mathematics. In sessions students work in groups and are encouraged to write down their thinking while solving problems and the steps they are taking to find the solution. We choose problems that have the “potential to promote mathematical activity and thinking” (Jaworski, 2007) and stimulate collaboration where discussions and sharing thinking is meaningful. The students find it difficult as they are not used to discuss their thinking and write down their reflections on their learning. We believe that it is crucial for all teachers to be able to reflect on their teaching and learning.

- What was my first thought? What did I do next?
- What new ideas emerged in discussions with students in the group?
- How did I contribute to the discussions?
- Did my ideas help developing the discussions?
- Did I learn something from other students’ ideas and explanations?

Finally the groups report their solution strategies to the class and we discuss different approaches. In what ways are their approaches different? Is one approach better or more sophisticated than the others? Is it important to try different ways of approaching problems? Why do we approach the problems in different ways? The students write notes from the discussions in their diary and together we add to our web-based diary that is accessible to all students in the class.

When we discuss the research on children’s and teenagers mathematical learning they refer to their own exploration and their reflection on the children’s learning help them reflect on their own learning. During their in-service training we see that these explorations reflect in their teaching in the way that they dare to rely on their students’ strengths and encourage them to explore into mathematics on their own premises.
Assessment

The assessed work is participation in classroom activities, reports on fieldwork with students, written assignment about one research article and finally discussion about own learning during the course – their conclusions from own diary. Writing about their reflection on their own learning is difficult but most of the students find it helpful to write about their reflections. Baldur wrote:

One of my main problems in mathematics is really how lazy I’m. I believe it is because I got away with it in school not to do my best. I only did enough to get good grades and nothing more. This is a general problem in schools. It has to do with how we evaluate our students. We give them tests and they get their grades. This gives them the message that it’s the grades that are the most important in school. We need to find a more effective way to help the students to learn to appreciate their learning more than the grades.

In his conclusions he writes:

Finally I want to add that this assignment has been very helpful. It is important to take time to think what one has been doing during the winter. I have discovered things about myself, that I of course had some vague ideas about, but is important to write down because then it somehow becomes more real. What I have discussed here does not only relate to my mathematics learning but gives a good picture of me as a person. Therefore my reflection on my way of studying mathematics has helped me to understand my way of learning not only mathematics but in general.

Erla described how she had solved a problem in a geometry class she was taking the same semester. One of the tasks was to prove that the angles in an equilateral triangle are equal. She wrote:

The first thing I did was to draw a picture because I find that helps me to organize in my mind what to do next.

Then she described thoroughly how she went on and included her drawings. Finally she wrote:

As you can see I think the proof in many small steps. First you need to figure out what is given in the task and try to use it to prove the rule. I used my previous knowledge to build up the proof. When proving you need to find your own way, there is no given rule for how to solve the task. In this task I went my own way and others could have done it differently.
The examples chosen here are representative for the learning we have seen taking place at the course. The students have developed in thinking about their own learning and that has helped them reflect on children’s learning and their teaching.

**Teaching mathematics and the professional development of mathematics teachers**

The aim of the course is to challenge the student teachers in studying and discussing issues related to the teaching and learning of mathematics. This is considered as important for the student teachers who are building up their professional identity and going through a learning process in order to be able to take responsibility for their own professional growth. The course is intertwined with teaching practice in grades 7-10. Emphasis is on connection to the field, especially to the schools where the student teachers are going have their teaching practice.

The course is organized in three periods. It begins with a seven-week period where the student teachers make observations in classrooms, study teaching materials, get introduced to journals and web-based materials for mathematics teachers, and read about research in mathematics education. This is done along with preparations for the teaching practice. They are encouraged to work on this question: What ideas do I want to try out in my teaching practice? The focus is on the teaching and learning of algebra to create a common ground for discussions on different teaching approaches. Everyone is supposed to prepare at least two weeks period of algebra teaching in one class.

The practice period is five weeks and the student teachers work in small groups in cooperation with their practice teachers. They are encouraged to try out new ideas and ideas they have read about in the course literature. They are supposed to include the use of ICT in their teaching. Both the practice teachers and the teacher educators support the student teachers when they are developing their ideas and finding ways to implement them in practice. The collaboration with their fellow students, both during the preparations and when it comes to the actual teaching is also very important. This process gives the student teachers many opportunities of having discussions with their practice teacher, their fellow students and the teacher educators.

The last two weeks of the term the student teachers evaluate and reflect on their experience of the teaching practice. They read about other teachers experiences and relate them to their own experiences. The focus is on the teacher role and how important it is for teachers to be a part of a progressive environment where they can reflect on their practice and develop ideas in cooperation with other teachers. The students discuss the idea that a teacher education is a lifelong
learning-process and that they have to take responsibility for their own professional development.

The choice of course literature is of great importance. The course literature will influence the students’ beliefs about the subject and what issues they find important to take up for discussion. In teacher education student teachers get to know some journals and magazines written for mathematics teachers and are as well introduced to examples of research and how it can support teachers in practice. It is of great value for them to have the opportunity to reflect on and discuss the literature in a learning community.

The focus in the literature of the course has been on different teaching approaches, the teacher’s role and developmental work in mathematics teaching. The student teachers have studied the book *Connecting Mathematical Ideas* (Boaler & Humphreys, 2005). The book and the accompanying classroom CD is a result of cooperation between the mathematics teacher Cathy Humphreys and the researcher Jo Boaler for a whole school year. All lessons in two seventh-grade mathematics classes were video-taped and eight lessons were chosen to be presented for the readers. The teacher Cathy writes about her aims and reflections while designing a lesson as well as her reflections after teaching each lesson. Jo Boaler writes some comments on relevant issues. The book has been a good source for learning for the student teachers. They see how it is possible to study, organize and reflect on a lesson. It has helped them to understand how mathematics can be taught differently from what they have experienced themselves. They also get some examples of what the teacher needs to think about when setting up a teaching environment. The use of the book has changed the way the students discuss teaching. They used to focus on the time schedule and felt they needed to come to the class with many different and entertaining tasks. Now they focus more on the class they are going to teach and make a deeper analysis of the tasks they are going to use. They see how one task can be used to challenge different pupils on different mathematical ideas. They also see that they need to take many issues into considerations when designing series of lessons. The benefits of cooperation and mutual support between teachers become evident to the students and they also get ideas about how teachers and researchers can work together, which can help them bridging the gap between theory and practice.

**Assessment**

From Jo’s and Cathy’s book the student teachers get ideas of how it is possible to structure and implement change in mathematics education. In their course assignments they are supposed to work further with those ideas. In order to help them understanding classroom-practice the first assignment in the course is to visit mathematics class for one lesson and write down a detailed description of how the lesson proceeds. This gives a good background for their discussion of Cathy’s lessons. In the second assignment they work in groups and write a jour-
nal about their discussions and reflections while studying the book and watching the CD. They are supposed to reflect both upon Cathy’s experiences and thoughts and on Jo’s comments. Her comments add more theoretical concepts to their discussions. Jo’s analysis and use of theory in the comments helps the students to make connections and to use Cathy’s case as an example.

The third assignment is a teaching plan and evaluation of the plan. In the teaching plan the student teachers are supposed to combine theoretical and practical knowledge. They have to show that they have a good understanding of the content and of how pupils build up their understanding of important concepts in that area. They also have to argue for their choice of teaching approaches. The series of lessons the students plan are supposed to be holistic, coherent and adapted to the pupils and their circumstances. They teach in small groups and they reflect both on their own teaching and on the teaching of their partners in discussions with the practice teacher. In the evaluation of the teaching plan the student teachers analyze their teaching and reflect on their experiences.

The fourth assignment is supposed to give each student teacher an opportunity to gather her/his knowledge, ideas and thinking and argue for her/his views on mathematics teaching and learning. By attending various courses in the teacher education the students have developed their knowledge of schools and teaching. At this point they should be able to present their own professional theory based on their theoretical knowledge and experiences. They are asked to select between some projects.

The assignment should reflect that the student teachers have developed the competence to discuss mathematics education as professionals and are aware of their opportunities and responsibilities for their own professional development.

**Example of assignment 4**

1. You are supposed to write an article in a journal for mathematics teachers about reforms in mathematic education in the last decade. You should choose to use as an example:

   a) One particular content area

   b) Processes and approaches

   c) Ideas about mathematical competencies

2. You recommend to a group of teachers that they form a reading group around the book Connecting Mathematical Ideas. Argue for your choice of book and give a picture of Jo Boaler as an experienced researcher. You can use her web page as a source for information.

   [http://www.sussex.ac.uk/education/profile205572.html](http://www.sussex.ac.uk/education/profile205572.html)
3. You are supposed to make a booklet or a webpage for parents about recent reforms in mathematics teaching and learning in grades 7-10 according to a new curriculum and new teaching materials. What is important to address in such a booklet and why? Give an idea of the structure of the booklet and write a chapter on important changes of emphasis and content.

4. In the book Developing Thinking in Algebra you have been challenged by many algebraic tasks. How has this work influenced you as a teacher of algebra? Has this work affected your beliefs and ideas on the content and ways of working in algebra? Would you recommend this book to other mathematics teachers? Why?

The assignments should help student teachers to build up their competence and to recognize that teaching is a complicated task. They should understand that being a teacher means that you need to research your own teaching and constantly seek ways to develop and improve it. You also need to be aware of new research and development in the field and create and use opportunities for continuous professional development.

**Teaching approach**

The teaching approach used in the course focuses on creating a learning community among the students. The teacher educators play an active role in this community as participants in discussions and as group leaders or experts. The student teachers are supposed to be active and bring in their knowledge, views and thoughts about mathematics education. One way to support the building of a learning community is the use of ideas from *Lesson Study* (Lewis, 2002). The ideas behind Lesson Study are clear and give all participants in the group opportunity to open up for their considerations about teaching and at the same time the strength in the whole group is used.

The main idea of Lesson Study is that a group of teachers with support from an expert develop a teaching plan for one lesson. The structure of the lesson and the role of the teacher are often in focus. The Lesson Study process can be described as a cycle:
In the first phase the group has to discuss what the content of the lesson should be. When the content has been defined participants are meant to explore it, both in regard with what it means to acquire understanding of the content and how it can be approached in teaching. The participants are in control and they make decisions about the process. One important factor is that the participants deepen their knowledge of the content and possible teaching approaches. They communicate, do research, work together, take decisions, plan teaching and experience the advantages of participating in a learning community. The research lesson is taught by one of the participants where the other participants are present and take notes. After the lesson the experience is discussed and an expert joins the group-discussions. On that basis the lesson plan is revised and the lesson taught again for a similar group. The cycle can be repeated several times.

The student teachers have used the Lesson Study cycle when preparing for teaching practice and the teacher educators have played the role of the expert. This has given discussions of teaching experiences a different focus. The teaching of the content and the interactions between the pupils and the teacher become more central. More student teachers are involved in the same lesson practice which gives them opportunities to discuss the connection between theory and praxis from a different perspective. Thinking through one lesson in details and look at it from many different angles is helpful at this stage. The interaction with pupils where the teacher is expected to promote the pupils learning is often thought to be the most difficult part of the teacher’s role. Many student teachers use most of their energy to carry out their lesson plan and find it difficult to interact with the class and respond to individual pupils at the same time.
The Lesson Study approach gave the student teachers opportunity to concentrate more on the pupils and interactions in the classroom during the planning, teaching and evaluation of the lesson. Moreover, it helped them to take their own teaching seriously and to look at it as an experiment that one can learn from. In teacher education the students are building their identity as teachers, especially in mathematics, and develop awareness of their own views and beliefs on teaching and learning. Their competence to be analytical and to combine their theoretical and practical knowledge is growing. The Lesson Study approach implies that learning to teach is a lifelong process that develops in collaboration.

Discussion
Teaching about teaching is challenging. In analysing the development of our teaching we have found that theories and research findings have affected our ways of thinking about mathematics teaching and learning. We have found it rewarding to build our work on research in mathematics education. Gradually we have realised how important it is for teacher educators to understand that pedagogy of teacher education must go beyond the transmission of information about teaching. Student teachers need not only to concentrate on learning what is being taught but also the way in which that teaching is conducted. We see teaching as a complex process that can not be learned once and for all and that it is important in teacher education to open the doors to research in the field. Loughran (2006) addresses this in his book Developing a pedagogy of teacher education and gives support to teacher educators in shaping learning about teaching. He discusses the complexity of teaching and that teaching can appear to be “a well ordered technically proficient and purposefully directed routine” (p. 30), but when teaching is unpacked by teachers then the constant tensions are made clear for all to see. In order to be able to respond to the demanding situations that awaits them and work as professional mathematics teachers student teachers must learn to research their own practice and develop in their professional life. In our teaching we therefore emphasise that the student teachers develop tools they can use to learn to learn to teach.

Our choice of teaching approaches creates opportunities for student teachers to do, study and reflect in collaboration with others both on experience in class and in teaching practice. The Lesson Study cycle is a tool that has been effective in giving the student teachers space to develop their knowledge and to reflect on their own ideas and experience. The cycle supports the building of a learning community where the students both develop their content knowledge and pedagogical knowledge.

Research in teacher education is voluminous both in general teacher education and mathematics teacher education and provides extensive results and evidence about approaches to teacher development as addressed by Wood & Berry
This knowledge has given teacher educators material to build on and ideas they can use in developing their own practice, knowing that teaching mathematics is a complex task. Collaborating in mathematics teacher education has urged us to refine and reflect on our teaching and course content. We discuss our practice and the students’ responses to the tasks we give them. Our identity as professional teachers has grown through this work and the three threads (focus on creativity and communication, and connection to the field and research) emphasised in the education at the Iceland University of Education have supported us in developing our teaching as teacher educators.

References


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